2020/07 McKay Correspondence, Mutation and Related Topics

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The motivic McKay Correspondence in arbitrary characteristics

Takehiko Yasuda (Osaka Univ.)

/ $\mathbb{C}$

$$\begin{cases} G \subset SL_d(\mathbb{C}) \text{ finite subgp} \\ X = \mathbb{C}^d/G \\ Y \to X : \text{a crepant resol.} \end{cases}$$

$$\Rightarrow \quad [Y] = \sum_{[g] \in \mathrm{Conj}(G)} \mathbb{L}^{d-\mathrm{age}(g)} \qquad \text{[Batyrev, Denef-Loeser]}$$

the e_{top} realization

In particular,

$$e_{top}(Y) = \# \mathrm{Conj}(G)$$

More generally, with or without a crep. res.,

$$M_{st}(X) = \sum_{[g] \in \mathrm{Conj}(G)} \mathbb{L}^{d-\mathrm{age}(g)} \qquad \text{[B, D-L]}$$

(holds also for a small subgp $G < GL_d(\mathbb{C})$ [Y])

($\nexists$ pseudo-reflection)

/ k (k: $\forall$ field)

$G \subset GL_d(k)$: small subgp

/ k (k : field)

$G \subset GL_d(k)$: small subgp

$X = \mathbb{A}^d_k / G$

$$\Rightarrow \quad M_{st}(X) = \int_{\Delta_G} \mathbb{L}^{d-v} \qquad [Y, '19]$$

Δ_G : the moduli sp. of G-torsors over Spec $k((t))$

$v : \Delta_G \longrightarrow \frac{1}{\# G}\mathbb{Z}$

Rem
- The point-counting version holds also in mixed char. (e.g. $/\mathbb{Z}_p$)
- $e_{l\text{-adic}}(Y) = \# \mathrm{Conj}(G)$ does not hold. [Yamamoto]
- The above result is a special case of :
  $(\mathcal{X}, D), (\mathcal{Y}, E)$: K-equiv. stacky log pairs
  $\Rightarrow M_{st}(\mathcal{X}, D) = M_{st}(\mathcal{Y}, D)$

Proof Uses the motivic integration over Deligne-Mumford stacks.

twisted arcs $\quad \mathcal{D} \longrightarrow \mathcal{X}$

tw. formal disk (stacky ver. of Spec $k[[t]]$) $\qquad$ target DM stack

$\{$ twisted formal disks $\} \longleftrightarrow \{$ Gal. exts of $k((t))$ $\}$

Arithmetics of local fields

An important case $\quad S_n \curvearrowright \mathbb{A}^{2n}_k = (\mathbb{A}^2_k)^n$

$$X = \mathbb{A}^{2n}_k / S_n = S^n \mathbb{A}^2_k$$

$$\rightsquigarrow \quad v = a : \Delta_{S_n} \to \mathbb{Z}$$

Artin conductor

$\exists$ crep. resol. $\quad \mathrm{Hilb}^n(\mathbb{A}^2_k) \longrightarrow X$

$$\Rightarrow \quad [\mathrm{Hilb}^n(\mathbb{A}^2_k)] = \int_{\Delta_{S_n}} \mathbb{L}^{2n-a}$$

Cell decomp. $\rightsquigarrow$ $\|$ $\qquad$ $/\!/$ $\leftarrow$ the motivic ver. of Bhargava's mass formula

$$\sum_{i=0}^{n-1} \underline{P(n, n-i)} \, \mathbb{L}^{2n-i}$$

\# of partitions of n into exactly $n-i$ parts

An application to singularities

In char. 0, quotient sing's are log terminal (also called KLT).

Reid-Shepherd-Barron-Tai criterion:

determines when they happen to be in classes of milder sing's (terminal or canonical)

$\downarrow$ refinement & generalization

<u>Cor</u> $\quad X = \mathbb{A}^d_k / G$: as before

$$\mathrm{discrep}(\text{centers} \subset X_{sing}; X)$$
$$= d - 1 - \max\left\{ \dim X_{sing},\ \dim \int_{\Delta_G \setminus \{\text{triv. torsor}\}} \mathbb{L}^{d-v} \right\}$$

<u>Rem</u> For $G = \mathbb{Z}/p\mathbb{Z}$ [Y], $\mathbb{Z}/p^2\mathbb{Z}$ [Tanno-Y],
$\exists$ more explicit criteria.

Can you solve the following problem?

<u>Problem</u> Generalize the derived McKay corresp. to arbitratry characteristics, in particular, the wild case.

Some sort of infinities would appear.

Evidences:
- The ∞-dim'l sp. Δ_G replaces the fin. set $\mathrm{Conj}(G)$.
- $\mathrm{gl.dim}\, k[\underline{x}] * G = \infty$