# Minimal gauged $\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}$ model and leptogenesis 

K．Asai，K．Hamaguchi，N．Nagata，S．－Y．Tseng，K．Tsumura，arXiv：1811．07571，PhysRevD．99．055029 K．Asai，K．Hamaguchi，N．Nagata，S．－Y．Tseng，in preparation

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- Introduction
- Minimal models
- Neutrino physics
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- Summary


## Introduction

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- Extending the gauge sector of SM



## Introduction

- Extending the gauge sector of SM
$\underbrace{{\mathrm{SU}(3)_{\mathrm{C}}}_{\mathrm{SUSU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}} \times \mathrm{U}(1)_{\mathrm{L}_{\alpha}-\mathrm{L}_{\beta}}}_{\mathrm{G}_{\mathrm{SM}}} \underset{\substack{\text { R. Foot, Mod.Phys.Lett. A6 (1991) 527-530 } \\ \text { X.-G. He et al, Phys. Rev. D 43, R22 }}}{\text { X. }}$

$$
\mathrm{U}(1)_{\mathrm{L}_{\mathrm{e}}-\mathrm{L}_{\mu}} \quad \mathrm{U}(1)_{\mathrm{L}_{\mathrm{e}}-\mathrm{L}_{\tau}} \quad \mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}
$$

- Transforming the lepton sector nontrivially compatible with the existing experimental data?


## Introduction

- Light neutrino mass $\rightarrow$ adding RH neutrinos

$$
M_{v}=-M_{D} M_{R}^{-1} M_{D}^{T}
$$

## Seesaw mechanism

P. Minkowski, Phys. Lett. B67 (1977) 421-428
T. Yanagida, Conf. Proc. C7902131 (1979) 95-99
! mass terms tightly restricted by the $\mathrm{U}(1)_{\mathrm{L}_{\alpha}-\mathrm{L}_{\beta}}$ symmetry

## Introduction

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! mass terms tightly restricted by the $\mathrm{U}(1)_{\mathrm{L}_{\alpha}-\mathrm{L}_{\beta}}$ symmetry

$$
\left(\begin{array}{ccc}
* & 0 & 0 \\
0 & 0 & * \\
0 & * & 0
\end{array}\right) \simeq\left(\begin{array}{lll}
* & 0 & 0 \\
0 & * & 0 \\
0 & 0 & *
\end{array}\right)\left(\begin{array}{ccc}
* & 0 & 0 \\
0 & 0 & * \\
0 & * & 0
\end{array}\right)\left(\begin{array}{lll}
* & 0 & 0 \\
0 & * & 0 \\
0 & 0 & *
\end{array}\right) \quad \text { block-diagonal }
$$

this simple structure fails to explain the sizable neutrino mixing...

## Introduction

- Spontaneous breaking of $U(1)_{\mathrm{L}_{\alpha}-\mathrm{L}_{\beta}}$ symmetry
introduce only one additional scalar field

$$
M_{v}^{-1} \simeq\left(\begin{array}{ccc}
* & * & * \\
* & 0 & * \\
* & * & 0
\end{array}\right) \quad \text { or } \quad M_{v} \simeq\left(\begin{array}{ccc}
* & 0 & * \\
0 & 0 & * \\
* & * & *
\end{array}\right)
$$

Two-zero structure leads into strong predictive power!

## Minimal models

K. Asai, K. Hamaguchi, N. Nagata, S.-Y. Tseng, K. Tsumura, arXiv:1811.07571, PhysRevD.99.055029

## A brief summary

The Standard Model $\otimes\left\{\begin{array}{l}U(1)_{L_{e}-L_{\mu}} \\ U(1)_{L_{e}-L_{\tau}} \\ U(1)_{L_{\mu}-L_{\tau}}\end{array}\right.$
$\otimes 3 \mathrm{RH}$ neutrinos
$\otimes\binom{$ an extra scalar singlet }{ an extra scalar doublet }
$\otimes\binom{$ normal mass ordering }{ inverted mass ordering }

## A brief summary

## The Standard Model

## $\otimes 3$ RH neutrinos

$\otimes\binom{$ an extra scalar singlet }{ an extra scalar doublet }
excluded by neutrino oscillation data, cosmological bound on $\sum_{i} m_{i}$, etc

- Charge assignment

| $L_{\mu}-L_{\tau}$ | $L_{e, \mu, \tau}$ | $e_{R}, \mu_{R}, \tau_{R}$ | $N_{e, \mu, \tau}$ | $\sigma$ | $H$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{U}(1)_{\mathrm{Y}}$ | $-\frac{1}{2}$ | -1 | 0 | 0 | $+\frac{1}{2}$ |
| $\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}$ | $0,+1,-1$ | $0,+1,-1$ | $0,+1,-1$ | +1 | 0 |
| $\mathrm{SU}(2)$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ |

- Charge assignment

| $L_{\mu}-L_{\tau}$ | $L_{e, \mu, \tau}$ | $e_{R}, \mu_{R}, \tau_{R}$ | $N_{e, \mu, \tau}$ | $\sigma$ | $H$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{U}(1)_{\mathrm{Y}}$ | $-\frac{1}{2}$ | -1 | 0 | 0 | $+\frac{1}{2}$ |
| $\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}$ | $0,+1,-1$ | $0,+1,-1$ | $0,+1,-1$ | +1 | 0 |
| $\mathrm{SU}(2)$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ |

$$
\begin{aligned}
\Delta \mathcal{L}= & -y_{e} e_{R}^{c} L_{e} H^{\dagger}-y_{\mu} \mu_{R}^{c} L_{\mu} H^{\dagger}-y_{\tau} \tau_{R}^{c} L_{\tau} H^{\dagger} \\
& -\lambda_{e} N_{e}^{c}\left(L_{e} \cdot H\right)-\lambda_{\mu} N_{\mu}^{c}\left(L_{\mu} \cdot H\right)-\lambda_{\tau} N_{\tau}^{c}\left(L_{\tau} \cdot H\right) \\
& -\frac{1}{2} M_{e e} N_{e}^{c} N_{e}^{c}-M_{\mu \tau} N_{\mu}^{c} N_{\tau}^{c}-\lambda_{e \mu} \sigma N_{e}^{c} N_{\mu}^{c}-\lambda_{e \tau} \sigma^{*} N_{e}^{c} N_{\tau}^{c}+\text { h.c. }
\end{aligned}
$$

## Neutrino physics

## After the SSB of $\sigma$

- Lepton mass matrices


## After the SSB of $\sigma$

- Lepton mass matrices
neutrino sector

$$
M_{D}=\frac{v}{\sqrt{2}}\left(\begin{array}{ccc}
\lambda_{e} & 0 & 0 \\
0 & \lambda_{\mu} & 0 \\
0 & 0 & \lambda_{\tau}
\end{array}\right) \quad M_{R}=\left(\begin{array}{ccc}
M_{e e} & \lambda_{e \mu}\langle\sigma\rangle & \lambda_{e \tau}\langle\sigma\rangle \\
\lambda_{e \mu}\langle\sigma\rangle & 0 & M_{\mu \tau} \\
\lambda_{e \tau}\langle\sigma\rangle & M_{\mu \tau} & 0
\end{array}\right)
$$

## After the SSB of $\sigma$

- Lepton mass matrices
neutrino sector

$$
M_{D}=\frac{v}{\sqrt{2}}\left(\begin{array}{ccc}
\lambda_{e} & 0 & 0 \\
0 & \lambda_{\mu} & 0 \\
0 & 0 & \lambda_{\tau}
\end{array}\right) \quad M_{R}=\left(\begin{array}{ccc}
M_{e e} & \lambda_{e \mu}\langle\sigma\rangle & \lambda_{e \tau}\langle\sigma\rangle \\
\lambda_{e \mu}\langle\sigma\rangle & 0 & M_{\mu \tau} \\
\lambda_{e \tau}\langle\sigma\rangle & M_{\mu \tau} & 0
\end{array}\right)
$$

charged lepton sector

$$
M_{\ell}=\frac{v}{\sqrt{2}}\left(\begin{array}{ccc}
y_{e} & 0 & 0 \\
0 & y_{\mu} & 0 \\
0 & 0 & y_{\tau}
\end{array}\right)
$$

## Analysis of $M_{v}$

$$
M_{D}=\frac{v}{\sqrt{2}}\left(\begin{array}{ccc}
\lambda_{e} & 0 & 0 \\
0 & \lambda_{\mu} & 0 \\
0 & 0 & \lambda_{\tau}
\end{array}\right) \quad M_{R}=\left(\begin{array}{ccc|}
M_{e e} & \lambda_{e \mu}\langle\sigma\rangle & \lambda_{e \tau}\langle\sigma\rangle \\
\lambda_{e \mu}\langle\sigma\rangle & 0 & M_{\mu \tau} \\
\lambda_{e \tau}\langle\sigma\rangle & M_{\mu \tau} & 0
\end{array}\right)
$$

## Analysis of $M_{v}$



## Analysis of $M_{v}$



## Analysis of $M_{v}$



## Input parameters

global-fitting group
NuFIT 4.1 (2019)

|  |  | Normal Ordering (best fit) |  | Inverted Ordering ( $\Delta \chi^{2}=10.4$ ) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | bfp $\pm 1 \sigma$ | $3 \sigma$ range | bfp $\pm 1 \sigma$ | $3 \sigma$ range |
|  | $\sin ^{2} \theta_{12}$ | $0.310_{-0.012}^{+0.013}$ | $0.275 \rightarrow 0.350$ | $0.310_{-0.012}^{+0.013}$ | $0.275 \rightarrow 0.350$ |
|  | $\theta_{12} /^{\circ}$ | $33.82_{-0.76}^{+0.78}$ | $31.61 \rightarrow 36.27$ | $33.82_{-0.75}^{+0.78}$ | $31.61 \rightarrow 36.27$ |
|  | $\sin ^{2} \theta_{23}$ | $0.563_{-0.024}^{+0.018}$ | $0.433 \rightarrow 0.609$ | $0.565_{-0.022}^{+0.017}$ | $0.436 \rightarrow 0.610$ |
|  | $\theta_{23} /{ }^{\circ}$ | $48.6_{-1.4}^{+1.0}$ | $41.1 \rightarrow 51.3$ | $48.8{ }_{-1.2}^{+1.0}$ | $41.4 \rightarrow 51.3$ |
|  | $\sin ^{2} \theta_{13}$ | $0.02237_{-0.00065}^{+0.00066}$ | $0.02044 \rightarrow 0.02435$ | $0.02259_{-0.00065}^{+0.00065}$ | $0.02064 \rightarrow 0.02457$ |
| 令 | $\theta_{13} /{ }^{\circ}$ | $8.60_{-0.13}^{+0.13}$ | $8.22 \rightarrow 8.98$ | $8.64{ }_{-0.13}^{+0.12}$ | $8.26 \rightarrow 9.02$ |
| 要 | $\delta_{\mathrm{CP}} /{ }^{\circ}$ | $221_{-28}^{+39}$ | $144 \rightarrow 357$ | $282_{-25}^{+23}$ | $205 \rightarrow 348$ |
|  | $\frac{\Delta m_{21}^{2}}{10^{-5} \mathrm{eV}^{2}}$ | $7.39_{-0.20}^{+0.21}$ | $6.79 \rightarrow 8.01$ | $7.39_{-0.20}^{+0.21}$ | $6.79 \rightarrow 8.01$ |
|  | $\frac{\Delta m_{3 \ell}^{2}}{10^{-3} \mathrm{eV}^{2}}$ | $+2.528_{-0.031}^{+0.029}$ | $+2.436 \rightarrow+2.618$ | $-2.510_{-0.031}^{+0.030}$ | $-2.601 \rightarrow-2.419$ |

## Results

Minimal gauged $\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}$ model, normal mass ordering



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Minimal gauged $\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}$ model, normal mass ordering



## Leptogenesis in the minimal gauged $\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}$ model

K. Asai, K. Hamaguchi, N. Nagata, S.-Y. Tseng, in preparation

## Leptogenesis

- Baryon asymmetry in the universe $Y_{\Delta B} \equiv \frac{n_{B}}{s} \simeq 8.7 \times 10^{-11}$


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- Baryon asymmetry in the universe $Y_{\Delta B} \equiv \frac{n_{B}}{s} \simeq 8.7 \times 10^{-11}$
- Decays of RH neutrinos generate lepton asymmetry

$$
\epsilon=\frac{\Gamma(N \rightarrow l H)-\Gamma\left(N \rightarrow \bar{l} H^{\dagger}\right)}{\Gamma(N \rightarrow l H)+\Gamma\left(N \rightarrow \bar{l} H^{\dagger}\right)}
$$

## Leptogenesis

- Baryon asymmetry in the universe $Y_{\Delta B} \equiv \frac{n_{B}}{s} \simeq 8.7 \times 10^{-11}$
- Decays of RH neutrinos generate lepton asymmetry

$$
\begin{array}{r}
\frac{1}{(8 \pi)} \frac{1}{\left[\lambda^{\dagger} \lambda\right]_{11}} \sum_{j} \operatorname{Im}\left\{\left[\left(\lambda^{\dagger} \lambda\right)_{1 j}\right]^{2}\right\} g\left(x_{j}\right) \\
\lambda: \text { Yukawa coupling }
\end{array},
$$

## Leptogenesis

- Baryon asymmetry in the universe $Y_{\Delta B} \equiv \frac{n_{B}}{s} \simeq 8.7 \times 10^{-11}$
- Decays of RH neutrinos generate lepton asymmetry
- Sphaleron processes
V. A. Kuzmin et al., Phys. Lett. 155B (1985) 36

$$
n_{B}=\frac{28}{79} \times\left(n_{B}-n_{L}\right)
$$



## Right-handed neutrino sector

- RH neutrino mass matrix $\quad M_{R}=-M_{D}^{T} M_{v}^{-1} M_{D}$


## Right-handed neutrino sector

- RH neutrino mass matrix $\quad M_{R}=-\sqrt{M_{D}^{T}} M_{v}^{-1} M_{D}$
- Parameters $\quad\left(\lambda_{e}, \lambda_{\mu}, \lambda_{\tau}\right) \equiv \lambda(\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)$


## Right-handed neutrino sector

- RH neutrino mass matrix $\quad M_{R}=-M_{D}^{T} M_{v}^{-1} M_{D}$
- Parameters

$$
\left(\lambda_{e}, \lambda_{\mu}, \lambda_{\tau}\right) \equiv \lambda(\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)
$$

$$
\begin{aligned}
\Delta \mathcal{L} & =-\sum_{\alpha=e, \mu, \tau} \sum_{i=1}^{3} \hat{\lambda}_{\alpha i}\left(L_{\alpha} \cdot H\right) \hat{N}_{i}^{c}-\frac{1}{2} \sum_{i=1}^{3} M_{R i}^{\mathrm{diag}} \hat{N}_{i}^{c} \hat{N}_{i}^{c}+\text { h.c. } \\
M_{R} & =\Omega^{*} M_{R}^{\mathrm{diag}} \Omega^{\dagger}, \hat{N}_{i}^{c}=\sum_{\alpha} \Omega_{i \alpha}^{\dagger} N_{\alpha}^{c}, \hat{\lambda}_{\alpha i}=\sum_{\beta} \lambda_{\alpha \beta} \Omega_{\beta i}
\end{aligned}
$$

## Non-thermal leptogenesis

- RH neutrino non-thermal decay : $T_{R}=\left(\frac{90}{\pi^{2} g_{*}}\right)^{\frac{1}{4}} \sqrt{\Gamma_{\sigma} M_{P}}<M_{1}$


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- RH neutrino non-thermal decay : $T_{R}=\left(\frac{90}{\pi^{2} g_{*}}\right)^{\frac{1}{4}} \sqrt{\Gamma_{\sigma} M_{P}}<M_{1}$
- Pairs of RH neutrinos generated from inflaton decays
- Taking $\sigma$ as inflaton

$$
\begin{aligned}
\Delta \mathcal{L}= & -y_{e} e_{R}^{c} L_{e} H^{\dagger}-y_{\mu} \mu_{R}^{c} L_{\mu} H^{\dagger}-y_{\tau} \tau_{R}^{c} L_{\tau} H^{\dagger} \\
& -\lambda_{e} N_{e}^{c}\left(L_{e} \cdot H\right)-\lambda_{\mu} N_{\mu}^{c}\left(L_{\mu} \cdot H\right)-\lambda_{\tau} N_{\tau}^{c}\left(L_{\tau} \cdot H\right) \\
& -\frac{1}{2} M_{e e} N_{e}^{c} N_{e}^{c}-M_{\mu \tau} N_{\mu}^{c} N_{\tau}^{c}-\lambda_{e}, \sqrt[\sigma]{\sigma} N_{e}^{c} N_{\mu}^{c}-\lambda_{e} \overparen{\sigma} N_{e}^{c} N_{\tau}^{c}+\text { h.c. }
\end{aligned}
$$

## Non-thermal leptogenesis

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$$
\mathcal{L}_{\sigma}=\frac{\left|\partial_{\mu} \sigma\right|^{2}}{\left(1-|\sigma|^{2} / \Lambda^{2}\right)^{2}}-\kappa\left(|\sigma|^{2}-\langle\sigma\rangle^{2}\right)^{2}
$$

## Non-thermal leptogenesis

- RH neutrino non-thermal decay : $T_{R}=\left(\frac{90}{\pi^{2} g_{*}}\right)^{\frac{1}{4}} \sqrt{\Gamma_{\sigma} M_{P}}<M_{1}$
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- Taking $\sigma$ as inflaton

$$
\varphi \equiv \sqrt{2} \operatorname{Re}(\sigma)
$$

$$
\mathcal{L}_{\sigma}=\frac{1}{2}(\partial \widetilde{\varphi})^{2}-\kappa \Lambda^{4}\left[\tanh ^{2}\left(\frac{\widetilde{\varphi}}{\sqrt{2} \Lambda}\right)-\left(\frac{\langle\sigma\rangle}{\Lambda}\right)^{2}\right]^{2} \quad \frac{\varphi}{\sqrt{2} \Lambda} \equiv \tanh \left(\frac{\widetilde{\varphi}}{\sqrt{2} \Lambda}\right)
$$

## Non-thermal leptogenesis

- RH neutrino non-thermal decay : $T_{R}=\left(\frac{90}{\pi^{2} g_{*}}\right)^{\frac{1}{4}} \sqrt{\Gamma_{\sigma} M_{P}}<M_{1}$
- Pairs of RH neutrinos generated from inflaton decays
- Taking $\sigma$ as inflaton

$$
m_{\sigma} \simeq 3.4 \times 10^{13} \times(\langle\sigma\rangle / \Lambda) \mathrm{GeV}
$$

## Non-thermal leptogenesis

- Inflaton $\sigma$

$$
m_{\sigma} \simeq 3.4 \times 10^{13} \times(\langle\sigma\rangle / \Lambda) \mathrm{GeV}
$$

- Pairs of RH neutrinos generated from inflaton decays
- RH neutrino non-thermal decay : $T_{R}=\left(\frac{90}{\pi^{2} g_{*}}\right)^{\frac{1}{4}} \sqrt{\Gamma_{\sigma} M_{P}}<M_{1}$
- Baryon asymmetry

$$
Y_{B}=\frac{n_{B}}{s}=\frac{n_{\phi}}{s} \times \frac{n_{N}}{n_{\phi}} \times \frac{n_{L}}{n_{N}} \times \frac{n_{B}}{n_{L}}
$$

## Non-thermal leptogenesis



## Non-thermal leptogenesis



## Non-thermal leptogenesis

$$
Y_{B}=\frac{n_{B}}{S}=\frac{n_{\phi}}{s} \times \frac{n_{N}}{n_{\phi}} \times \frac{n_{L}}{n_{N}} \times \frac{n_{B}}{n_{L}}
$$

## Result



- $\lambda=0.01, \theta=60^{\circ}, \phi=30^{\circ}$
- all neutrino oscillation parameters determined by the NuFIT inputs


## Result



- $\lambda=0.01, \theta=60^{\circ}, \phi=30^{\circ}$
- all neutrino oscillation parameters determined by the NuFIT inputs


## It works!

## Summary

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- Among minimal $U(1)_{\mathrm{L}_{\alpha}-\mathrm{L}_{\beta}}$ models, only one survives constraints $\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}$ extension with an extra $\operatorname{SU}(2)_{\mathrm{L}}$ scalar singlet
- Minimal gauged $\mathrm{U}(1)_{\mathrm{L}_{\alpha}-\mathrm{L}_{\beta}}$ models are driven into a corner
- Potential of successful leptogenesis


## Backup

## Inflaton mass

$$
\begin{aligned}
& \zeta=-\frac{H_{\mathrm{inf}}}{\dot{\phi}} \delta \phi \\
& \mathcal{P}_{\zeta}(k)=\frac{H_{\mathrm{inf}}^{4}}{4 \pi^{2}(\dot{\phi})^{2}}=\frac{V^{3}}{12 \pi^{2}\left(V^{\prime}\right)^{2} M_{G}^{6}} \\
& =\frac{\kappa \Lambda^{6}}{96 \pi^{2} M_{P}^{6}}\left\{1-\left[\frac{\kappa}{\Lambda} \operatorname{coth}\left(\frac{\widetilde{\varphi}}{\sqrt{2} \Lambda}\right)\right]^{2}\right\}^{4} \sinh ^{4}\left(\frac{\widetilde{\varphi}}{\sqrt{2} \Lambda}\right) \tanh ^{2}\left(\frac{\widetilde{\varphi}}{\sqrt{2} \Lambda}\right) \\
& \simeq \frac{\kappa}{6 \pi^{2}}\left[\frac{N_{e}\left(\Lambda^{2}-\langle\sigma\rangle^{2}\right)}{M_{P} \Lambda}\right]^{2} . \\
& m_{\sigma}^{2}=\left.\frac{d^{2} V}{d \widetilde{\varphi}^{2}}\right|_{\widetilde{\varphi}=\widetilde{\varphi}_{\text {min }}}=4 \kappa\langle\sigma\rangle^{2}\left(1-\frac{\langle\sigma\rangle^{2}}{\Lambda^{2}}\right)^{2} \\
& P_{\zeta} \simeq 2.1 \times 10^{-9} \quad m_{\sigma} \simeq 3.4 \times 10^{13} \times(\langle\sigma\rangle / \Lambda) \mathrm{GeV} \\
& \kappa \simeq 2.9 \times 10^{-2} \times\left(\frac{50}{N_{e}}\right)^{2} \times\left(\frac{10^{14} \mathrm{GeV}}{\Lambda}\right)^{2} \times\left(1-\frac{\langle\sigma\rangle^{2}}{\Lambda^{2}}\right)^{-2}
\end{aligned}
$$

## Non-thermal leptogenesis

- Taking $\sigma$ as inflaton

$$
\mathcal{L}_{\sigma}=\frac{1}{2}(\partial \widetilde{\varphi})^{2}-\kappa \Lambda^{4}\left[\tanh ^{2}\left(\frac{\widetilde{\varphi}}{\sqrt{2} \Lambda}\right)-\left(\frac{\langle\sigma\rangle}{\Lambda}\right)^{2}\right]^{2} \quad \frac{\varphi}{\sqrt{2} \Lambda} \equiv \tanh \left(\frac{\widetilde{\varphi}}{\sqrt{2} \Lambda}\right)
$$




## Inflaton potential



## Cosmic string

$$
\begin{aligned}
\mathcal{L} & =D_{\mu} \Phi D^{\mu} \Phi^{\dagger}-\frac{1}{4} F_{\mu \nu} F^{\mu \nu}-\lambda\left(\Phi^{\dagger} \Phi-\sigma^{2} / 2\right)^{2} \\
\mu & \equiv \frac{d H}{d z}=\int r d r d \theta \mathcal{H}=-\int r d r d \theta \mathcal{L} \\
= & \int_{0}^{\infty} \int_{0}^{2 \pi} r d r d \theta\left[\left|\frac{\partial \Phi}{\partial r}\right|^{2}+\left|\frac{1}{r} \frac{\partial \Phi}{\partial \theta}-i e A_{\theta} \Phi\right|^{2}+V(\Phi)+\frac{B^{2}}{2}\right] \\
& \mu \simeq \pi \sigma^{2}
\end{aligned}
$$

## Cosmic string

$$
\begin{gathered}
\mu \simeq \pi \sigma^{2} \\
G \mu<2.4 \times 10^{-7} \\
\sigma<3.4 \times 10^{15} \mathrm{GeV}
\end{gathered}
$$

## What are the contents?

- The SM

|  | $L=\binom{\nu_{L}}{e_{L}}$ | $e_{R}$ | $Q=\binom{u_{L}}{d_{L}}$ | $u_{R}$ | $d_{R}$ | $H$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{SU}(3)_{\mathrm{C}}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}$ |
| $\mathrm{SU}(2)_{\mathrm{L}}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ |
| $\mathrm{U}(1)_{\mathrm{Y}}$ | $-\frac{1}{2}$ | -1 | $\frac{1}{6}$ | $\frac{2}{3}$ | $-\frac{1}{3}$ | $\frac{1}{2}$ |

## What are the contents?

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{SU}(3)_{\mathrm{C}}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}$ |
| $\mathrm{SU}(2)_{\mathrm{L}}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ |
| $\mathrm{U}(1)_{\mathrm{Y}}$ | $-\frac{1}{2}$ | -1 | $\frac{1}{6}$ | $\frac{2}{3}$ | $-\frac{1}{3}$ | $\frac{1}{2}$ |

- Three RH neutrinos


## What are the contents?

- The SM

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{SU}(3)_{\mathrm{C}}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}$ |
| $\mathrm{SU}(2)_{\mathrm{L}}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ |
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- Three RH neutrinos
- One extra scalar field
- Case (i)
one $\operatorname{SU}(2)_{\mathrm{L}}$ scalar singlet $\sigma$ with $\mathrm{Y}=0$ and $\mathrm{Q}\left(\mathrm{U}(1)_{\mathrm{L}_{\alpha}-\mathrm{L}_{\beta}}\right)=+1$
- Case (ii)
one $\operatorname{SU}(2)_{\mathrm{L}}$ scalar doublet $\Phi_{1}$ with $\mathrm{Y}=+1 / 2$ and $Q\left(\mathrm{U}(1)_{\mathrm{L}_{\alpha}-\mathrm{L}_{\beta}}\right)=+1$
- Case (iii)
one $\operatorname{SU}(2)_{\mathrm{L}}$ scalar doublet $\Phi_{1}$ with $\mathrm{Y}=+1 / 2$ and $\mathrm{Q}\left(\mathrm{U}(1)_{\mathrm{L}_{\alpha}-\mathrm{L}_{\beta}}\right)=-1$

| $\mathrm{L}_{\mu}-\mathrm{L}_{\tau}$ | $L_{e, \mu, \tau}$ | $e_{R}, \mu_{R}, \tau_{R}$ | $N_{e, \mu, \tau}$ | $\sigma$ | $\Phi_{1,2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{U}(1)_{\mathrm{Y}}$ | $-\frac{1}{2}$ | -1 | 0 | 0 | $+\frac{1}{2}$ |
| $\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}$ | $0,+1,-1$ | $0,+1,-1$ | $0,+1,-1$ | +1 | $\pm 1,0$ |
| $\mathrm{SU}(2)$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ |

- Case (i)

$$
\begin{aligned}
\Delta \mathcal{L}= & -y_{e} e_{R}^{c} L_{e} H^{\dagger}-y_{\mu} \mu_{R}^{c} L_{\mu} H^{\dagger}-y_{\tau} \tau_{R}^{c} L_{\tau} H^{\dagger} \\
& -\lambda_{e} N_{e}^{c}\left(L_{e} \cdot H\right)-\lambda_{\mu} N_{\mu}^{c}\left(L_{\mu} \cdot H\right)-\lambda_{\tau} N_{\tau}^{c}\left(L_{\tau} \cdot H\right) \\
& -\frac{1}{2} M_{e e} N_{e}^{c} N_{e}^{c}-M_{\mu \tau} N_{\mu}^{c} N_{\tau}^{c}-\lambda_{e \mu} \sigma N_{e}^{c} N_{\mu}^{c}-\lambda_{e \tau} \sigma^{*} N_{e}^{c} N_{\tau}^{c}+\text { h.c. }
\end{aligned}
$$

- Case (ii)

$$
\begin{aligned}
\Delta \mathcal{L}= & -y_{e} e_{R}^{c} L_{e} \Phi_{2}^{\dagger}-y_{\mu} \mu_{R}^{c} L_{\mu} \Phi_{2}^{\dagger}-y_{\tau} \tau_{R}^{c} L_{\tau} \Phi_{2}^{\dagger}-y_{\mu e} e_{R}^{c} L_{\mu} \Phi_{1}^{\dagger}-y_{e \tau} \tau_{R}^{c} L_{e} \Phi_{1}^{\dagger} \\
& -\lambda_{e} N_{e}^{c}\left(L_{e} \cdot \Phi_{2}\right)-\lambda_{\mu} N_{\mu}^{c}\left(L_{\mu} \cdot \Phi_{2}\right)-\lambda_{\tau} N_{\tau}^{c}\left(L_{\tau} \cdot \Phi_{2}\right) \\
& -\lambda_{\tau e} N_{e}^{c}\left(L_{\tau} \cdot \Phi_{1}\right)-\lambda_{e \mu} N_{\mu}^{c}\left(L_{e} \cdot \Phi_{1}\right)-\frac{1}{2} M_{e e} N_{e}^{c} N_{e}^{c}-M_{\mu \tau} N_{\mu}^{c} N_{\tau}^{c}+\text { h.c. }
\end{aligned}
$$

- Case (iii)

$$
\begin{aligned}
\Delta \mathcal{L}= & -y_{e} e_{R}^{c} L_{e} \Phi_{2}^{\dagger}-y_{\mu} \mu_{R}^{c} L_{\mu} \Phi_{2}^{\dagger}-y_{\tau} \tau_{R}^{c} L_{\tau} \Phi_{2}^{\dagger}-y_{\tau e} e_{R}^{c} L_{\tau} \Phi_{1}^{\dagger}-y_{e \mu} \mu_{R}^{c} L_{e} \Phi_{1}^{\dagger} \\
& -\lambda_{e} N_{e}^{c}\left(L_{e} \cdot \Phi_{2}\right)-\lambda_{\mu} N_{\mu}^{c}\left(L_{\mu} \cdot \Phi_{2}\right)-\lambda_{\tau} N_{\tau}^{c}\left(L_{\tau} \cdot \Phi_{2}\right) \\
& -\lambda_{\mu e} N_{e}^{c}\left(L_{\mu} \cdot \Phi_{1}\right)-\lambda_{e \tau} N_{\tau}^{c}\left(L_{e} \cdot \Phi_{1}\right)-\frac{1}{2} M_{e e} N_{e}^{c} N_{e}^{c}-M_{\mu \tau} N_{\mu}^{c} N_{\tau}^{c}+\text { h.c. }
\end{aligned}
$$

## After the SSB...

- Case (i)

$$
M_{D}=\frac{v}{\sqrt{2}}\left(\begin{array}{ccc}
\lambda_{e} & 0 & 0 \\
0 & \lambda_{\mu} & 0 \\
0 & 0 & \lambda_{\tau}
\end{array}\right) \quad M_{R}=\left(\begin{array}{ccc}
M_{e e} & \lambda_{e \mu}\langle\sigma\rangle & \lambda_{e \tau}\langle\sigma\rangle \\
\lambda_{e \mu}\langle\sigma\rangle & 0 & M_{\mu \tau} \\
\lambda_{e \tau}\langle\sigma\rangle & M_{\mu \tau} & 0
\end{array}\right) \quad M_{\ell}=\frac{v}{\sqrt{2}}\left(\begin{array}{ccc}
y_{e} & 0 & 0 \\
0 & y_{\mu} & 0 \\
0 & 0 & y_{\tau}
\end{array}\right)
$$

- Case (ii)

$$
M_{D}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
\lambda_{e} v_{2} & \lambda_{e \mu} v_{1} & 0 \\
0 & \lambda_{\mu} v_{2} & 0 \\
\lambda_{\tau e} v_{1} & 0 & \lambda_{\tau} v_{2}
\end{array}\right) \quad M_{R}=\left(\begin{array}{ccc}
M_{e e} & 0 & 0 \\
0 & 0 & M_{\mu \tau} \\
0 & M_{\mu \tau} & 0
\end{array}\right) \quad M_{\ell}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
y_{e} v_{2} & 0 & y_{e \tau} v_{1} \\
y_{\mu e} v_{1} & y_{\mu} v_{2} & 0 \\
0 & 0 & y_{\tau} v_{2}
\end{array}\right)
$$

- Case (iii)

$$
M_{D}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
\lambda_{e} v_{2} & 0 & \lambda_{e \tau} v_{1} \\
\lambda_{\mu e} v_{1} & \lambda_{\mu} v_{2} & 0 \\
0 & 0 & \lambda_{\tau} v_{2}
\end{array}\right) \quad M_{R}=\left(\begin{array}{ccc}
M_{e e} & 0 & 0 \\
0 & 0 & M_{\mu \tau} \\
0 & M_{\mu \tau} & 0
\end{array}\right) \quad M_{\ell}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
y_{e} v_{2} & y_{e \mu} v_{1} & 0 \\
0 & y_{\mu} v_{2} & 0 \\
y_{\tau e} v_{1} & 0 & y_{\tau} v_{2}
\end{array}\right)
$$

## After the SSB...

! cases (ii), (iii),

$$
v=\sqrt{v_{1}^{2}+v_{2}^{2}} \simeq 246 \mathrm{GeV}
$$

- Case (i)

$$
M_{D}=\frac{v}{\sqrt{2}}\left(\begin{array}{ccc}
\lambda_{e} & 0 & 0 \\
0 & \lambda_{\mu} & 0 \\
0 & 0 & \lambda_{\tau}
\end{array}\right) \quad M_{R}=\left(\begin{array}{ccc}
M_{e e} & \lambda_{e \mu}\langle\sigma\rangle & \lambda_{e \tau}\langle\sigma\rangle \\
\lambda_{e \mu}\langle\sigma\rangle & 0 & M_{\mu \tau} \\
\lambda_{e \tau}\langle\sigma\rangle & M_{\mu \tau} & 0
\end{array}\right) \quad M_{\ell}=\frac{v}{\sqrt{2}}\left(\begin{array}{ccc}
y_{e} & 0 & 0 \\
0 & y_{\mu} & 0 \\
0 & 0 & y_{\tau}
\end{array}\right)
$$

- Case (ii)

$$
M_{D}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
\lambda_{e} v_{2} & \lambda_{e \mu} v_{1} & 0 \\
0 & \lambda_{\mu} v_{2} & 0 \\
\lambda_{\tau e} v_{1} & 0 & \lambda_{\tau} v_{2}
\end{array}\right) \quad M_{R}=\left(\begin{array}{ccc}
M_{e e} & 0 & 0 \\
0 & 0 & M_{\mu \tau} \\
0 & M_{\mu \tau} & 0
\end{array}\right) \quad M_{\ell}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
y_{e} v_{2} & 0 & y_{e \tau} v_{1} \\
y_{\mu e} v_{1} & y_{\mu} v_{2} & 0 \\
0 & 0 & y_{\tau} v_{2}
\end{array}\right)
$$

- Case (iii)

$$
\text { (iii) } M_{D}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
\lambda_{e} v_{2} & 0 & \lambda_{e \tau} v_{1} \\
\lambda_{\mu e} v_{1} & \lambda_{\mu} v_{2} & 0 \\
0 & 0 & \lambda_{\tau} v_{2}
\end{array}\right) \quad M_{R}=\left(\begin{array}{ccc}
M_{e e} & 0 & 0 \\
0 & 0 & M_{\mu \tau} \\
0 & M_{\mu \tau} & 0
\end{array}\right) \quad M_{\ell}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
y_{e} v_{2} & y_{e \mu} v_{1} & 0 \\
0 & y_{\mu} v_{2} & 0 \\
y_{\tau e} v_{1} & 0 & y_{\tau} v_{2}
\end{array}\right)
$$

## Introduction

- Possible extension of SM

- $\mathrm{U}(1)_{L_{\mu}-L_{\tau}}$ models



## Leptogenesis





$$
Y_{\Delta B} \equiv \frac{n_{B}}{s} \simeq 8.7 \times 10^{-11}>0
$$

$$
\frac{n_{B}}{n_{L}}=-\frac{28}{79}
$$

$$
\epsilon_{1}<0
$$

## Leptogenesis




## $\mathrm{U}(1)_{\mathrm{L}_{\alpha}-\mathrm{L}_{\beta}}$ bosons in cases (ii) and (iii) <br> 

## LFV process in cases (ii) and (iii)

$$
\begin{aligned}
& \Gamma\left(\tau \rightarrow e Z^{\prime}\right)=\frac{g_{Z^{\prime}}^{2} m_{\tau}}{128 \pi} \sin ^{2} 2 \theta_{L}\left(2+\frac{m_{\tau}^{2}}{m_{Z^{\prime}}^{2}}\right)\left(1-\frac{m_{Z^{\prime}}^{2}}{m_{\tau}^{2}}\right)^{2} \\
& \left|\sin 2 \theta_{L}\right|< \begin{cases}7 \times 10^{-5} & \text { for } m_{Z^{\prime}}=100 \mathrm{MeV} \text { and } g_{Z^{\prime}}=10^{-3} \\
1 \times 10^{-5} & \text { for } m_{Z^{\prime}}=10 \mathrm{MeV} \text { and } g_{Z^{\prime}}=5 \times 10^{-4}\end{cases} \\
& \theta_{L, R} \simeq 0 \text { or } \underbrace{\pi / 2} \longrightarrow U_{L, R} \simeq\left(\begin{array}{lll}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{array}\right) \Longrightarrow Q_{\mu-\tau}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{array}\right) \rightarrow\left(\begin{array}{ccc}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{array}\right)=Q_{\mu-e} \\
& M_{l}=U_{L}\left(\begin{array}{ccc}
m_{e} & 0 & 0 \\
0 & m_{\mu} & 0 \\
0 & 0 & m_{\tau}
\end{array}\right) U_{R}^{T} \square \text { Symmetry group } \mathrm{S}_{3}: D_{3}(g)\left(\begin{array}{ccc}
m_{e} & 0 & 0 \\
0 & m_{\mu} & 0 \\
0 & 0 & m_{\tau}
\end{array}\right) D_{3}^{T}(g) \\
& g=\underbrace{g_{e \mu \tau}, g_{e \tau \mu}}_{\mathrm{U}(1)_{L_{\mu}-L_{\tau}}}, \underbrace{g_{\mu e \tau}, g_{\mu \tau e}}_{\left.\mathrm{U}(1)_{L_{e}-L_{\tau}}\right\rangle}, \underbrace{g_{\tau \tau \mu}, g_{\tau \mu e}}_{\mathrm{U}(1)_{\iota_{e}-L_{\mu}}}
\end{aligned}
$$

## LFV process



## LFV decay

$$
M_{\ell}=U_{L}^{*}\left(\begin{array}{ccc}
m_{e} & 0 & 0 \\
0 & m_{\mu} & 0 \\
0 & 0 & m_{\tau}
\end{array}\right) U_{R}^{T} \quad U_{L, R}=\left(\begin{array}{ccc}
\cos \theta_{L, R} & 0 & e^{-i \phi} \sin \theta_{L, R} \\
0 & 1 & 0 \\
-e^{i \phi} \sin \theta_{L, R} & 0 & \cos \theta_{L, R}
\end{array}\right)
$$

$$
\begin{aligned}
& \mathcal{L}_{Z^{\prime}}=g_{Z^{\prime}} \bar{\ell}^{\prime} \gamma^{\mu}\left[U_{L}^{\dagger} Q_{\mu-\tau} U_{L} P_{L}+U_{R}^{\dagger} Q_{\mu-\tau} U_{R} P_{R}\right] \quad \ell^{\prime} Z_{\mu}^{\prime} \quad \frac{\tan \theta_{R}}{\tan \theta_{L}}=\frac{m_{e}}{m_{\tau}} \\
& Q_{\mu-\tau}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{array}\right) \quad\left|y_{e \tau} v_{1}\right|=\frac{\left(m_{\tau}^{2}-m_{e}^{2}\right) \sin 2 \theta_{L}}{\sqrt{\left(m_{\tau}^{2}+m_{e}^{2}\right)+\left(m_{\tau}^{2}-m_{e}^{2}\right) \cos 2 \theta_{L}}} \\
& \Gamma\left(\tau \rightarrow e Z^{\prime}\right)=\frac{g_{Z^{\prime}}^{2} m_{\tau}}{32 \pi}\left[\left|\left(U_{L}^{\dagger} Q_{\mu-\tau} U_{L}\right)_{13}\right|^{2}+\left|\left(U_{R}^{\dagger} Q_{\mu-\tau} U_{R}\right)_{13}\right|^{2}\right]\left(2+\frac{m_{\tau}^{2}}{m_{Z^{\prime}}^{2}}\right)\left(1-\frac{m_{Z^{\prime}}^{2}}{m_{\tau}^{2}}\right)^{2} \\
& \operatorname{BR}(\tau \rightarrow e X) \lesssim 2.7 \times 10^{-3} \\
& \text { Z.Phys. C68 (1995) 25-28 } \\
& \operatorname{BR}\left(\tau^{-} \rightarrow e^{-} \mu^{+} \mu^{-}\right)<2.7 \times 10^{-8} \\
& \text { PLB 687:139-143,2010 } \\
& \mathrm{BR}(\tau \rightarrow e \gamma)<3.3 \times 10^{-8} \\
& \text { PRL 104:021802,2010 }
\end{aligned}
$$

## $\mu-e$ mixing

```
Massless m}\mp@subsup{m}{\mp@subsup{Z}{}{\prime}}{
BR}(\mu->eX)\lesssim1\mp@subsup{0}{}{-6
    Phys. Rev. D 34, }196
```

$13 \mathrm{MeV} \lesssim m_{Z^{\prime}} \lesssim 80 \mathrm{MeV}$
$\mathrm{BR}(\mu \rightarrow e X) \lesssim 10^{-5}$
Phys. Rev. D 91, 052020
$m_{Z^{\prime}} \lesssim 100 \mathrm{MeV}$
$\mathrm{BR}(\mu \rightarrow e X) \lesssim 10^{-4}$
Phys. Rev. Lett. 57, 2787

Z' contributes to $\mu \rightarrow e \gamma$ loop corrections

$$
\operatorname{BR}(\mu \rightarrow e \gamma) \lesssim 10^{-13}
$$

Eur. Phys. J. C (2016) 76: 434

## Analysis of $M_{v}$ : Model 1



## Analysis of $M_{\nu}$ : Model 1



## Analysis of $M_{v}$ : Model 2



## Basic Idea of the analysis

$$
\operatorname{SU}(3)_{C} \times \operatorname{SU}(2)_{L} \times \mathrm{U}(1)_{Y} \times \mathrm{U}(1)_{L_{\alpha}-L_{\beta}}
$$



## Analysis of $M_{v}$

## Scalar-doublet case



## Analysis of $M_{v}$ : case(iii)

$$
\begin{aligned}
& M_{\nu}=-\left(\begin{array}{ccc}
\frac{\lambda_{e}^{2} v_{2}^{2}}{2 M_{e e}} & \frac{\lambda_{e \tau} \lambda_{\mu} v_{1} v_{2}}{2 M_{\mu \tau}}+\frac{\lambda_{e} \lambda_{\mu e} v_{1} v_{2}}{2 M_{e e}} & 0 \\
\frac{\lambda_{e \tau} \lambda_{\mu} v_{1} v_{2}}{2 M_{\mu \tau}}+\frac{\lambda_{e} \lambda_{\mu e} v_{1} v_{2}}{2 M_{e e}} & \frac{\lambda_{\mu e}^{2} v_{1}^{2}}{2 M_{e e}} & \frac{\lambda_{\mu} \lambda_{\tau} v_{2}^{2}}{2 M_{\mu \tau}} \\
0 & \frac{\lambda_{\mu} \lambda_{T} v_{2}^{2}}{2 M_{\mu \tau}} & 0
\end{array}\right) \\
& {\left[D_{3}(g) U_{\text {PMNS }}^{*} \operatorname{diag}\left(m_{1}, m_{2}, m_{3}\right) U_{\text {PMNS }}^{\dagger} D_{3}^{T}(g)\right]_{e \tau}=0} \\
& {\left[D_{3}(g) U_{\text {PMNS }}^{*} \operatorname{diag}\left(m_{1}, m_{2}, m_{3}\right) U_{\text {PMNS }}^{\dagger} D_{3}^{T}(g)\right]_{\tau \tau}=0}
\end{aligned}
$$

## Analysis of $M_{\nu}$ : case (ii)

$$
\begin{align*}
& M_{D}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
\lambda_{e} v_{2} & \lambda_{e \mu} v_{1} & 0 \\
0 & \lambda_{\mu} v_{2} & 0 \\
\lambda_{\tau e} v_{1} & 0 & \lambda_{\tau} v_{2}
\end{array}\right) \quad M_{R}=\left(\begin{array}{ccc}
M_{e e} & 0 & 0 \\
0 & 0 & M_{\mu \tau} \\
0 & M_{\mu \tau} & 0
\end{array}\right) \\
& \text { Two-zero texture } \\
& M_{\nu}=\left(\begin{array}{ccc}
* & \boxed{0} & * \\
0 & 0 & * \\
* & * & *
\end{array}\right) \\
& \text { Predictions of } \sum_{i} m_{i}, \delta, \alpha_{2}, \alpha_{3} \ldots \\
& {\left[D_{3}(g) U_{P M N S}^{*} M_{\nu}^{\mathrm{d}} U_{\text {PMNS }}^{\dagger} D_{3}^{T}(g)\right]_{e \mu}=0} \\
& {\left[D_{3}(g) U_{P M N S}^{*} M_{\nu}^{\mathrm{d}} U_{\mathrm{PMNS}}^{\dagger} D_{3}^{T}(g)\right]_{\mu \mu}=0} \\
& f\left(\theta_{12}, \theta_{23}, \theta_{13}, \Delta m_{21}^{2}, \Delta m_{31}^{2}\right)
\end{align*}
$$

## Neutrino phenomenology : case (i)

$\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}$ normal ordering :

$\left.\begin{array}{l}\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}} \text { inverted ordering } \\ \mathrm{U}(1)_{\mathrm{L}_{\mathrm{e}}-\mathrm{L}_{\mu}}, \mathrm{U}(1)_{\mathrm{L}_{\mathrm{e}}-\mathrm{L}_{\tau}}\end{array}\right\}$ incorrect mass orderings, no real $\delta_{\mathrm{CP}}$ solutions

## Neutrino phenomenology: case (i)

$\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}$ normal ordering :


The strongest bound set
by KamLAND-Zen exp.
$\left\langle m_{\beta \beta}\right\rangle<61 \sim 165 \mathrm{meV}$
PRL 117, 082503 (2016)

$\Gamma \propto \mathrm{G}_{\mathrm{F}}^{4} \times\left\langle\mathrm{m}_{\beta \beta}\right\rangle^{2} \times\left|\mathrm{M}_{\mathrm{nucl}}\right|^{2}$
$\left\langle m_{\beta \beta}\right\rangle \equiv\left|\sum_{i}\left(U_{\mathrm{PMNS}}\right)_{e i}^{2} m_{i}\right|=\left|c_{12}^{2} c_{13}^{2} m_{1}+s_{12}^{2} c_{13}^{2} e^{i \alpha_{2}} m_{2}+s_{13}^{2} e^{i\left(\alpha_{3}-2 \delta\right)} m_{3}\right|$

## Neutrino phenomenology: cases (ii), (iii)

$\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}$ normal ordering with $\mathrm{U}(1)_{\mathrm{L}_{\mu}-\mathrm{L}_{\tau}}\left(\Phi_{1}\right)=+1$ :


The tight bound set by Planck exp.
$\sum_{i} m_{i}<0.12 \mathrm{eV}$
Planck, arXiv:1807.06209
$\mathrm{U}(1)_{L_{\alpha}-L_{\beta}}$ models with an extra scalar doublet are excluded

## Update of the results for case (i)

- The latest result of NuFIT

NuFIT 4.1 released in 2019

- Adopting different constraint on $\sum_{i} m_{i}$
degenerate $m_{i}$ s are assumed in the Planck results
- From a new analysis,
$\sum_{i} m_{i}<0.121 \mathrm{eV}$ (degenerate)
$\sum_{i} m_{i}<0.146 \mathrm{eV}$ (normal) We need this one.
$\sum_{i} m_{i}<0.172 \mathrm{eV}$ (inverted) arXiv:1907.12598 [astro-ph.CO]


## Mass ordering



## Majorana phase




