Temporal Bell Inequality Violations in Cosmological Perturbations

Kenta Ando
ICRR, University of Tokyo

in collaboration with
Vincent Vennin, APC
(ongoing)

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Outline

1. Introduction
2. Bell Inequalities
3. Quantum State of Cosmological Perturbations
4. Bell Experiments in Cosmology
5. Results
6. Summary

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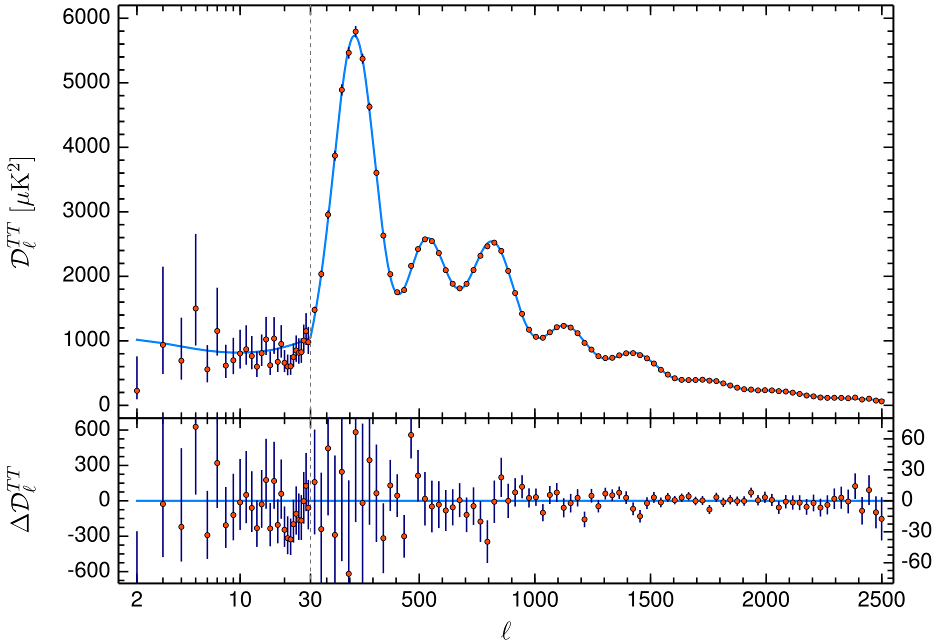
Motivation 1

Inflation + Quantum theory $\rightarrow$ CMB / LSS



Planck 2018

If we can see some quantum features in cosmological observations themselves, it will be a stronger evidence that the cosmological perturbations have a quantum origin.

$$\text{CMB / LSS} \xrightarrow{?} \text{Quantum feature}$$

Violation of Bell inequality ?

Motivation 2

Quantum state of cosmological perturbation is two-mode squeezed state $|\Psi_{2\mathrm{sq}}\rangle$

Squeezing parameter $\begin{cases} \text{Inflation} & r \sim N \sim 50 \\ \text{Lab} & r \sim 1.7 \end{cases}$

Analogy? $\begin{cases} \text{CMB} & \text{: the best Black Body radiation} \\ \text{Perturbation} & \text{: the best Squeezed state} \end{cases}$

$$\text{cf.} \quad |\Psi_{2\mathrm{sq}}\rangle \to |\mathrm{EPR}\rangle = \sum_{n=0}^{\infty} |n\rangle_A \otimes |n\rangle_B = \int \mathrm{d}x \, |x\rangle_A \otimes |x\rangle_B \quad \text{for} \quad r \to \infty$$

Cosmology ⟷ Quantum Information
Quantum Optics

- Quantum nature of Inflation
- Inflation as a Quantum lab

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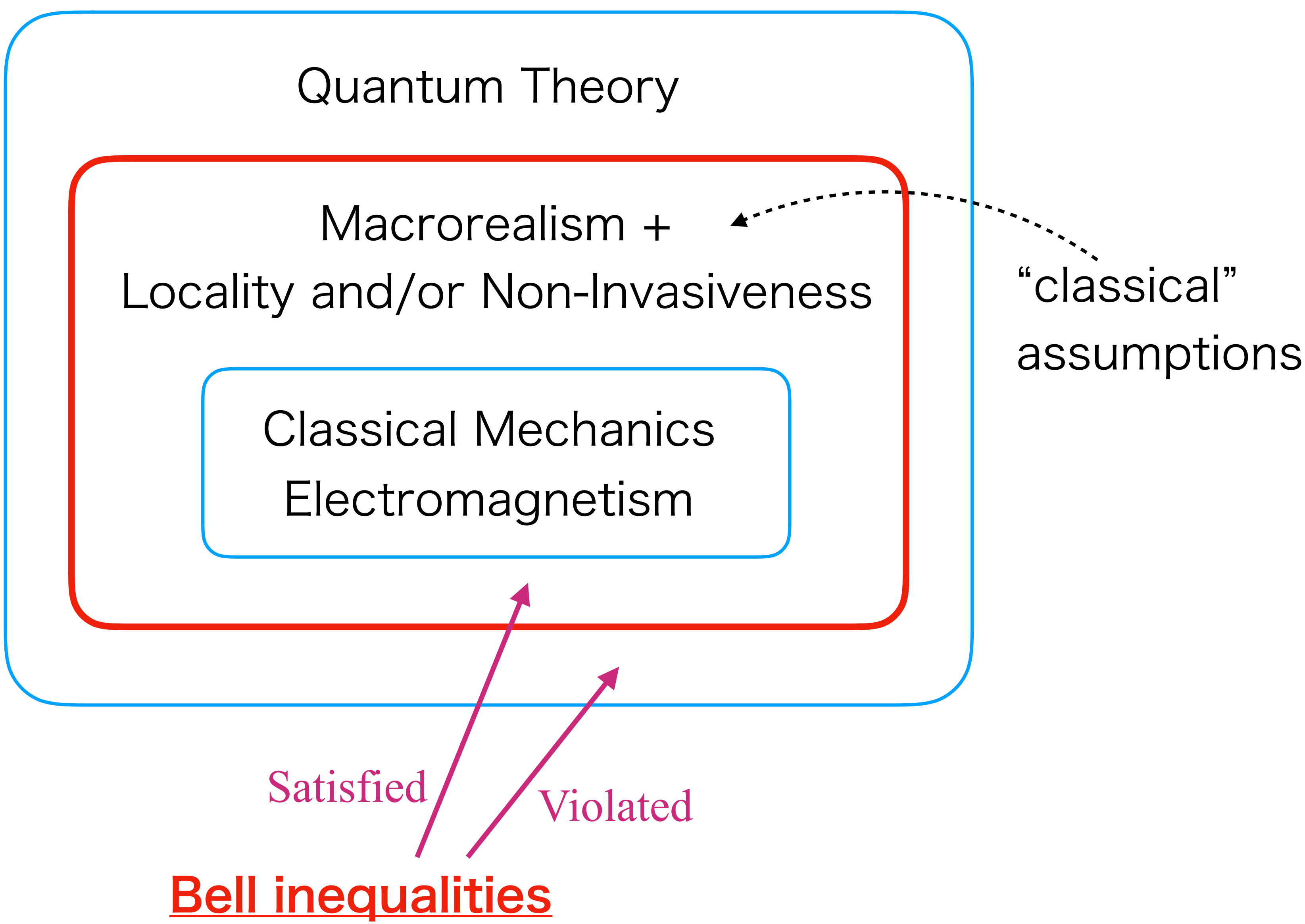
Bell Inequalities



CHSH Scenario

J. F. Clauser, M. A. Horne, A. Shimony, R. A. Holt (1969)

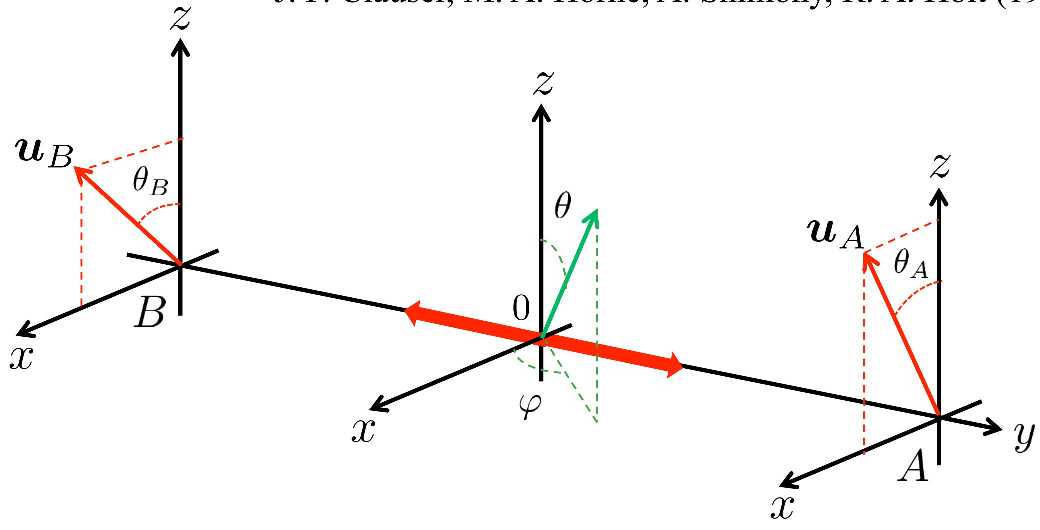


Fig. from J. Martin, V. Vennin (2017)

Bipartite system

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

Spin variables

$$S_A(\theta),\ S_B(\theta) = \pm 1$$

Algebraically,

$$-1 \leq a, a', b, b' \leq 1 \Rightarrow -2 \leq a(b+b') + a'(b-b') \leq 2$$

If the correlation function is described by stochastic average ←

- MacroRealism
- Locality

$$\langle a, b \rangle = \int \mathrm{d}\lambda\, a(\lambda) b(\lambda) P(\lambda)$$

λ : hidden variable

Then,

$$-2 \leq B \leq 2$$

$$B \equiv \langle S_A(\theta_a) S_B(\theta_b) \rangle + \langle S_A(\theta_a) S_B(\theta_b') \rangle + \langle S_A(\theta_a') S_B(\theta_b) \rangle - \langle S_A(\theta_a') S_B(\theta_b') \rangle$$

(Spatial) CHSH inequality

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Cosmological Perturbations

Mukhanov-Sasaki action

$$S = \frac{1}{2} \int \mathrm{d}^4 x \left[(v')^2 - \delta^{ij} \partial_i v \partial_j v + \frac{z''}{z} v^2 \right] \qquad \text{where} \qquad z(\eta) = a M_P \sqrt{2\epsilon}$$

➡ Hamiltonian of cosmological perturbations

$$\hat{H} = \frac{1}{2} \int_{\mathbb{R}^3} \mathrm{d}^3 k \left[\underbrace{k \left(\hat{c}_{\boldsymbol{k}} \hat{c}^\dagger_{\boldsymbol{k}} + \hat{c}^\dagger_{-\boldsymbol{k}} \hat{c}_{-\boldsymbol{k}} \right)}_{\text{free field (harmonic oscillator)}} \underbrace{- i \frac{z'}{z} \left(\hat{c}_{\boldsymbol{k}} \hat{c}_{-\boldsymbol{k}} - \hat{c}^\dagger_{-\boldsymbol{k}} \hat{c}^\dagger_{\boldsymbol{k}} \right)}_{\text{coupling to gravity}} \right]$$

free field
(harmonic oscillator)

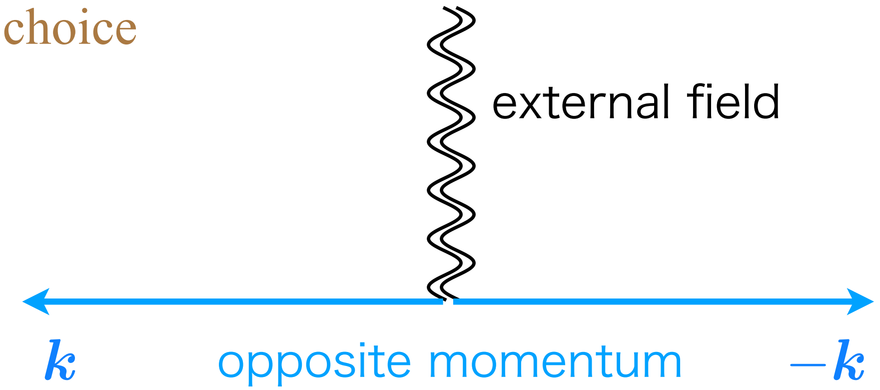
coupling to gravity

$$\left[\hat{c}_{\boldsymbol{k}}, \hat{c}^\dagger_{\boldsymbol{q}} \right] = \delta \left(\boldsymbol{k} - \boldsymbol{q} \right)$$

Hilbert space

$$\mathcal{H} = \prod_{\boldsymbol{k} \in \mathbb{R}^{3+}} \underline{\mathcal{H}_{\boldsymbol{k}} \otimes \mathcal{H}_{-\boldsymbol{k}}}$$

k and -k will be a natural choice
for a bipartite system



Cosmological Perturbations

$1 \leftrightarrow \boldsymbol{k}$

$2 \leftrightarrow -\boldsymbol{k}$

Time evolution operator $\hat{U}(t) = \hat{U}_S(t)\hat{R}(t)$

(Two-mode) Squeezing operator $\hat{U}_S(t) = \exp\left[re^{-2i\varphi}\hat{c}_1^\dagger\hat{c}_2^\dagger - re^{2i\varphi}\hat{c}_1\hat{c}_2\right]$

Rotation operator $\hat{R}(t) = \exp\left[i\theta\hat{c}_1^\dagger\hat{c}_1 + i\theta\hat{c}_2^\dagger\hat{c}_2\right] = e^{i\theta\hat{n}_1}e^{i\theta\hat{n}_2}$

time dependence of $r,\ \theta,\ \varphi$:

$$\begin{cases} \dfrac{\mathrm{d}r}{\mathrm{d}\eta} = \dfrac{z'}{z}\cos(2\varphi) \\ \dfrac{\mathrm{d}\varphi}{\mathrm{d}\eta} = k - \dfrac{z'}{z}\coth(2r)\sin(2\varphi) \\ \dfrac{\mathrm{d}\theta}{\mathrm{d}\eta} = -k + \dfrac{z'}{z}\tanh(r)\sin(2\varphi) \end{cases}$$

Heisenberg picture

$$\hat{c}_{\boldsymbol{k}}(t) \equiv \hat{U}^\dagger(t)\hat{c}_{\boldsymbol{k}}\hat{U}(t)$$
$$= e^{i\theta}\cosh r\,\hat{c}_{\boldsymbol{k}} + e^{-i\theta-2i\varphi}\sinh r\,\hat{c}_{-\boldsymbol{k}}^\dagger$$

Bogoliubov transformation

Schrödinger picture

$$|\Psi(t)\rangle \equiv \hat{U}(t)\,|0,0\rangle$$
$$= \frac{1}{\cosh r}\sum_{n=0}^{\infty} e^{-2in\varphi}\tanh^n r\,|n,n\rangle$$

start from Bunch-Davis vacuum

Two-mode Squeezed state

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Pseudo-Spin Operator

Define the "position" and "momentum" operators

$$\hat{Q}_{\boldsymbol{k}} = \frac{1}{\sqrt{2}}\left(\hat{c}_{\boldsymbol{k}} + \hat{c}_{\boldsymbol{k}}^{\dagger}\right)$$

$$\hat{P}_{\boldsymbol{k}} = -i\sqrt{\frac{1}{2}}\left(\hat{c}_{\boldsymbol{k}} - \hat{c}_{\boldsymbol{k}}^{\dagger}\right)$$

cf. $\hat{\zeta}_{\boldsymbol{k}} = \frac{1}{z\sqrt{2k}}\left(\hat{c}_{\boldsymbol{k}} + \hat{c}_{-\boldsymbol{k}}^{\dagger}\right)$

Continuous variables to Dichotomic variables

- Larsson's spin operators $\hat{S}_x, \hat{S}_y, \underline{\hat{S}_z}$

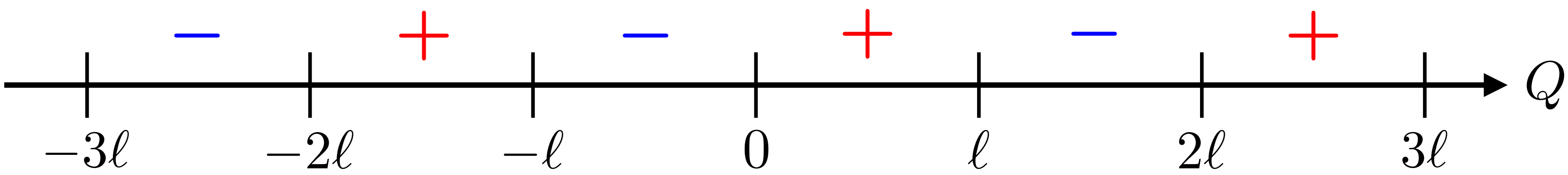
$$\hat{S}_z(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{n\ell}^{(n+1)\ell} \mathrm{d}Q \, |Q\rangle \langle Q|$$

J. A. Larsson (2004)

- easy to relate with measurements
- "proper" operator $\rightarrow$ observable in cosmology

In $\ell \rightarrow \infty$ limit

$$\hat{S}_z = \mathrm{sign}\left(\hat{Q}\right)$$



Obstacles

Are the pseudo-spin operators measurable in Cosmology? J. Martin, V. Vennin (2017)

In Cosmology, we are "given" observations.

If we neglect the decaying modes, $\zeta_{\boldsymbol{k}} \leftrightarrow Q_{\boldsymbol{k}}$

We cannot measure all of the x, y, z components of the spin.

ex) Larsson's spin operator

$$\left[\hat{S}_z, \hat{Q}\right] = 0$$

$$\left[\hat{S}_x, \hat{Q}\right] \neq 0, \quad \left[\hat{S}_y, \hat{Q}\right] \neq 0$$

→ **two-time correlations are motivated**

CMB and LSS ?
finite thickness effects in CMB?

Temporal Bell Inequality

temporal CHSH inequality J. Martin (2019)

1, 2 : two subsystems

$$-2 \leq B \leq 2$$

$$B(t_a, t_b, t'_a, t'_b) \equiv E(t_a, t_b) + E(t_a, t'_b) + E(t'_a, t_b) - E(t'_a, t'_b)$$

$$E(t_a, t_b) \equiv \frac{1}{2} \left\langle \left\{ \hat{S}_1(t_a), \hat{S}_2(t_b) \right\} \right\rangle$$

polarization angle $\theta \leftrightarrow$ time t

In spatial CHSH inequality,

$$B \equiv \langle S_1(\theta_a) S_2(\theta_b) \rangle + \langle S_1(\theta_a) S_2(\theta'_b) \rangle + \langle S_1(\theta'_a) S_2(\theta_b) \rangle - \langle S_1(\theta'_a) S_2(\theta'_b) \rangle$$

Temporal Bell Inequality

temporal CHSH inequality   J. Martin (2019)   1, 2 : two subsystems

$$-2 \le B \le 2$$

$$B(t_a, t_b, t'_a, t'_b) \equiv E(t_a, t_b) + E(t_a, t'_b) + E(t'_a, t_b) - E(t'_a, t'_b)$$

$$E(t_a, t_b) \equiv \frac{1}{2} \left\langle \left\{ \hat{S}_1(t_a), \hat{S}_2(t_b) \right\} \right\rangle$$

What we did:

Search for violations of the temporal CHSH inequality
for a general two-mode squeezed state

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Correlation Function

1, 2 : two subsystems
a, b : two times

$$E(t_a, t_b) = \frac{1}{2} \left\langle \left\{ \hat{S}_1(t_a), \hat{S}_2(t_b) \right\} \right\rangle$$
$$= \Re\mathrm{e} \left\langle 0, 0 \left| \hat{U}^\dagger(t_a) \hat{S}_1 \hat{U}(t_a) \hat{U}^\dagger(t_b) \hat{S}_2 \hat{U}(t_b) \right| 0, 0 \right\rangle$$

5 parameters for the system : $r_a,\ r_b,\ \varphi_a,\ \varphi_b,\ \Delta\theta\,(=\theta_a - \theta_b)$

1 parameters for the spin operator : ℓ

Note

The rotation angle θ vanishes in a state evolved from the vacuum state

$$\hat{R}(t)\,|0,0\rangle = |0,0\rangle$$

However, it exists in a time correlation due to projection.

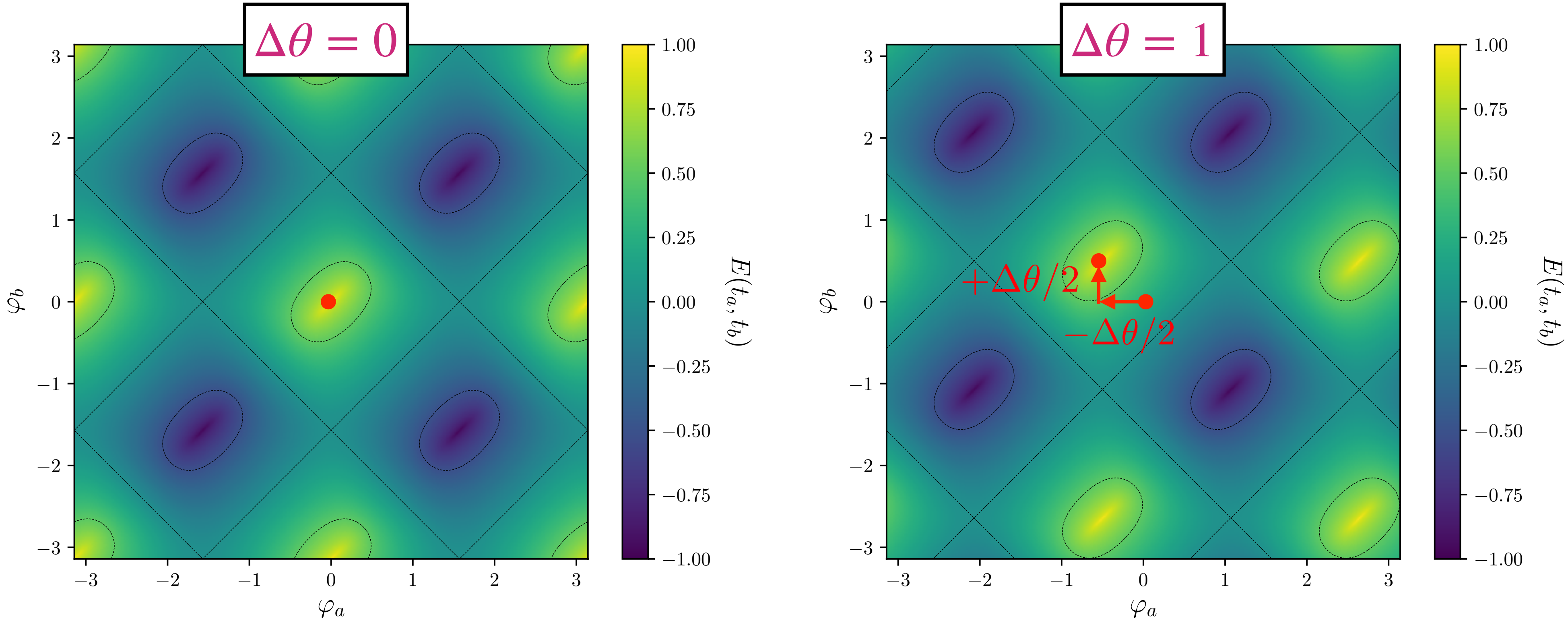
$$\left\langle \hat{O}(t_a)\hat{O}(t_b) \right\rangle = \left\langle 0, 0 \left| \hat{R}^\dagger(t_a)\hat{O}\hat{R}(t_a)\hat{R}^\dagger(t_b)\hat{O}\hat{R}(t_b) \right| 0, 0 \right\rangle = \left\langle 0, 0 \left| \hat{O}\hat{R}(t_a)\hat{R}^\dagger(t_b)\hat{O} \right| 0, 0 \right\rangle$$

Correlation Function - Infinite ℓ

$$E(t_a, t_b) = \frac{1}{2}\left\langle\left\{\hat{S}_1(t_a), \hat{S}_2(t_b)\right\}\right\rangle$$

1, 2 : two subsystems
a, b : two times

Large squeezing ($r_a, r_b \to \infty$)
Sign operator ($\ell \to \infty$)



contour: $E(t_a, t_b) = 0.5, 0, -0.5$

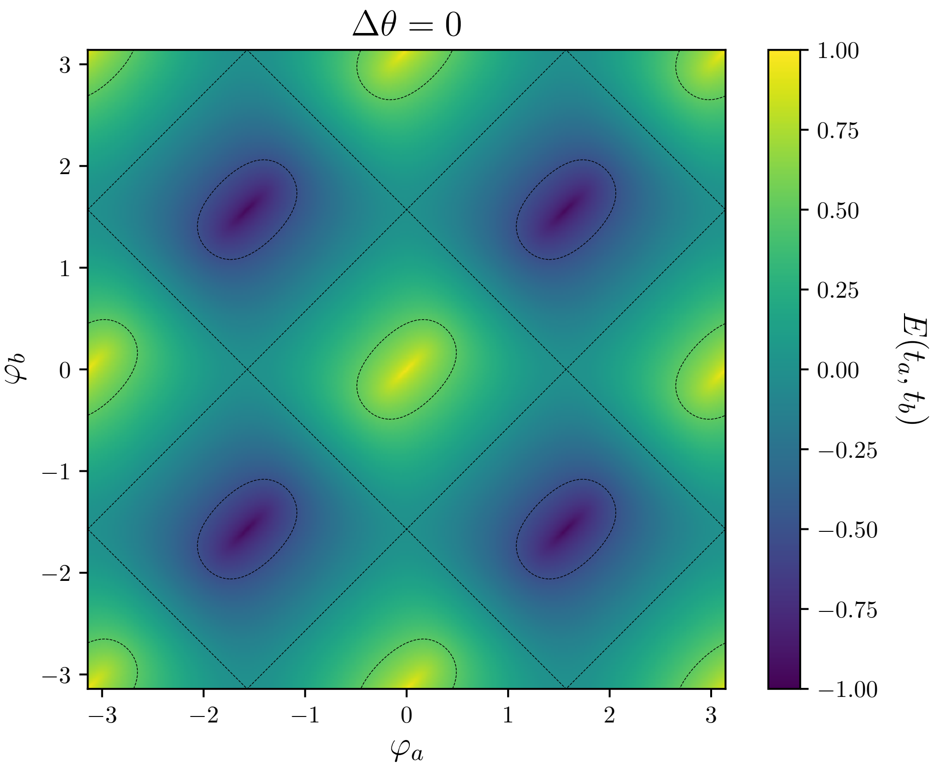
$$E(t_a, t_b) = 1 \quad \text{at} \quad (\varphi_a, \varphi_b) = \left(-\frac{\Delta\theta}{2}, \frac{\Delta\theta}{2}\right)$$

Correlation Function - Finite ℓ

$$E(t_a, t_b) = \frac{1}{2} \left\langle \left\{ \hat{S}_1(t_a), \hat{S}_2(t_b) \right\} \right\rangle$$

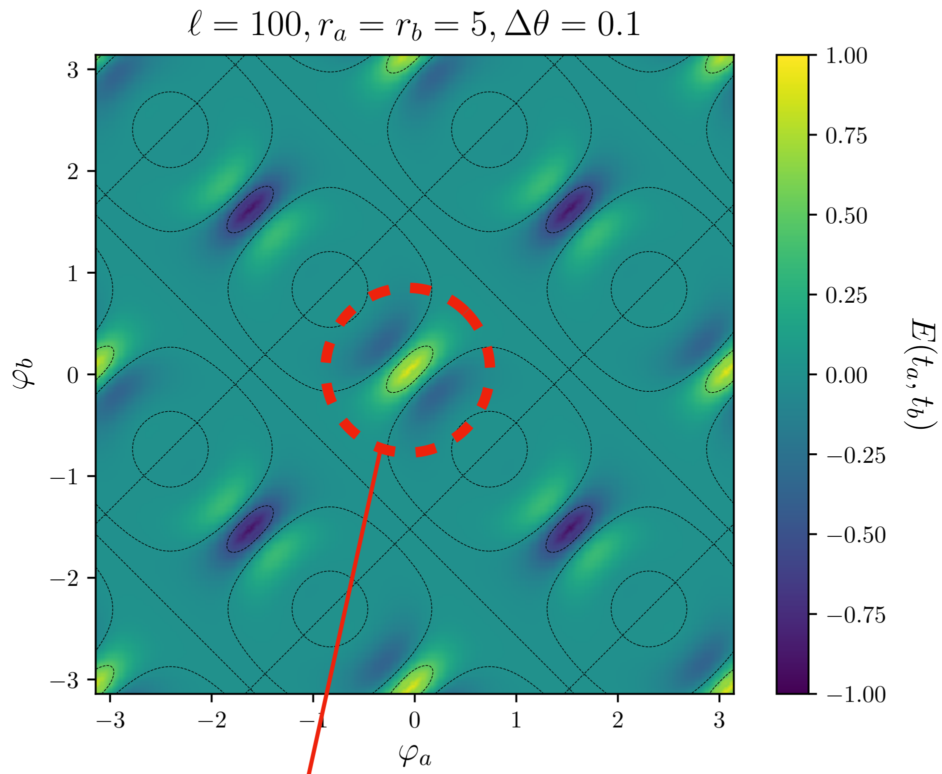
1, 2 : two subsystems
a, b : two times

Large squeezing ($r_a, r_b \to \infty$)
Sign operator ($\ell \to \infty$)



contour: $E(t_a, t_b) = 0.5, 0, -0.5$

Large squeezing ($r_a = r_b = 5$)
Finite ℓ ($\ell = 100$)

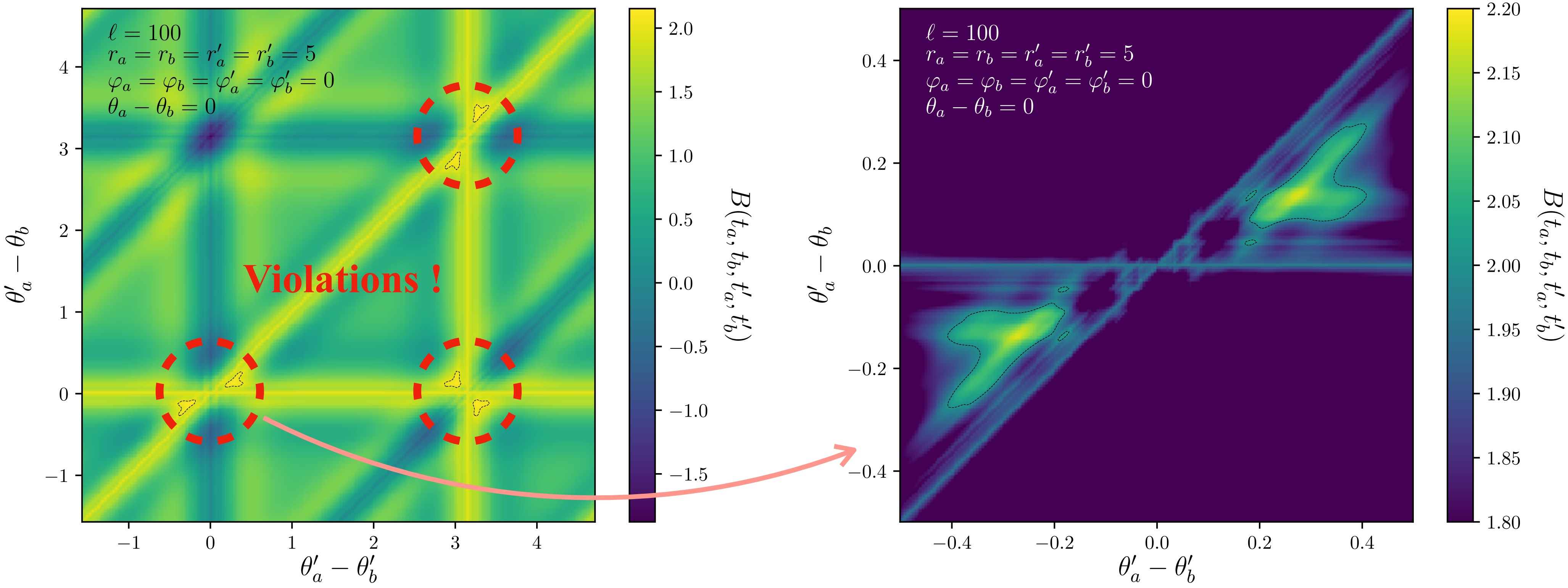


local minimums appear around the maximum

$\rightarrow$ crucial for violations of Bell inequalities

Bell Inequality Violations

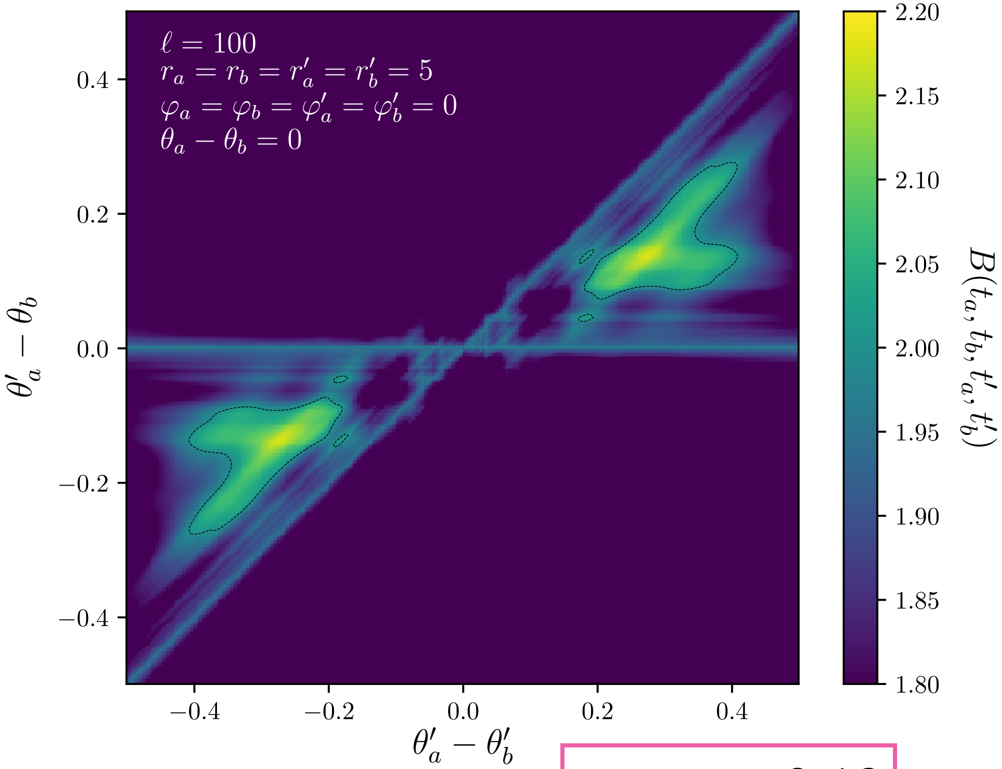
$$B(t_a, t_b, t'_a, t'_b) \equiv E(t_a, t_b) + E(t_a, t'_b) + E(t'_a, t_b) - E(t'_a, t'_b)$$



Contour: $B = 2$

ℓ dependence

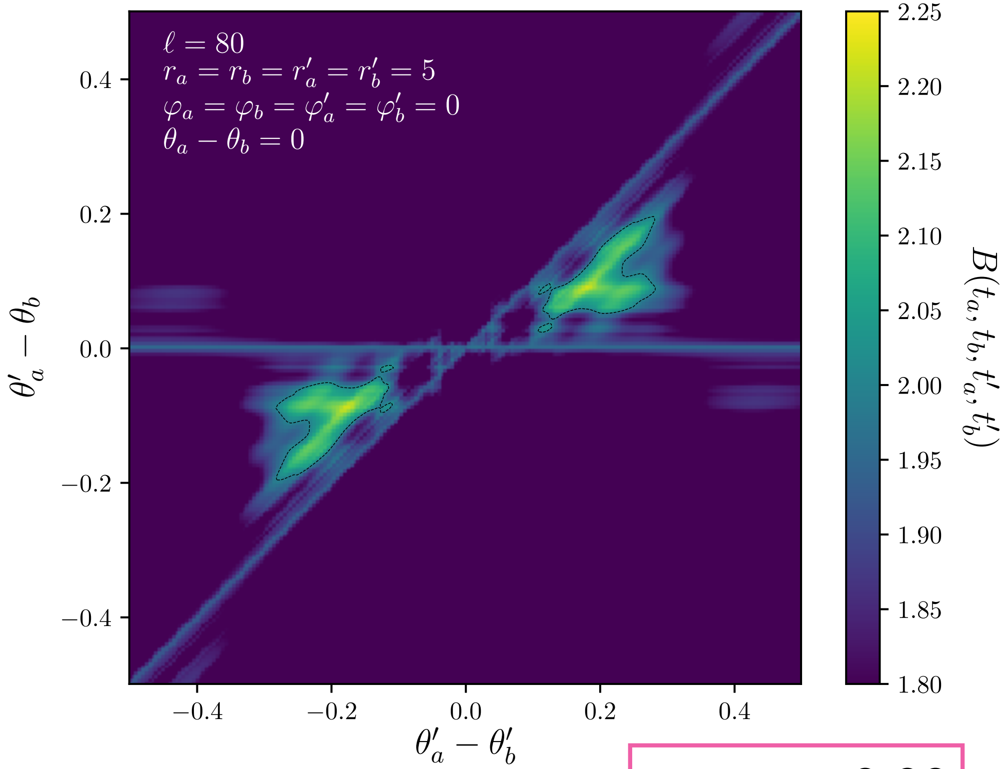
$r = 5$
$\ell = 100$



max $B = 2.18$

$r = 5$
$\ell = 80$

Contour: $B = 2$



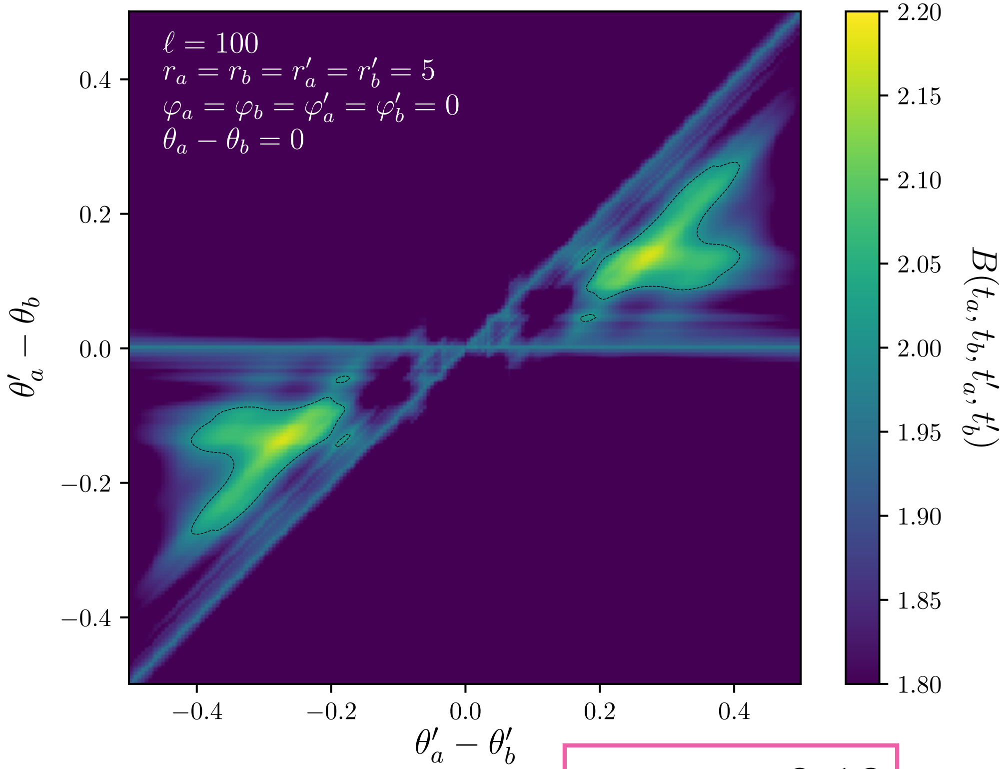
max $B = 2.22$

small ℓ $\leftrightarrow$ narrow range of θ $\leftrightarrow$ large violation

$\ell \lesssim e^r$ for Bell inequality violations ($e^r \simeq 150$ for $r = 5$)

r dependence

$r = 5$
$\ell = 100$

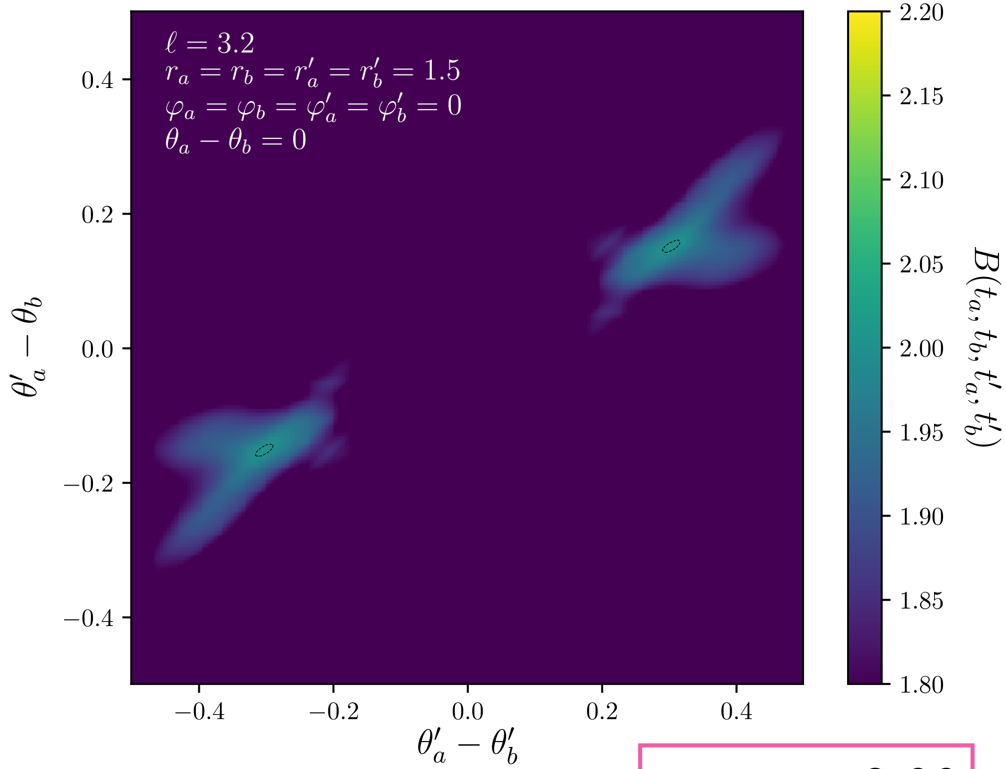


max $B = 2.18$

optimized for violations

$r = 1.5$
$\ell = 3.2$

Contour: $B = 2$



max $B = 2.00$

$r \gtrsim 1.5$ for Bell inequality violations

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Summary

- Bell experiments in cosmology may be able to give a strong evidence that cosmological perturbations have a quantum origin.
- The quantum state of the cosmological perturbations is a two-mode squeezed state and it is highly squeezed during inflation.
- The rotation angle disappears in a state evolved from the vacuum state but appears in temporal correlations.
- Temporal correlations would be the only way for Bell experiments in cosmology.
- A two-mode squeezed state shows violations of the temporal CHSH inequality for the Larsson's spin operator.
- Necessary conditions for the violations: $\ell \lesssim e^r,\ r \gtrsim 1.5$