Flowing to the Bounce

So Chigusa

Department of Physics, University of Tokyo

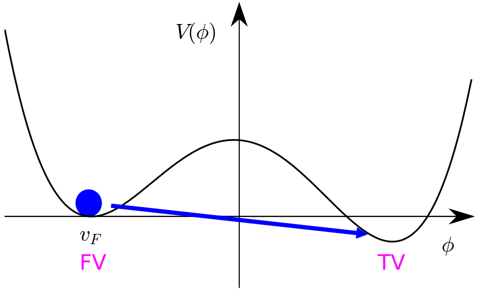
2020/1/14 @ Berkeley Week

SC, Takeo Moroi, and Yutaro Shoji

PLB **800** (2020) 135115 [1906.10829]

Vacuum Decay

Scalar potential with more than one local minima



Tunneling effect allows false vacuum decaying into true vacuum

Example

SM：Due to the RGE effect, $\lambda < 0$ for $Q > 10^{10}$ GeV

MSSM：When scalar tri-linear coupling huge（$A \gtrsim \sqrt{6} M_{\mathrm{SUSY}}$）

2HDM, etc...

Calculation of decay rate

Master Formula

$$\gamma = A e^{-S[\bar{\phi}]}$$

$S[\phi]$ is action、A is pre-factor

S. R. Coleman (1977), C. G. Callan^{+} (1977)

Bounce configuration $\bar{\phi}$

– Solution of EoM

$$\left.\frac{\delta S[\phi]}{\delta \phi}\right|_{\phi \to \bar{\phi}} = -\partial_r^2 \bar{\phi} - \frac{D-1}{r}\partial_r \bar{\phi} + \left.\frac{\partial V}{\partial \phi}\right|_{\phi \to \bar{\phi}} = 0$$

– Boundary conditions

$$\partial_r \bar{\phi}(r=0) = 0, \quad \bar{\phi}(r \to \infty) = v_F$$

– Spherically symmetric

S. R. Coleman^{+} (1978), K. Blum^{+}(2017)

Calculation of decay rate

Bounce should be calculated precisely

– LO contribution to γ

– $r \to \infty$ behavior of bounce is used for evaluation of A

– Numerical calculation is required for more complex models

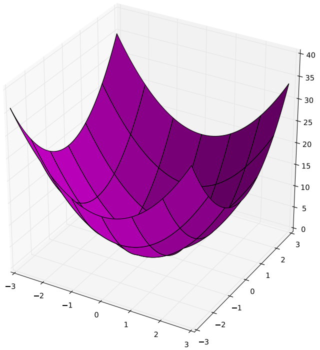
(cf) Calculation of A is a hard work ... **Example**: SM

G. Isidori$^+$(2001), G.Degrassi$^+$(2012), A. Andreassen$^+$(2017), S. Chigusa$^+$(2017, 18)

$$\log_{10} \gamma_{\rm SM} \times {\rm Gyr\,Gpc}^3 = -582^{+237}_{-397}$$

Steepest decent for local minimum search

Example: Local minimum of multi-scalar potential



s ⋯ "flow time"

Local minimum search by rolling down the slope

$$\partial_s \Phi_A(s) = -\left.\frac{\partial V}{\partial \phi_A}\right|_{\phi \to \Phi(s)}$$

Same idea can be applied to local minimum search of functional;

$$\partial_s \Phi_A(s,x) = -\left.\frac{\delta S[\phi_1(x), \phi_2(x), \ldots]}{\delta \phi_A(x)}\right|_{\phi(x) \to \Phi(s,x)}$$

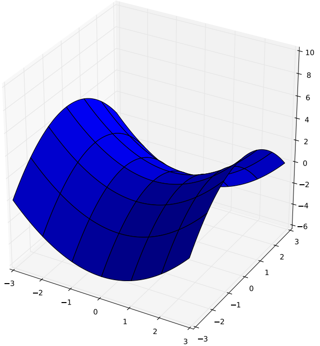
Similar to gradient flow!

Numerical calculation of bounce in multi-scalar models

Local minimum is a stable fixed point of the flow equation

$\Updownarrow$

Bounce configuration is a saddle-point of action



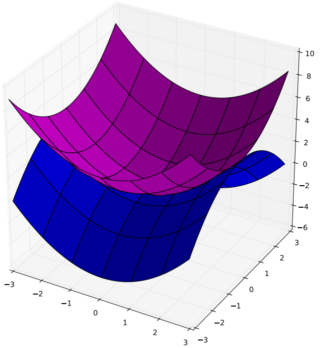
unstable fixed point

Numerical calculation of bounce in multi-scalar models

Local minimum is a stable fixed point of the flow equation

$\Updownarrow$

Bounce configuration is a saddle-point of action



Modification of “gradient”

may stabilize the bounce!

$\Updownarrow$

unstable fixed point

Action around the bounce

Fluctuation operator around the bounce

$$\mathcal{M}_{AB} \equiv \frac{\delta^2 S}{\delta\phi_A \delta\phi_B}\bigg|_{\phi\to\bar{\phi}} = -\left(\partial_r^2 + \frac{D-1}{r}\partial_r\right)\delta_{AB} + \frac{\partial^2 V}{\partial\phi_A \partial\phi_B}\bigg|_{\phi\to\bar{\phi}}$$

Eigenfunctions $\chi_{n,A}(r)$ $(n = -1, 1, 2, \cdots)$

$$\mathcal{M}_{AB}\chi_{n,B} = \lambda_n \chi_{n,A} \quad \text{w/} \quad \begin{cases} \lambda_n < 0 & (n = -1) \\ \lambda_n > 0 & (n \geq 1) \end{cases}$$

$$\partial_r \chi_{n,A}(r=0) = 0 \quad ; \quad \chi_{n,A}(r\to\infty) = 0$$

Ortho-normality of eigenfunctions

$$\langle \chi_n \,|\, \chi_m \rangle = \delta_{n,m}$$

$$\langle f \,|\, f' \rangle \equiv \int_0^\infty dr\, r^{D-1} f_A(r) f'_A(r)$$

Modified flow equation for bounce

Flow equation

$$\partial_s \Phi_A(r,s) = F_A(r,s) - \beta \langle F \,|\, g \rangle \, g_A(r,s)$$
$$F_A \equiv -\frac{\delta S[\Phi]}{\delta \Phi_A}$$

– $\beta \neq 1$ controls size of modification

$\beta = 0$ corresponds to the usual steepest descent

– $g_A(r,s)$ is a normalized function with suitable BCs

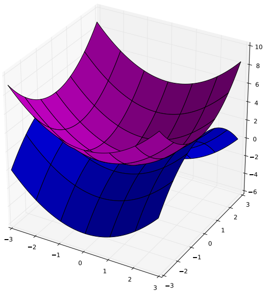
$$g_A(r,s) = \sum_{n=-1,1,2,\ldots} c_n(s) \chi_{n,A}(r) \quad ; \quad \langle g \,|\, g \rangle = \sum_n c_n^2 = 1$$

– Fixed points ($\partial_s \Phi_A = 0$) = solutions of EoM ($F_A = 0$)

proof

$$\langle \partial_s \Phi \,|\, g \rangle = (1-\beta) \cancel{\langle F \,|\, g \rangle} = 0$$
$$\partial_s \Phi_A = F_A - \cancel{\beta \langle F \,|\, g \rangle \, g_A} = 0$$

Stability?



Expansion around bounce

$$\Phi_A(r,s) = \bar{\phi}_A(r) + \sum_n a_n(s)\chi_{n,A}$$

Linearlized flow equation

$$\dot{a}_n \simeq -\lambda_n a_n + \beta \sum_m c_n c_m \lambda_m a_m + \mathcal{O}(a_n^2)$$

Asymptotic behavior of linearlized equation

Linearlized flow equation

$$\dot{a}_n \simeq -\lambda_n a_n + \beta \sum_m c_n c_m \lambda_m a_m \equiv -\sum_m \Gamma_{nm}(\beta) a_m$$

Condition for stability of bounce = $a_n \to 0$ at $s \to \infty$

Let $\{\gamma_\alpha\}$ be the set of eigenvalues of Γ

Iff $\mathrm{Re}\,\gamma_\alpha > 0$ for $\forall\alpha$, $a_n \to 0$

(cf) Case with $\beta = 0$

- $\Gamma(\beta = 0) = \mathrm{diag}(\lambda_{-1}, \lambda_1, \dots)$
- $\{\gamma_\alpha\} = \{\lambda_n\}$
- Existence of $\lambda_{-1} < 0$ destabilizes the bounce

Suitable choice of β and g_A

One of necessary conditions

$$\det \Gamma = (1-\beta) \prod_n \lambda_n > 0$$

– $\beta > 1$ is suitable for bounce search (with $\lambda_{-1} < 0$)

– For $\beta > 1$, all FV configurations are destabilized

It also helps fast convergence (though not necessary)

$$\langle g \,|\, \mathcal{M} g \rangle = \sum_n \lambda_n c_n^2 < 0$$

$$\frac{1}{c_{-1}^2} < \beta < \frac{1}{\max_{n\geq 1} c_n^2}$$

Guideline for modification

g_A should contain large fraction of $\chi_{-1,A}$

$$c_{-1}^2 \gg c_{n\geq 1}^2$$

Example of g_A

A good choice of g_A we used

$$g_A(r,s) \propto r\partial_r \Phi(r,s)$$

– Direction of scaling transf., along which $\delta^2 S/\delta\phi^2 < 0$
– Behavior of g_A around bounce

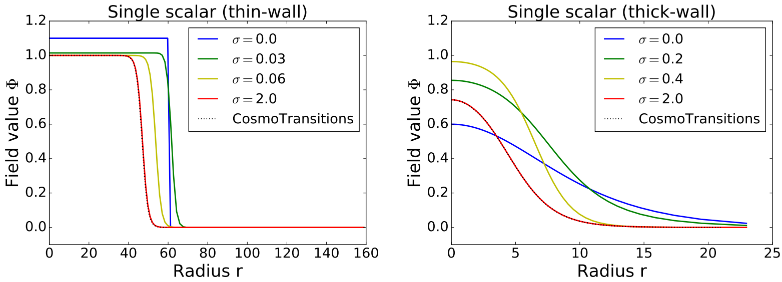
$$\langle g \,|\, \mathcal{M} g\rangle|_{\Phi\to\bar{\phi}} < 0 \quad (D > 2)$$

Flowing to the Bounce

For $D > 2$, $\beta \gtrsim 1$, and $g_A(r,s) \propto r\partial_r\Phi(r,s)$, stable fixed points are always identified as non-trivial solutions of the EoM such as the bounce

Numerical results I: single scalar model

$$V(\phi) = \frac{1}{4}\phi^4 + \frac{c_1 + 1}{3}\phi^3 + \frac{c_1}{2}\phi^2$$



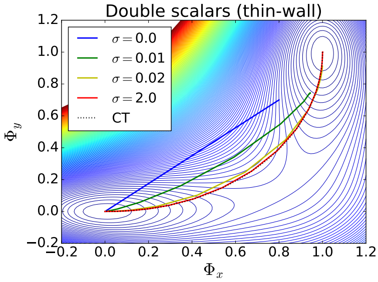
Thin-wall: $c_1 = 0.47$ Thick-wall: $c_1 = 0.2$

– Results agrees well with `CosmoTransitions` C. L. Wainwright (2012)

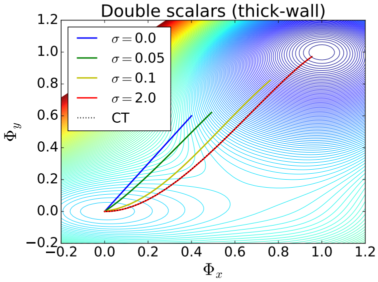
– $r \to \infty$ behavior can be better understood in our method

Numerical results II: multi-scalar model

$$V(\phi) = \left(\phi_x^2 + 5\phi_y^2\right)\left[5\left(\phi_x - 1\right)^2 + \left(\phi_y - 1\right)^2\right] + c_2\left(\frac{1}{4}\phi_y^4 - \frac{1}{3}\phi_y^3\right)$$



Thin-wall: $c_2 = 2$



Thick-wall: $c_2 = 80$

Numerical results III: comparison of bounce action

Table of $S[\bar{\phi}]$ for each model

Model	Our method	`CosmoTransitions`
Single scalar thin-wall	1086.6	1092.8
Single scalar thick-wall	6.6360	6.6490
Multi-scalar thin-wall	1763.7	1767.7
Multi-scalar thick-wall	4.4585	4.4661

– Consistent decay rate can be derived

Conclusion

We proposed a new way of calculating multi-field bounce configuration using steepest descent method.

– Applicable to general multi-scalar potential
– Better way to understand $r \to \infty$ behavior of bounce compared with other existing methods

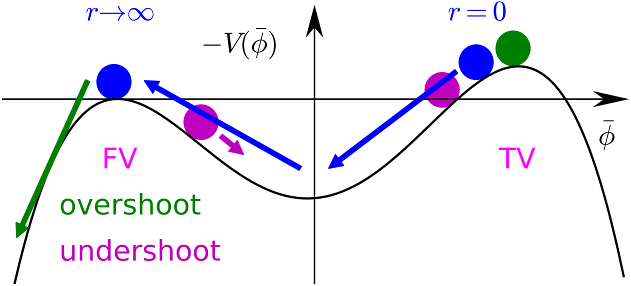
– Applicable to the search of saddle points in general

Backup slides

Single scalar model（Example：SM）

$$\underbrace{\partial_r^2 \bar{\phi}}_{\text{acceleration}} + \underbrace{\frac{D-1}{r}\partial_r \bar{\phi}}_{\text{friction term}} = \underbrace{\left.\frac{\partial V}{\partial \phi}\right|_{\phi \to \bar{\phi}}}_{\text{potential } -V[\bar{\phi}]}$$

EoM for an object moving under the potential $-V[\bar{\phi}]$



This method cannot be applied for multi-scalar case

Fixed point of flow

Flow equation

$$\partial_s \Phi_A(r,s) = F_A(r,s) - \beta \langle F \,|\, g \rangle\, g_A(r,s)$$

$$\langle \partial_s \Phi \,|\, g \rangle = (1-\beta) \langle F \,|\, g \rangle \quad \cdots (*)$$

$\Phi(r, s \to \infty)$ is a solution of EoM ($F_A = 0$) for $\beta \neq 1$

Proof:

1. Fixed point satisfies $\partial_s \Phi_A = 0$
2. $F_A = 0$ or F_A should be parallel to g_A ($\langle F \,|\, g \rangle \neq 0$)
3. $(*)$ confirms $F_A = 0$

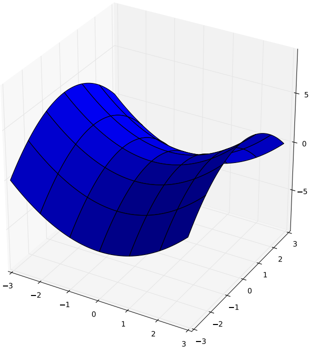
Asymptotic behavior of linearlized equation

Linearlized flow equation

$$\dot{a}_n \simeq -\lambda_n a_n + \beta \sum_m c_n c_m \lambda_m a_m \equiv -\sum_m \Gamma_{nm}(\beta) a_m$$

Illustration of our method

(bounce) $\beta = 0$ (FV)



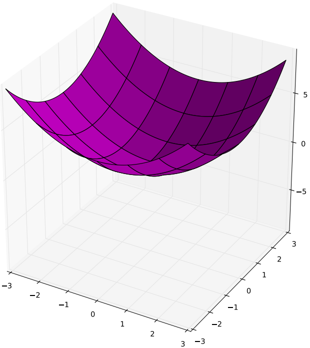


Illustration of our method

(bounce) $\quad 0 < \beta < 1 \quad$ (FV)

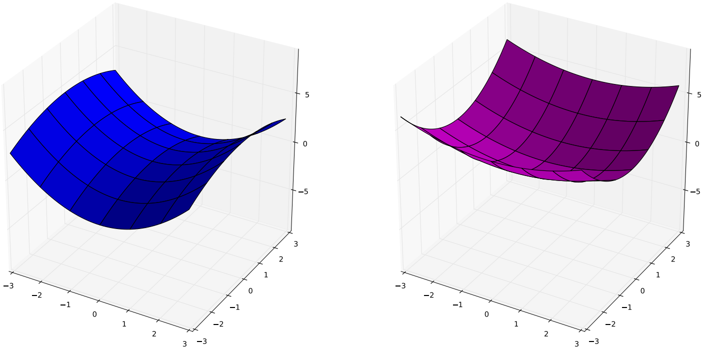


Illustration of our method

(bounce) $\beta \gtrsim 1$ (FV)

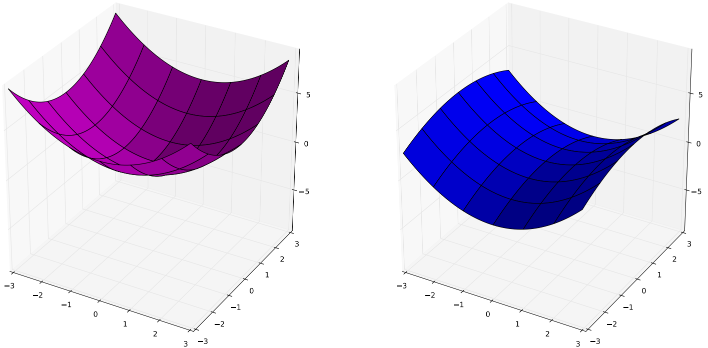
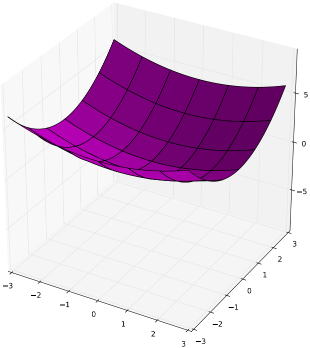


Illustration of our method

(bounce) $\beta \gtrsim 1$ (FV)



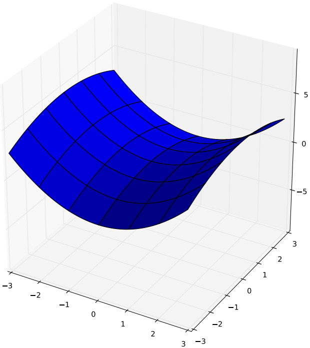
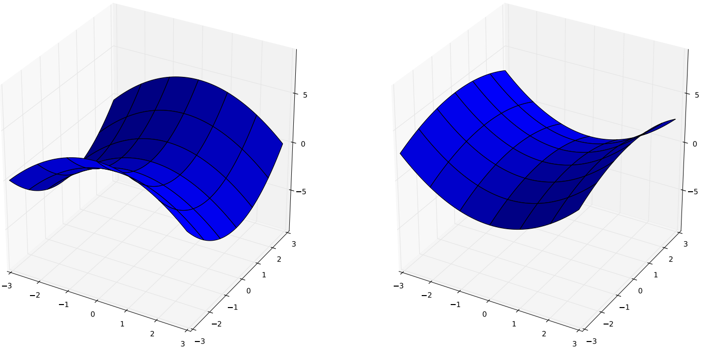


Illustration of our method

(bounce) $\beta \gg 1$ (FV)



Requires $\beta_{\text{stabilization}} < \beta < \beta_{\text{destabilization}}$

memo

– Dilatation maximization : After minimize the kinetic energy T for some $V < 0$, perform the dilatation transformation and maximize the action.
– Improved action : Use $T + 2V$ to modify the action without changing the position of bounce. However, the bounce position is just a local minimum and we should carefully choose the BC.
– Squared EoM : Minimize the $(\mathrm{EoM})^2$ which should become zero if the configuration is a solution of EoM.
– Back step : Combine small steepest descents and a large steepest 'assent'. Tendency to converge both local minimum and maximum (including saddle points).
– Improved potential :
– Path deformation : Devide the "gradient force" into the path direction and path deformation (perpendicular to path) direcction. On a path, the EoM is just the same as the overshoot and undershoot problem.
– Perturbative method : Semi-analytic calculation using some ansatz plus perturbation.
– Multiple shooting : Improved version of overshoot / undershoot for multifield case.
– Tunneling potential : A short cut to evaluate bounce action using modified potential.
– Polygon approximation : Approximate the potential with combination of several patches of linear potential. Then we can derive some analyitical expression to approximate the bounce configuration.
– Machine learning : Use image recognition.