UNIVERSITÄT BERN
AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

Asymptotic safety and walking dynamics at large charge

Susanne Reffert
University of Bern

based on arXiv:1505.01537, 1610.04495, 1707.00710, 1809.06371, 1902.09542, 1905.00026, 1909.02571, 1909.08642, and work in progress with:

L. Alvarez-Gaume (SCGP), D. Banerjee (Humboldt U.), Sh. Chandrasekharan (Duke), S. Favrod (Bern), S. Hellerman (IPMU), O. Loukas, D. Orlando (INFN Torino), F. Sannino (Odense), M. Watanabe (Weizmann)

Introduction

Strongly coupled physics is notoriously difficult to access.
We do not have small parameters in which to do a perturbative expansion. Our most basic notions of field theory are of a perturbative nature.

Make use of symmetries, look at special limits/ subsectors where things simplify.

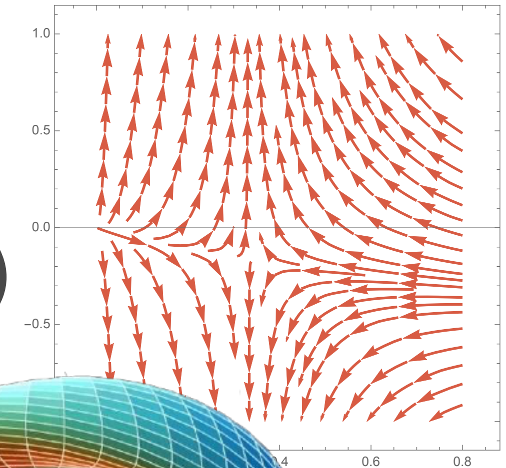
Here: study theories with a global symmetry group.
Hilbert space of the theory can be decomposed into sectors of fixed charge Q under the action of the global symmetry group.
Study subsectors with large charge Q.

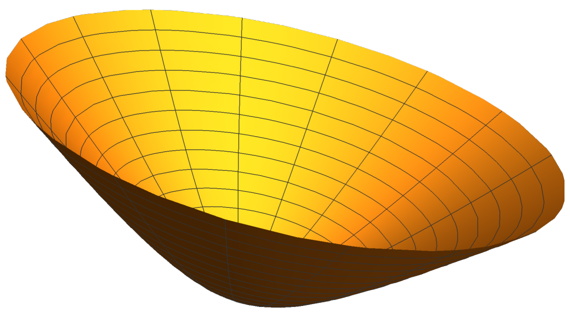
Large charge Q becomes controlling parameter in a perturbative expansion!

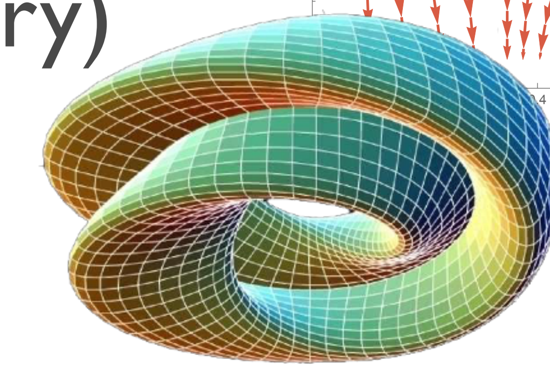
Introduction

CFTs play an important role in theoretical physics:

- fixed points in RG flows
- critical phenomena
- quantum gravity (via AdS/CFT)
- string theory (WS theory)







Conformal field theories (CFTs) do not have any intrinsic scales, most have by naturalness couplings of O(1).

Possibilities: analytic (2d), conformal bootstrap (d>2), lattice calculations, non-perturbative methods…

Prime candidate for the large-charge approach.

(Also: they come with a lot of space-time symmetry that will help us in practice to constrain the eff. action.)

Beyond O(2)

Where else can apply the large-charge expansion?

Try out other known CFTs/assume they exist.

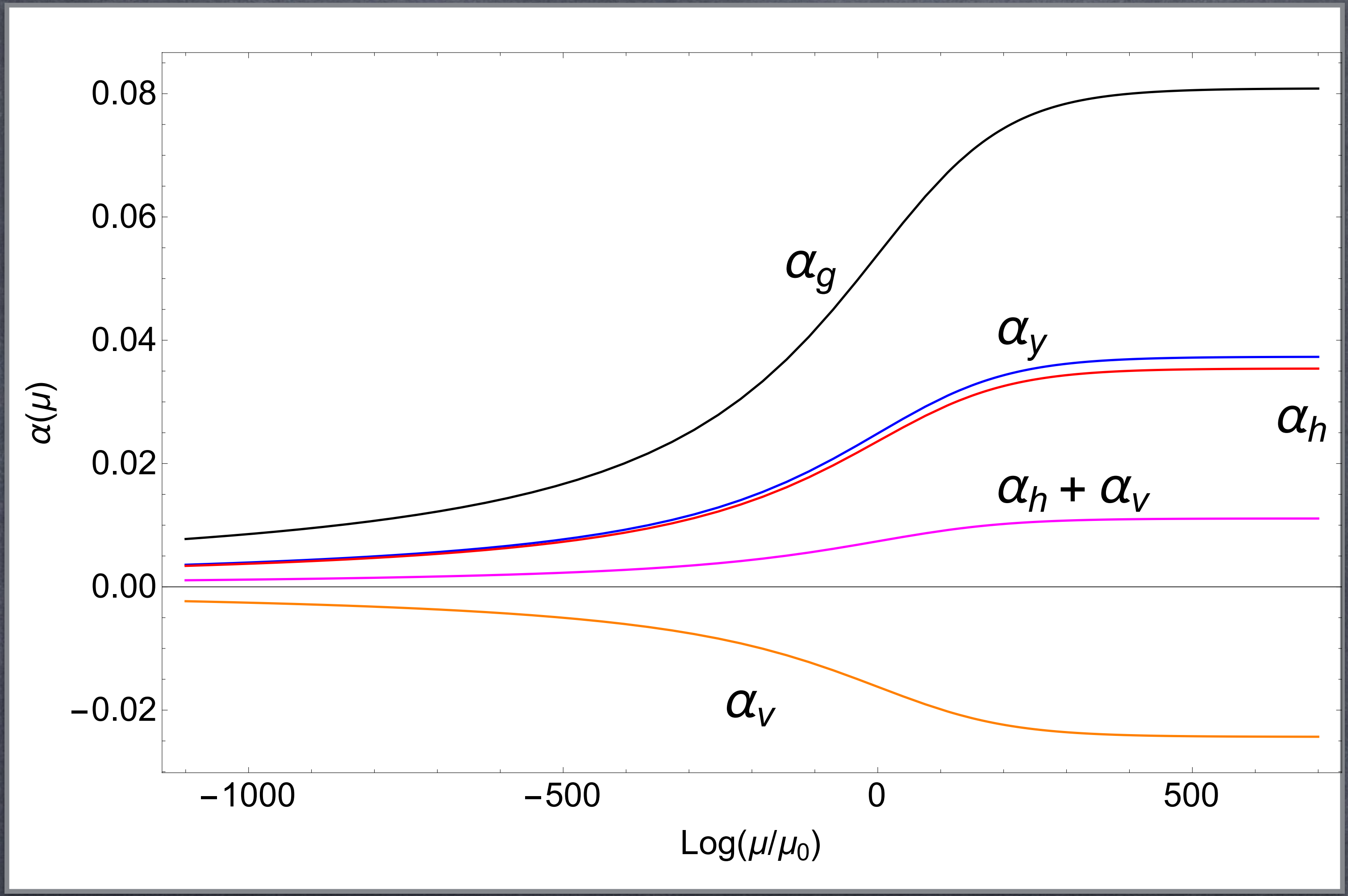
Obvious generalization in 3d: O(2n) vector model
non-Abelian global symmetry group: new effects

SU(N) matrix model in 3d.

Not many examples of (non-susy, non-fermionic) CFTs known in 4d.
Asymptotically safe CFT (UV fixed point)

Superconformal CFTs in 3d and 4d. Cases with moduli space work differently!

Non-relativistic CFTs (Schrödinger symmetry) in 3d, 4d



An asymptotically safe CFT in 4d

An asymptotically safe CFT

Look for CFTs with bosons in 4D. Start with a QCD-inspired theory with quarks, gluons and scalars:

gauge group $SU(N_C)$ — N_F flavors of fermions — $N_F \times N_F$ matrix of cplx scalars

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \mathrm{Tr}\big(\bar{Q}i\not{D}Q\big) + y\,\mathrm{Tr}\big(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\big)$$
$$+\,\mathrm{Tr}\big(\partial_\mu H^\dagger \partial^\mu H\big) - u\,\mathrm{Tr}\big(H^\dagger H\big)^2 - v(\mathrm{Tr}\,H^\dagger H)^2 - \frac{R}{6}\,\mathrm{Tr}\big(H^\dagger H\big)$$

$Q_{L/R} = \frac{1}{2}(1 \pm \gamma_5)Q$

Rescaled couplings

$$\alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \qquad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}, \qquad \alpha_h = \frac{u N_F}{(4\pi)^2}, \qquad \alpha_v = \frac{v N_F^2}{(4\pi)^2}$$

Control parameter $\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$

In the limit $N_F \to \infty,\ N_C \to \infty$ with N_F/N_C fixed: asymptotically safe.

Litim, Sannino

Perturbatively controlled UV fixed point with

$$\alpha_g^* = \frac{26}{57}\epsilon + \dots \qquad \alpha_y^* = \frac{4}{19}\epsilon + \dots \qquad \alpha_h^* = \frac{\sqrt{23}-1}{19}\epsilon + \dots \qquad \alpha_{v1}^* = -0.1373\,\epsilon + \dots$$

An asymptotically safe CFT

Study this theory at large charge.

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \mathrm{Tr}\big(\bar{Q}i\not{D}Q\big) + y\,\mathrm{Tr}\big(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\big)$$
$$+\,\mathrm{Tr}\big(\partial_\mu H^\dagger \partial^\mu H\big) - u\,\mathrm{Tr}\big(H^\dagger H\big)^2 - v(\mathrm{Tr}\,H^\dagger H)^2 - \frac{R}{6}\,\mathrm{Tr}\big(H^\dagger H\big)$$

Global symmetry: $SU(N_F)_L \times SU(N_F)_R \times U(1)_B$

New elements compared to vector model:

- H is a matrix field, large non-Abelian global symmetry
- fermions and gluons are present
- 4D, different scalings
- UV fixed point, perturbatively controlled, trustable LSM

Large-charge expansion: focus on scalar sector

An asymptotically safe CFT

Noether currents:

$$J_L = \frac{i}{2}\left(\mathrm{d}H\, H^\dagger - H\,\mathrm{d}H^\dagger\right), \qquad J_R = -\frac{i}{2}\left(H^\dagger\,\mathrm{d}H - \mathrm{d}H^\dagger\, H\right)$$

Corresponding charges:

$$\mathcal{Q}_L = \int \mathrm{d}^3x\, J^0_L, \qquad \mathcal{Q}_R = \int \mathrm{d}^3x\, J^0_R$$

$$\mathrm{spec}(\mathcal{Q}_L) = \{J^L_1, J^L_2, \ldots, J^L_{N_F}\} \qquad \mathrm{spec}(\mathcal{Q}_R) = \{J^R_1, J^R_2, \ldots, J^R_{N_F}\}$$

Ansatz for homogeneous ground state:

Cartan subalgebra

self-adjoint

$$H_0(t) = e^{iM_L t} B e^{-iM_R t}$$

Impose charge conservation:

$$\dot{\mathcal{Q}}_L = -iVe^{iM_L t}\left(\left[M_L^2, BB^\dagger\right] - 2\left[M_L, BM_RB^\dagger\right]\right)e^{-iM_L t} = 0,$$

$$\dot{\mathcal{Q}}_R = iVe^{iM_R t}\left(\left[M_R^2, B^\dagger B\right] - 2\left[M_R, B^\dagger M_L B\right]\right)e^{-iM_R t} = 0$$

M commutes or anti-comm with B

diagonal

$$\Rightarrow H_0 = e^{2iMt} B$$

An asymptotically safe CFT

We find: $\mathcal{Q}_L = -2VMB^2, \qquad \mathcal{Q}_R = 2VB^2M = -\mathcal{Q}_L$

Simple choice for charges:

$$M = \mu\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & -\mathbb{1} \end{array}\right), \qquad B = b\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & \mathbb{1} \end{array}\right) \qquad \mathcal{Q}_L = J\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & -\mathbb{1} \end{array}\right)$$

in su(N), traceless

$$J = 2Vb^2\mu$$

EOM on ansatz $H_0 = e^{2iMt}B$:

$$2\mu^2 = (u + vN_F)b^2 + \frac{R}{12}$$

Assume J large, expand in series:

$$\mu = \left(\frac{2\pi^2}{V}\right)^{1/3}\mathcal{J}^{1/3} + \frac{R}{72}\left(\frac{V}{2\pi^2}\right)^{1/3}\mathcal{J}^{-1/3} + \mathcal{O}\left(\mathcal{J}^{-5/3}\right)$$

Natural expansion parameter:

$$\mathcal{J} = J\frac{(u + vN_F)}{8\pi^2} = 2J\frac{\alpha_h + \alpha_v}{N_F} = 2J_{\text{tot}}\frac{\alpha_h + \alpha_v}{N_F^2} \gg 1$$

$J_{\text{tot}} = JN_F$

Consistent for $J_{\text{tot}} \gg \frac{N_F^2}{\epsilon}$ (N_F^2: huge; ϵ: tiny)

An asymptotically safe CFT

Ground-state energy:

not universal

$$E = \frac{3}{2}\frac{N_F^2}{\alpha_h + \alpha_v}\left(\frac{2\pi}{V}\right)^{1/3}\left[\mathcal{J}^{4/3} + \frac{R}{36}\left(\frac{V}{2\pi^2}\right)^{2/3}\mathcal{J}^{2/3} - \frac{1}{144}\left(\frac{R}{6}\right)^2\left(\frac{V}{2\pi^2}\right)^{4/3}\mathcal{J}^0 + \mathcal{O}\left(\mathcal{J}^{-2/3}\right)\right]$$

Specialize to 3-sphere: $E = \frac{3}{2r_0}\frac{N_F^2}{\alpha_h + \alpha_v}\left[\mathcal{J}^{4/3} + \frac{1}{6}\mathcal{J}^{2/3} - \frac{1}{144}\mathcal{J}^0 + \mathcal{O}\left(\mathcal{J}^{-2/3}\right)\right]$

Classical result. What about Goldstone contributions, what about fermions, gluons?

At large charge, the fermions receive large masses and decouple:

kinetic term

Yukawa term

$$m_\psi = (\mu^2 + y^2 b^2)^{1/2} = \left(\frac{2\pi^2}{V}\right)^{1/3}\left(1 + 2\frac{N_F}{N_c}\frac{\alpha_y}{\alpha_h + \alpha_v}\right)^{1/2}\mathcal{J}^{1/3} + \mathcal{O}\left(\mathcal{J}^{-1/3}\right)$$

Below the fermion mass scale, also gluons decouple.

Gap:

$$\Lambda_{YM} = m_\psi \exp\left[-\frac{3}{22\alpha_g(m_\psi)}\right]$$

$\approx \mathcal{O}(\epsilon)$

Low-energy physics described by Goldstones only!

An asymptotically safe CFT

Symmetry-breaking pattern: $H_0 = e^{2iMt}B$

$$SU(N_F)\times SU(N_F)\times U(1) \xrightarrow{\text{exp.}} SU(N_F/2)\times SU(N_F/2)\times U(1)^2\times SU(N_F)$$
$$\xrightarrow{\text{spont.}} SU(N_F/2)\times SU(N_F/2)\times U(1)^2$$

Expect $\dim(SU(N_F)) = N_F^2 - 1$ Goldstone DoF

Do quadratic expansion of the Lagrangian around the ground state, find dispersion relations.

$$\omega = \frac{p^2}{4\mu} + \ldots$$ $(N_F/2)^2$ type II Goldstone modes

$$\omega = \frac{p}{\sqrt{3}} + \ldots$$ conformal Goldstone (type I)

$$\omega = \sqrt{\frac{\alpha_h}{3\alpha_h + 2\alpha_v}}\,p + \ldots$$ $N_F^2/2 - 2$ type I Goldstones

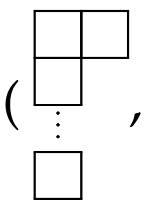
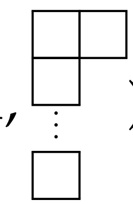
causality constraint: $0 < \alpha_h/(3\alpha_h + 2\alpha_v) < 1$

Constraint satisfied at fixed point.

An asymptotically safe CFT

Goldstones are organized in reps of the unbroken symmetry group:

adjoint   bifundamental

$SU(N_F/2)\times SU(N_F/2)$ representation	$(\mathbf{1},\mathbf{1})$	( , $\mathbf{1}$)	($\mathbf{1}$, )	$(\Box,\Box)$
type	I	I	I	II
DOF	1	$N^2/4-1$	$N^2/4-1$	$2\times N^2/4$
velocity	$1/\sqrt{3}$	$\sqrt{\frac{\alpha_h}{3\alpha_h+2\alpha_v}}$	$\sqrt{\frac{\alpha_h}{3\alpha_h+2\alpha_v}}$	n/a

Vacuum energy of the type I Goldstones:

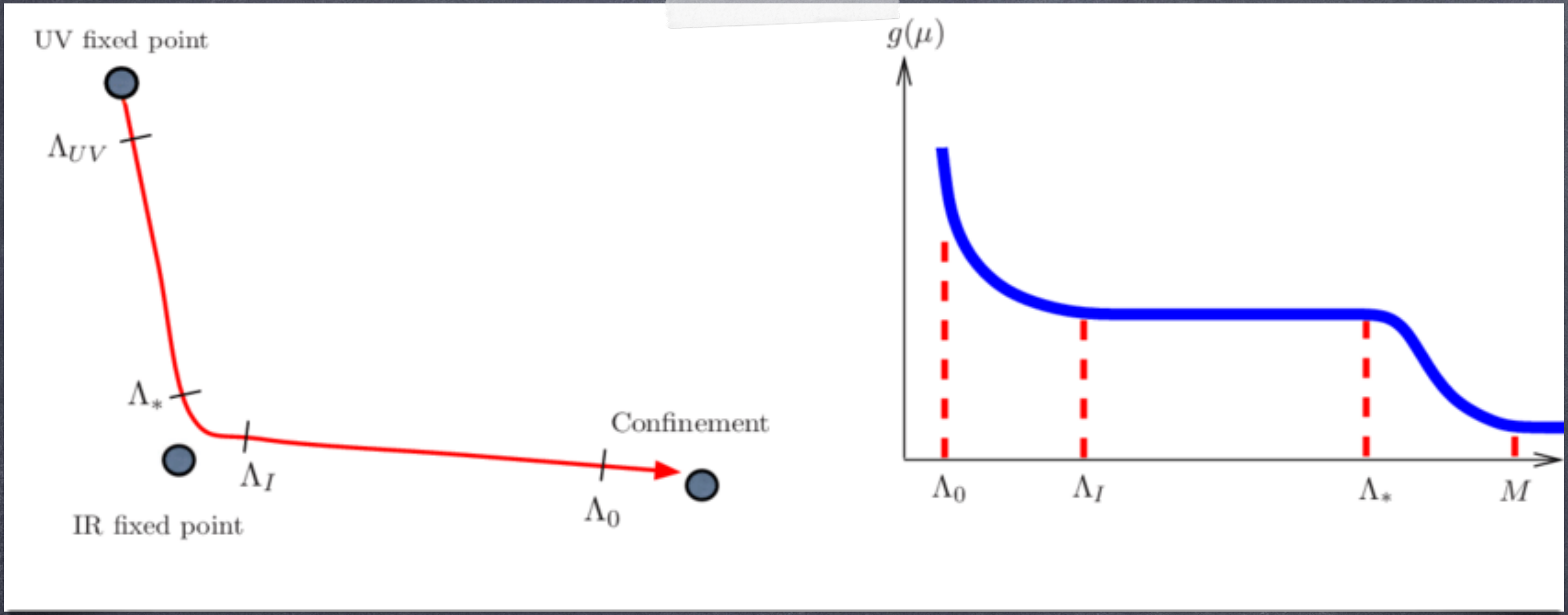
$$\zeta(-1/2|S^3) = -\frac{0.414\ldots}{r_0}$$

$$E_0 = \frac{1}{2}\left(2\times\left(\frac{N_F^2}{4}-1\right)\sqrt{\frac{\alpha_h}{3\alpha_h+2\alpha_v}}+\frac{1}{\sqrt{3}}\right)\zeta(-1/2|M_3).$$

Conformal dimension (via state-operator corr.):

$$\Delta(J) = r_0 E(S^3) = \frac{3}{2}\frac{N_F^2}{\alpha_h+\alpha_v}\left[\mathcal{J}^{4/3}+\frac{1}{6}\mathcal{J}^{2/3}-\frac{1}{144}\mathcal{J}^0+\mathcal{O}\left(\mathcal{J}^{-2/3}\right)\right]$$
$$-\left(\left(\frac{N_F^2}{2}-2\right)\sqrt{\frac{\alpha_h}{3\alpha_h+2\alpha_v}}+\frac{1}{\sqrt{3}}\right)\times 0.212\ldots$$

D.Orlando, S.R., F. Sannino, arXiv:1905.00026



Leaving the conformal point

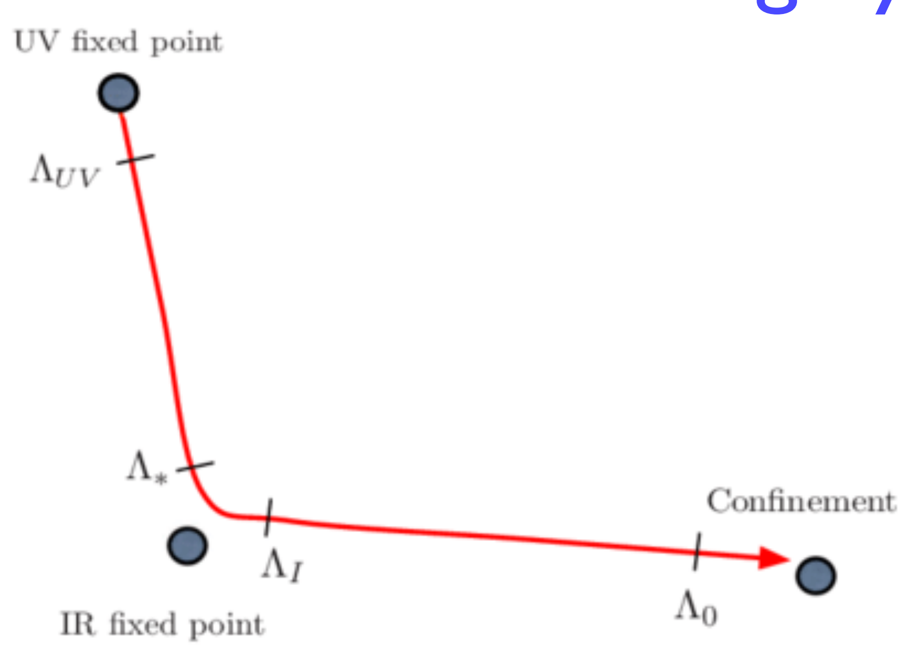
Leaving the conformal point

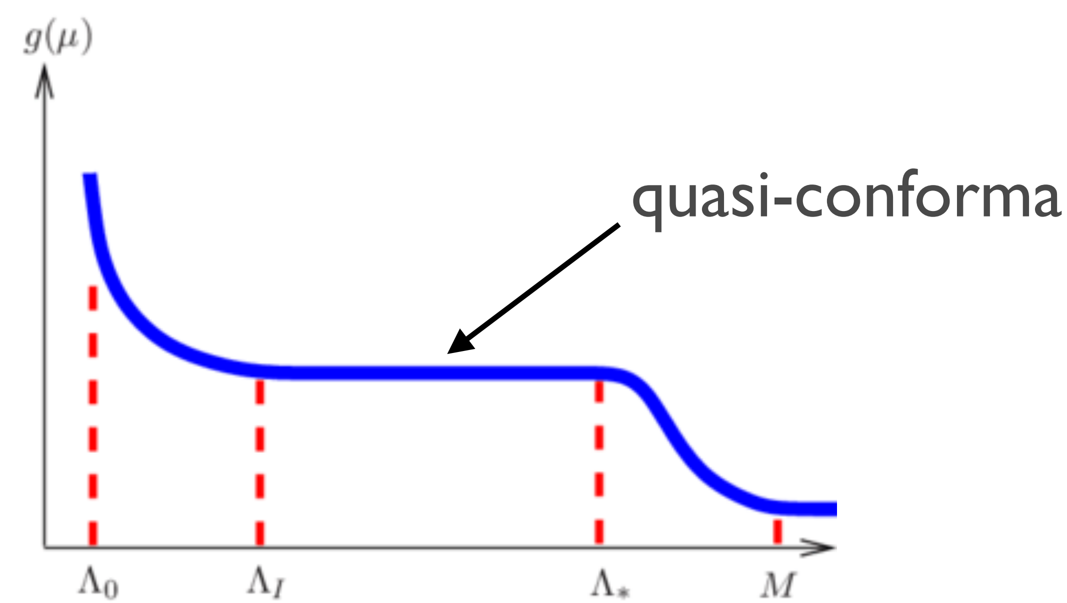
There is no reason why the large-charge approach should not work for general QFTs.

Of course, there are many practical advantages in working at conformality (restricting the form of terms appearing in the eff. action, state/op. correspondence…)

First step: work near enough a conformal point that it still dominates the dynamics.

Possible scenario: walking dynamics





Leaving the conformal point

Consider simple case with a global U(1) at large charge in 4D.

The leading term in the effective action (on torus) is given in terms of the Goldstone,

$$L_{NLSM}[\chi] = k_4(\partial_\mu\chi\,\partial_\mu\chi)^2$$

Class. ground state: $\chi = \mu t \qquad \mu = (4k_4 Q)^{1/3}/L$

Start differently: two-derivative EFT for Goldstone:

$$L_2[\chi] = \frac{f_\pi^2}{2}\partial_\mu\chi\,\partial_\mu\chi - C^4$$

dim[1] constants

Introduce new field σ to non-linearly realize conformal invariance. σ acts as the massive Goldstone of broken conformal symmetry.

Leaving the conformal point

Under dilatations: $x \to e^{\alpha} x :\qquad \sigma \to \sigma - \alpha/f$ ← [f]= -1

To non-linearly realize conformal symmetry, dress all operators:

$[\mathcal{O}_k] = k$

$$\mathcal{O}_k \to e^{(k-4)f\sigma}\mathcal{O}_k$$

kin. term, conf. coupling

$$L_{CFT}[\chi,\sigma] = \frac{1}{2} g^{\mu\nu} f_\pi^2 e^{-2\sigma f}\,\partial_\mu\chi\,\partial_\nu\chi - C^4 e^{-4\sigma f} + \frac{1}{2} e^{-2\sigma f}\left(g^{\mu\nu}\,\partial_\mu\sigma\,\partial_\nu\sigma - \frac{\xi R}{f^2}\right) + \mathcal{O}(R^2)$$

Introduce complex field: $\Sigma = \sigma + i f_\pi \chi$

Recast action as

$\varphi = \frac{1}{\sqrt{2}f} e^{-f\Sigma}$ $\qquad u = 4C^4 f^4$

$$L[\varphi] = \partial_\mu\varphi^*\,\partial^\mu\varphi - \xi R\varphi^*\varphi - u(\varphi^*\varphi)^2 + \mathcal{O}(R^2)$$

LSM model action, σ appears as radial mode!

$$\varphi_{IR} = a\, e^{ib\chi}$$

Leaving the conformal point

Fixed-charge ground state:

$$\chi = \mu t, \qquad \sigma = \frac{1}{f}\log(v),$$

$$\mu = 4c_{4/3}\Lambda_Q/3, \qquad v = 2f_\pi\sqrt{c_{4/3}/3}/\Lambda_Q,$$

$$c_{4/3} = 3(C/(2f_\pi))^{4/3}, \qquad \Lambda_Q = Q^{1/3}/L$$

Expanding the fields around this vacuum, we find (as expected) a massless and a massive mode (which decouples in the EFT) ⇒ go back to NLSM

Can use it to explicitly break conformal invariance: add a (small!) mass for σ.

$$L_m[\chi,\sigma] = L_{CFT}[\chi,\sigma] - U_m(\sigma)$$

$$U_m(\sigma) = \frac{m_\sigma^2}{16f^2}(e^{-4\sigma f} + 4\sigma f - 1)$$

Energy-momentum tensor no longer traceless:

$$T^\mu{}_\mu = \frac{m_\sigma^2}{f}\sigma$$

Leaving the conformal point

What is the signature of this mass term at large charge?

Action admits same type of fixed-charge ground state solution.

Energy:
$$E = c_{4/3}\frac{Q^{4/3}}{L} - \frac{m_\sigma^2 L^3}{12 f^2}\log(Q) + c_0$$

Dispersion relations of the two modes:

$$\omega = \frac{1}{\sqrt{3}}\left(1 + \frac{m_\sigma^2}{9 c_{4/3} f^2 \Lambda_Q^4}\right) p,$$
$$\omega = b c_{4/3}\sqrt{\frac{32}{3}}\Lambda_Q + \frac{5}{8\sqrt{6} b c_{4/3}\Lambda_Q}\left(1 - \frac{m_\sigma^2}{20 c_{4/3} f^2 \Lambda_Q^4}\right) p^2.$$

Near the conformal point, physics is still governed by fixed point. Makes sense to study conformal dimension.

Leaving the conformal point

Calculate 2-point fn on the cylinder and map it to flat space via Weyl-rescaling:

$$\langle \mathcal{O}_Q(t_0, \mathbf{n}_0) \mathcal{O}_{-Q}(t_1, \mathbf{n}_1) \rangle_{\text{cyl}} = \int \mathcal{D}\chi \mathcal{D}\sigma \exp[Q \log(\varphi(t_0, \mathbf{n}_0) \bar{\varphi}(t_1, \mathbf{n}_1)) - \int \mathrm{d}t \mathrm{d}\Omega L_m[\chi, \sigma]]$$

Large Q: integral is dominated by saddle point,

$$\chi = i\mu t, \qquad \sigma = \text{const.}$$

$$\langle \mathcal{O}_Q(t_0, \mathbf{n}_0) \mathcal{O}_{-Q}(t_1, \mathbf{n}_1) \rangle_{\text{cyl}} \approx e^{-E_{\text{cyl}} |t_1 - t_0|}$$

$$r_0 E_{\text{cyl}} = \frac{c_{4/3}}{(4\pi^2)^{1/3}} Q^{4/3} + c_{2/3} Q^{2/3} + c_0 - \frac{\pi^2 m_\sigma^2 r_0^4}{3 f^2} \log Q + \dots$$

$$c_{2/3} = (\pi / (f_\pi \Lambda^2))^{2/3} / (2 f^2)$$

Map to flat space:

$$\langle \mathcal{O}_Q(t_0, \mathbf{n}_0) \mathcal{O}_{-Q}(t_1, \mathbf{n}_1) \rangle_{\text{flat}} = \frac{c_Q}{|x|^{\Delta^* + r_0 E_{\text{cyl}}}} = \frac{c_Q}{|x|^{2\Delta}}$$

$$\Delta = \Delta^* \left(1 - \frac{m_\sigma^2}{24 c_{4/3} f^2 \Lambda_Q^4} \log Q + \dots \right)$$

D.Orlando, S.R., F. Sannino, arXiv:1909.08642

Summary

Summary

We studied various CFTs in sectors of large global charge

Concrete examples where a (strongly-coupled) CFT simplifies in a special sector.

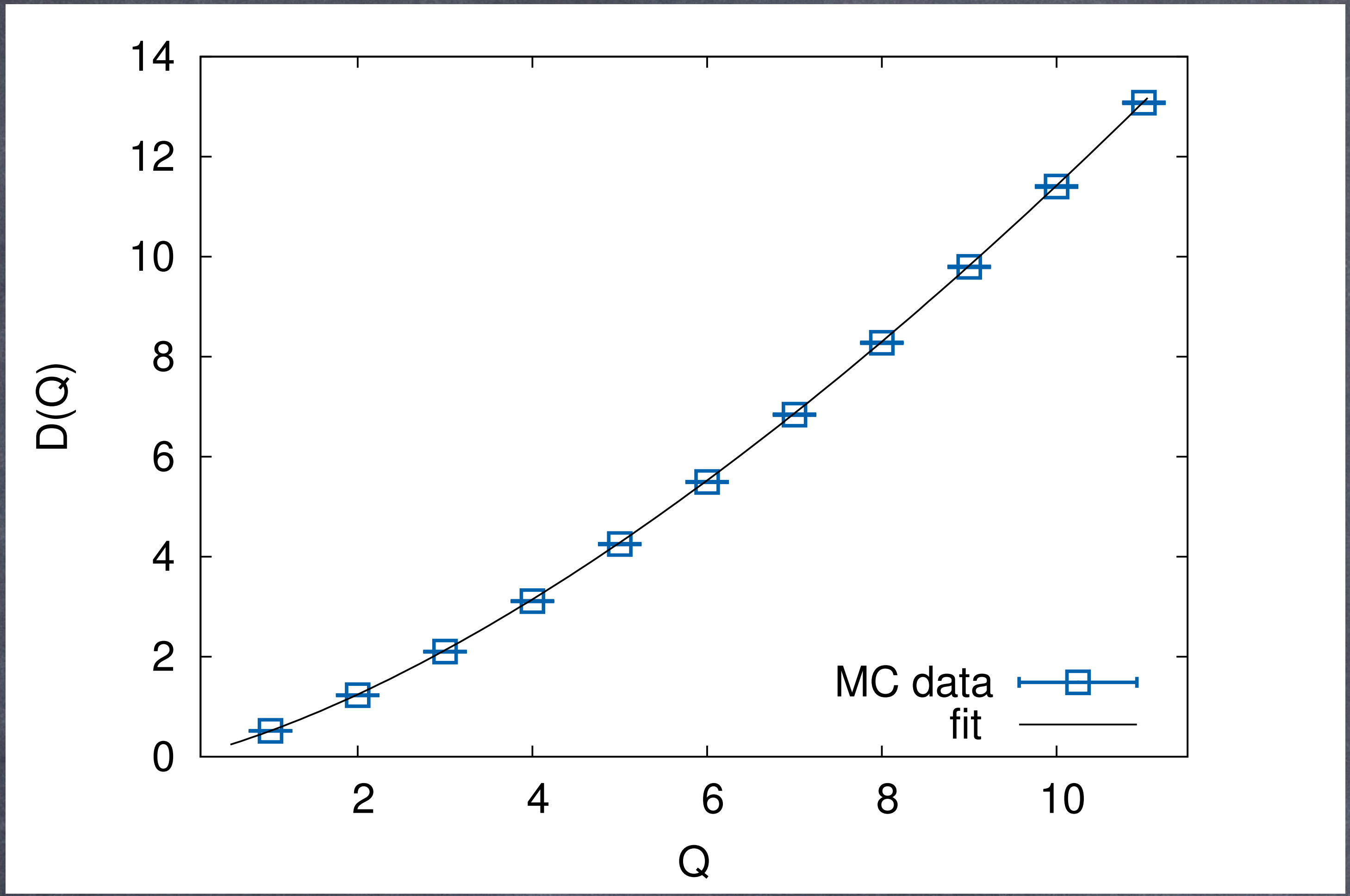
- O(2N) model in 3d: in the limit of large U(1) charge Q, we computed the conformal dimensions in a controlled perturbative expansion:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

  - Excellent agreement with lattice results for O(2), O(4)
- Can be applied beyond vector model: SU(N) matrix models, SCFT

Summary

- Asymptotically safe CFT in 4d (scalars, fermions and gauge fields). Controllable UV fixed point.
  - fermions and gluons decouple
  - large-charge expansion for scalar sector
  - interesting Goldstone spectrum
- near-conformal/walking dynamics:
  - radial mode can be reinterpreted as dilaton of spontaneously broken conformal symmetry.
  - Explicitly break conformality by adding mass term for dilaton.
  - log(Q)-term appears in ground state energy: signature of massive dilaton



Thank you for your attention!