QCD Axion Dark Matter from a Late Time Phase Transition

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QCD Axion Dark Matter

- Attractive dark matter candidate
  - Automatically arises from solving the Strong CP problem of the Standard Model via the PQ mechanism

$$\mathcal{L} \supset \frac{g^2}{32\pi^2}\left(\bar{\theta} + C\frac{a}{f_a}\right)F^{a\mu\nu}\tilde{F}^a_{\mu\nu}$$

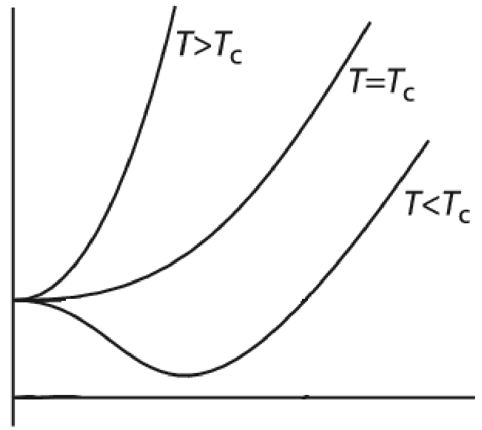
  - The mass is tied directly to the decay constant:

$$m_a \sim 6 \times 10^{-6}\ \mathrm{eV}\ \frac{10^{12}\ \mathrm{GeV}}{f_a/N}$$

  - Natural Production Mechanisms: Misalignment, Cosmic Strings, Domain Walls
  - Can be a good cold dark matter candidate for $f_a \sim 10^{11}\ \mathrm{GeV}$ using these production mechanisms.
  - Number of variations and other production scenarios have been developed, some of which allow for axion DM with $f_a \sim 10^9\ \mathrm{GeV}$.
  - Here we will focus on axions produced after a late time phase transition.

Axions from Cosmic Strings

- Consider a complex scalar field P with a global U(1) symmetry that is spontaneously broken as by the vev $\langle P \rangle = f_a/\sqrt{2}$
- Assuming some coupling to the SM, the scalar field acquires thermal corrections such that the phase transition occurs at T=T_c



- Since the vacuum manifold $\mathcal{M}$ is a circle , $\pi_1(\mathcal{M}) = \mathbb{Z}$, and topological defects in the form of strings can form at the phase transition

Axions from Cosmic Strings

- Kibble-Zurek Mechanism:
  - Consider a scalar field with a thermal potential:

$$V \supset \lambda^2 (T^2 - T_c^2)|P|^2$$

  - As T → T_c, the effective mass goes to zero and the correlation length

$$r \sim \lambda^{-1}(T^2 - T_c^2)^{-1/2}$$

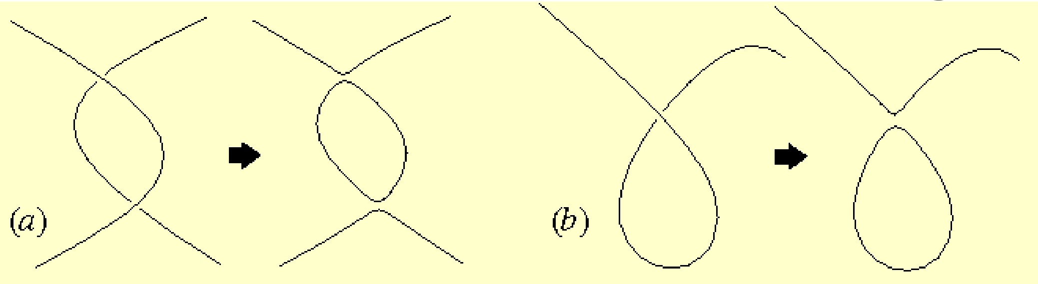
  diverges.
  - In reality, there exists a temperature at which the field cannot keep up with the temperature change of the bath and fluctuations freeze on the length scale

$$r_c \sim \frac{1}{\lambda^{2/3} T_c} \left(\frac{T_C}{H_{PT}} \right)^{1/3} \qquad H_{PT} = H(T_c)$$

  - About 1 string forms in each r_c^3 volume – the field configuration is very inhomogeneous right after the phase transition

Axions from Cosmic Strings

- Intercommutation and self-intersection events quickly bring string network into the so-called scaling regime:



$$\frac{\rho_{str}}{\rho_{rad}} \sim 30 \frac{32\pi}{3} G\mu \qquad \mu \sim f_a^2$$

- To maintain scaling, string energy density gets dumped into axions. These axions can be cold dark matter for $f_a \sim 10^{11}$ GeV
- There is more to the story – uncertainties in numerical simulations & domain walls at the QCD phase transition

Axions from (early) Cosmic Strings

- The axion population analyzed in this way occurs late in the network's evolution. What about early network dynamics?

$$\rho_{str}^{early} \sim \frac{\mu r_c}{r_c^3} \sim \frac{f_a^2}{r_c^2} \qquad \rho_{str}^{late} \sim \frac{f_a^2}{r_H^2}$$

- Postulate that early energy density goes into axions with wavelength on the order of r_c
- Then we have an axion number density from the inhomogeneous configuration

$$n_a^{inh} \sim \frac{f_a^2}{r_c}$$

- We want to try and make these early era axions dark matter, so we will consider a late time phase transition.

The Model

- Consider a complex scalar field P that couples to a pair of (KSVZ) fermions:

$$\mathcal{L} \supset yP\psi\bar{\psi}$$

- Then the scalar field gets a thermal mass:

$$V \supset (y^2T^2 - m^2)|P|^2$$

- and critical temperature

$$T_c = \frac{m}{y}$$

- We consider T_c far below the PQ symmetry breaking scale f_a, so $m \ll f_a$

The Model

- This hierarchy is natural in supersymmetric scenarios where P is stabilized by higher dimensional interactions:

$$V = \left(\frac{2^{n-2} m_s^2}{n(n-1) f_a^{2n-2}} \right) |P|^{2n} - \frac{m_s^2}{2n-2} |P|^2 + \frac{m_s^2 f_a^2}{4n}$$

- or running of its soft mass:

$$V = \frac{m_s^2}{2} |P|^2 \left(\ln \frac{2|P|^2}{f_a^2} - 1 \right) + \frac{1}{4} m_s^2 f_a^2$$

- m_s is the saxion mass and is proportional to the parameter m.
- The axions produced from the early inhomogeneous configuration are independent of the details of the potential, but we will need a specific form for the potential later.

The Model – Axions from Early String Network

- We assume that the energy of P at the origin, $V(0) = m_s^2 f_a^2$, is greater than the radiation energy density at T=T_c. This is true provided

$$y \gtrsim \sqrt{\frac{m}{f_a}}$$

- Therefore, a period of thermal inflation takes place. The Hubble scale at the phase transition is then

$$H_{PT} \sim \frac{m_s f_a}{M_{pl}}$$

- Thus we find a critical correlation length

$$r_c \sim \frac{1}{m_s} \left(\frac{M_{pl}}{f_a} \right)^{1/3}$$

The Model – Axions from Early String Network

- The axion number density from the inhomogeneous field configuration:

$$n_a^{inh} \sim \frac{f_a^2}{r_c} = m_s f_a^2 \left(\frac{f_a}{M_{pl}}\right)^{1/3}$$

- The vacuum energy largely goes into a saxion energy density . Thus we can form the redshift-invariant quantity

$$\frac{n_a^{inh}}{\rho_s} = \frac{1}{m_s}\left(\frac{f_a}{M_{pl}}\right)^{1/3}$$

and yield

$$Y_a^{inh} = \frac{n_a^{inh}}{\rho_s}\frac{\rho_s}{s} \sim \left(\frac{f_a}{M_{pl}}\right)^{1/3}\frac{T_{RH}}{m_s}$$

- However, there is another effect – parametric resonance!

Parametric Resonance

- For the moment, consider the Mathieu equation

$$\chi_k'' + (A_k - 2q\cos(2z))\chi_k = 0 \qquad A_k \propto k^2$$

- This equation has exponential instabilities

$$\chi_k \sim \exp\left(\mu_k^{(n)} z\right)$$

  in a set of resonance bands $\Delta k^{(n)}$ labeled by an integer n

- Under certain conditions, the resonance occurs in narrow bands that satisfy

$$A_k = l^2$$
$$l = 1, 2, \ldots$$

- If we consider the χ_k to be the Fourier modes of a quantum field, then the resonance corresponds to an exponential growth in occupation numbers

Parametric Resonance

- As the saxion oscillates, exponential growth of axion modes can occur
- Consider the simple potential

$$V = \alpha(|P|^2 - f_a^2)^2 \qquad P = f_a + s + ia$$

- The equation of motion for a is

$$\ddot{a} - \nabla^2 a + \frac{m^2}{f_a} s a + \mathcal{O}(a^3) = \ddot{a} - \nabla^2 a + \frac{m^2 S_0}{f_a} \sin(mt) a + \mathcal{O}(a^3) = 0$$

- Dropping the nonlinear term, Fourier transforming, and implementing some redefinitions, we get a differential equation for the axion modes:

$$\ddot{a}_k + (A_k - 2q\cos(2x))a_k = 0$$

$$A_k = \frac{4k^2}{m^2} \qquad q = \frac{2S_0}{f_a}$$

- Exponential growth occurs if

$$qm \geq H \rightarrow S_0 \geq \frac{f_a}{2}\frac{f_a}{M_{pl}}$$

The Model - Axions from Parametric Resonance

- Similar results hold for the potentials discussed earlier - parametric resonance does occur and efficiently produces axions with momenta peaked at $m_s/2$.
- The axion growth continues until the axion energy density is roughly equal to the initial saxion energy density $m_s^2 f_a^2$.
- Thus we get a population of PR produced axions:

$$\frac{n_a^{PR}}{\rho_s} \sim \frac{1}{m_s}$$

- Note that in this scenario, parametric resonance is occurring without the large field displacements of [3]

Axions as Dark Matter

- PR axions far outnumber early string axions – this is the dominant contribution.
- Neglecting the axions produced by the early field inhomogeneities, the axion yield is

$$Y_a = \frac{T_{RH}}{m_s}$$

- To get the observed dark matter abundance, the reheat temperature must be at least

$$T_{DM} \sim 0.7 \text{ GeV} \left(\frac{m_s}{10 \text{ MeV}} \right) \left(\frac{f_a}{10^9 \text{ GeV}} \right)$$

Axions as Dark Matter: Thermalization

- The axions are never thermalized. The thermalization rate for the axion is
$$\Gamma_a = b\frac{k_a^2}{f_a^2}T$$
- During matter domination era of saxion oscillations, $k_a^3/\rho_s = \text{constant}$, so
$$k_a \sim \left(\frac{m_s \rho_s}{f_a^2}\right)^{1/3}$$
- The energy density of the thermal bath never exceeds that of the saxion, so
$$\frac{\Gamma_a}{H} < b\frac{m_s^{2/3}\rho_s^{5/12}M_{pl}}{f_a^{10/3}} < b\frac{m_s^{3/2}M_{pl}}{f_a^{5/2}}$$
- Thus the late time phase transition is critical for ensuring that the axions are never thermalized – if $m_s \sim f_a$, then the thermalization would be effective.

Cosmological Constraints

- Axion Warmness
  - The ratio of axion momentum to the cube root of axion number density is constant. For the two potentials specified earlier,

$$\frac{k_a}{n_a^{1/3}} = \left(\frac{n}{2}\right)^{1/3} \left(\frac{m_s}{f_a}\right)^{2/3}$$

  - Using the observed dark matter abundance,

$$v_a \simeq 6 \times 10^{-4} n^{1/3} \left(\frac{f_a}{10^9 \text{ GeV}}\right)^{2/3} \left(\frac{m_s}{\text{GeV}}\right) \left(\frac{T}{\text{eV}}\right)$$

  which must be less than 10^{-4} at T = 1 eV

  - This gives us a bound on the saxion mass:

$$m_s \leq 30 \text{ MeV} \left(\frac{3}{n}\right)^{1/2} \left(\frac{10^9 \text{ GeV}}{f_a}\right)$$

Cosmological Constraints

- Stellar Cooling
  - The axion coupling to electrons and nucleons can give rise to rapid cooling in stars
  - For Red Giant and Horizontal Branch stars, the energy loss rate due to axions must be less than

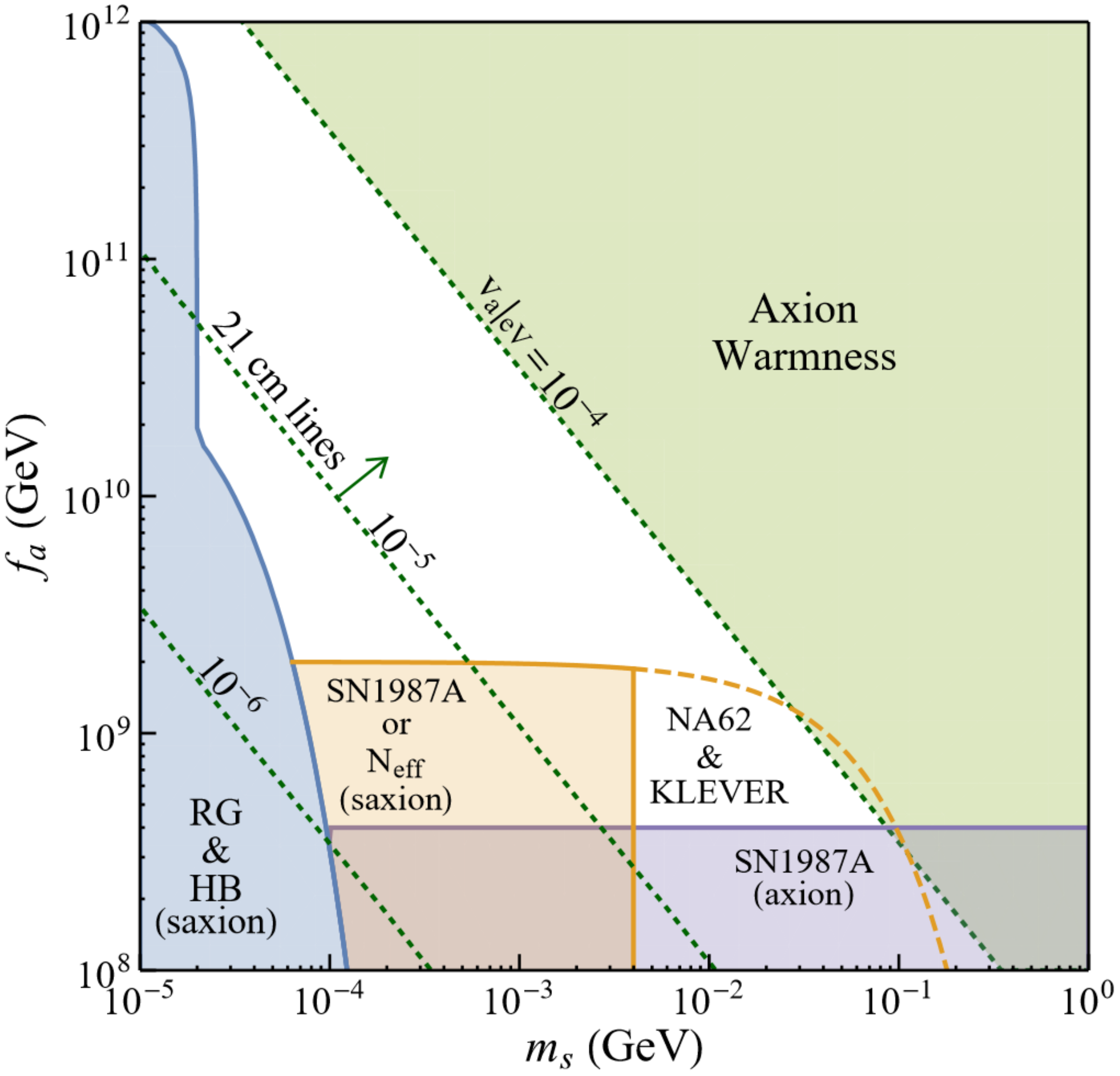
$$\epsilon < 10 \text{ erg/g/s}$$

- Supernovae - 1987A
  - The energy loss for new particles in supernovae is constrained by the 1987A observations to be

$$\epsilon < 10^{19} \text{ erg/g/s}$$

  - However, there are is at least an O(10) degree of uncertainty regarding this constraint.[4]
  - Furthermore, this constraint can be bypassed if one assumes a strong enough coupling between the saxion and SM particles.

Plots



- We consider n=3 for definiteness
- The slanted, dashed green lines are contours for the axion velocity at T = 1 eV
- The region under the dashed orange curve is unconstrained if the saxion has sufficient coupling to the SM Higgs such that it is in the trapping regime.
- If one does not assume strong coupling to the SM Higgs, then it would appear that fa = 10^9 GeV is ruled out. However, this is too strong a statement given the uncertainty in the supernovae constraints

Cosmological Constraints

- Supernovae Constraints & the Trapping Regime
  - If the coupling between the saxion and SM Higgs is large, then the saxions do not efficiently carry away energy since they get ‘trapped’.
- Relativistic Degrees of Freedom
  - If we are in the trapping regime, then the large saxion-Higgs coupling can keep the saxion in thermal equilibrium with electrons even after the neutrinos decouple.
  - Thus the depletion of saxion energy heats up the photons, resulting in $N_{eff} < 3$
  - Assuming the neutrinos suddenly decouple at T = 2 MeV, the saxion mass must satisfy

$$m_s > 4 \text{ MeV}$$

Experimental signatures

- 21 cm lines & structure
  - The high axion velocity makes this a warm dark matter scenario
  - WDM has a distinctive matter power spectrum
  - Future observations of the 21cm should probe m_{wdm} < 10-20 keV, which corresponds to $v > 10^{-5}$. Our parameter space will be explored.
- NA62 & KLEVER
  - Assuming a large saxion-Higgs coupling, one gets rare Kaon decays

$$K^+ \to \pi^+ S$$
$$K_L \to \pi^0 S$$
$$K_S \to \pi^0 S$$

  - The NA62 and KLEVER experiments measure Kaon decay branching ratios to 10%, so the above can lead to observable deviations from the SM predictions.

Conclusions

- QCD axion dark matter can be produced by a late time phase transition
    - Two mechanisms contribute – early cosmic string network dynamics and parametric resonance. Parametric resonance dominates over the axions from cosmic strings
- Features:
    - The parametric resonance does not require large field displacement, in contrast to previous scenarios
    - Low values of the axion decay constant are permitted, especially if large saxion-Higgs mixing is introduced or one relaxes supernovae bounds.
    - The axion dark matter is warmer than other scenarios – should leave detectable imprints on structure formation visible in future 21 cm line studies
    - If the saxion is in the trapping regime, there should be signals from rare Kaon decays at the NA62 and KLEVER experiments.

References

- [1]. Topological Dark matter – H. Murayama, J. Shu
- [2]. String intercommutation and self-intersection graphic:http://www.damtp.cam.ac.uk/research/gr/public/cs_interact.html
- [3]. QCD Axion Dark Matter with a Small Decay Constant – R. Co, L. Hall, K. Harigaya
- [4]. Light Dark Matter: Models and Constraints – S. Knapen, T. Lin, K. Zurek and references [54-59] of our paper.
- [5]. Towards the theory of Reheating after Inflation – L. Kofman, A. Linde, A. Starobinsky

Axions from Parametric Resonance

- Does Parametric Resonance Really Occur?
  - Our scenario differs from typical parametric resonance production mechanisms since the field P has a very inhomogeneous configuration during the oscillating regime
  - However, parametric resonance produces axions with wavelengths that are sharply peaked at $1/m_s$. This is much shorter than r_c, which is the length scale on which the PQ scale is correlated.
  - Therefore, we expected parametric resonance to take place even thought a dense network of cosmic strings exists.

Reheating & Thermalization

- The saxion must be thermalized at or above TDM. We could consider a coupling between P and a new pair of fermions as

$$\mathcal{L} \supset \frac{\mu}{f_a} P f \bar{f}$$

- Then the saxions would thermalize at a rate $\sim 0.1 T\mu^2/f_a^2$, which leads to a reheat temperature

$$T_{RH} \sim 100 \text{ GeV} \left(\frac{\mu}{100 \text{ GeV}}\right)^2 \left(\frac{10^9 \text{ GeV}}{f_a}\right)^2$$

- If the fermions have SM charges, μ must be greater than 100 GeV. This results in a reheat temperature that is larger than T_{DM}.
- To get the right reheat temperature, one can consider coupling to SM particles