Aspects of Nonlinear Effect on Black Hole Superradiance

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Outline

- Introduction
- Nonlinear effect
- Examples

Introduction

Black hole superradiance

What is BH superradiance? Saul, Teukolsky 72; Bekenstein 73

- Thermodynamic process to lose energy/charges by emitting particles for the BH with charges
  - Not the Hawking radiation, the entropy-decreasing process

BH superradiance - example

Kerr BH: $dM = \frac{\kappa}{8\pi} dA + \Omega\, dJ$

$$\left\{ \begin{array}{c} \Omega = a/(r_+^2 + a^2) \\ r_+ = GM + \sqrt{(GM)^2 - a^2} \\ \tilde{a} = a/GM, 0 \le \tilde{a} \le 1 \\ J = aM \end{array} \right.$$

Q. Can be the particle with $\frac{\Delta J}{\Delta M} = \frac{m}{\omega}$ emitted?

A. $\Delta A = \frac{8\pi(\omega - m\Omega)}{\omega\kappa} \Delta M$: Allowed if $\omega < m\Omega$

(For $\tilde{a} \sim 1$, $GM_{BH}\omega \lesssim m$)

For $m = 1$, $\omega \sim 10^{-10}(M_{\odot}/M_{BH})\, eV$

→relevant to light d.o.f

BH superradiance rate

- The SR rate for Boson is estimated as a scattering process

$(J, E) = (m, \omega)$

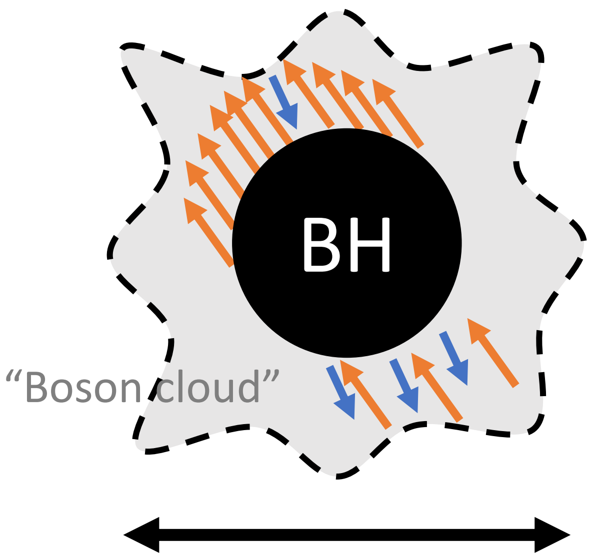
(Quantum fluctuation)

- This process is linear in time – not so fast
  - Slower than BH accretion

BH superradiant instability

- The particle may form a bound state with BH

If it is a Boson,



$\sim \alpha^{-1}\mu^{-1}, \alpha = MG\mu$

Exponentially enhanced mode (= $\mathrm{Im}\,\omega > 0$) exists in the bound state spectrum if Bosonic

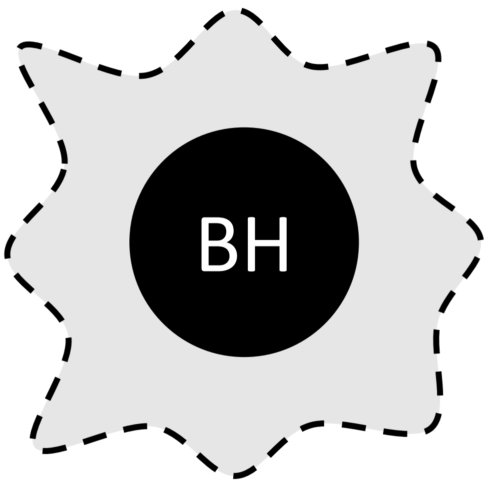
$\mathrm{Im}\,\omega$ is minimized when $\alpha \sim 0.5$ and otherwise exponentially small

Kerr BH loses its angular momentum and rot. E in constant time if a Boson w/ $M\omega \sim 1$ exists

Press, Teukolsky 72; Damour, Deruelle, Ruffini 76

Nonlinear Effect

How dense is the cloud?



BH angular momentum: $J = G\tilde{a}M^2$
Cloud volume: $\mathcal{V} \sim \pi(\alpha\mu)^{-3}$
Cloud energy density: $\rho \sim J\mu/\mathcal{V}$

In terms of the field amplitude ($\rho \sim \mu^2\phi_0^2$),

$$\phi_0^2 \sim 8\tilde{a}\alpha^5 M_{Pl}^2$$

The field amplitude is $\sim \mathbf{0.1}\ \boldsymbol{M_{Pl}}$!

HF and Nakayama 19

Large field amplitude

- Field amplitude close to M_{Pl} itself ***does not*** mean the theory is invalid
  - Recall the inflation theory
- It rather mean the potential may be distorted
  - Recall the inflation theory again!
- We treated a free theory, so the discussion on the non-linear effect on BH superradiance is important

Nonlinearity on superradiance

- The possible consequences of the nonlinearity are
    1. The spectrum may be distorted
    2. The particle w/ high p may be produced
    3. Some another particle may be produced
- The spectrum change: the effective mass change
    - If it changes by $\mathcal{O}(1)$, the other two are already serious

Why is the particle production important?

- Because the SR instability is an exponential process

$$\frac{dM_{cloud}}{dt} \sim \omega_I M_{cloud} - \frac{dM_{p.p.}}{dt}$$

Two balances →

exponential growth stop and

BH stops to lose E/J

in constant time any more!

Accretion, superradiance and nonlinearity

$$\dot{M}_{cloud} \sim \omega_I M_{cloud} - \dot{M}_{p.p.}$$

$$\dot{M}_{cloud} \to 0 \text{ and } M_{cloud} \to M_{cloud}^{\infty} = \text{const.}$$

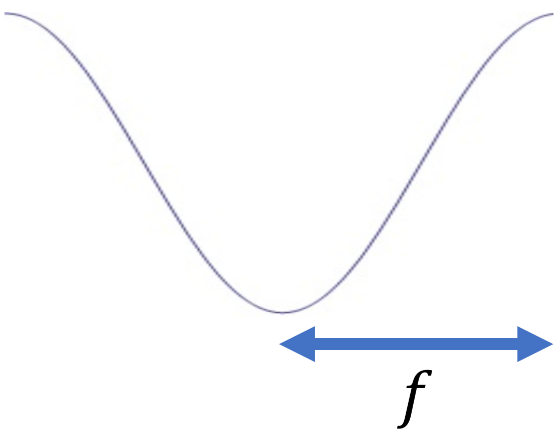
$$\dot{M}_{BH} \sim -\omega_I M_{cloud}^{\infty} + \dot{M}_{acc}$$

- BH energy/angular momentums decreased **linearly in time** by $\omega_I M_{cloud}^{\infty} = \dot{M}_{p.p.}$
  - $\Delta M_{BH} \sim \exp(-\omega_I t) \to \Delta M_{BH} \sim -\dot{M}_{p.p.} t$

Examples

Axion (like particles)

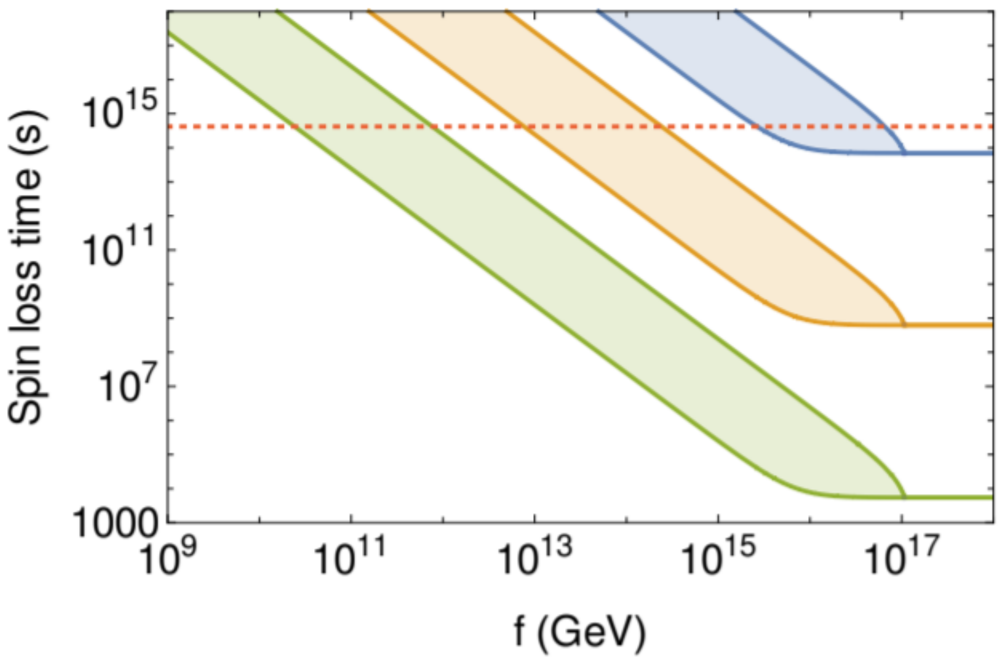
- $V = -\mu^2 f^2 \cos a/f$ → mass: μ
  - For $10^9 M_\odot > M_{BH} > M_\odot$, $10^{-11} > \mu/\mathrm{eV} > 10^{-20}$



- Definitely, $a < f$
- What if $a \sim f$?

Instability stops, but BH J may still escape linearly!

Result



Blue: $\mu = 10^{-20}$ eV
Orange: $\mu = 10^{-15}$ eV
Green: $\mu = 10^{-10}$ eV

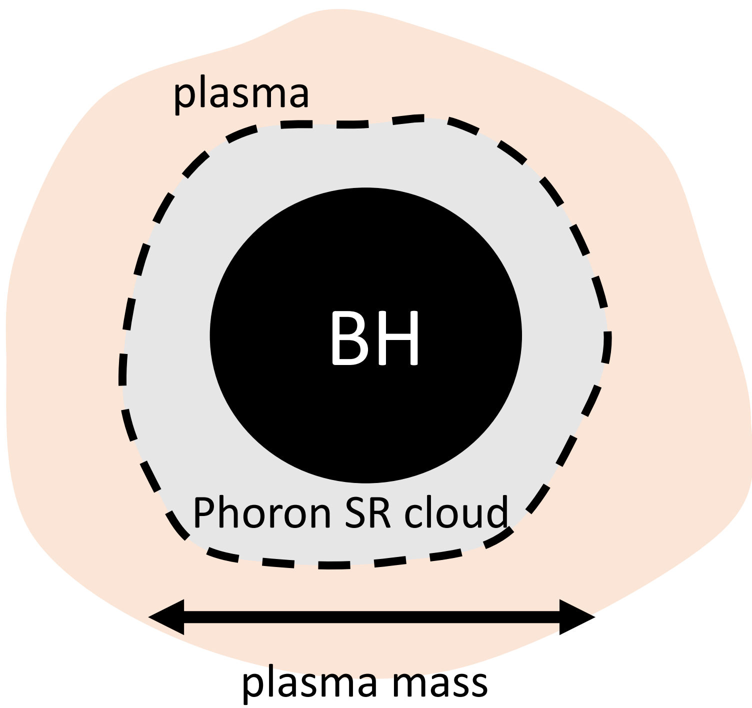
M_{BH} is set so that $GM_{BH}\mu = 0.5$

The band corresponds to the efficiency of the particle production

The dotted line is the accretion time scale estimated from the Eddington limit

Standard model photon and the primodial BH

- SM photon in the early Universe has a plasma mass

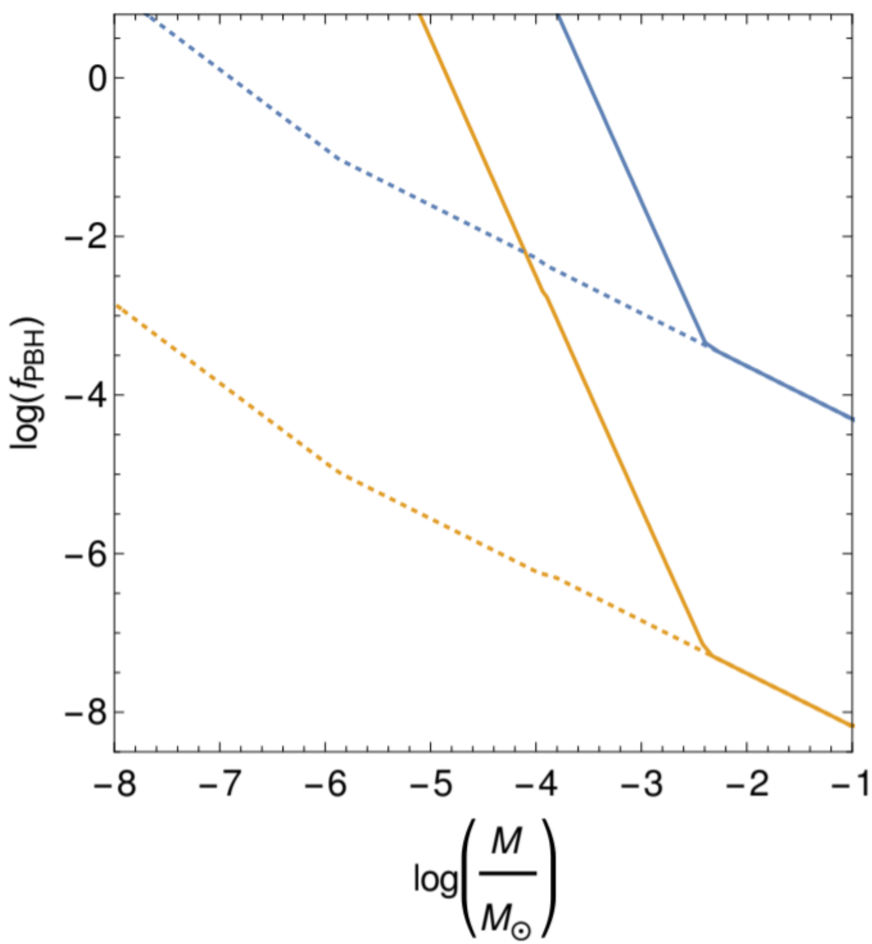


- Photon may form SR cloud around the primordial BH
- When the mass changes, the photon is released
  - constrained from CMB distortion

Pani, Loeb 13

- However, large photon amplitude results the Schwinger pair production

Result



Blue: COBE, Orange: PIXIE

Dotted: previous estimation
Solid: our estimation

The regions above the line is constrained

Summary

- In superradiance cloud, the field amplitude becomes as large as the Planck scale and the nonlinearity plays an important role
- Depending on models, superradiance effects are less efficient than expected before