Recent Advances in the Black Hole Information Paradox

Liz Wildenhain (UC Berkeley)

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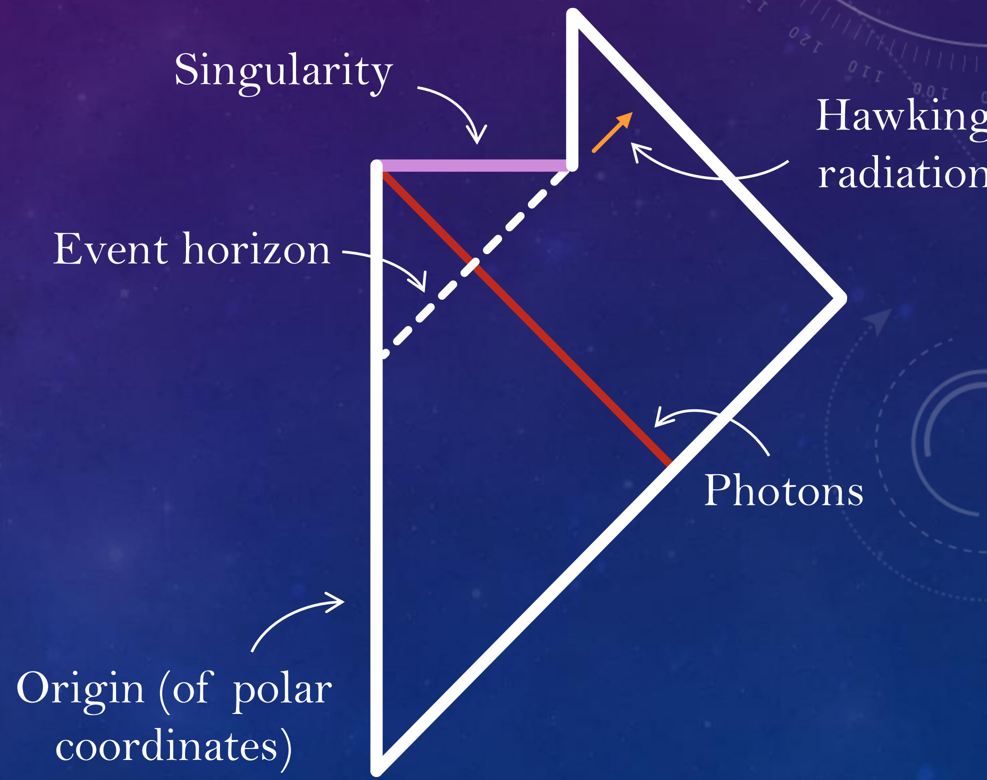
The Information Paradox: Black Hole Evaporation

Hawking (1975) showed that, semiclassically, black holes should emit radiation and eventually evaporate. This radiation comes out in a mixed state.



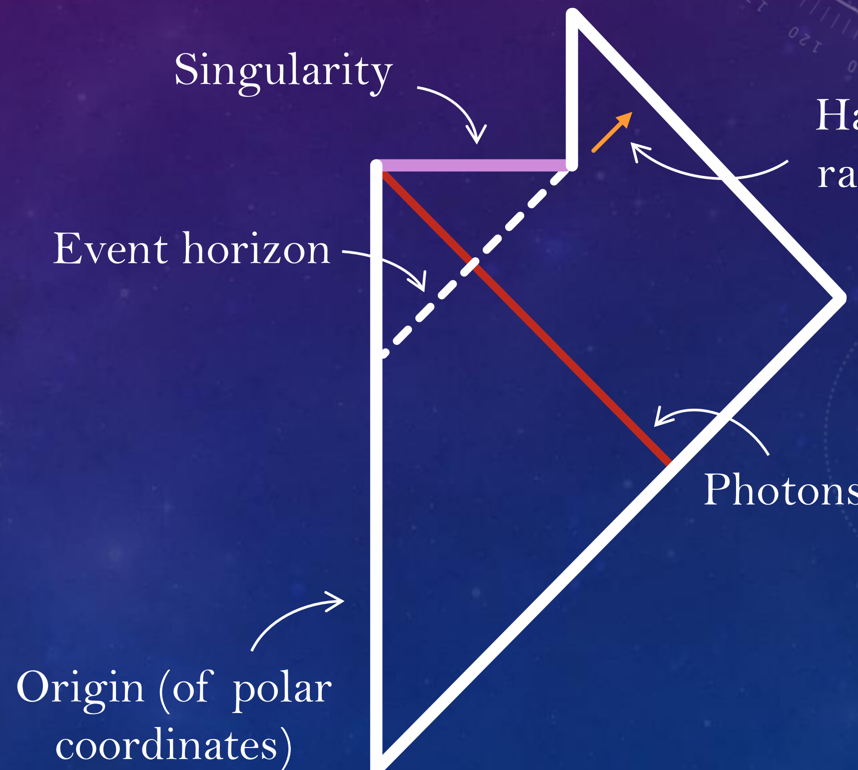
The Information Paradox: Black Hole Evaporation

The Minkowski vacuum can be decomposed along any plane into an entangled pure state:

$$|\Omega\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\pi\omega_i} |i^*\rangle_A |i\rangle_B$$

Tracing out region A → thermal state in region B:

$$\rho_B = \frac{1}{Z} \sum_i e^{-2\pi\omega_i} |i\rangle_B \langle i|$$



The Information Paradox: The S-Matrix

Initial State → Linear Map → *Final State*

$$P(\text{out}|\text{in}) = |\langle \text{out}|S|\text{in}\rangle|^2$$

If the S-Matrix is unitary…

Pure Initial State → Linear Map → *Pure Final State*

The Information Paradox: A Statement of the Paradox

1. The (unitary) S-matrix maps pure in-states to pure out-states. In other words, no information is lost.
2. We expect a process in asymptotically-flat quantum gravity to be described by an S-matrix.
3. According to Hawking's calculation, a black hole formed from matter in a pure state will evaporate into radiation in a mixed state.
4. If a black hole is formed from matter in a pure state and evaporates into a mixed state, the process cannot be described by a unitary S-matrix. The whole process involves information loss.

Why Favor Unitarity Nonetheless?

1. We think QFT works in weakly-gravitating regions.

2. The AdS/CFT Correspondence.

The AdS/CFT Correspondence

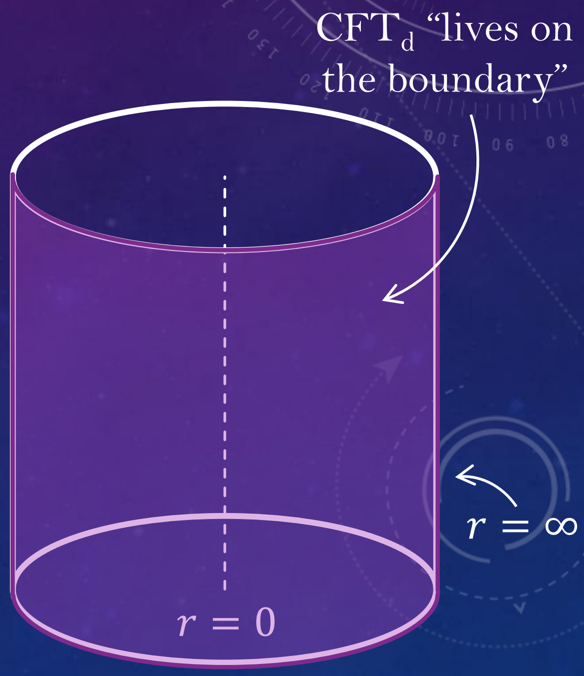
Quantum Gravity in $\mathbf{AdS_{d+1}}$

↕ *physically equivalent*

Non-Gravitational $\mathbf{CFT_d}$

Anti de Sitter space:

Maximally symmetric solution to Einstein's equations with negative cosmological constant



AdS Space

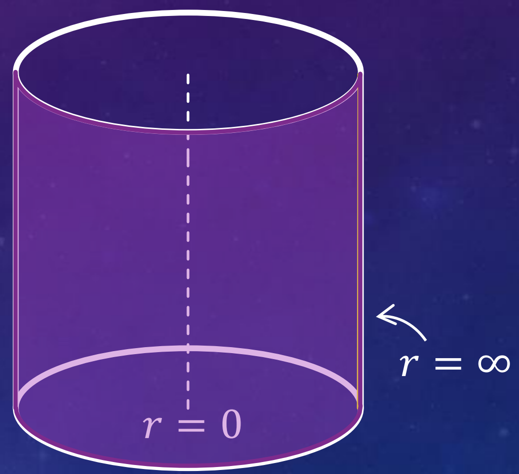
CFT is unitary, so black hole evaporation should be unitary

The AdS/CFT Correspondence

Quantities in the bulk

Holographic dictionary

Quantities on the boundary



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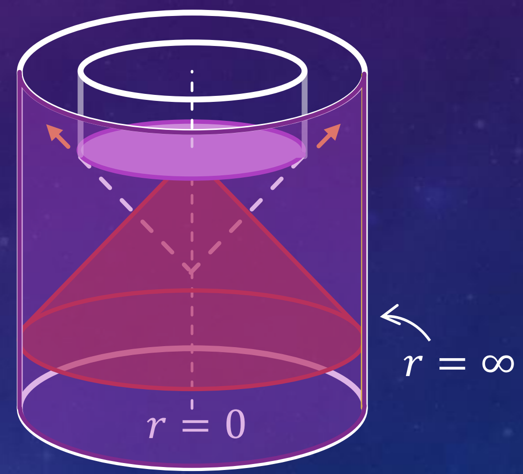

AdS Space with evaporating black hole

THE ADS/CFT CORRESPONDENCE

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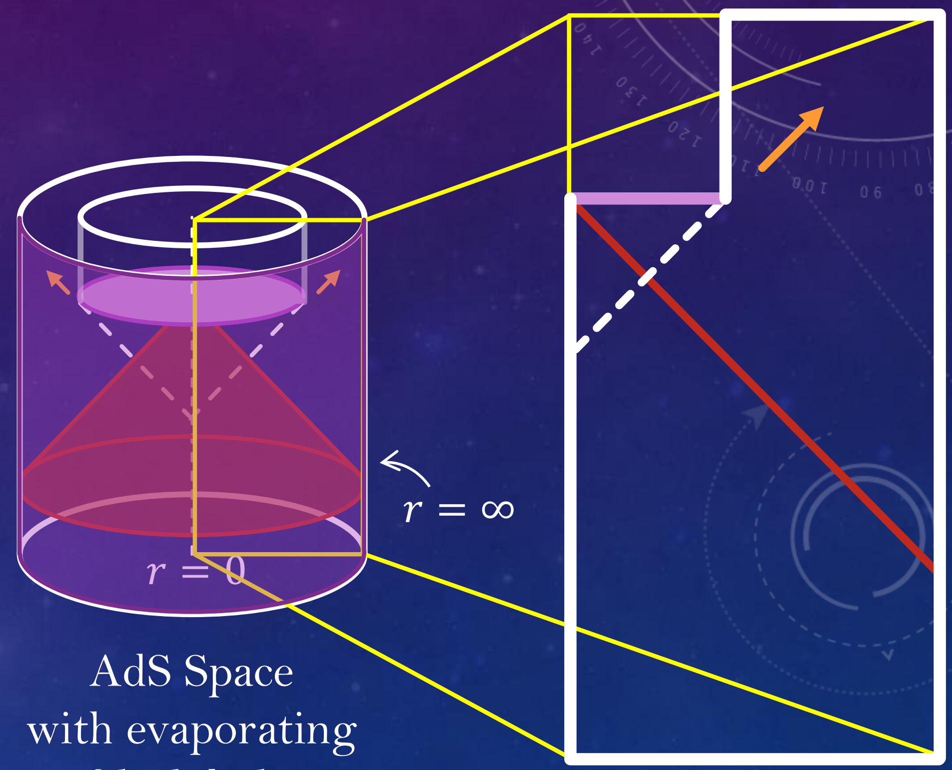

AdS Space
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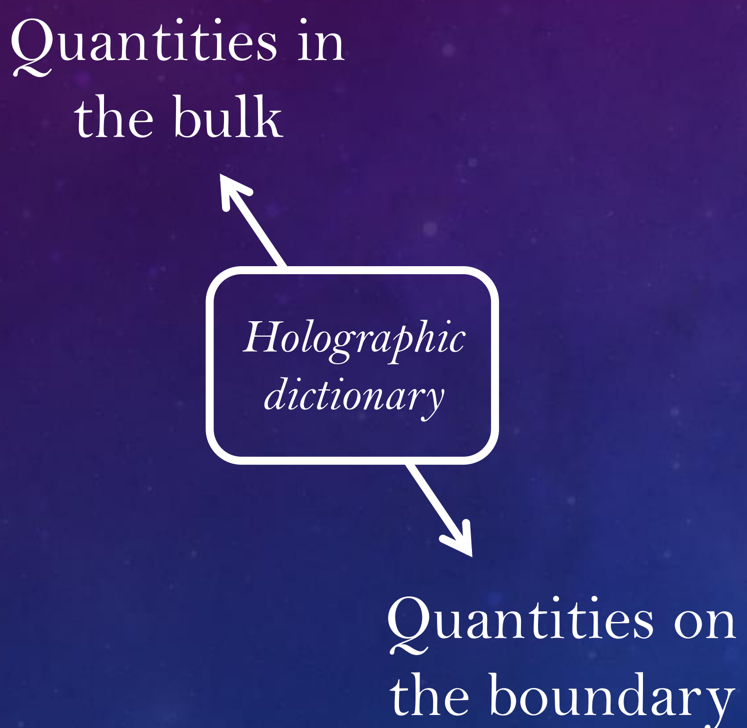
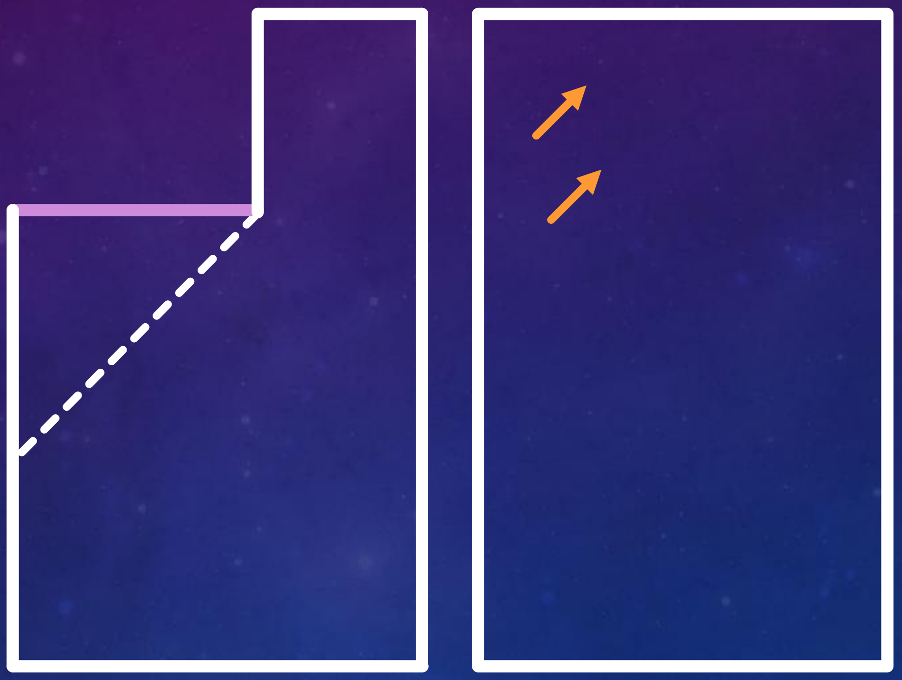
THE ADS/CFT CORRESPONDENCE



Evaporating black hole in AdS space

A Recent Look at Evaporating BH's: The Setup

JT (1+1D) gravity with Hawking State



Auxiliary System

arXiv:1905.08255
arXiv:1905.08762

Tracking Information in Black Hole Evaporation

Von Neumann Entropy:

$S(\rho) \equiv -\mathrm{tr}(\rho \log \rho)$

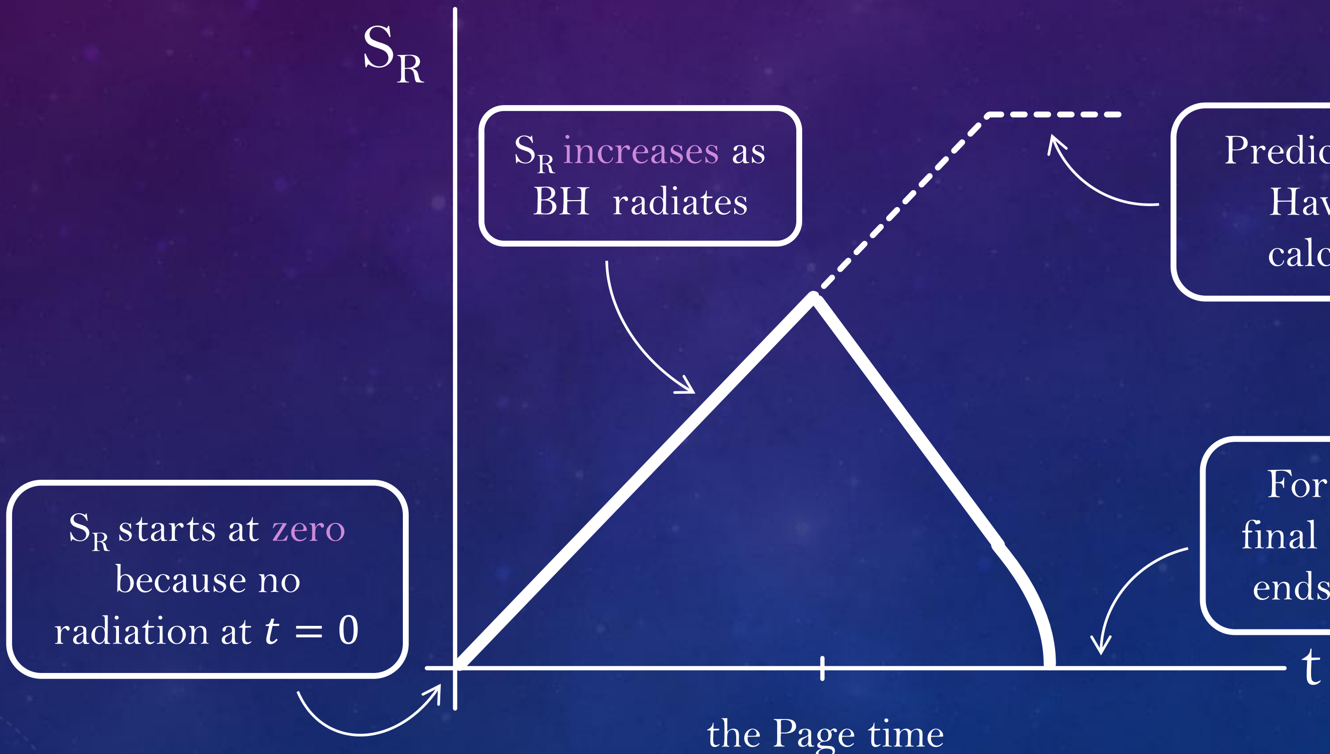
Pure total state implies $S_A = S_B$

Entanglement Entropy:

$S_A \equiv -\mathrm{tr}(\rho_A \log \rho_A)$

$\langle a'|\rho_A|a\rangle \equiv \sum_b \langle a'b|\rho_{AB}|ab\rangle$

THE PREDICTION OF UNITARY EVAPORATION: THE PAGE CURVE



A Useful Holographic Relationship: The (H)RT Prescription and Quantum Extremal Surfaces

Generalized Entropy:

$$S_{\text{gen}}[\sigma] = \frac{\mathcal{A}(\sigma)}{4G\hbar} + S[\text{Ext}(\sigma)]$$

Quantum Extremal Surface (QES):

A surface whose generalized entropy is stationary with respect to all deformations.

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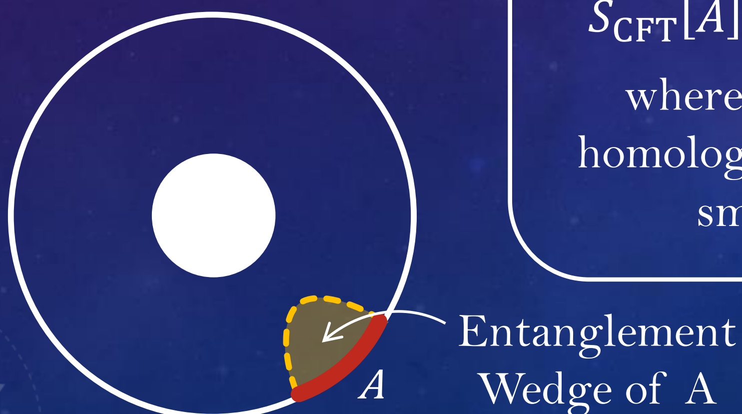


Quantum (H)RT Prescription:

$$S_{\text{CFT}}[A] = S_{\text{gen}}\big[\gamma_{\text{RT}}[A]\big],$$

where γ_{RT} is the QES homologous to A with the smallest S_{gen}.

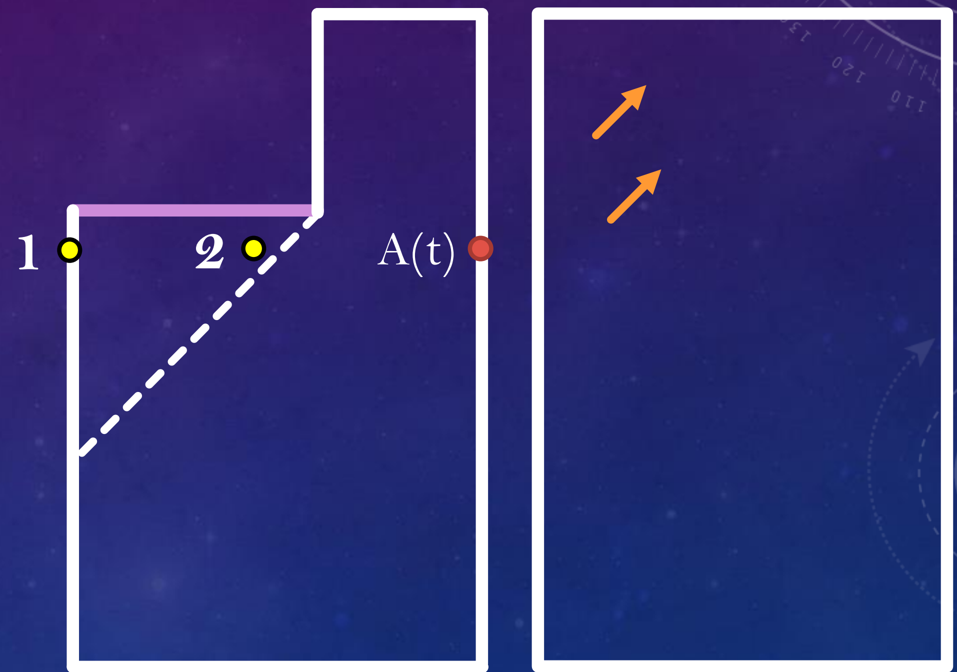
Quantum Extremal Surface (QES):
A surface whose generalized entropy is stationary with respect to all deformations.

A Recent Look at Evaporating BH's: The Calculation

General Approach: Taking the boundary region (A) to be the whole boundary at a particular time (t), find the QES's.

The QES's:

1. The empty surface
2. $\gamma(t)$



JT gravity with Hawking State

Auxiliary System

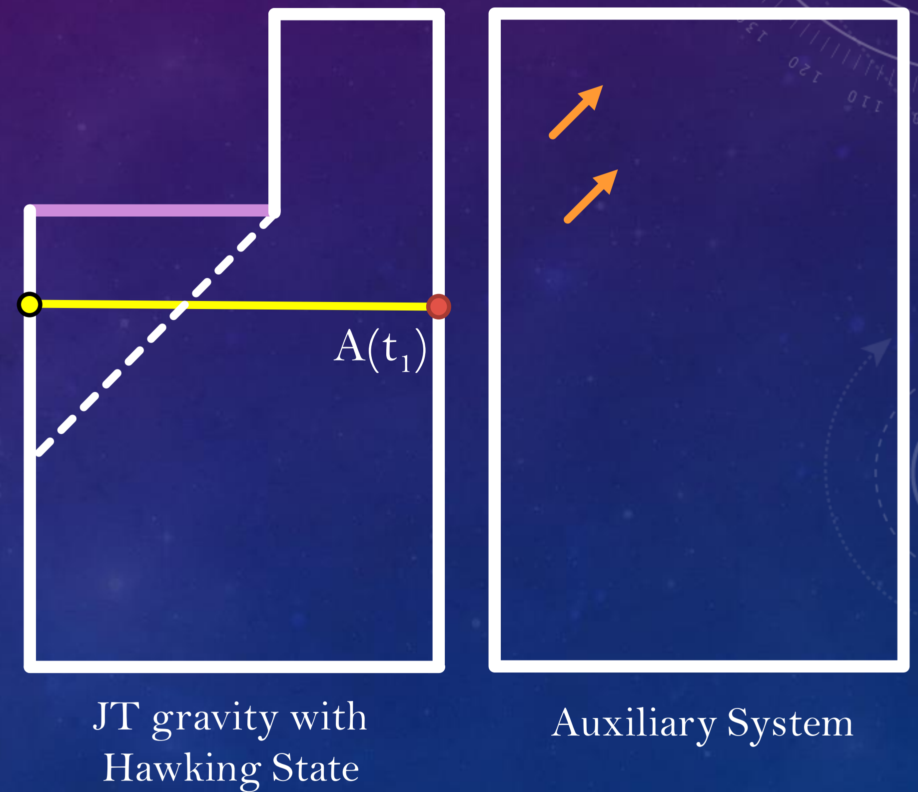
A Recent Look at Evaporating BH's: The Calculation

Before the Page Time:

Minimal QES = empty surface

$$S_{\mathrm{CFT}} = 0 + S_{\mathrm{bulk}} = S_{\mathrm{interior}}$$
$$S_{\mathrm{interior}} = S_{\mathrm{R}}$$

S_{CFT} increases monotonically before the Page time



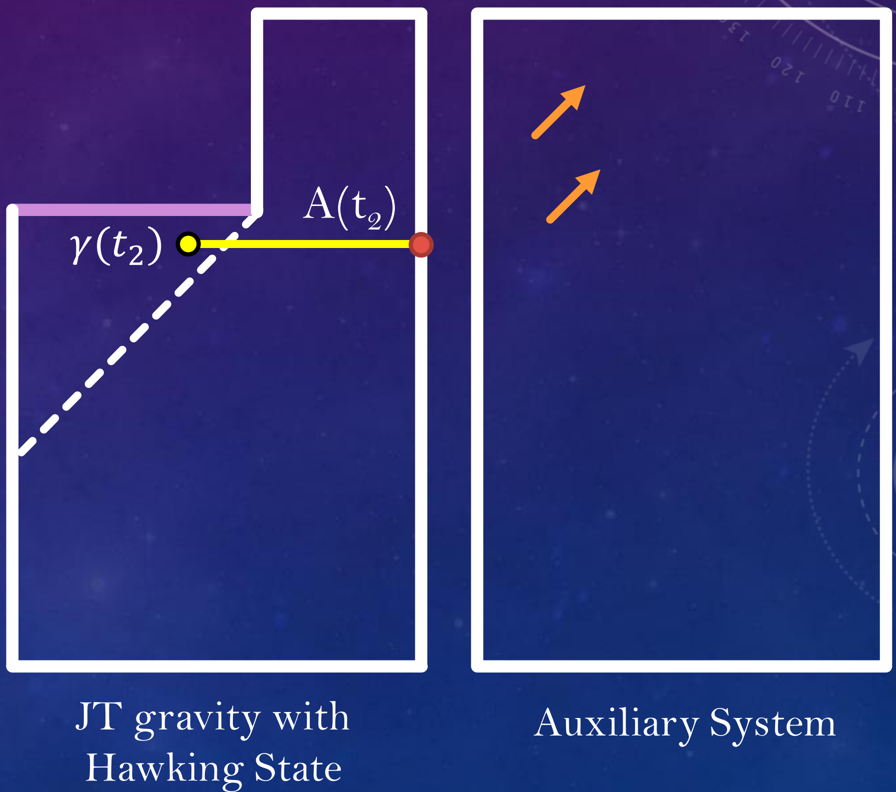
A Recent Look at Evaporating BH's: The Calculation

After the Page Time:

Minimal QES = $\gamma(t)$

$$S_{\text{CFT}} = \frac{\mathcal{A}(t)}{4G\hbar} + 0$$

S_{CFT} decreases monotonically after the Page time



A Recent Look at Evaporating BH's: The Results

S_{CFT} starts at zero and increases monotonically until the Page time.

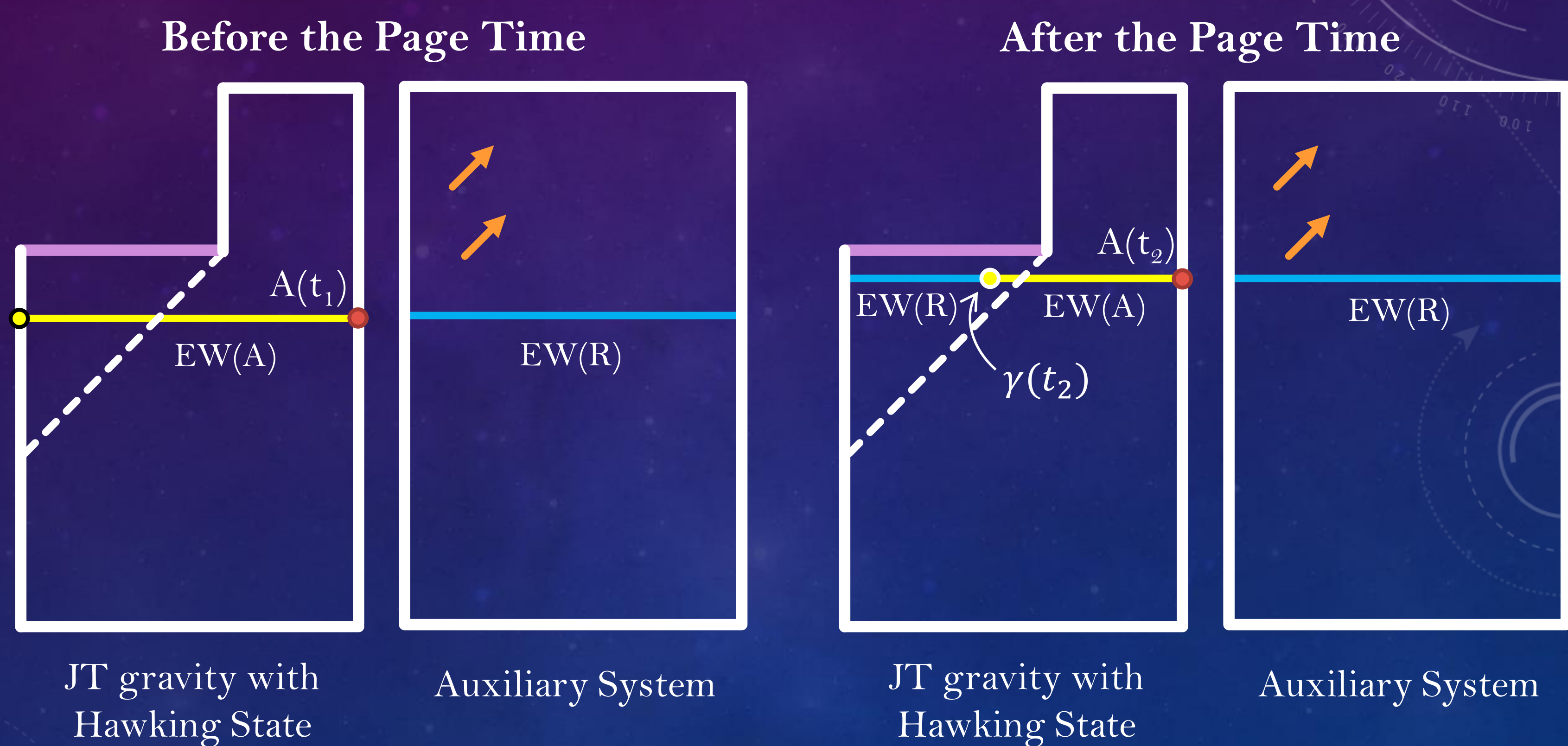
After the Page time, S_{CFT} decreases monotonically until it reaches zero.

Therefore, S_{CFT} follows the Page Curve.

This is surprising because the calculation was semiclassical (Hawking's calculation) in the bulk.

WHAT ABOUT THE RADIATION?

Entanglement Wedge Complementarity



An "island" appears in the entanglement wedge

Some Puzzles and Further Relevant Work

What does the observer detect? Information loss or not?

Contradiction between mixed out-state and purity of the CFT? Significance of ensemble of CFTs on the boundary? *arXiv:1911.06305*.

Computing the Page curve for the radiation with double holography: *arXiv:1908.10996*.

Applying the gravitational path integral (via replica wormholes) to this setup: *arXiv:1911.12333*, *arXiv:1911.11977*.

Double-holography in higher dimensions: *arXiv:1911.09666*.

References

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T. Takayanagi. "Holographic dual of BCFT," *Phys. Rev. Lett.* **107** (2011) 101602. arXiv:1105.5165.

Thank You for Your Attention!

Extra Slides

AdS Geometry

Metric of AdS_{d+1}

$$ds^2 = \ell^2\left(-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho d\Omega_{d-1}^2\right)$$

Construction of Penrose diagram

$$d\sigma = \frac{d\rho}{\cosh\rho} \rightarrow \sigma = 2\tan^{-1}\tanh\left(\frac{\rho}{2}\right)$$

ρ runs from 0 to ∞, σ runs from 0 to $\frac{\pi}{2}$.

JT Gravity and the SYK Model

JT Gravity

Action of JT gravity: $I = I_0 + I_G + I_M$, where

$$I_0 = \frac{\phi_0}{16\pi G_N}\left[\int_{\mathcal{M}} d^2x\,\sqrt{-g}R + 2\int_{\partial\mathcal{M}} K\right]$$

$$I_G = \frac{1}{16\pi G_N}\left[\int_{\mathcal{M}} d^2x\sqrt{-g}\phi(R+2) + 2\int_{\partial\mathcal{M}} \phi_b K\right]$$

$$I_M = I_{CFT}[g]$$

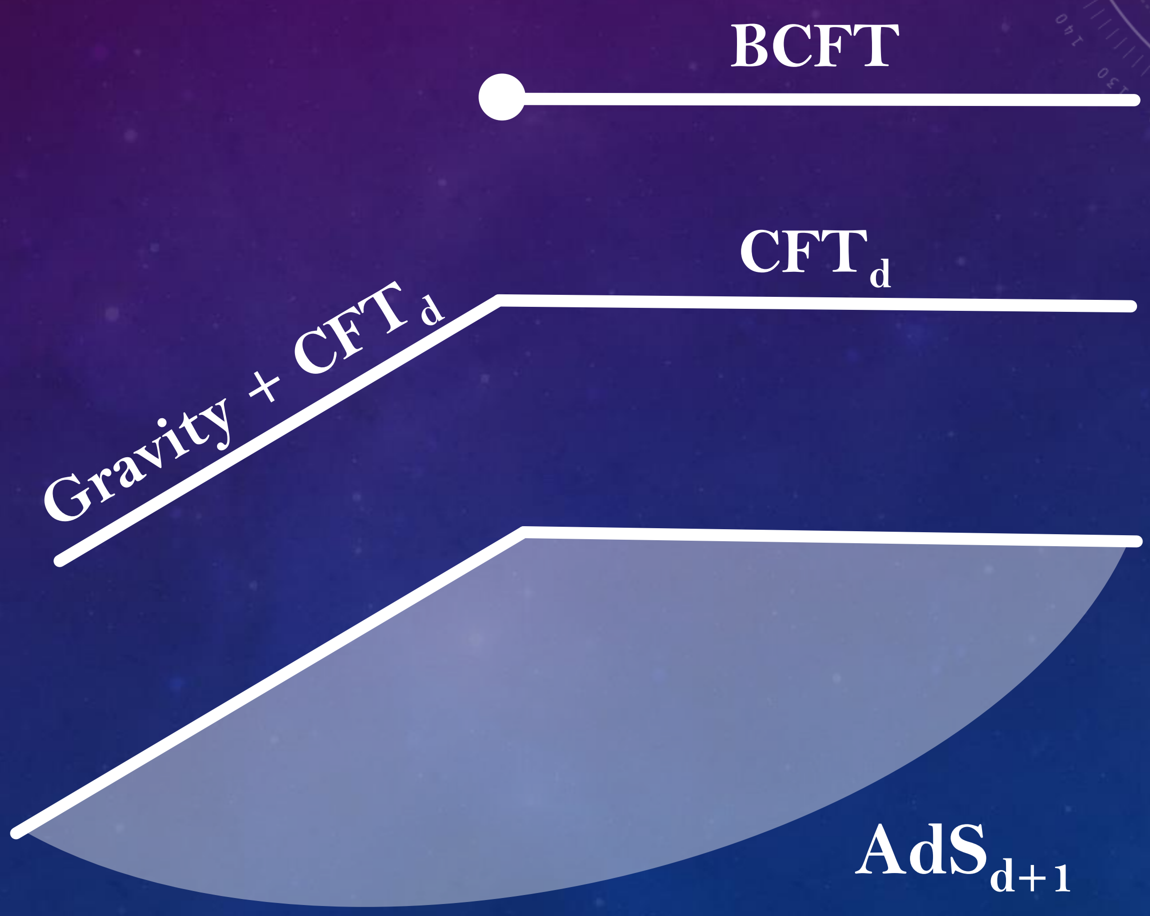
Solving the action yields a 1+1D metric that is locally AdS_2.

SYK Model

SKY model: 0+1D QM model of N Majorana fermions (ψ_i).

Hamiltonian: $H = \sum_{iklm} j_{iklm}\psi_i\psi_k\psi_l\psi_m$, where j_{iklm} are independent Gaussian random couplings.

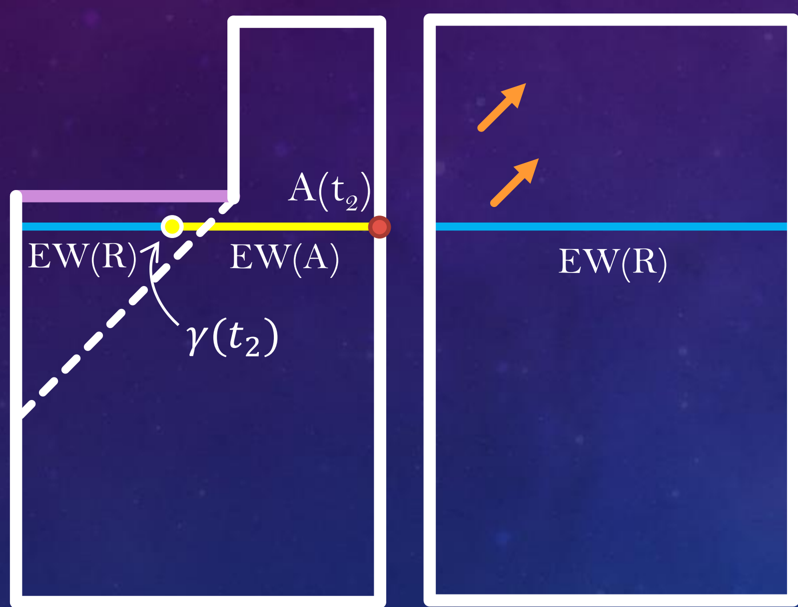
DOUBLE-LAYER HOLOGRAPHY: THE BATH AS A CFT



arXiv:1908.10996

Double-Layer Holography: The Bath as a CFT

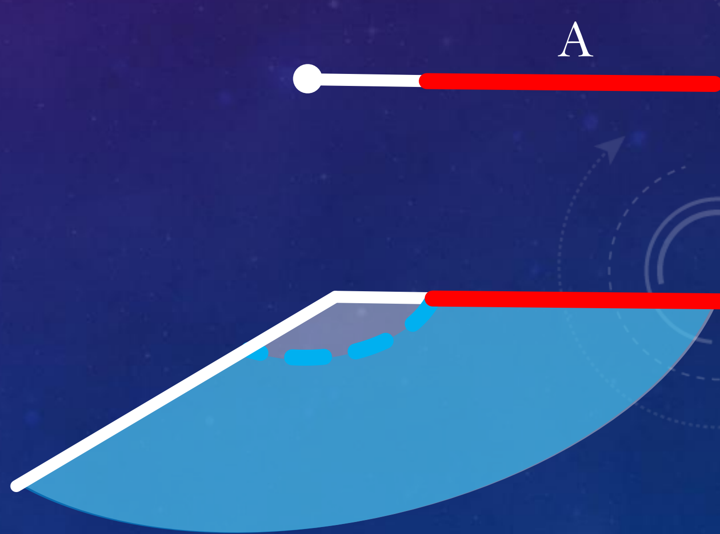
Entanglement Wedge Complementarity



JT gravity with Hawking State

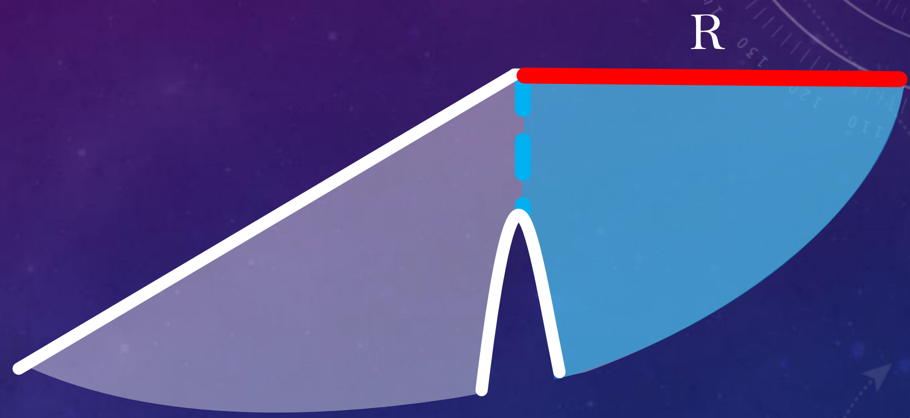
Auxiliary System

RT for BCFTs



DOUBLE-LAYER HOLOGRAPHY: THE PAGE CURVE OF THE RADIATION

Before
the Page Time



After
the Page Time

Double-Layer Holography: The Page Curve of the Radiation

The entropy of the radiation follows the Page curve.

A New Prescription:

$$S(A) = \underset{\mathcal{I}_g}{\text{Min}}\ \underset{\mathcal{I}_g}{\text{Ext}} \left[S_{\text{eff}}(A \cup \mathcal{I}_g) + \frac{\text{Area}[\partial \mathcal{I}_g]}{4G_N} \right]$$

Rindler Coordinates

Rindler Space metric:

$$ds^2 = dR^2 - R^2 d\eta^2, R > 0, \eta = \text{anything}$$

Coordinate transformation makes the Rindler patch more evident:

$$x = R\cosh\eta$$

$$t = R\sinh\eta$$

$$ds^2 = -dt^2 + dx^2, x > 0, |t| < x$$