New Type of Cosmic String Solutions with Long Range Forces

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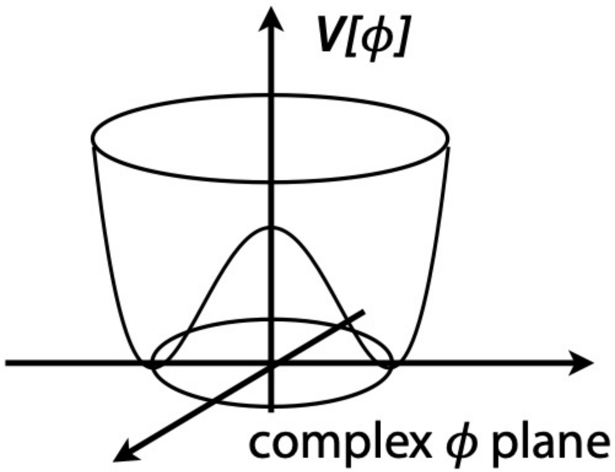
Takashi Hiramatsu, Motoo Suzuki and MI arXiv:1910.14321

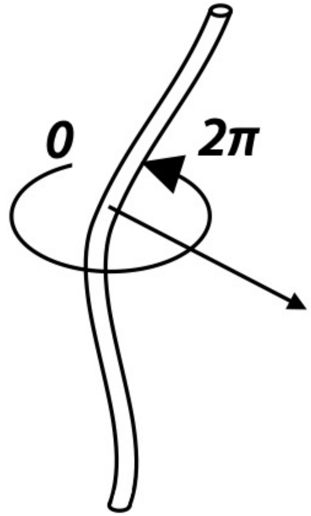
Cosmic String

- Cosmic strings appear when ***U(1)*** symmetries are spontaneously broken

[1973 Nielsen Olsen]

$$V[\phi] = \lambda(|\phi|^2 - v^2)^2$$





$$\phi(r \to 0) = 0$$

$$\phi(r \to \infty) = v\, e^{i\theta}$$

e.g.) winding number = 1

- An isolated cosmic string is stable due to the topological charge :

$$\Pi_1(U(1)) = Z$$

Cosmic String

✔ *U(1)* gauge symmetry → ***Local String***

cylindrically (r,θ,z) symmetric static solution

$$\begin{cases} \phi(r,\theta) = ve^{i\theta}h(r) \\ A_\theta(r) = \frac{1}{e}\xi(r) \end{cases}$$

$$h(r) \to 1 - \mathcal{O}\left(\frac{1}{\sqrt{r}}\right)\exp(-m_h r)$$

$$\xi(r) \to 1 - \mathcal{O}\left(\sqrt{r}\right)\exp(-m_A r)$$

$$m_h = 2\sqrt{\lambda}v \qquad m_A = \sqrt{2}ev$$

$$\partial_\theta \phi \to ive^{i\theta}$$

$$D_\theta \phi = \partial_\theta \phi - ieA_\theta \phi \to 0$$ (exponentially dumped)

✔ Local string has a finite tension

$$\mathcal{E} = \int d^2x \left[\frac{1}{4}F_{ij}F^{ij} + |D_i\phi|^2 + V(\phi)\right] = 2\pi v^2 \times \mathcal{F}(2\lambda/e^2)$$

$$(\mathcal{F}(1) = 1 \quad \mathcal{F}'(x) < 0)$$

A few local strings are formed in each Hubble volume at the phase transition.

Cosmic String

- ***U(1)*** global symmetry → ***Global String***

cylindrically (r,θ,z) symmetric static solution

$$\phi(r,\theta) = ve^{i\theta}h(r)$$

$$h(r) \to 1 - \mathcal{O}\left(\frac{1}{\sqrt{r}}\right)\exp(-m_h r)$$

$$m_h = 2\sqrt{\lambda}v$$

$$\partial_\theta\phi \to ive^{i\theta}$$

- String tension has a logarithmic divergence !

$$\mathcal{E} = \int d^2x\left[|\partial_i\phi|^2 + V(\phi)\right] \to 2\pi v^2 \int dr\frac{1}{r} \to \infty$$

The global string is also formed at the phase transition where
the divergence is cutoff by a typical distance between the strings.

New Types of Cosmic Strings ?

✔ Cosmic string solutions in a model with $U(1)_{local}$ x $U(1)_{global}$ symmetry

$U(1)_{global}$ emerges in the $U(1)_{local}$ symmetric two complex scalar models.

Two complex scalars with $U(1)_{local}$ gauge charges q_1 and q_2:

$$\phi_1(q_1) \qquad\qquad \phi_2(q_2)$$

(q_1, q_2 : positive and relatively prime integers)

The model can have an ***accidental global U(1) symmetry*** when the charge ratio is larger than ***3*** .

$$V(\phi_1, \phi_2) = V_s(\phi_1^\dagger \phi_1, \phi_2^\dagger \phi_2) + \frac{1}{M_*^{q_1+q_2-4}} \phi_1^{\dagger q_1} \phi_2^{q_2} + h.c.$$

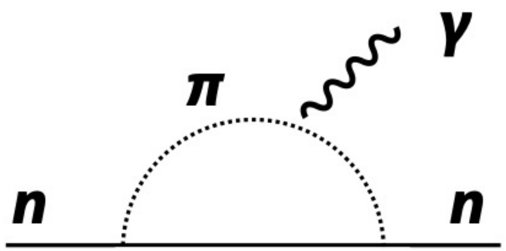
An additional *U(1)* symmetry emerges for $M_* \to \infty$.

U(1)$_{PQ}$ Quality & Domain Wall Problem

- ***QCD*** has its own ***CP***-violating parameter : ***θ***

$$\mathcal{L}_{\rm SM} \ni \frac{g_s^2}{32\pi^2}\theta G^{\mu\nu}\tilde{G}_{\mu\nu}$$

- Null observation of the ***neutron EDM*** :



$d_n/e \sim 10^{-15}\,\theta$ cm   [1979 Crewther, Veccia, Veneziano, Witten]

$d_n/e < 2.9 \times 10^{-26}$ cm @ 90%CL [hep-ex/0602020]   → $\theta < 10^{-11}$

$U(1)_{PQ}$ Quality & Domain Wall Problem

- θ can be nullified by the global $U(1)_{PQ}$ symmetry broken by ***QCD*** anomaly

[1977 Peccei & Quinn]

KSVZ axion model ***['79 Kim, '80 Shifman, Vainshtein, Zakharov]***

$\mathcal{L} = y\phi q_L \bar{q}_R + h.c.$ ϕ : singlet $(q_L, \bar{q}_R)$: N_f extra quarks

$U(1)_{PQ}$ symmetry :

$$\phi \to e^{i\alpha}\phi \qquad q_L\bar{q}_R \to e^{-i\alpha} q_L \bar{q}_R$$

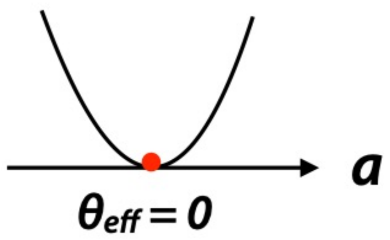
θ is unphysical due to the anomalous $U(1)_{PQ}$ rotation.

$$\theta \to \theta' = \theta - N_f \alpha$$

- Axion appears when $U(1)_{PQ}$ is broken by $\langle\phi\rangle = v$

$$a = f_a \arg\phi \qquad f_a = \sqrt{2}\langle\phi\rangle$$

- Axion obtains a scalar potential due to the ***QCD*** anomaly



The effective term is given by the VEV of the axion :

$$\theta_{\text{eff}} = \langle a/f_a \rangle = 0$$

$U(1)_{PQ}$ Quality & Domain Wall Problem

✔ Why is the ***PQ*** symmetry broken only by the ***QCD*** anomaly?

(The ***PQ*** symmetry cannot be an exact symmetry by definition)

Planck suppressed explicit breaking term distorts the axion potential

$$\Delta\mathcal{L} = \frac{\kappa\phi^m}{M_{\rm Pl}^{m-4}} + h.c. \qquad \mathcal{L} \simeq \frac{1}{2}m_a^2 a^2 + \frac{\delta|\kappa|f_a^m}{M_{\rm Pl}^{m-4}}\frac{a}{f_a} + \cdots \qquad \delta = 2\sin(\arg\kappa)$$

θ_{eff} is no more vanishing…

$$\theta_{\rm eff} \sim \frac{f_a^m}{N_f f_\pi^2 m_\pi^2 M_{\rm Pl}^{m-4}} \qquad m_a \sim \frac{N_f f_\pi}{f_a} m_\pi$$

If we require $\theta_{eff} \ll 10^{-11}$, we need to forbid terms with $m < 10$ for $f_a > 10^9$GeV.

✔ How about discrete Z_n gauge symmetry instead of global $U(1)_{PQ}$?

discrete Z_n gauge symmetry = exact symmetry

$\theta_{eff} \ll 10^{-11}$ is achieved for $n > 10$ *See e.g. [2009 Carpenter, Dine, Festuccia]*

✔ Z_n symmetric model has the axion domain wall problem when approximate $U(1)_{PQ}$ breaking takes place after inflation.

$U(1)_{PQ}$ Quality & Domain Wall Problem

✔ A model with $U(1)_{local}$ x $U(1)_{global}$ symmetry provides an alternative mechanism which provides an accidental $U(1)_{PQ}$ from $U(1)_{local}$.

[1992 Barr, Seckel] see also [2017, Fukda, Suzuki, Yanagida and MI]

$U(1)_{global}$ breaking is small enough when the charge ratio is larger than ***10***.

$$V(\phi_1, \phi_2) = V_s(\phi_1^\dagger \phi_1, \phi_2^\dagger \phi_2) + \frac{1}{M_*^{q_1+q_2-4}} \phi_1^{\dagger q_1} \phi_2^{q_2} + h.c.$$

(q_1, q_2 : positive and relatively prime integers → $q_1 + q_2 > 10$)

✔ What happens when both $<\phi_1>$ and $<\phi_2>$ obtain non-vanishing expectation value after inflation ?

What kind of string network is formed ?

Does the model have the axion domain wall problem?

Goldstone Decomposition

✔ A model with $U(1)_{local} \times U(1)_{global}$ symmetry :

$$V = \frac{\lambda_1}{4}(|\phi_1|^2 - \eta_1^2)^2 + \frac{\lambda_2}{4}(|\phi_2|^2 - \eta_2^2)^2 - \kappa(|\phi_1|^2 - \eta_1^2)(|\phi_2|^2 - \eta_2^2) \ ,$$

$$\lambda_{1,2} > 0 \qquad \lambda_1\lambda_2 > 4\kappa$$

$U(1)_{local}$ charges : $\phi_1(q_1) \quad \phi_2(q_2)$

(q_1, q_2 : positive and relatively prime integers)

At the vacuum, both ϕ_1 and ϕ_2 obtain the VEV

$$\langle\phi_n\rangle = \eta_n \ , \quad (n = 1, 2)$$

and hence, both $U(1)_{local} \times U(1)_{global}$ are broken.

In the following, we assume

$q_1 = 1$, $q_2 = N$ and $\eta_1 > \eta_2$

→ ***no discrete gauge symmetry*** remains after ϕ_1 obtains the VEV

Goldstone Decomposition

- Two goldstone modes :

$$\phi_1 = \frac{1}{\sqrt{2}} f_1\, e^{i\tilde{a}_1/f_1} \ , \quad \phi_2 = \frac{1}{\sqrt{2}} f_2\, e^{i\tilde{a}_2/f_2} \ , \qquad f_n = \sqrt{2}\eta_n$$

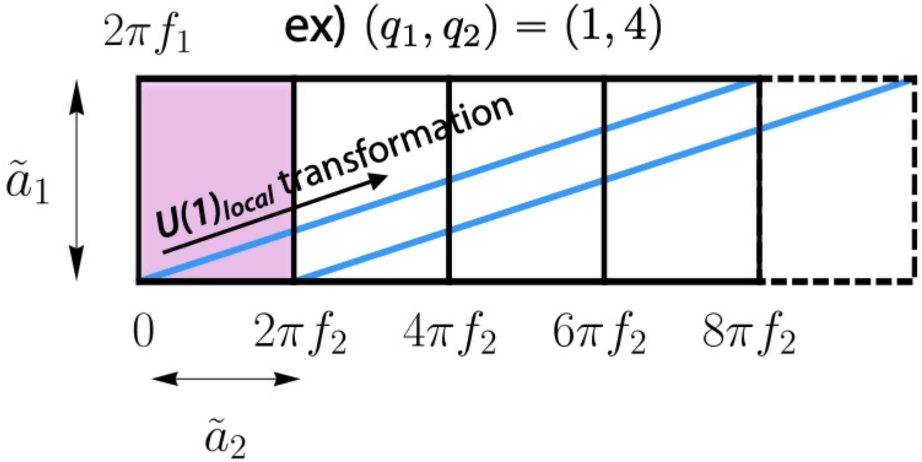
- The domain of the goldstone modes:

$$\tilde{a}_1/f_1 = [0, 2\pi) \ , \quad \tilde{a}_2/f_2 = [0, 2\pi) \ ,$$

- The *U(1)* symmetry is realized by:

$$\tilde{a}_1/f_1 \to \tilde{a}_1/f_1 + q_1\alpha \ , \quad \tilde{a}_2/f_2 \to \tilde{a}_2/f_2 + q_2\alpha \ ,$$

ex) $(q_1, q_2) = (1, 4)$



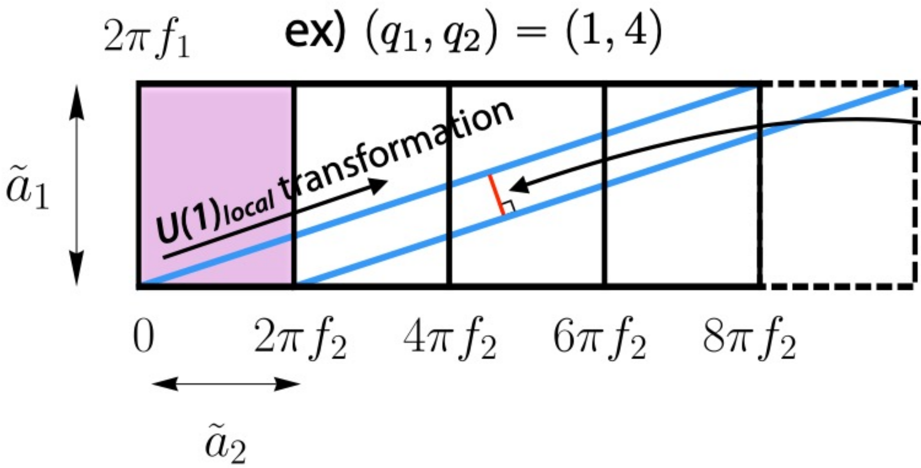
Goldstone Decomposition

- The gauge invariant axion :

$$\mathcal{L} = |\mathscr{D}_\mu \phi_1|^2 + |\mathscr{D}_\mu \phi_2|^2$$
$$\to \frac{1}{2}(\partial \tilde{a}_1)^2 + \frac{1}{2}(\partial \tilde{a}_2)^2 - eA_\mu(q_1 f_1 \partial^\mu \tilde{a}_1 + q_2 f_2 \partial^\mu \tilde{a}_2) - \frac{1}{2}e^2(q_1^2 f_1^2 + q_2^2 f_2^2)A_\mu^2$$
$$= \frac{1}{2}(\partial a)^2 + \frac{1}{2}m_A^2\left(A_\mu - \frac{1}{m_A}\partial_\mu b\right)^2 \qquad m_A^2 = e^2\left(q_1^2 f_1^2 + q_2^2 f_2^2\right)$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}} \begin{pmatrix} q_2 f_2 & -q_1 f_1 \\ q_1 f_1 & q_2 f_2 \end{pmatrix} \begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{pmatrix}$$

ex) $(q_1, q_2) = (1, 4)$



$a = [0, 2\pi F_a)$

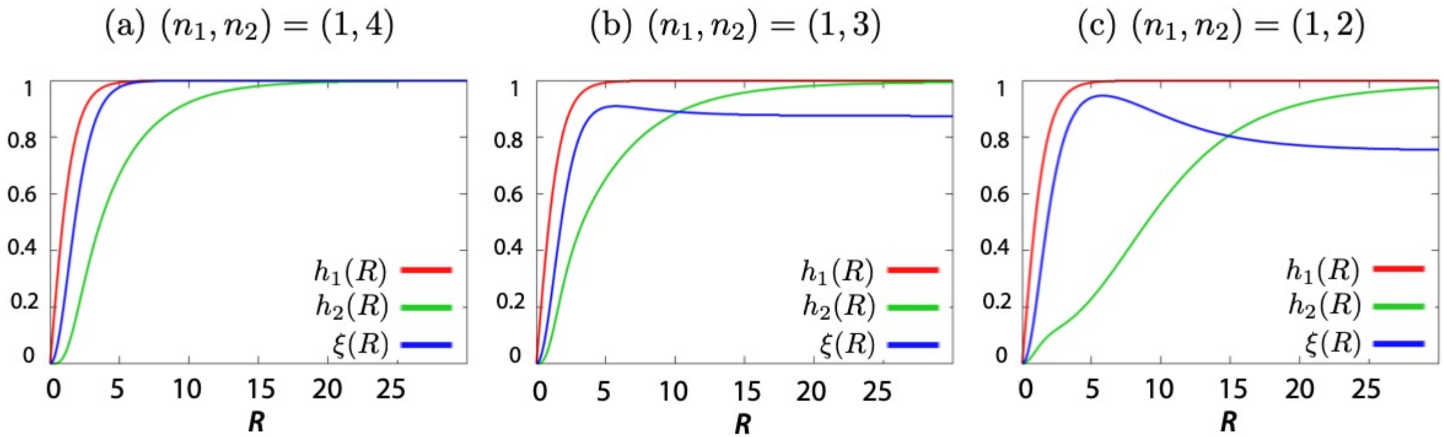
$$F_a = \frac{f_1 f_2}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}}$$

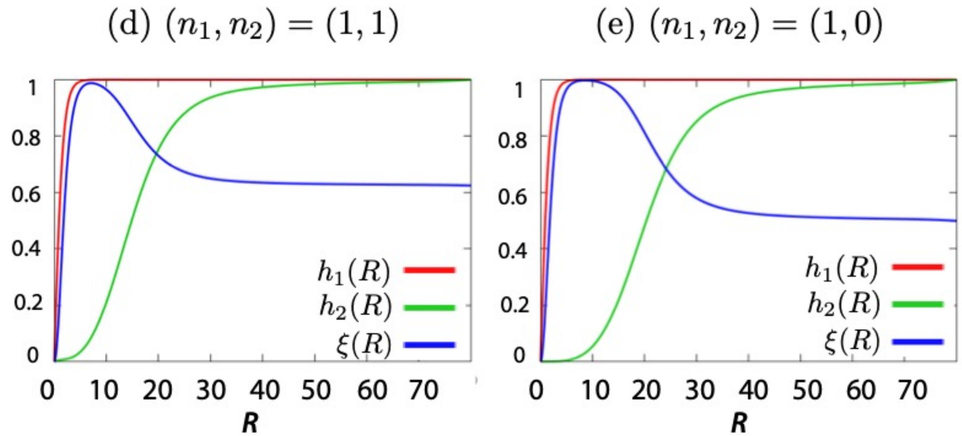
Static Solution

cylindrically (r,θ,z) symmetric static solutions **(n_1, n_2)** : winding number

$$\phi_1(r,\theta) = \eta_1 e^{in_1\theta} h_1(r) \ ,$$

$$\phi_2(r,\theta) = \eta_2 e^{in_2\theta} h_2(r) \ , \qquad A_\theta(r) = \frac{1}{e}\xi(r) \ , \ A_r = A_z = 0 \ ,$$





$(q_1, q_2) = (1, 4)$

$f_1/f_2 = 4$

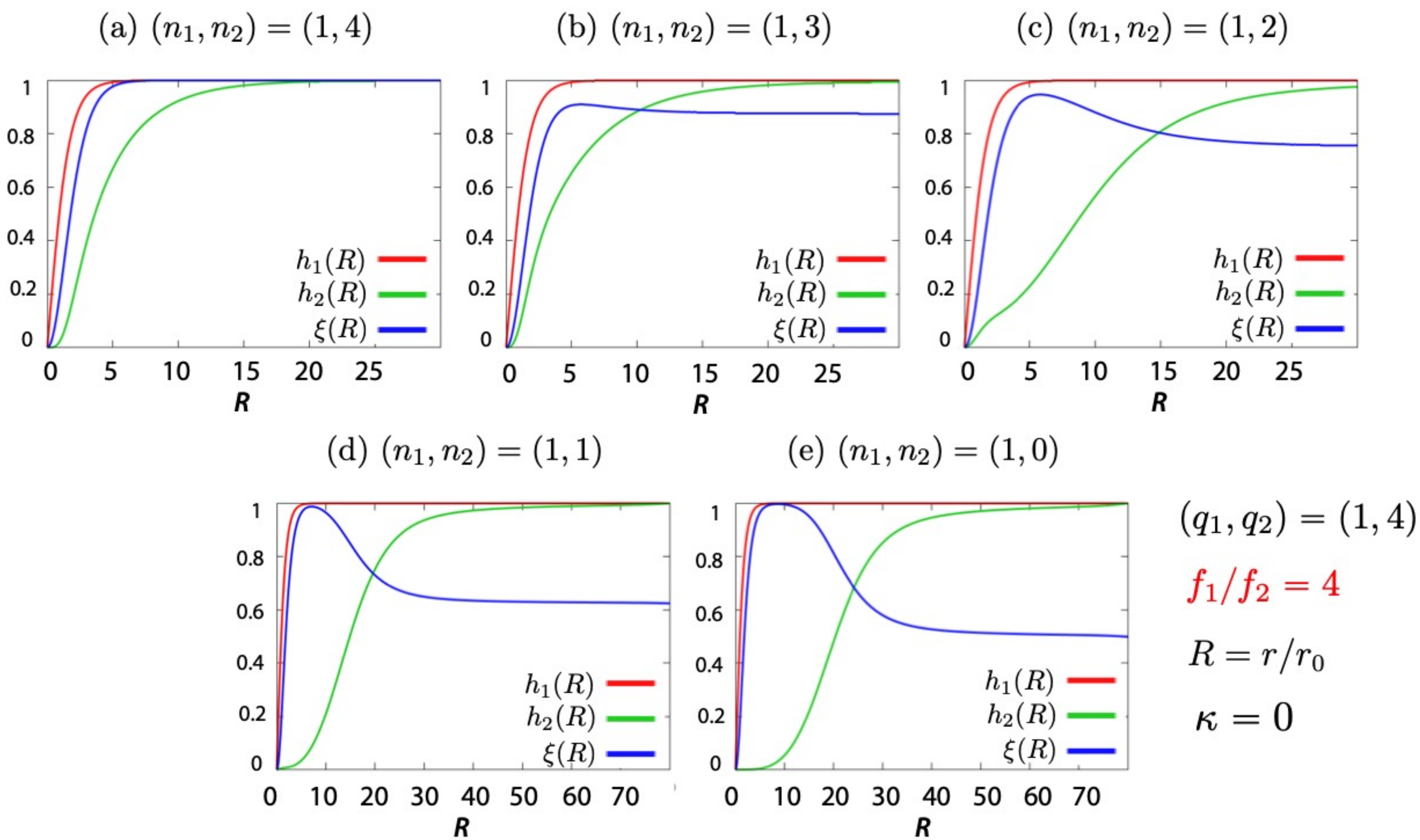
$R = r/r_0$

$\kappa = 0$

Static Solution

$$h_1(R\to\infty)=1 \qquad h_2(R\to\infty)=1 \qquad \xi(R\to\infty)=\frac{n_1q_1f_1^2+n_2q_2f_2^2}{q_1^2f_1^2+q_2^2f_2^2}$$

$$\mathscr{D}_\theta\phi_1 \to i\left(n_1-q_1\frac{n_1q_1\eta_1^2+n_2q_2\eta_2^2}{q_1^2\eta_1^2+q_2^2\eta_2^2}\right)\phi_1 \qquad \mathscr{D}_\theta\phi_2 \to i\left(n_2-q_2\frac{n_1q_1\eta_1^2+n_2q_2\eta_2^2}{q_1^2\eta_1^2+q_2^2\eta_2^2}\right)\phi_2$$

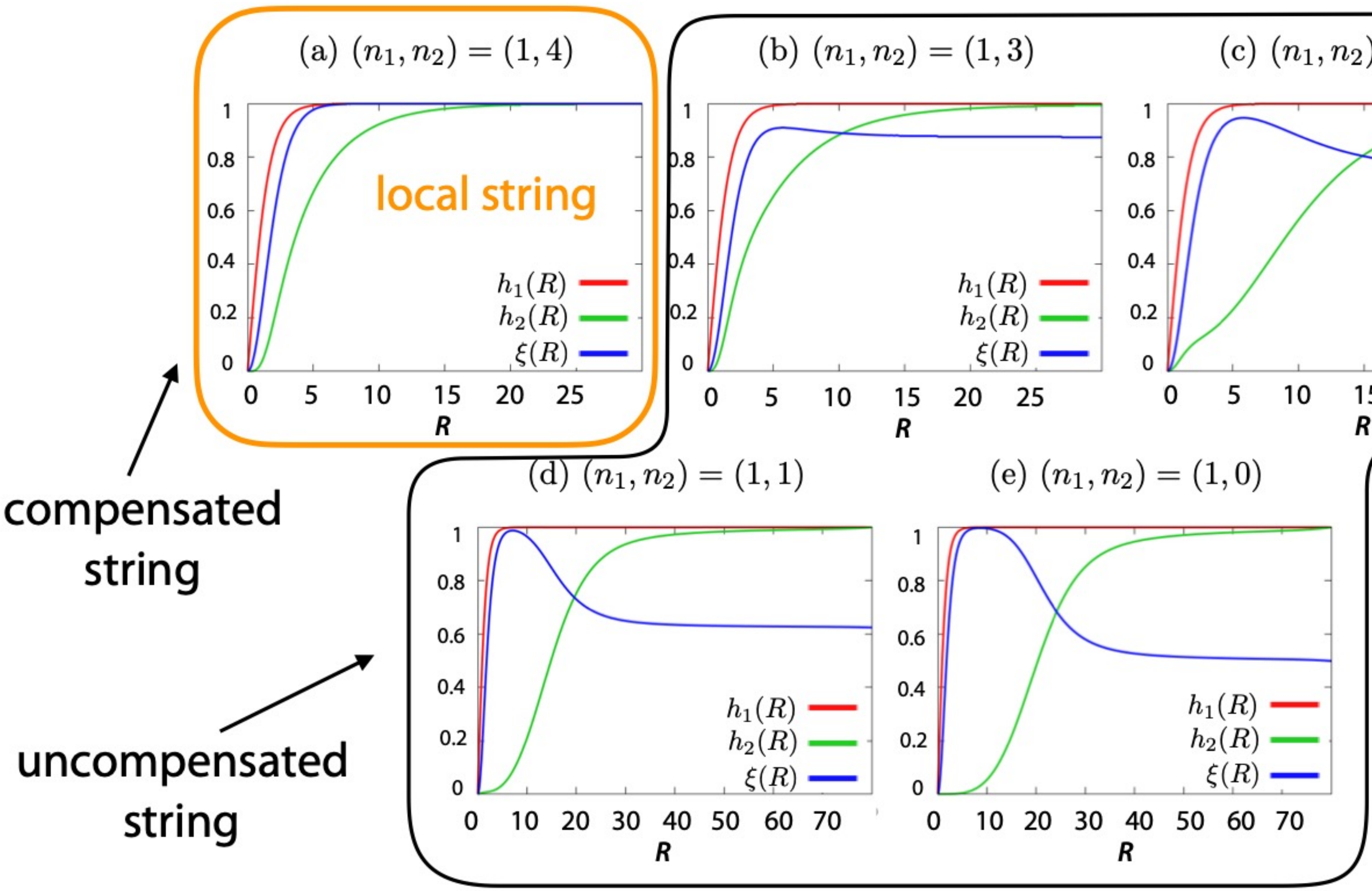


Static Solution

Both $\mathscr{D}_\theta \phi_1$ and $\mathscr{D}_\theta \phi_2$ vanish at $r \to \infty$ only for

$$n_1 = N_w \times q_1 \ , \quad n_2 = N_w \times q_2 \ , \quad N_w \in \mathbb{Z}$$

→ Finite tension string = local string



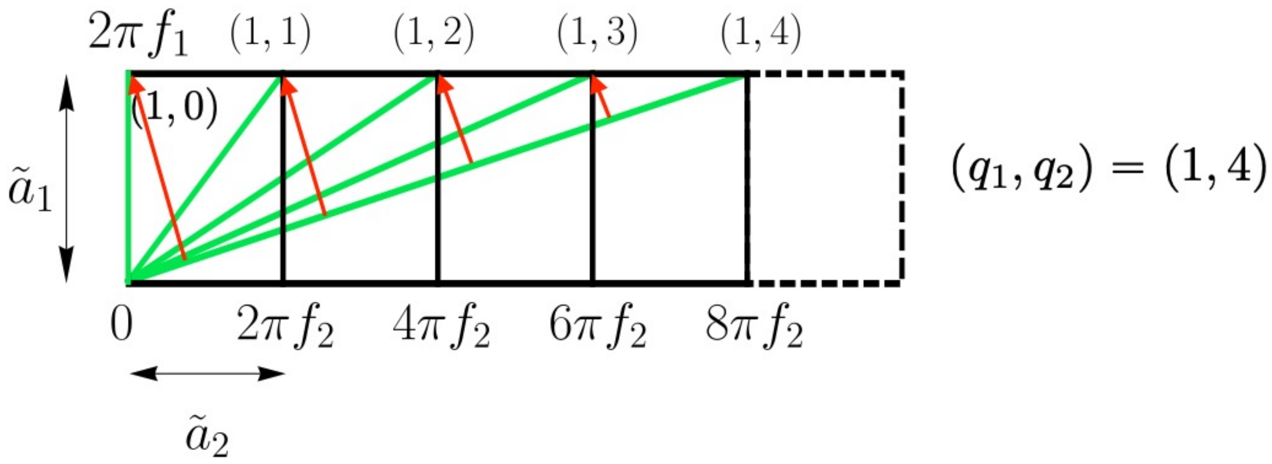
$(q_1, q_2) = (1, 4)$

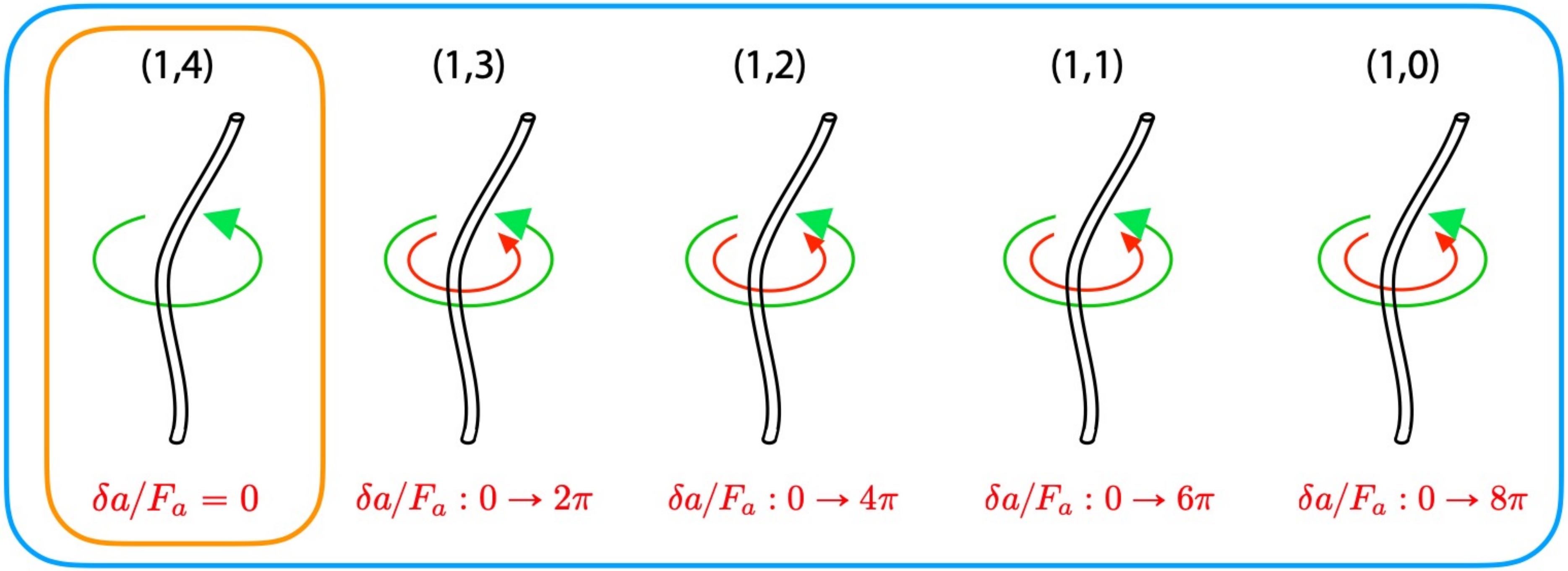
$f_1/f_2 = 4$

$R = r/r_0$

$\kappa = 0$

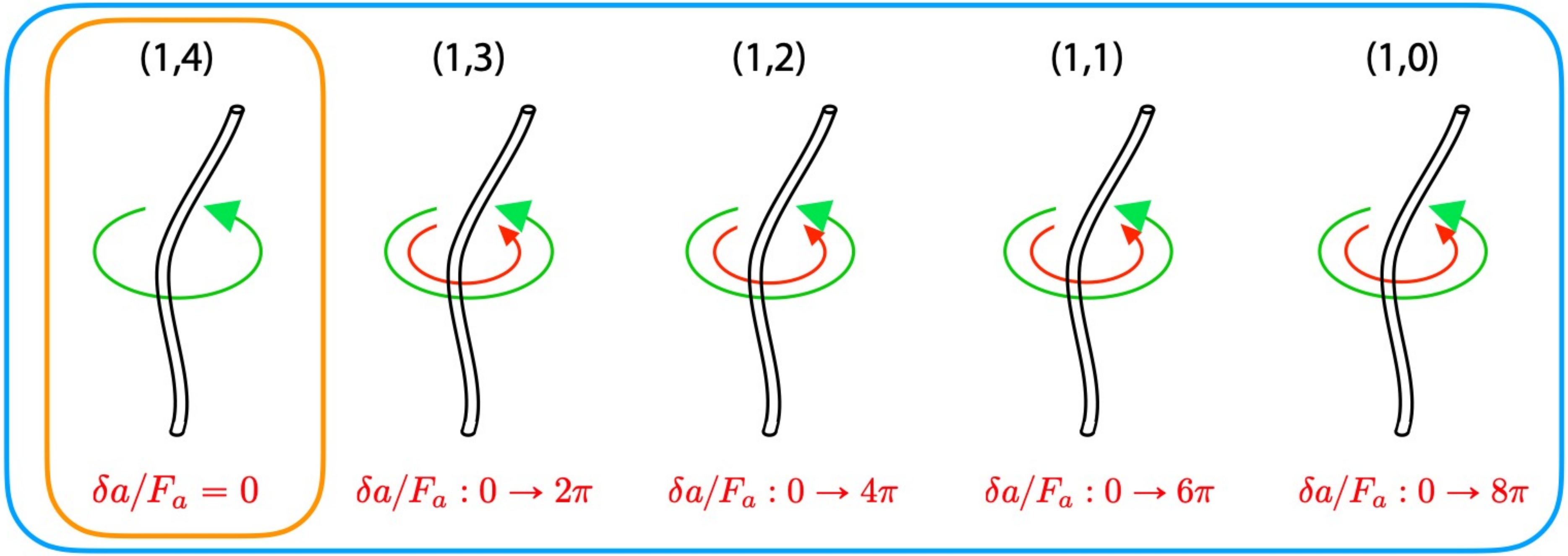
Axion windings





If only the compensated (local) string is formed, no axion domain wall appears !

Axion windings



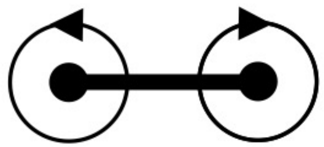
✔ Axion anomaly coupling

$$\mathcal{L} = y\phi_1 \underline{q_{1L}\bar{q}_{1R}} + y\phi_2^\dagger \underline{q_{2L}\bar{q}_{2R}} + h.c.$$

n (=4) flavors ***1*** flavors

$$\longrightarrow \mathcal{L}_{\rm QCD} = \frac{g_s^2}{32\pi}\left(\frac{q_2\tilde{a}_1}{f_1} - \frac{q_1\tilde{a}_2}{f_2}\right) G\tilde{G} = \frac{g_s^2}{32\pi}\frac{a}{F_a} G\tilde{G}$$

(***1***,***4***) strings and (***1***,***3***) strings do not cause the axion domain wall problem !

One axion winding is attached to just one wall : 

Formation of Cosmic Strings

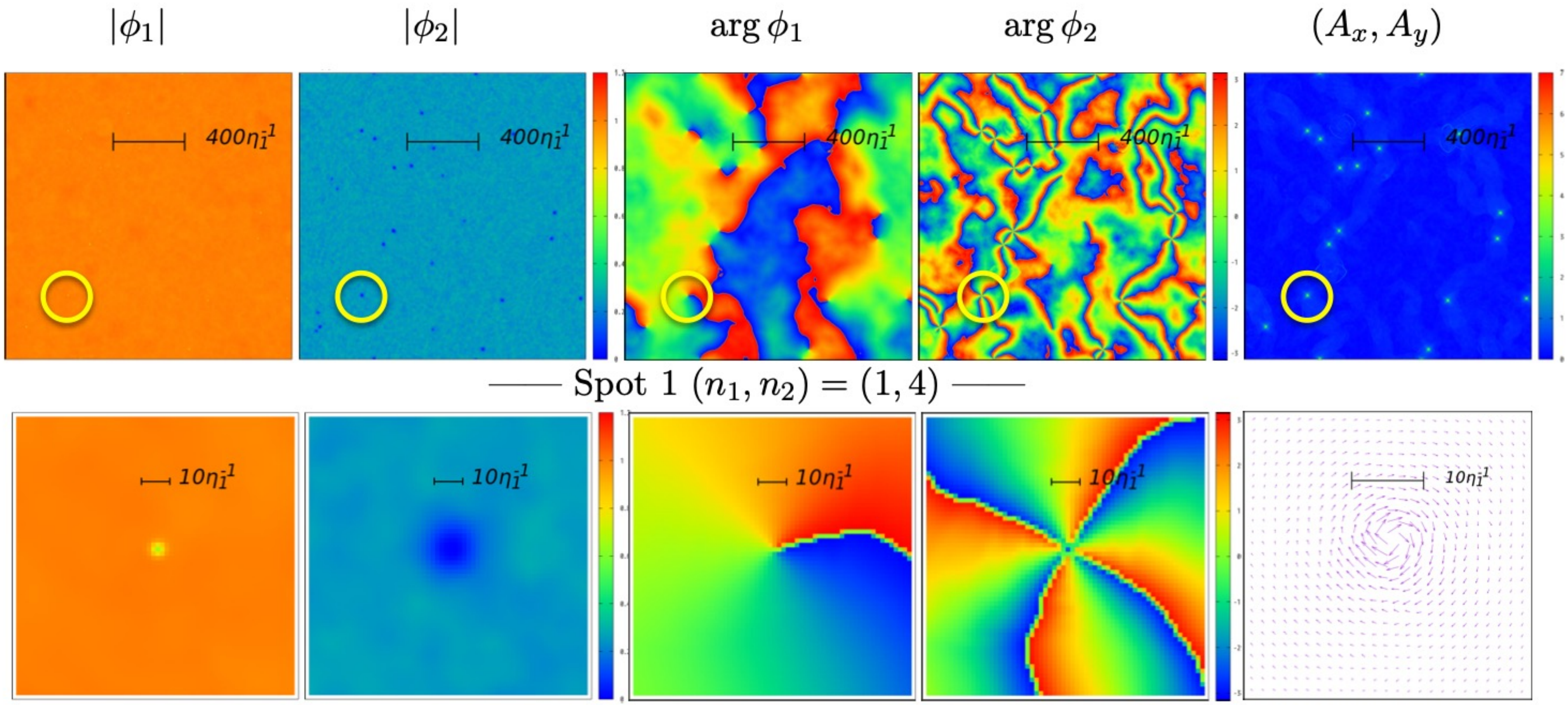
Classical lattice simulation
(translational invariance in z-direction)

$(q_1, q_2) = (1, 4) \qquad f_1/f_2 = 4$

Initial : $T_{in} = 3^{1/2}\eta_1$

$\phi_{1,2}, \dot{\phi}_{1,2}$: random fluctuation following the Planck distribution

Final : $T_{fin} = T_{in}/20$



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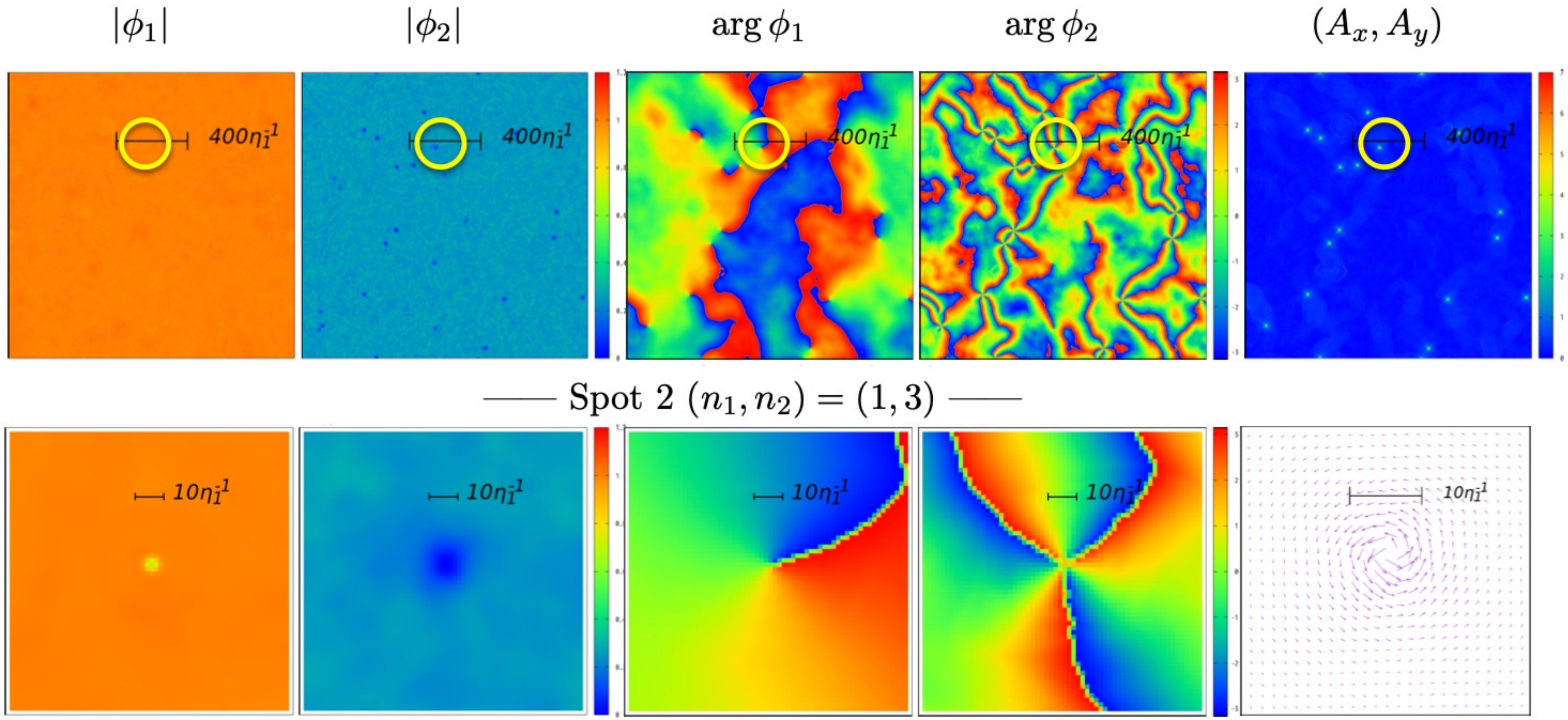

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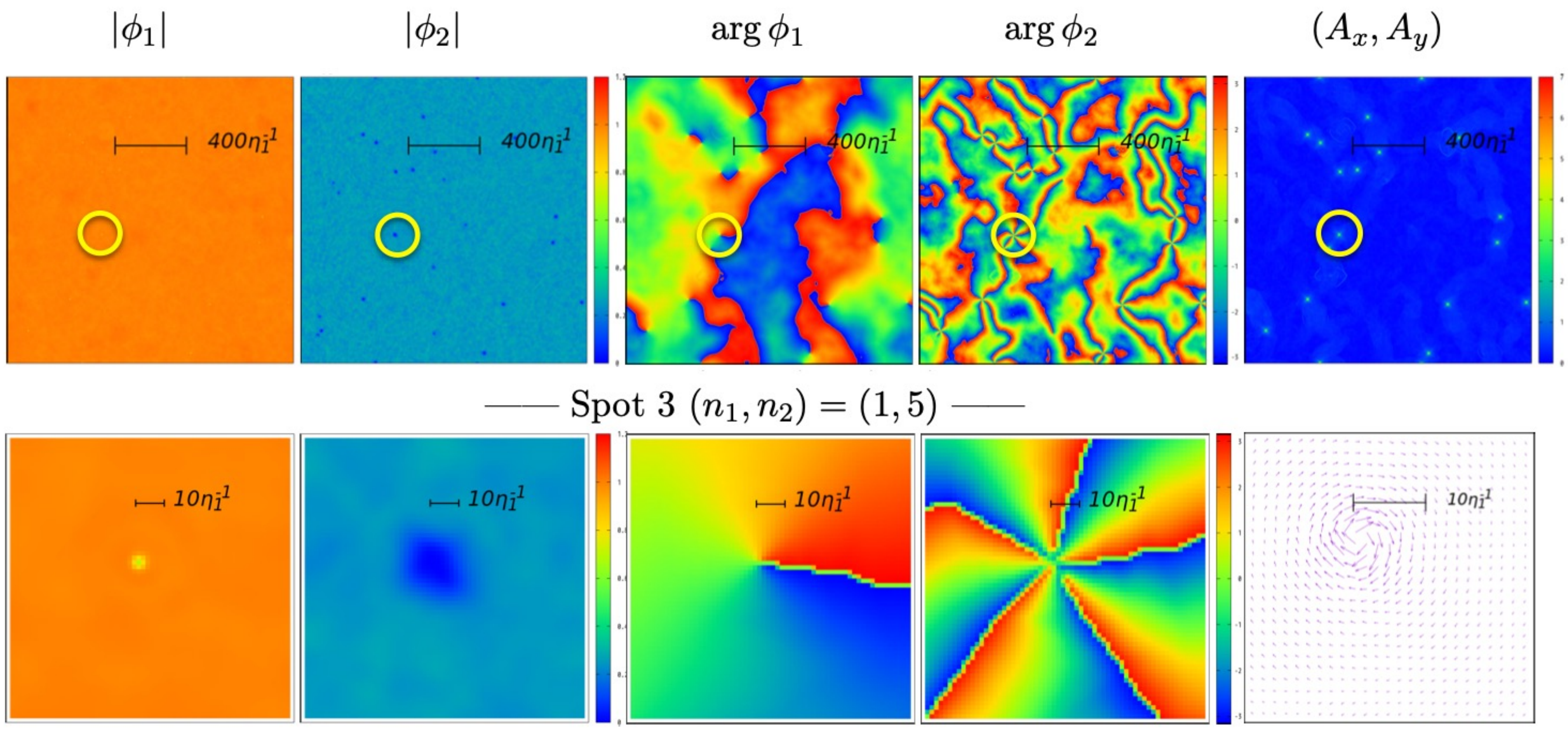

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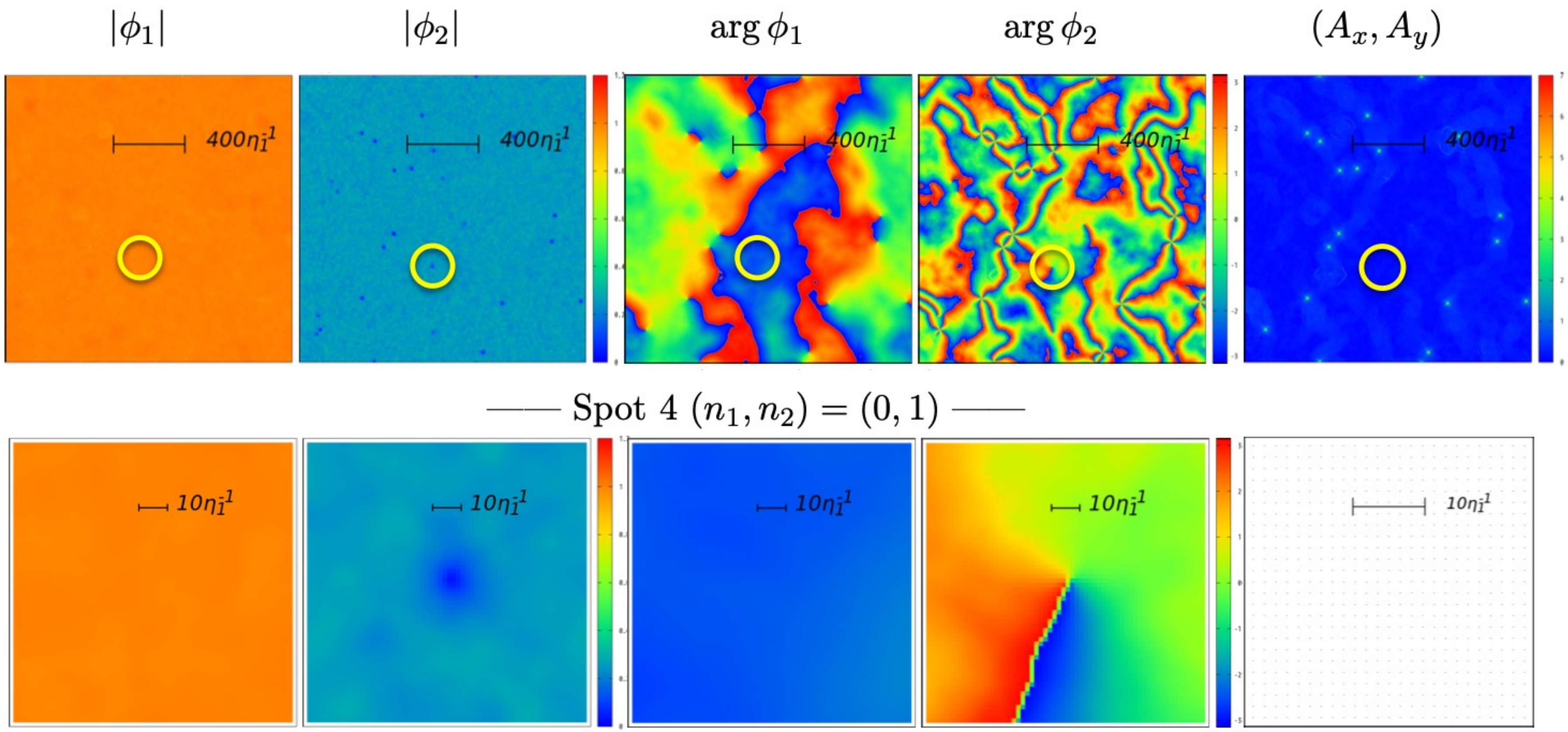


Formation of Cosmic Strings

For $(q_1, q_2) = (1,4)$ and $\eta_1 = 4\,\eta_2$

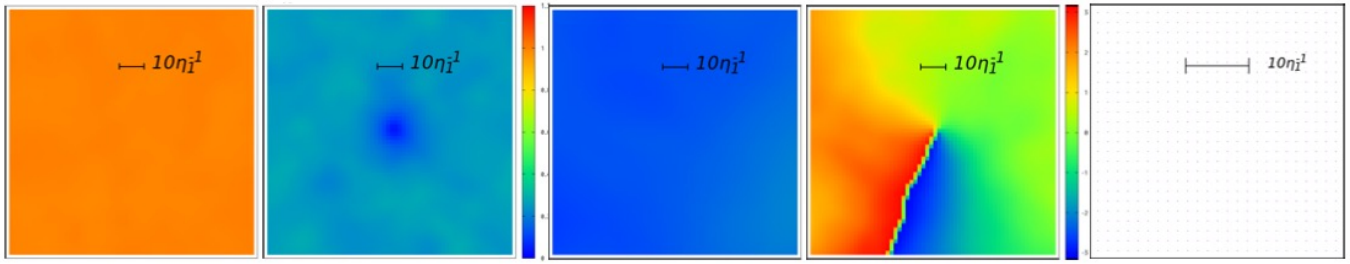
- ✔ Compensated strings (i.e. $(1,4)$ local strings) are formed.

  (No strings with $n_1 > 1$ is formed)

- ✔ Uncompensated strings (i.e. $(1, n_2 \neq 4)$ strings) are also formed.

- ✔ Global strings (i.e. $(0,1)$ strings) are also formed.

Spot 4 does not involve non-trivial A_θ configuration = global string



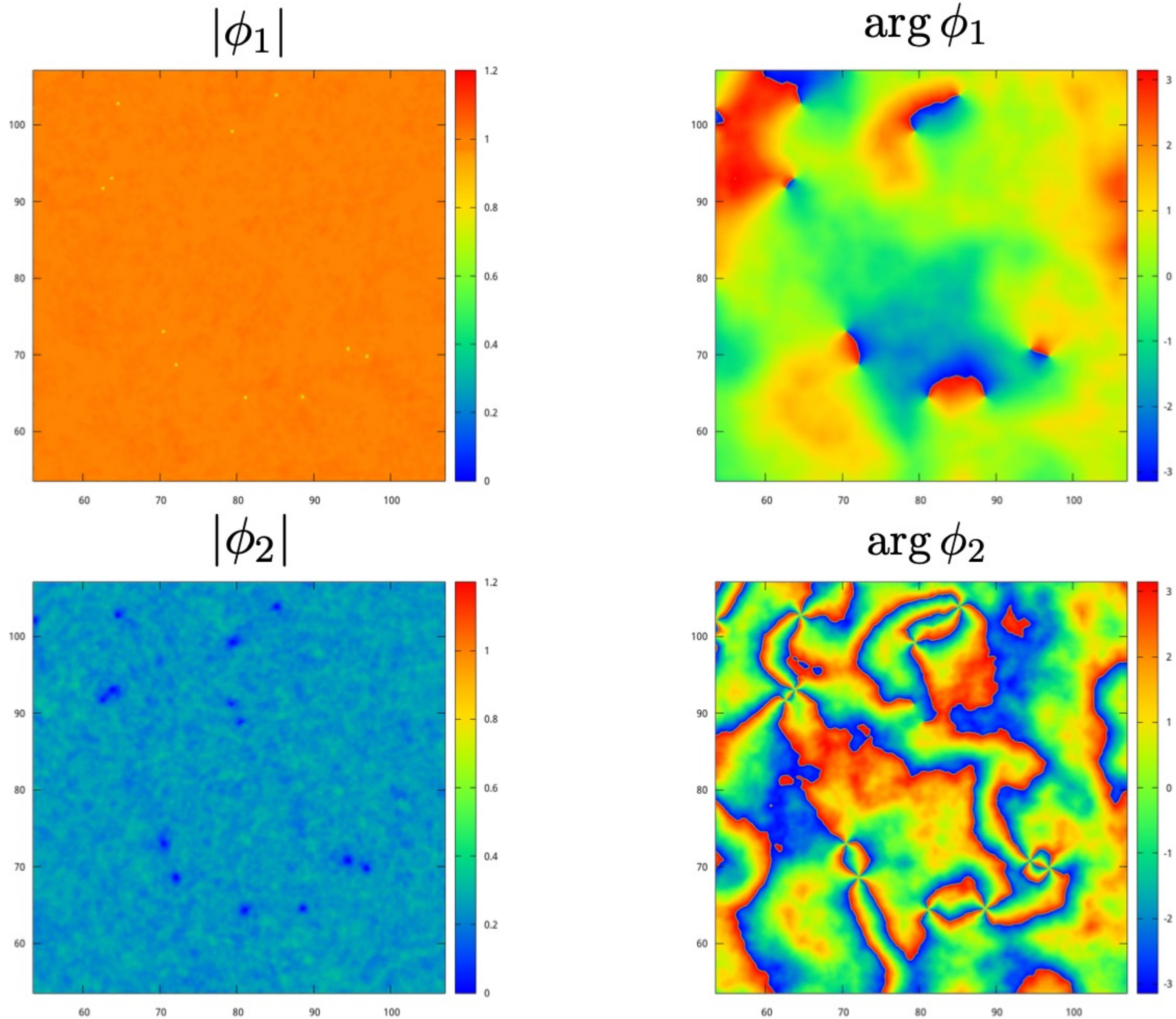
At around $T \sim \eta_1$, ϕ_1 obtains ***uniform*** $(n_1 = 0)$ VEV $\langle\phi_1\rangle = \eta_1$.

At around $T \sim \eta_2 = \eta_1/4$, ϕ_2 obtains VEV and forms in a global string.

At the later state, the gauge boson has been massive due to $\langle\phi_1\rangle = \eta_1$.

Evolution of Cosmic Strings

http://numerus.sakura.ne.jp/research/open/NewString/

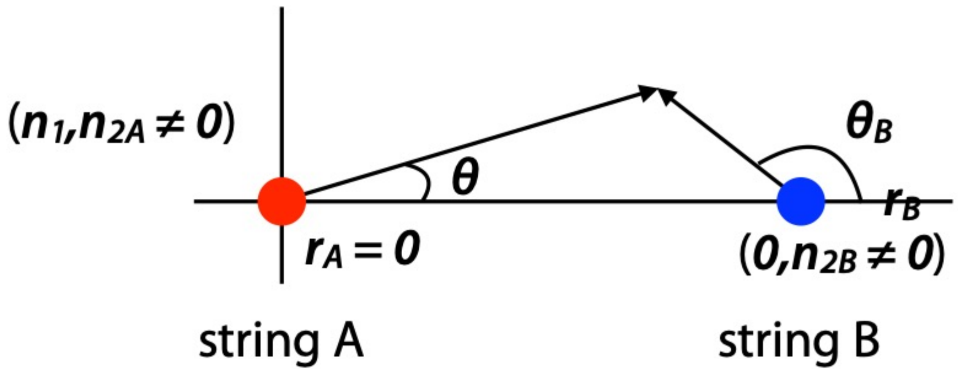


A global (***0,1***) string is absorbed by a (***1,3***) string and form a (***1,4***) string!

Evolution of Cosmic Strings

Long range force between an uncompensated string and a global string.

↔ logarithmic divergence of the string tension.



For $\eta_1 >> \eta_2$, the string A configurations of ϕ_1 and A_θ are barely affected by ϕ_2.

$$\phi_1(r,\theta) = \eta_1 e^{in_1\theta} h_1(r) \ , \quad A_\theta(r) = \frac{1}{e}\xi(r) \quad \xi(R\to\infty) = \frac{n_1 q_1 f_1^2 + n_2 q_2 f_2^2}{q_1^2 f_1^2 + q_2^2 f_2^2} \simeq n_1$$

The configuration of ϕ_2 can be approximated by,

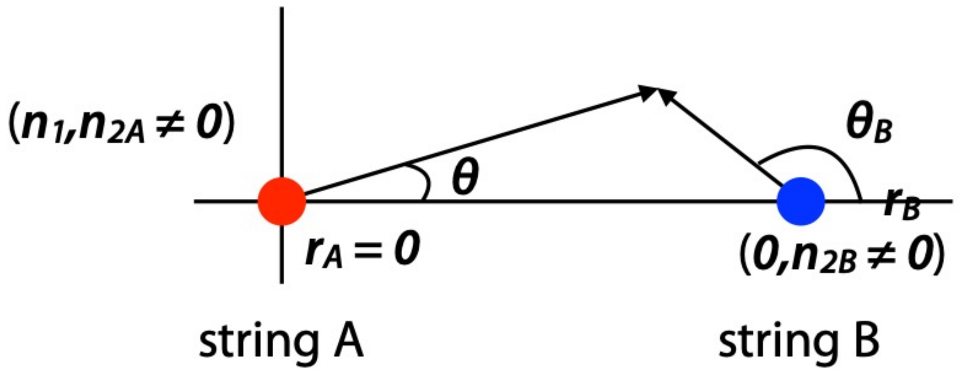
$$\phi_{2AB}(\mathbf{r}) \equiv \phi_2(\mathbf{r};\mathbf{r}_A,\mathbf{r}_B) = \frac{\phi_{2A}(\mathbf{r}-\mathbf{r}_A)\phi_{2B}(\mathbf{r}-\mathbf{r}_B)}{\eta_2}$$

$$\phi_{2AB}(\mathbf{r})|_{\mathbf{r}\sim\mathbf{r}_A} \sim \phi_{2A}(\mathbf{r}) \qquad \phi_{2AB}(\mathbf{r})|_{\mathbf{r}\sim\mathbf{r}_B} \sim \phi_{2B}(\mathbf{r}-\mathbf{r}_B)$$

Evolution of Cosmic Strings

Long range force between an uncompensated string and a global string.

$\leftrightarrow$ logarithmic divergence of the string tension.



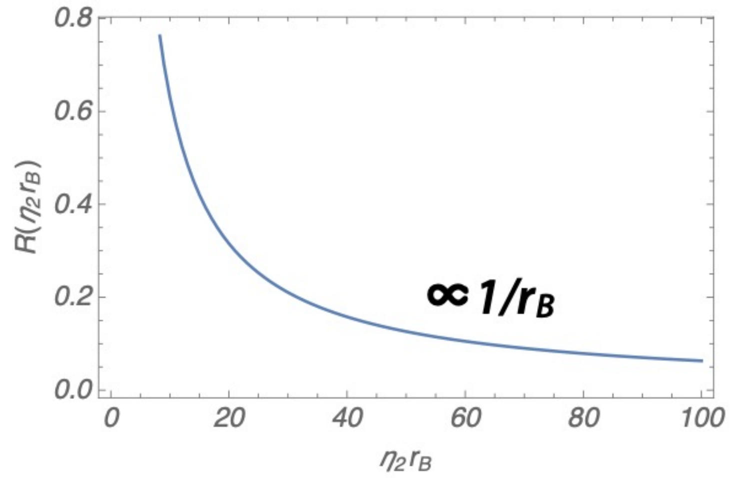
$$\Delta E(|\mathbf{r}_A - \mathbf{r}_B|) = E(\{\phi_{2AB}\}) - E(\{\phi_{2A}\}) - E(\{\phi_{2B}\})$$

$$E(\{\phi_2\}) = \int d^2\mathbf{r}\left[|\partial_r \phi_2|^2 + \frac{1}{r^2}|\partial_\theta \phi_2 - ieq_2 A_\theta \phi_2|^2 + \frac{\lambda_2}{4}\left(|\phi_2|^2 - \eta_2^2\right)^2\right]$$

$$\longrightarrow \quad \Delta E \simeq 2\, n_{2B}\left(n_{2A} - n_1 \frac{q_2}{q_1}\right)\eta_2^2 \int d^2\mathbf{r}\, \frac{1}{r^2}\frac{\partial \theta_B}{\partial \theta} \ .$$

$$\frac{\partial \theta_B}{\partial \theta} = \frac{r(r - |\mathbf{r}_A - \mathbf{r}_B|\cos\theta)}{|\mathbf{r} - \mathbf{r}_B|^2}$$

Evolution of Cosmic Strings



Long range force between an uncompensated string and a global string :

$$\mathbf{F}(\mathbf{r}_B) = n_{2B}\left(n_{2A} - n_q \frac{q_2}{q_1}\right)\eta_2^2 R(\eta_2 r_B)\mathbf{e}_{r_B}$$

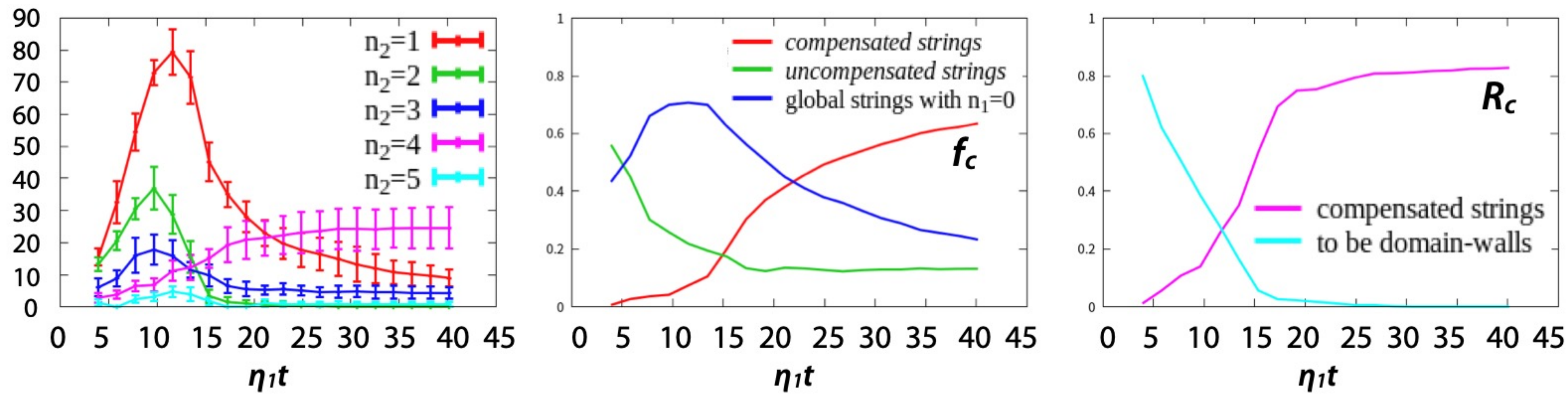
$$n_{2B}\left(n_{2A} - n_1 \frac{q_2}{q_1}\right) < 0$$ attractive

$$n_{2B}\left(n_{2A} - n_1 \frac{q_2}{q_1}\right) > 0$$ repulsive

Compensated string ($q_1 n_2 - q_2 n_1 = 0$) does not have $1/r_B$ force !

Evolution of Cosmic Strings $(q_1,q_2)=(1,4)$ $f_1/f_2=4$

(a) Number of strings (b) f_c, f_u and f_g (c) R_c, R_{dw}



Most (but not all) strings are combined into the compensated strings.

$$f_c = \frac{N_c}{N_c + N_u + N_{\rm global}} \to 0.6 \qquad R_c = \frac{N_c}{N_c + N_u} \to 0.8$$

For $(q_1,q_2)=(1,4)$ and for $f_1/f_2=4$ Remaining strings are

compensated $(1,4)$ string ~60% : $\delta a/F_a=0$

uncompensated $(1,3)$ string ~23% : $\delta a/F_a=0\text{-}2\pi$

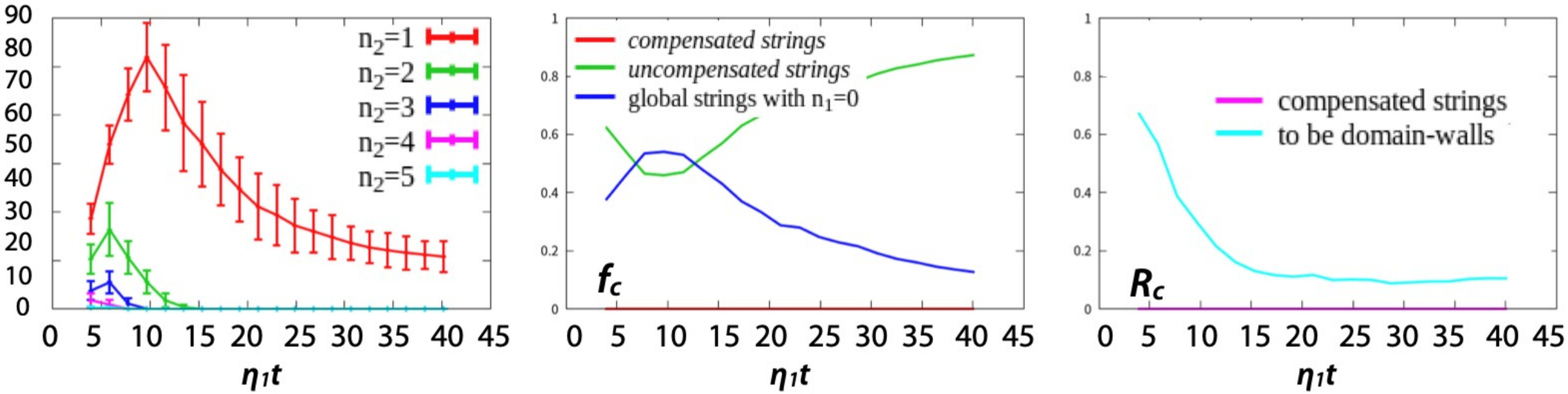
global $(0,1)$ string ~17% : $\delta a/F_a=0\text{-}2\pi$

→ these strings do not cause domain wall problem!

Domain wall problems are solved? 3D simulation is important.

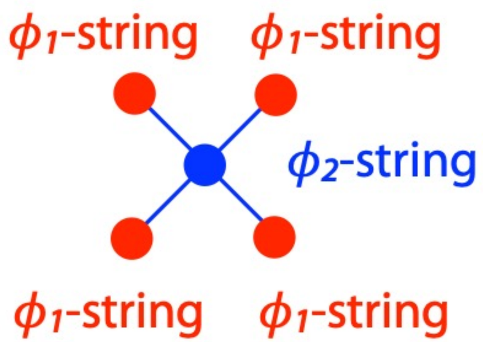
Evolution of Cosmic Strings $(q_1,q_2)=(4,1)$ $f_1/f_2=4$

(a) Number of strings (b) f_c, f_u and f_g (c) R_c, R_{dw}



For $(q_1,q_2)=(4,1)$ and for $f_1/f_2=4$, it is difficult to form compensated string.

$$f_c = \frac{N_c}{N_c + N_u + N_{\rm global}} \to 0 \qquad R_c = \frac{N_c}{N_c + N_u} \to 0$$



The compensated string consists of 4 windings of ϕ_1-string and 1 winding of ϕ_2-string.

For $f_1/f_2=4$, [# of ϕ_1-strings] > [# of ϕ_2-strings]

→ it is difficult to form compensated strings

This set up suffers from the axion domain wall problem.

Summary

✔ $U(1)_{local}$ gauge theory with a large charge hierarchy leads to an approximate global $U(1)$ symmetry appropriate for the PQ symmetry.

✔ Many types of cosmic strings are formed.

✔ For $(q_1, q_2)=(1,4)$ and for $f_1/f_2 = 4$, most of the strings are combined into the compensated string.

✔ Around the remaining strings in 2D simulation, the axion winds at most just once.

The axion domain wall problem might not occur?

→ 3D simulation is important ! (in preparation)

✔ For $(q_1, q_2)=(4,1)$ and for $f_1/f_2 = 4$, the compensated strings are rarely formed.

The axion winds more than once around the remaining string

The axion domain wall problem is unavoidable.

~ similar to Z_n model.

Backup Slides

Equations of Motion

$$h_1''(R) + \frac{h_1'(R)}{R} - \beta_1 h_1(R)^3 + \left(\beta_1 - \gamma_2\left(1 - h_2(R)^2\right) - \frac{n_1^2}{R^2}\left(1 - \frac{q_1}{n_1}\xi(R)\right)^2\right) h_1(R) = 0$$

$$h_2''(R) + \frac{h_2'(R)}{R} - \beta_2 h_2(R)^3 + \left(\beta_2 - \gamma_1\left(1 - h_1(R)^2\right) - \frac{n_2^2}{R^2}\left(1 - \frac{q_2}{n_2}\xi(R)\right)^2\right) h_2(R) = 0$$

$$\xi''(R) - \frac{\xi'(R)}{R} - 2c_1\left(\xi(R) - \frac{n_1}{q_1}\right) h_1(R)^2 - 2c_2\left(\xi(R) - \frac{n_2}{q_2}\right) h_2(R)^2 = 0$$

$$r_0 = \frac{1}{e\sqrt{q_1^2\eta_1^2 + q_1^2\eta_2^2}} \qquad \beta_n = \frac{\lambda_n \eta_n^2}{2e^2\left(q_1^2\eta_1^2 + q_2^2\eta_2^2\right)} \,, \quad \gamma_n = \frac{\kappa\, \eta_n^2}{e^2\left(q_1^2\eta_1^2 + q_2^2\eta_2^2\right)} \,, \quad c_n = \frac{q_n^2\eta_n^2}{q_1^2\eta_1^2 + q_2^2\eta_2^2} \,,$$

Boundary conditions

$h_1(R) = 0\ ,\ h_2(R) = 0\ ,\ \xi(R) = 0\ ,$ ***$n_{1,2} \neq 0$*** ***For $n_{1,2} = 0$, $h_{1,2}(R=0)$: Neumann***

$h_1(R) = 1\ ,\ h_2(R) = 1\ ,$

$U(1)_{PQ}$ Quality & Domain Wall Problem

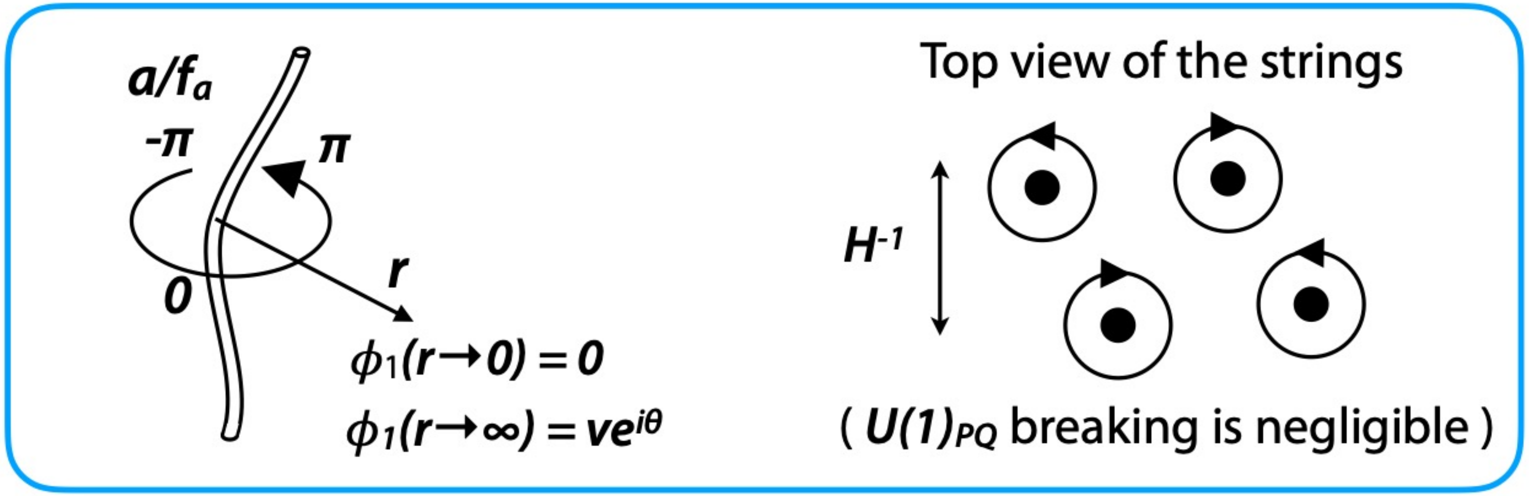
✔ Axion domain wall problem in a Z_n gauge symmetric model

Consider Z_n gauge symmetry obtained from $U(1)_{local}$ symmetry,

$$\phi_1(q_1) \qquad \phi_2(q_2) \qquad q_1 = 1, \quad q_2 = n$$

by taking $\langle\phi_2\rangle \to \infty$ with $\langle\phi_2\rangle/M_*$ kept tiny.

(1) When ϕ_1 obtains VEV after inflation, the ***approximate*** $U(1)_{PQ}$ is spontaneously broken, and a few cosmic strings are formed in one Hubble volume.



(2) The number of the strings in a Hubble volume is kept $O(1)$ in time evolution due to reconnection → the scaling solution : $\rho_{str} \propto H^2$

[1986 Davis, 1989 Davis&Shellard, see also 2018 Kawasaki et.al.]

$U(1)_{PQ}$ Quality & Domain Wall Problem

✓ Axion domain wall problem in Z_n gauge symmetric model

(3) At around the ***QCD*** scale, the axion feels its axion potential.

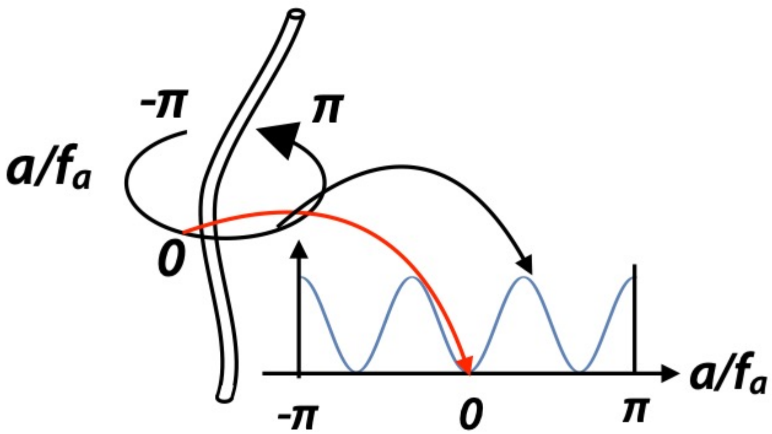
$$\mathcal{L} = \frac{g_s^2}{32\pi^2}\frac{N_f a}{f_a} G^{\mu\nu}\tilde{G}_{\mu\nu}$$

Anomaly free Z_n requires $N_f = kn$
see e.g. [1997, Csaki & Murayama]

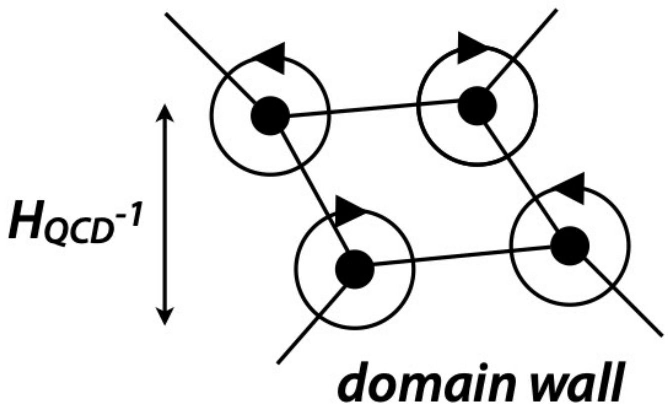
The axion potential potential has N_f periodicity in $a/f_a = [-\pi, \pi)$.

Non-trivial axion field values around the strings causes energy contrast

ex) $n = N_f = 3$



n-walls are attached



String-wall network immediately dominates the Universe

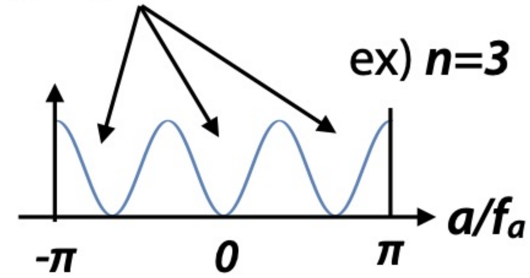
→***Z_n model suffers from domain wall problem***

Axion Quality & Domain Wall Problem

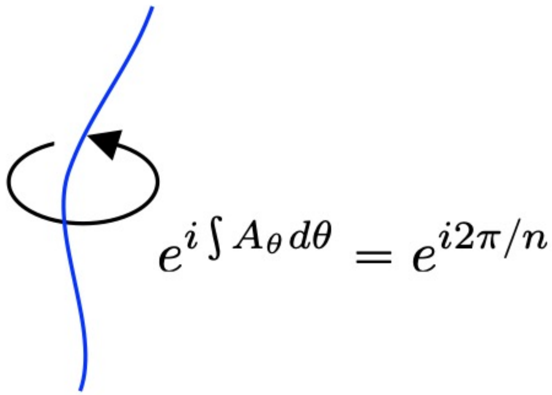
✔ Axion Domain wall problem in Z_n gauge symmetric model

Is the wall-string network stable ?

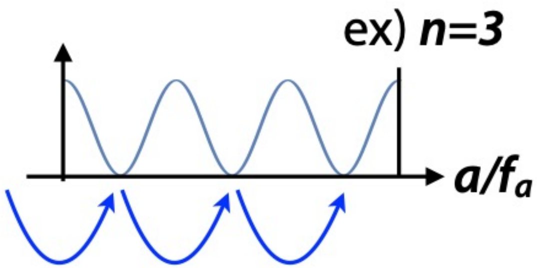
n-domains are Z_n equivalent



For $\langle\phi_2\rangle$ = finite but $\gg \langle\phi_1\rangle$, ϕ_2-string has a finite tension,



The phase of ϕ_1 changes by $e^{i2\pi/n}$ due to the Aharonov-Bohm effect

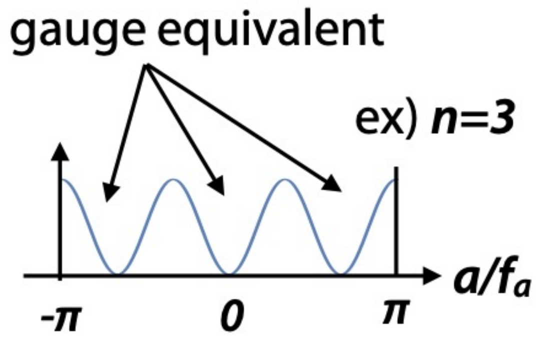


Axion Quality & Domain Wall Problem

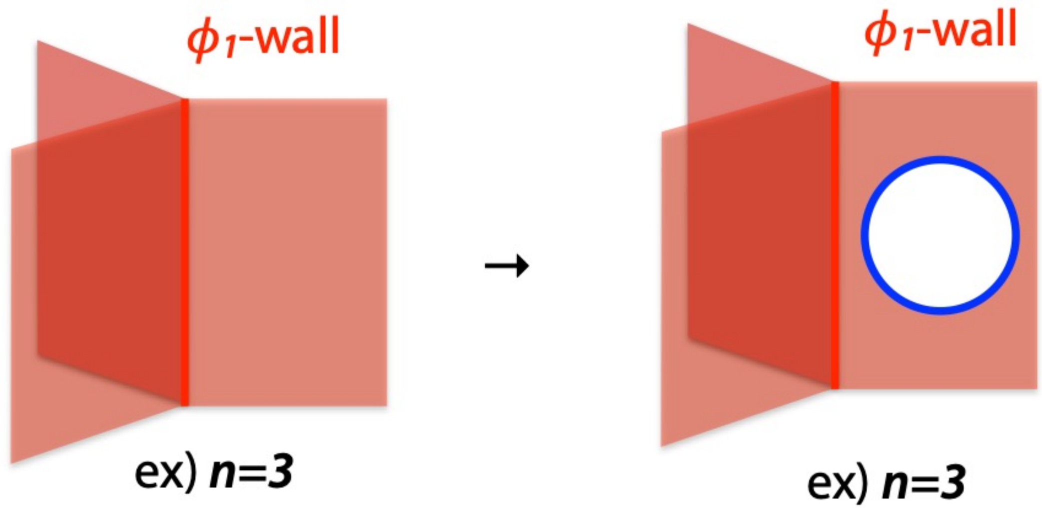
✔ Axion Domain wall problem in Z_n gauge symmetric model

Is the wall-string network stable ?

n-domains are Z_n equivalent



ϕ_1-wall can be pierced by a ϕ_2-string loop



The tunneling rate is quite low… $\Gamma \propto Exp[\,-f_2{}^4/(\Lambda_{QCD}{}^2 F_a T)\,]$

[e.g. 1982, Kibble, Lazarides, Shafi]

Lazarides-Shafi Model & Domain Wall Problem

Lazarides-Shafi Model ~ $U(1)_{PQ} \times G$ [1982 Lazarides & Shafi]

Assume $U(1)_{PQ}$ symmetry which is broken down to Z_N symmetry by QCD anomaly

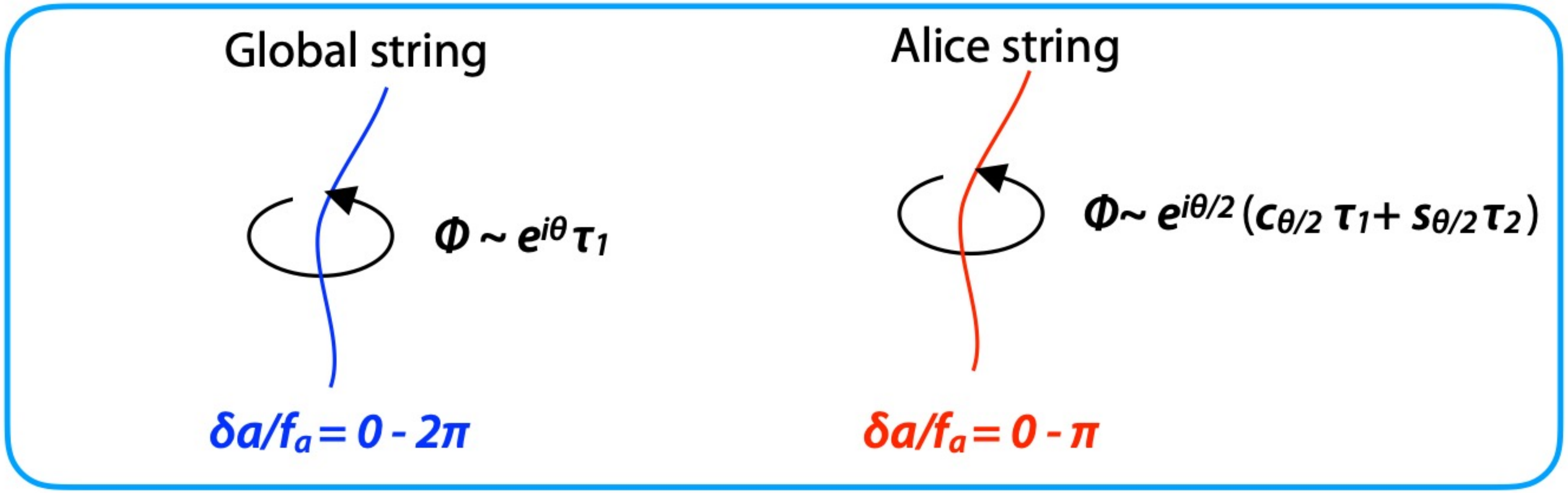
$$\mathcal{L} = \frac{g_s^2}{32\pi^2}\frac{Na}{f_a}G^{\mu\nu}\tilde{G}_{\mu\nu}$$

✔ Choose G symmetry whose center is also Z_n

ex) For $N = 2$, we may take $G = SU(2)$ (the original model is based on $SO(10)$)

Φ : $SU(2)$ triplet complex scalar $U(1)_{PQ}$: $\Phi \to e^{i\alpha}\Phi$

✔ $U(1)_{PQ} \times SU(2)$ is simultaneously broken by a VEV of Φ, $\langle\Phi\rangle \propto \tau_1$



(Both string tensions have logarithmically divergence.)

Lazarides-Shafi Model & Domain Wall Problem

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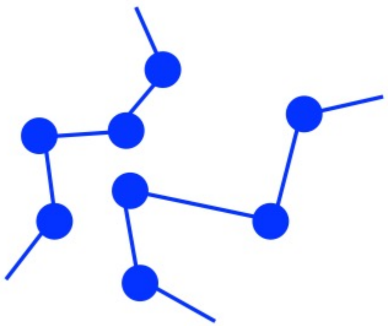
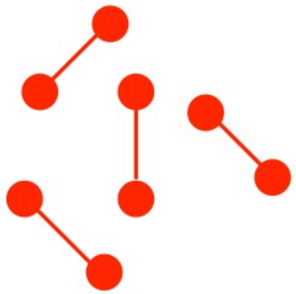
If both strings remain in the Universe until QCD scale, the global string causes the axion domain wall problem, while the Alice string does not.



Global string-wall network



Alice string-wall network

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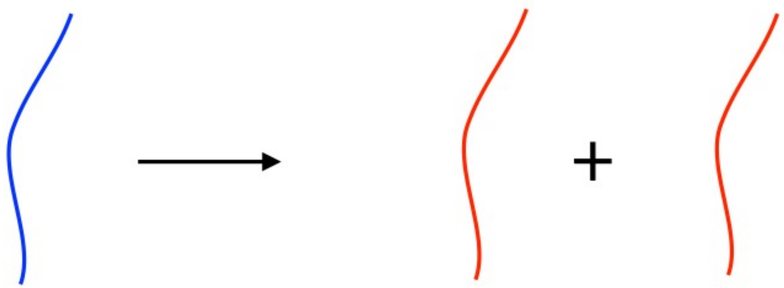
✔ $U(1)_{PQ} \times SU(2)$ is simultaneously broken by a VEV of Φ, $\langle\Phi\rangle \propto \tau_1$

Fortunately, the global string in this model breaks up into Alice strings well above the QCD scale → No domain wall problem.

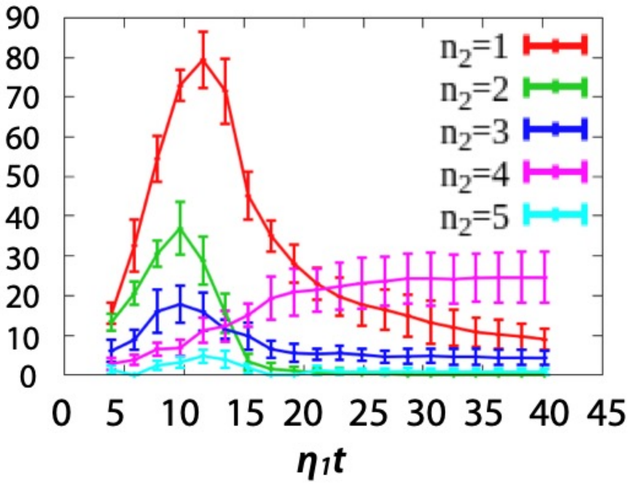


see also [2019 Chatterjee, Higaki, Nitta]

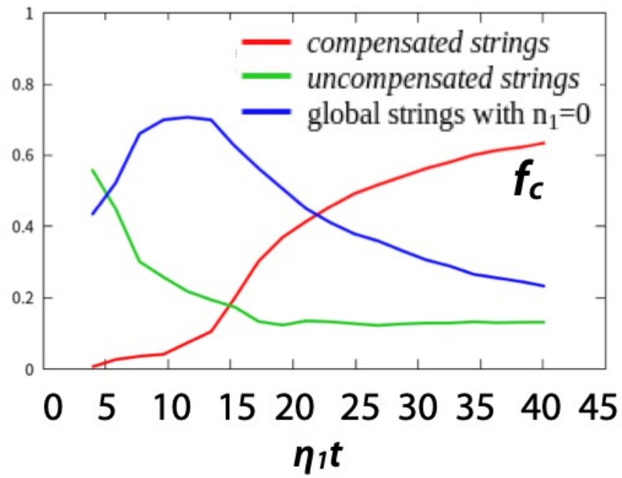
High quality accidental $U(1)_{PQ}$ seems difficult in the Lazarides-Shafi model.

Evolution of Cosmic Strings $(q_1,q_2)=(1,4)$ $f_1/f_2=4$

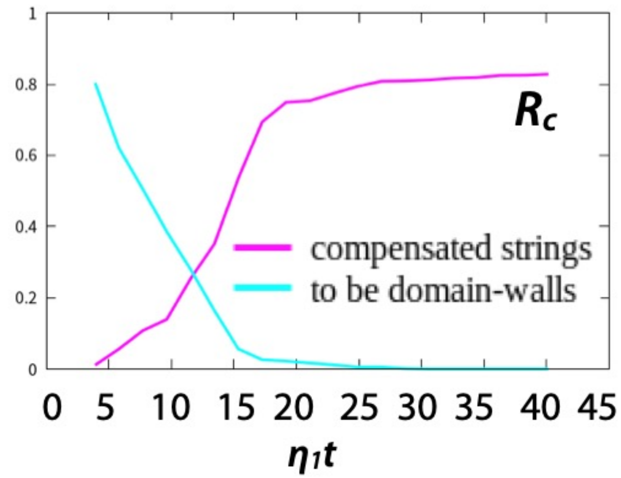
(a) Number of strings



(b) f_c, f_u and f_g



(c) R_c, R_{dw}



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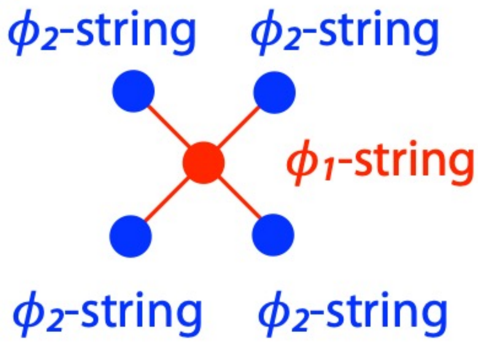

The compensated string consists of 1 winding of ϕ_1-string and 4 windings of ϕ_2-string.

For $f_1/f_2=4$, [# of ϕ_1-strings] > [# of ϕ_2-strings]

→ it is easy to form compensated strings

Domain wall problems might be solved? 3D simulation is important.