Hypersurface symplectic singularities

Quivers — Higgs branch — Coulomb branch

Amihay Hanany July 2020

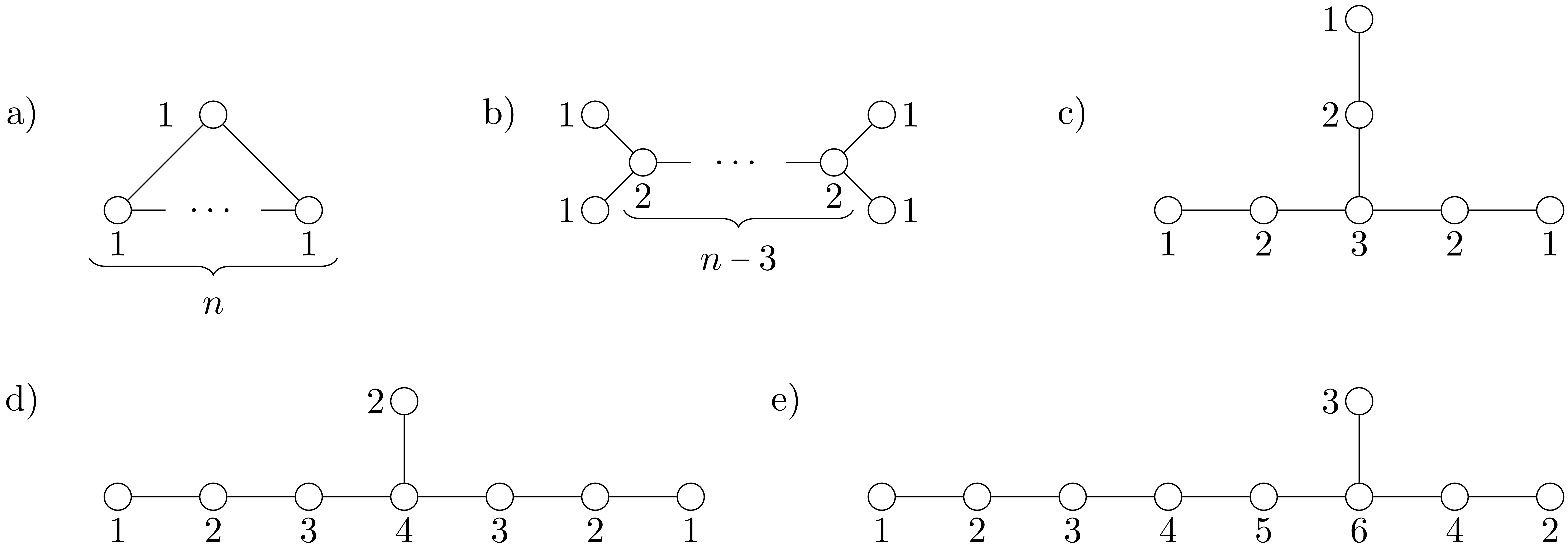
Quivers and symplectic singularities

Higgs branch — Coulomb branch

- In this talk we will look at moduli spaces which can be constructed from quivers similar to those which appear in the McKay correspondence
- Given a quiver, there are two types of moduli spaces (symplectic singularities)
- We will use here the physics terminology
- Higgs branch — these are known as Nakajima quiver varieties and are well studied
- Coulomb branch — these are relatively new objects (~90’s) and produce very interesting moduli spaces

Affine ADE quivers

Higgs branch — Klein singularity. Coulomb branch — minimal nilpotent orbit



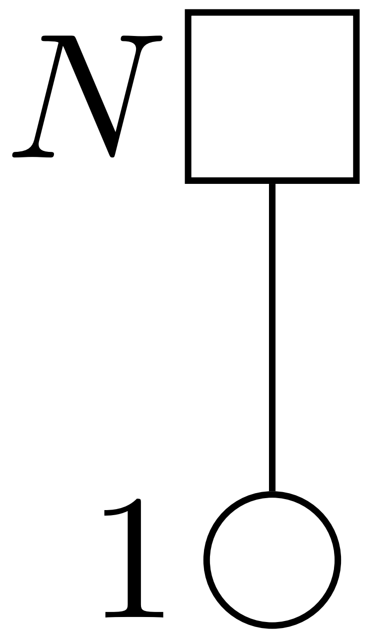
Coulomb branch

Recall the Oct 2014 McKay correspondence meeting

- In my talk at this meeting, I presented the formula
- So called “monopole formula”
- Computes the Hilbert Series of the Coulomb branch
- Extracted out of the combinatorial data of the quiver
- Progress in understanding of symplectic singularities
- An explicit evaluation for many moduli spaces
- Construction of known and also of new (previusly unknown) moduli spaces

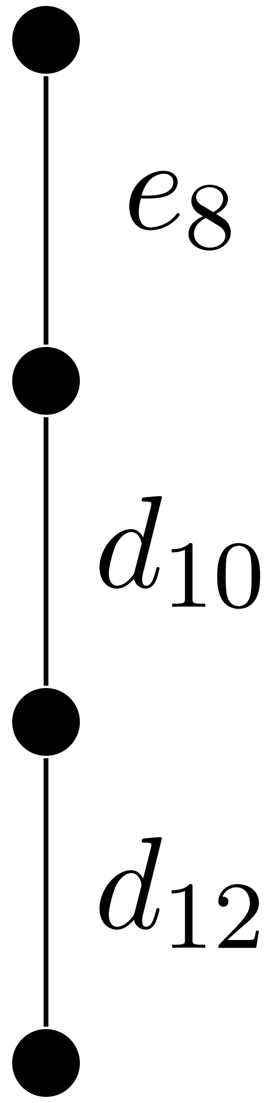
Example — known symplectic singularities

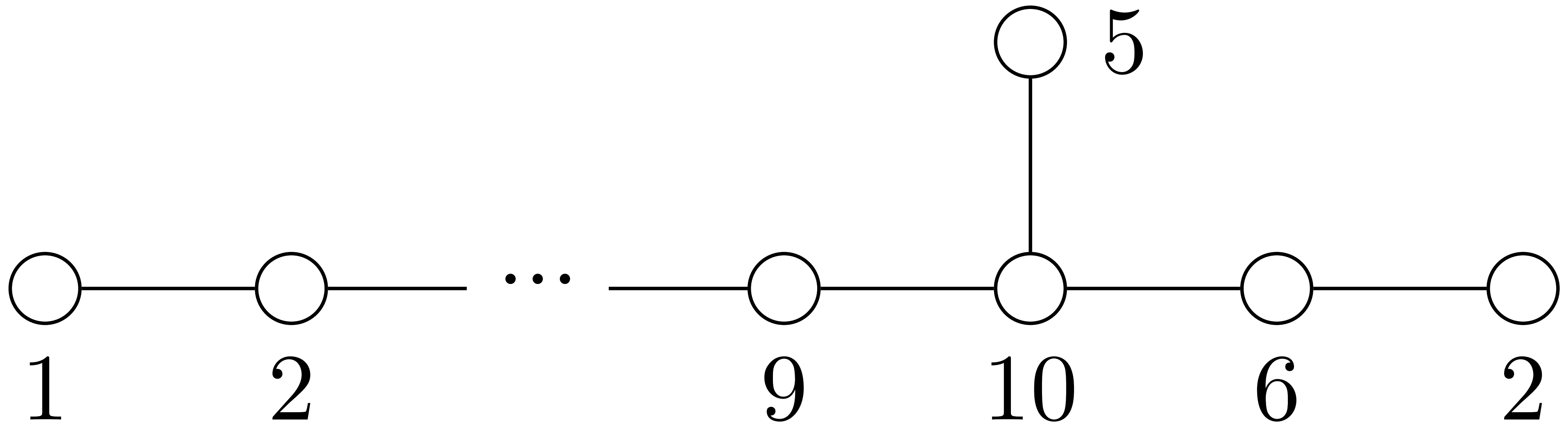
Coulomb branch — Klein singularity. Higgs branch — minimal SL(N) nilpotent orbit

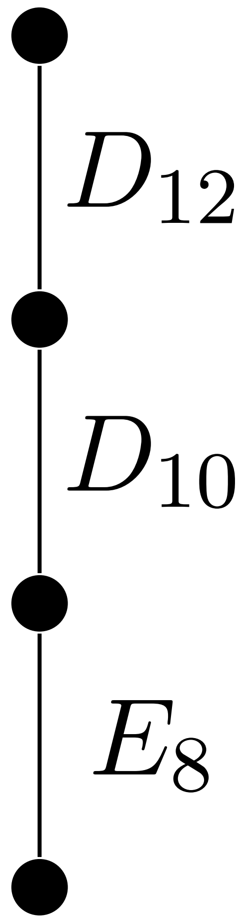


Example — new symplectic singularities

Structure of symplectic leaves — left: Coulomb branch; right: Higgs branch







Definition of the Coulomb branch

Braverman Finkelberg Nakajima

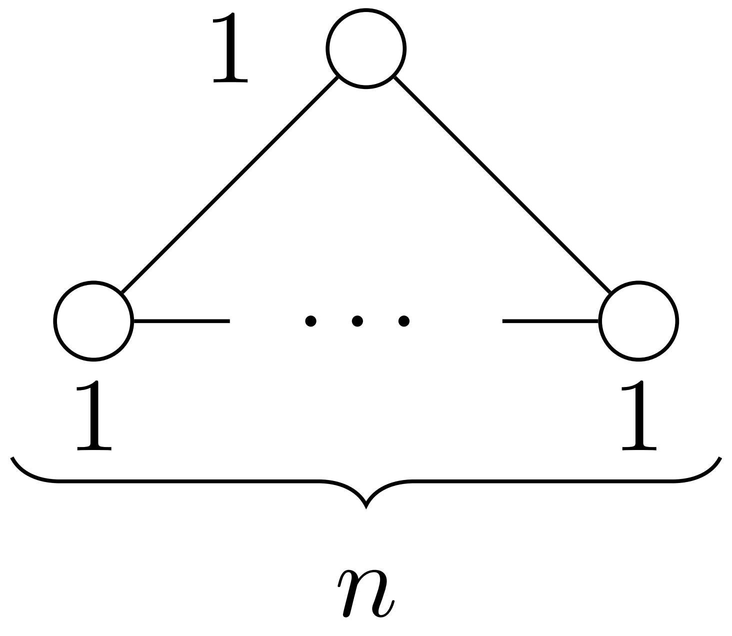
- In the following we will skip technical details and present results for computations of Higgs branch and Coulomb branch of some quivers

4 types of quivers (graphs)

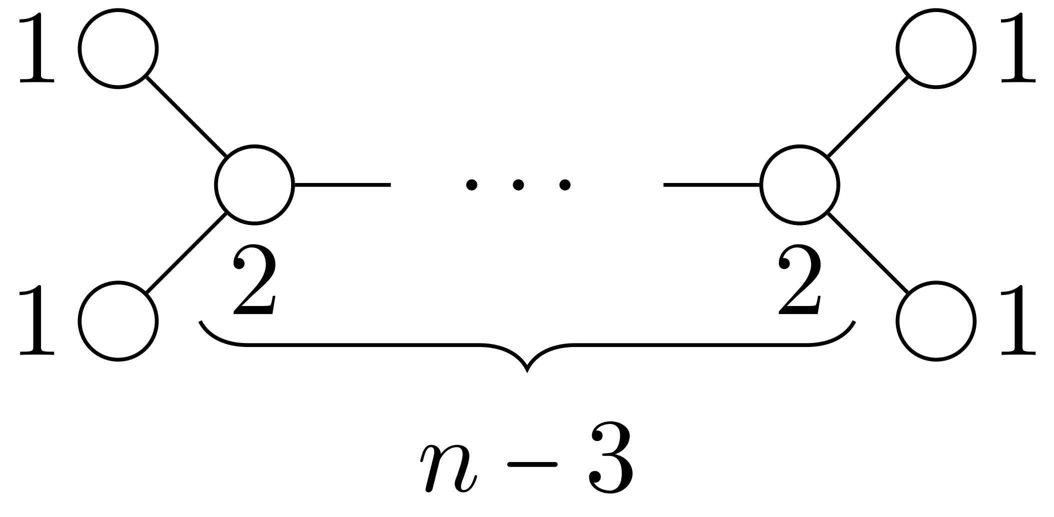
- quivers with unitary nodes — Nakajima quiver varieties
- orthosymplectic quivers — nodes alternate between symplectic and orthogonal gauge groups
- non simply laced quivers — nodes are unitary but some edges are not simply laced
- graphs made out of trivalent vertices

Examples of unitary quivers

a)



b)



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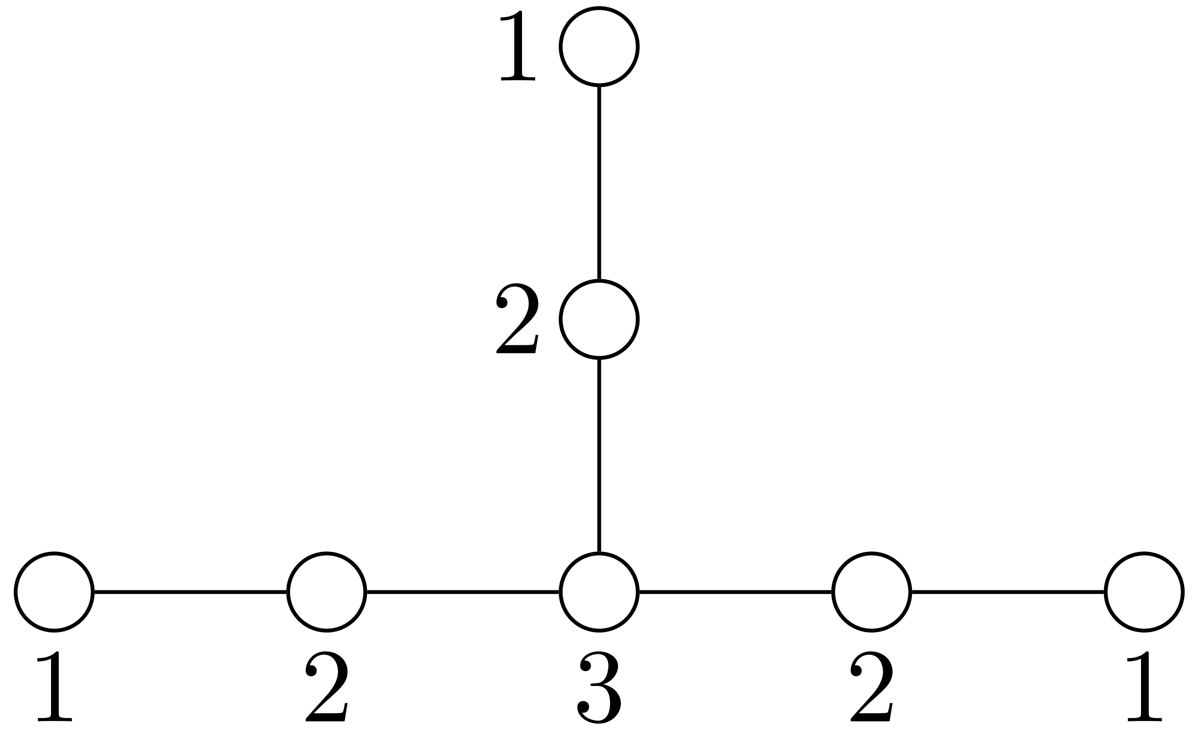

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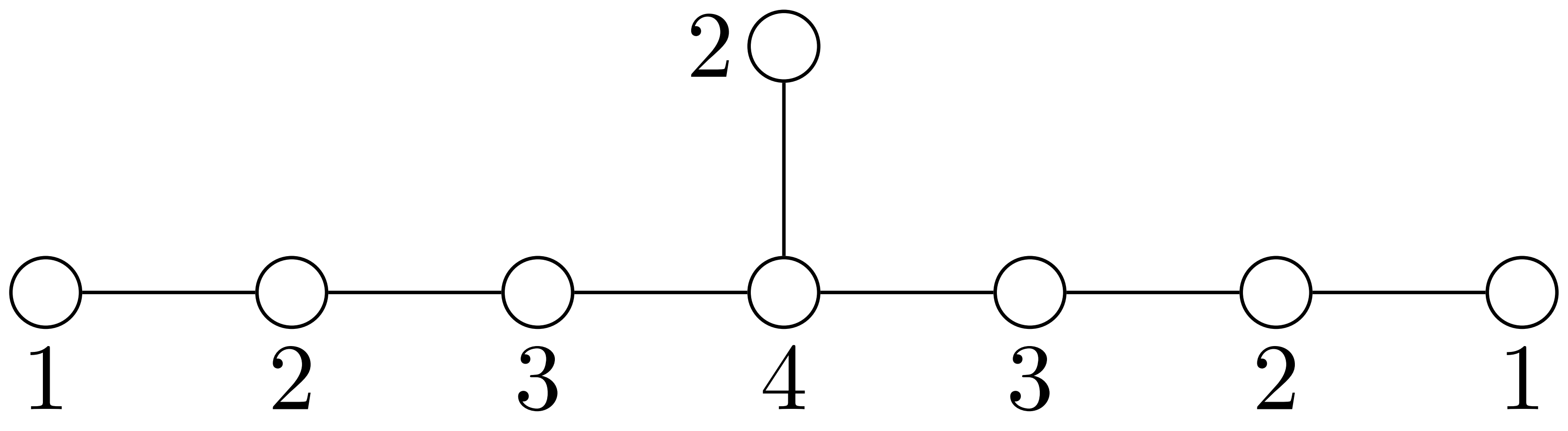

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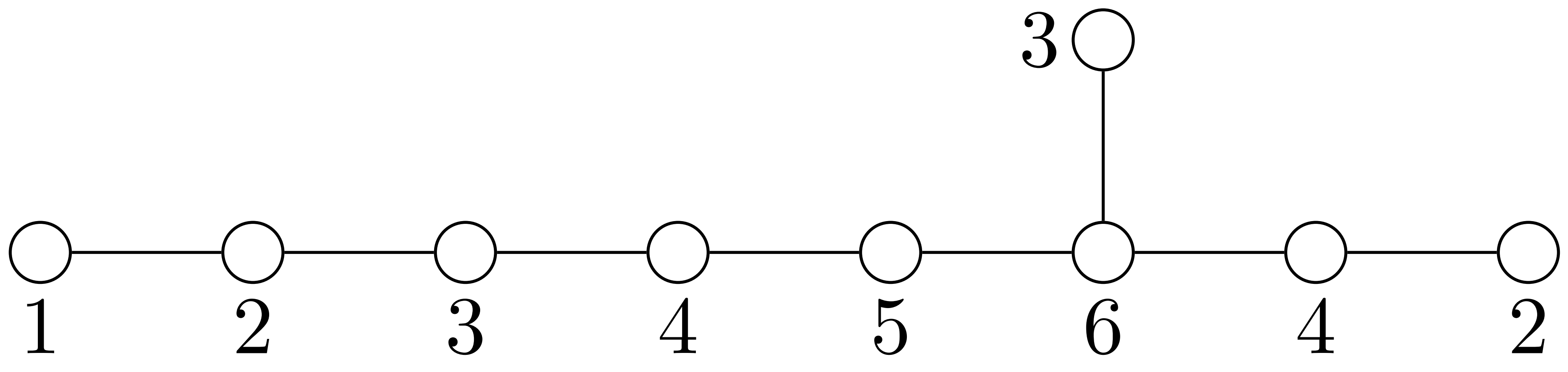
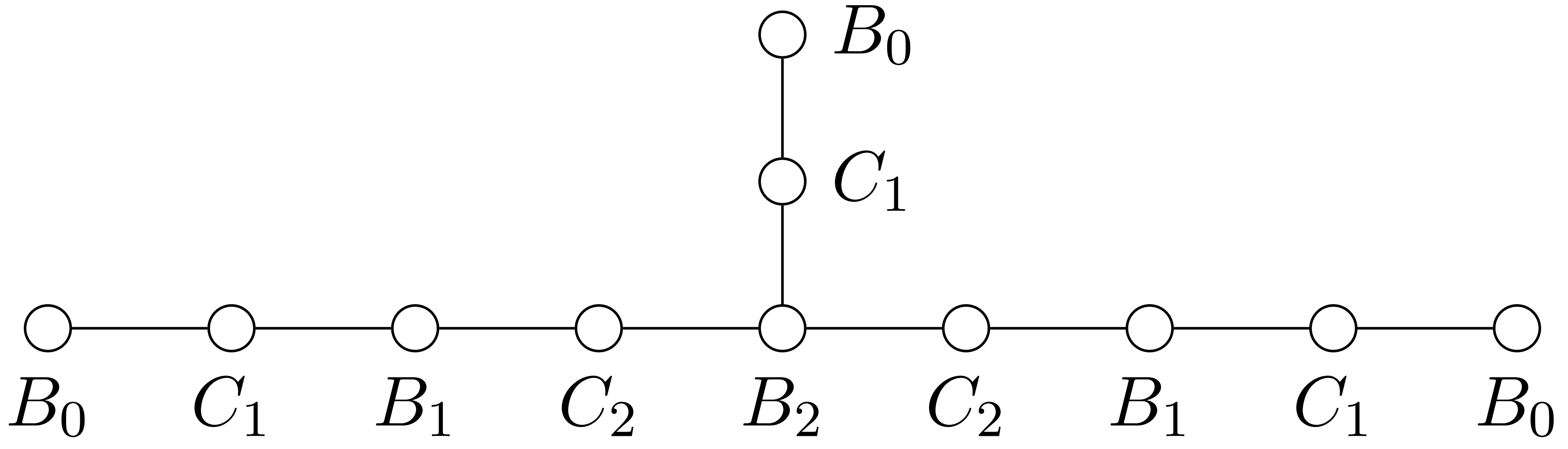
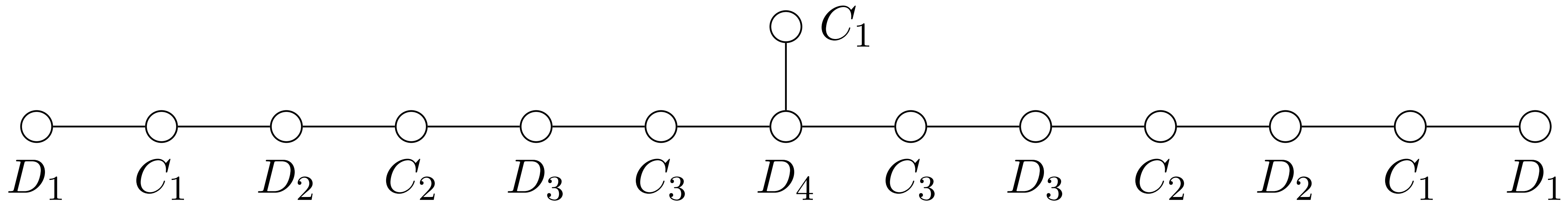
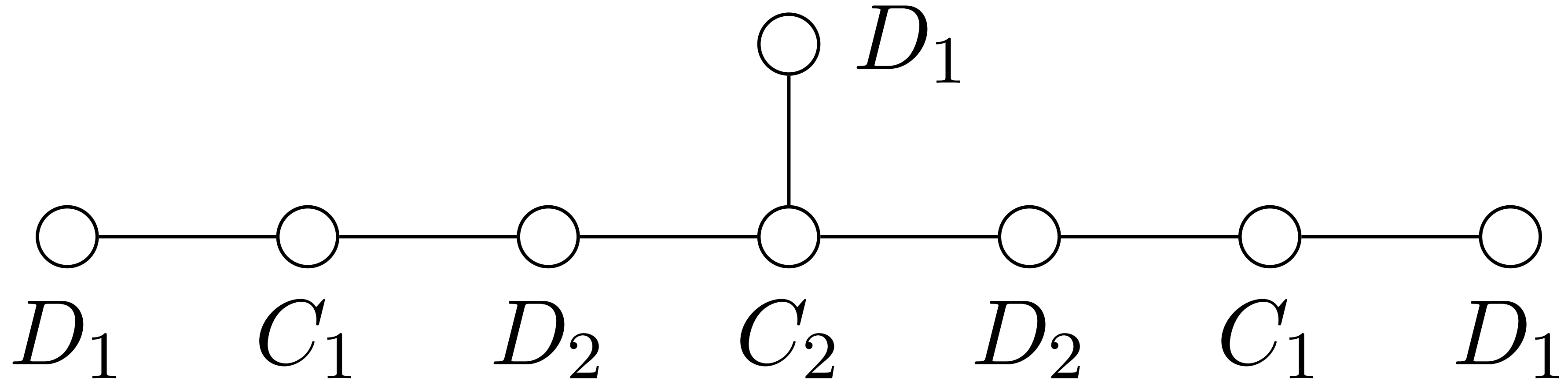
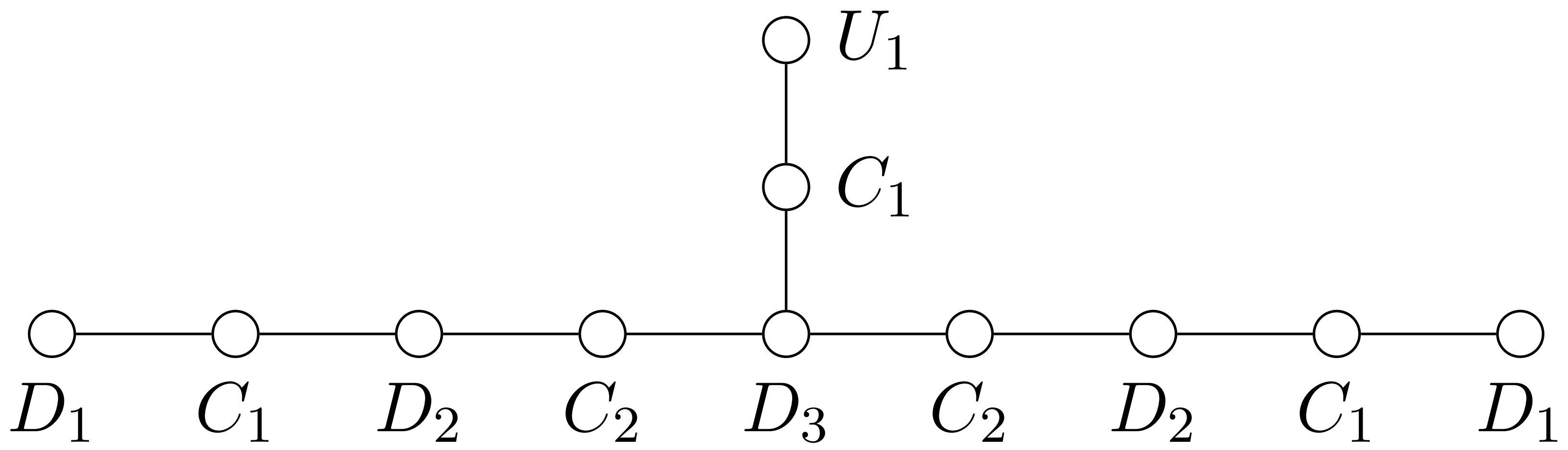


Examples of orthosymplectic quivers

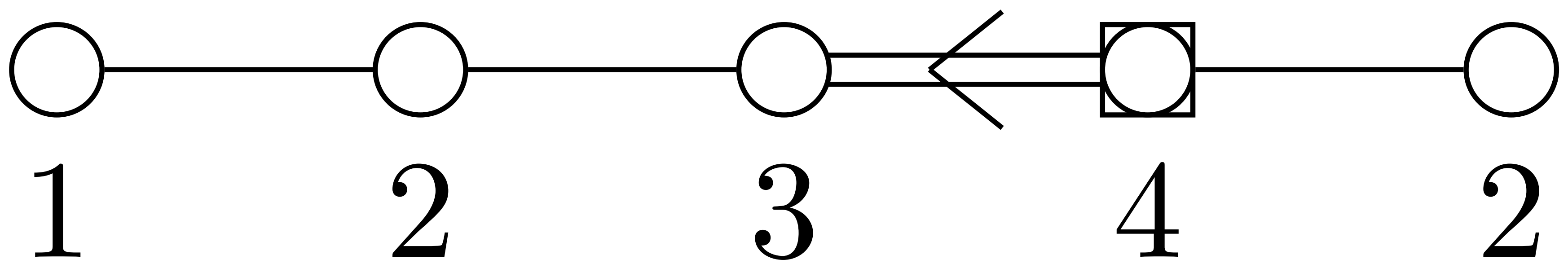


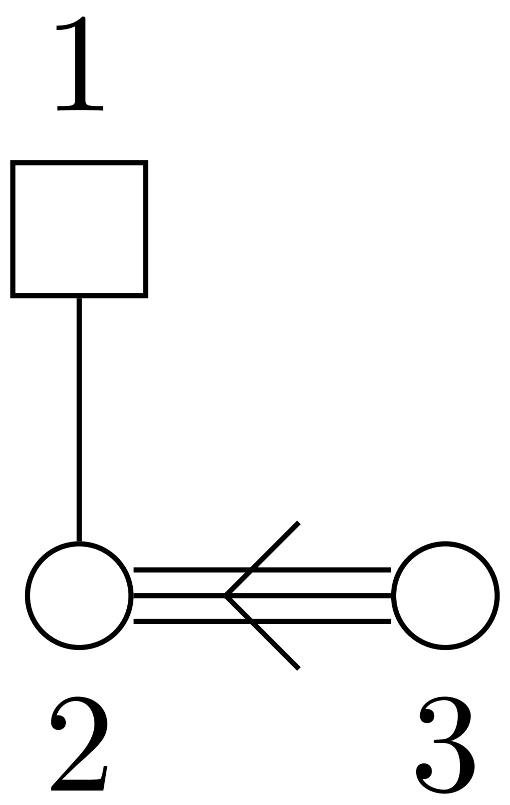






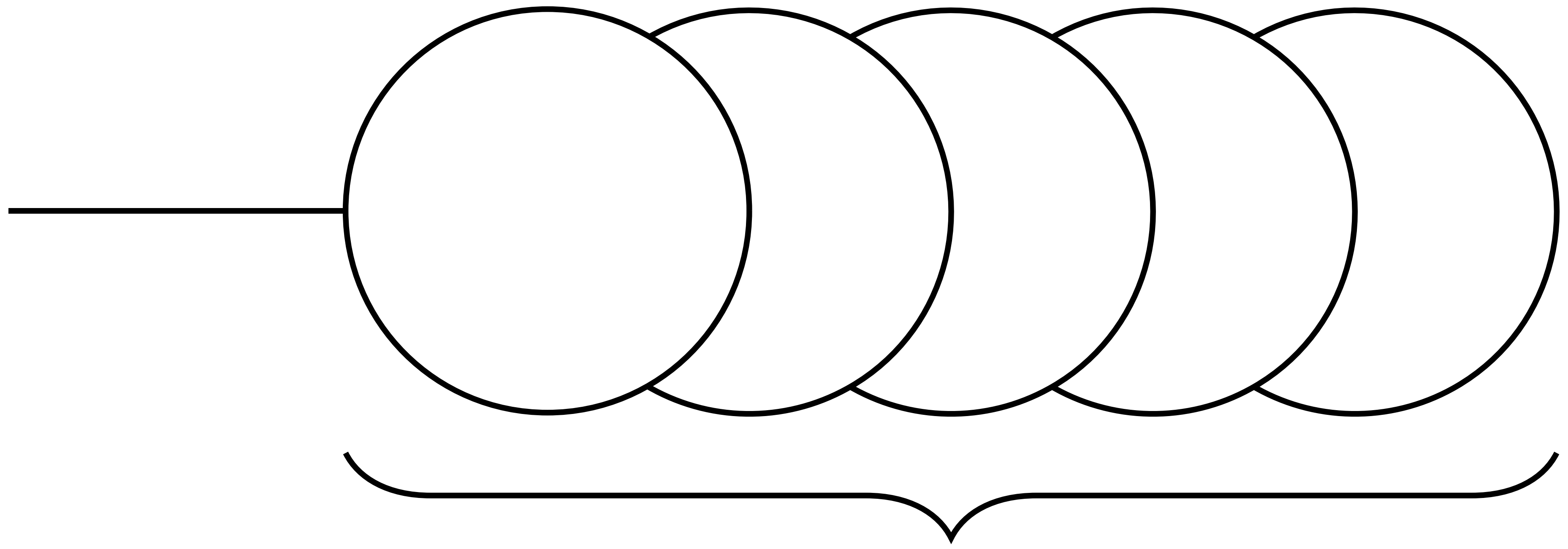
Examples of non simply laced quivers





Example of a trivalent graph

Each line SL(2), each vertex trifundamental; semi infinite — global



Hypersurface Symplectic Singularities

- We will use such quivers to construct a family of symplectic singularities which are also hypersurfaces
- Apparently, the combination of these two conditions is highly restrictive

Hypersurface Symplectic Singularities

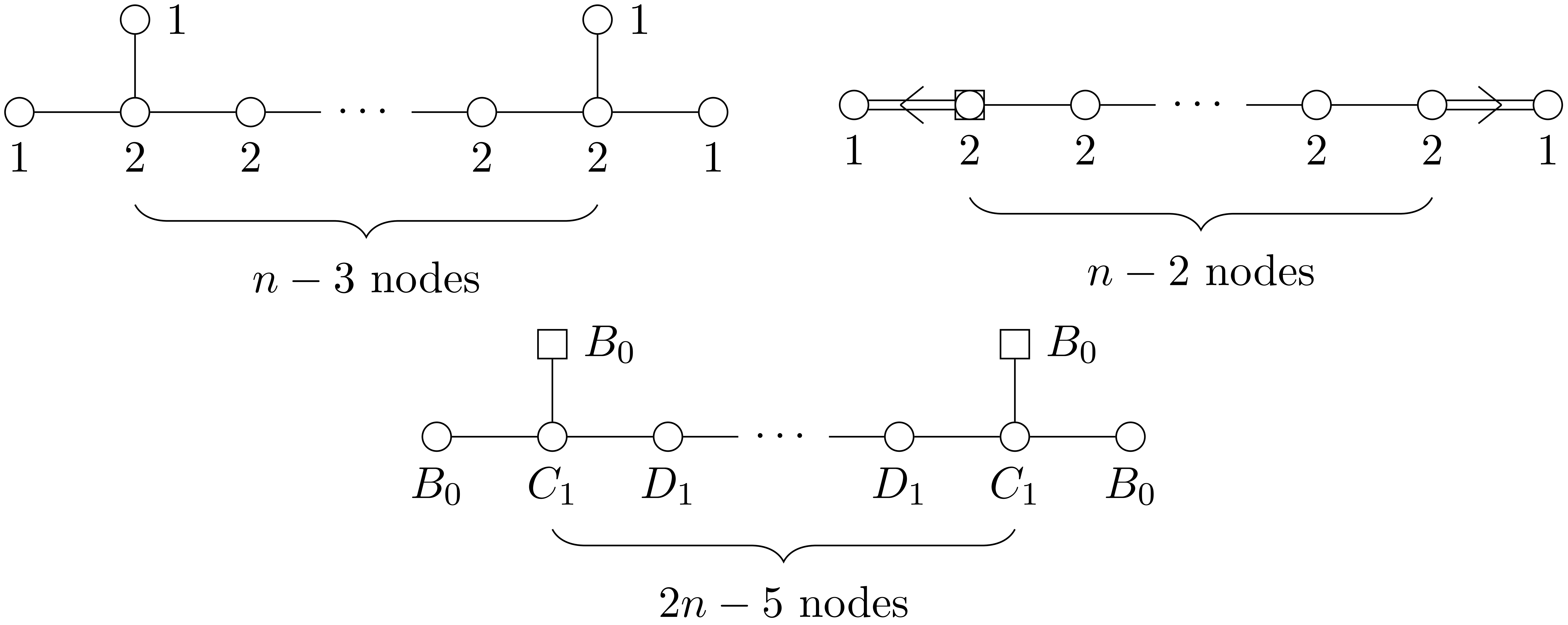
known singularities (Yamagishi Namikawa..) conjectured classification

- 1. ADE Klein singularities in 2 complex dimensions
- 2. Transverse slices in Sp(n) in 4 complex dimensions $\mathcal{S} \cap \mathcal{N}$
- where $\mathcal{S}$ is the slodowy slice to the orbit $2^{2n-2}1^2$ of Sp(n) and $\mathcal{N}$ is the nilpotent cone of Sp(n)
- 3. Transverse slice in 6 complex dimensions $\mathcal{S}' \cap \mathcal{N}'$
- where $\mathcal{S}'$ is the slodowy slice to the minimal nilpotent orbit of G2 and $\mathcal{N}'$ is the nilpotent cone of G2

Quivers for hyper surface symplectic singularities

D type

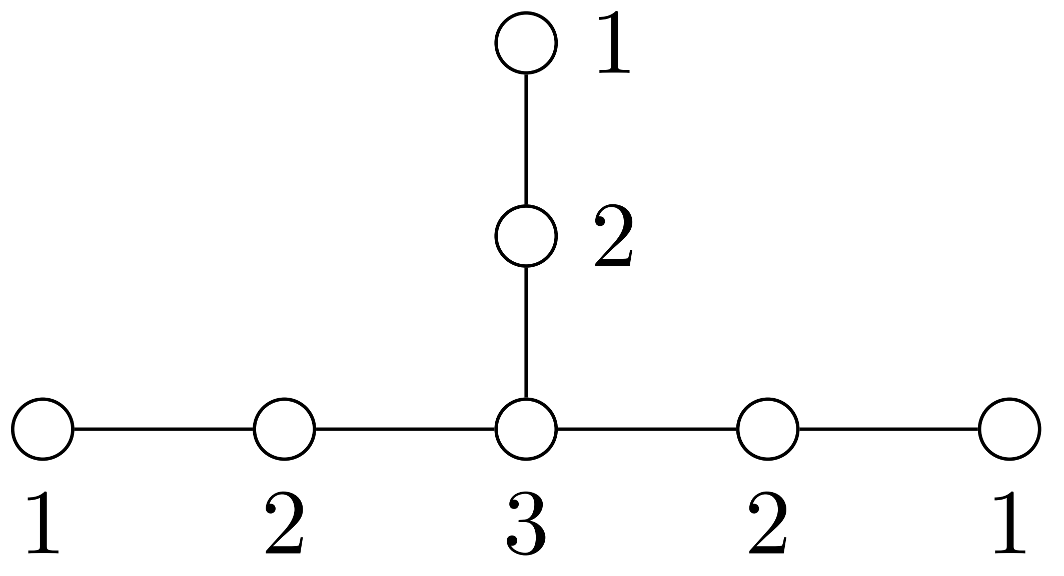
$\mathbb{C}^2/D_n$
$(n \geq 4)$
$\dim_{\mathbb{H}} = 1$

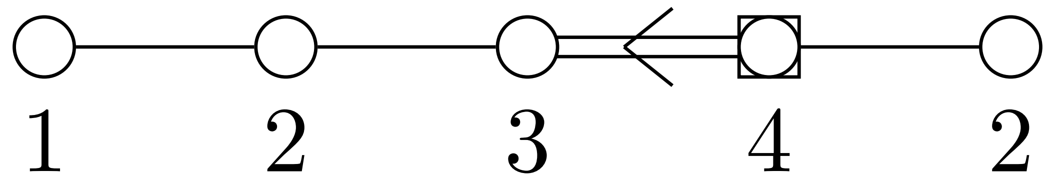


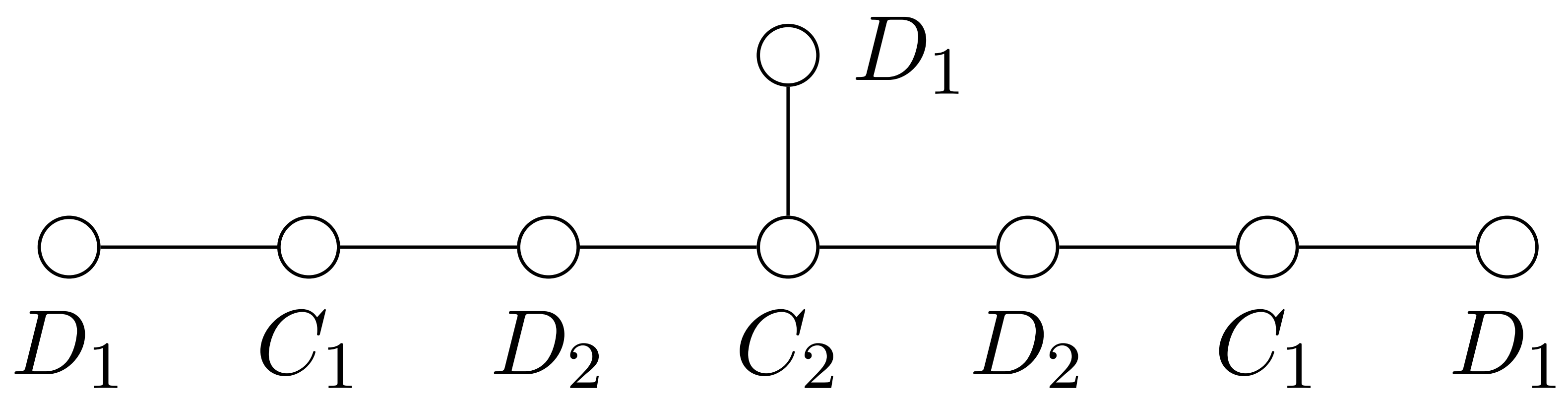
Quivers for hyper surface symplectic singularities

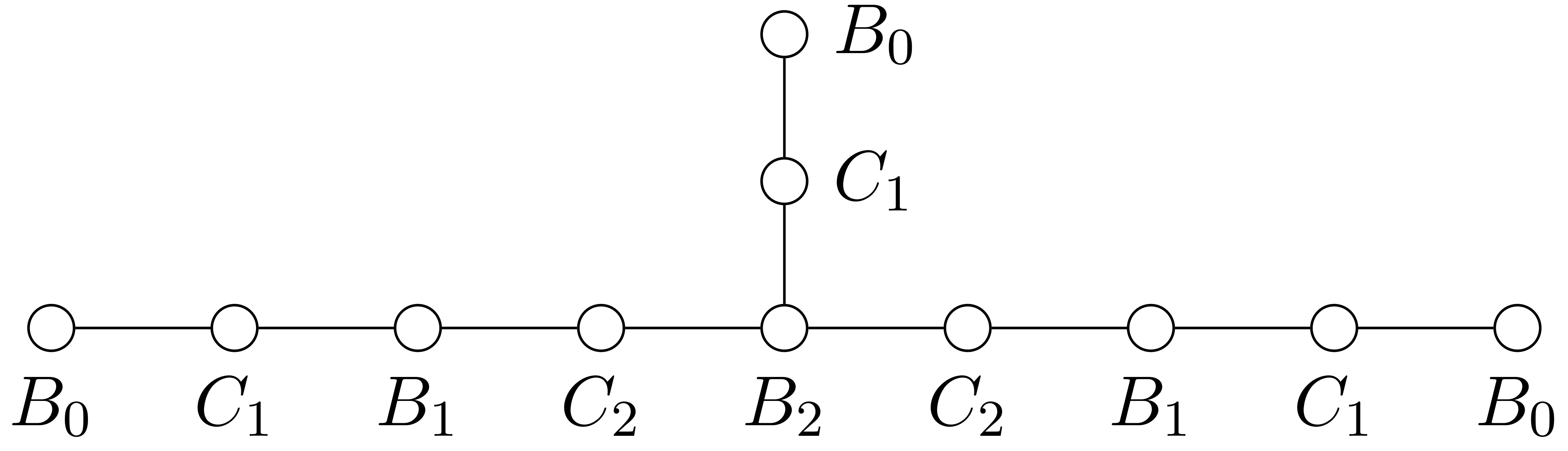
E6 type

$\mathbb{C}^2/E_6$
$\dim_{\mathbb{H}} = 1$





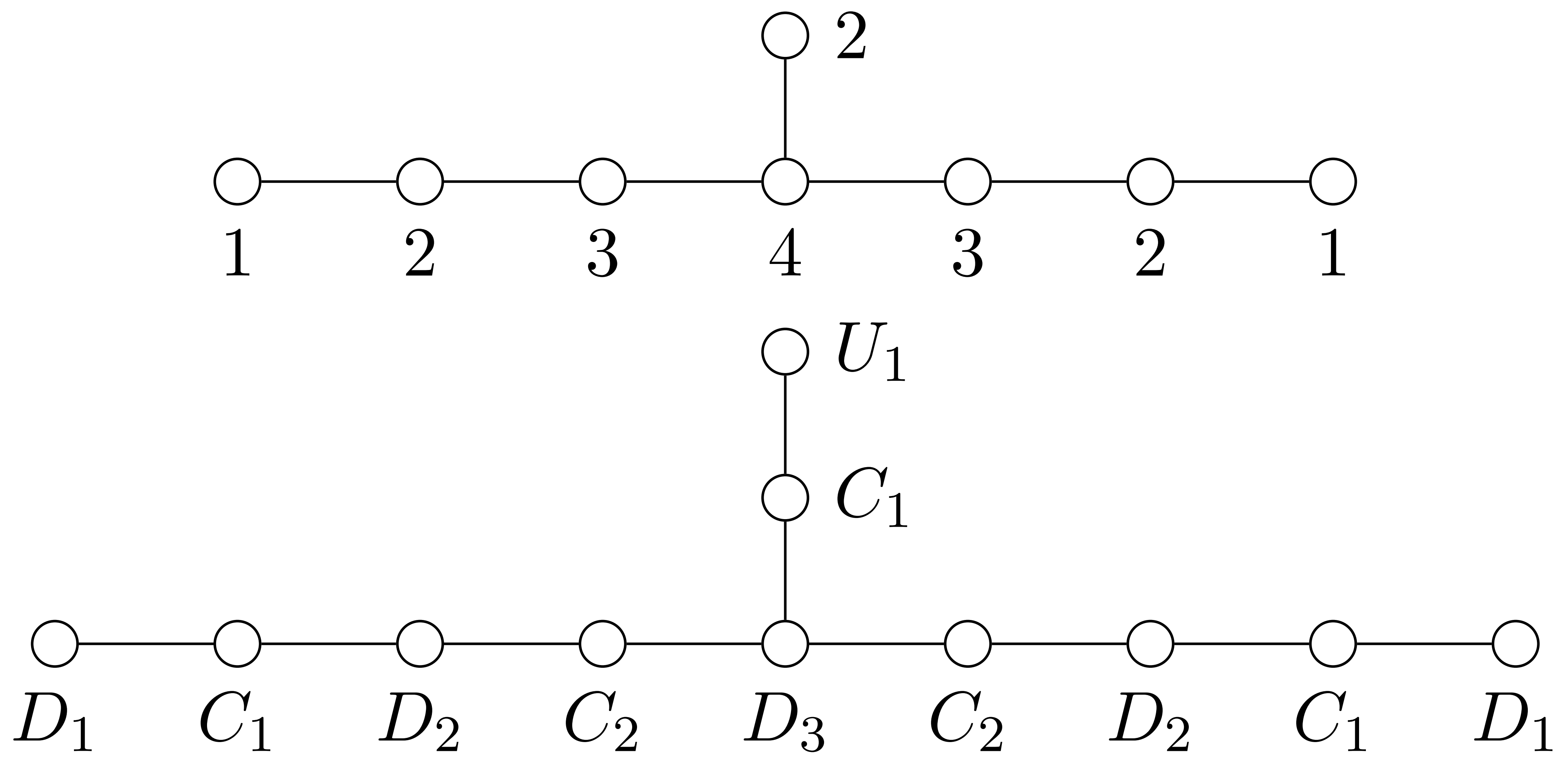




Quivers for hyper surface symplectic singularities

E7 type

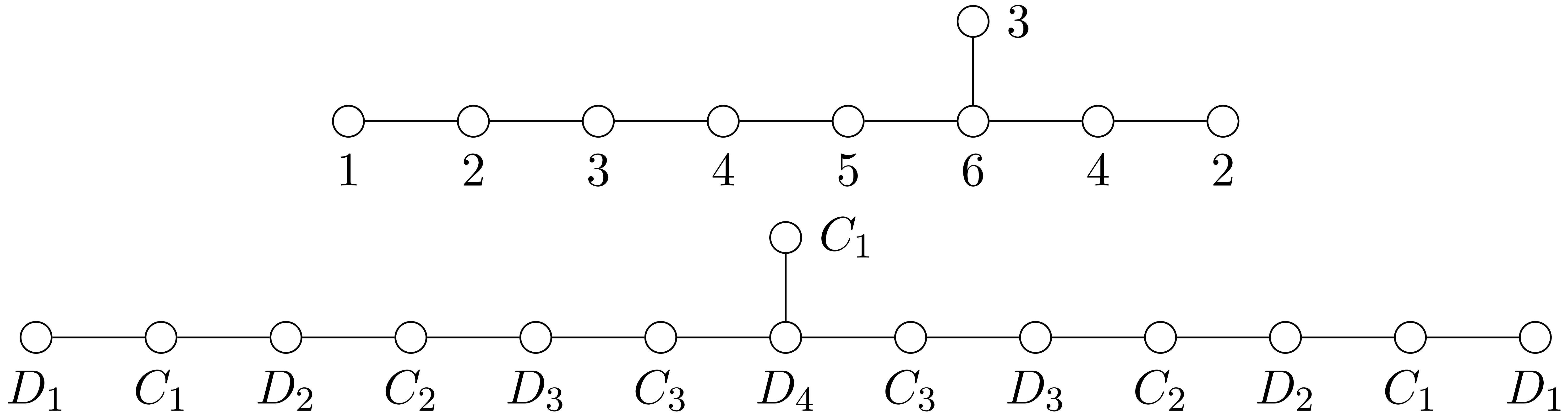
$\mathbb{C}^2/E_7$
$\dim_{\mathbb{H}} = 1$



Quivers for hyper surface symplectic singularities

E8 type

$\mathbb{C}^2/E_8$
$\dim_{\mathbb{H}} = 1$



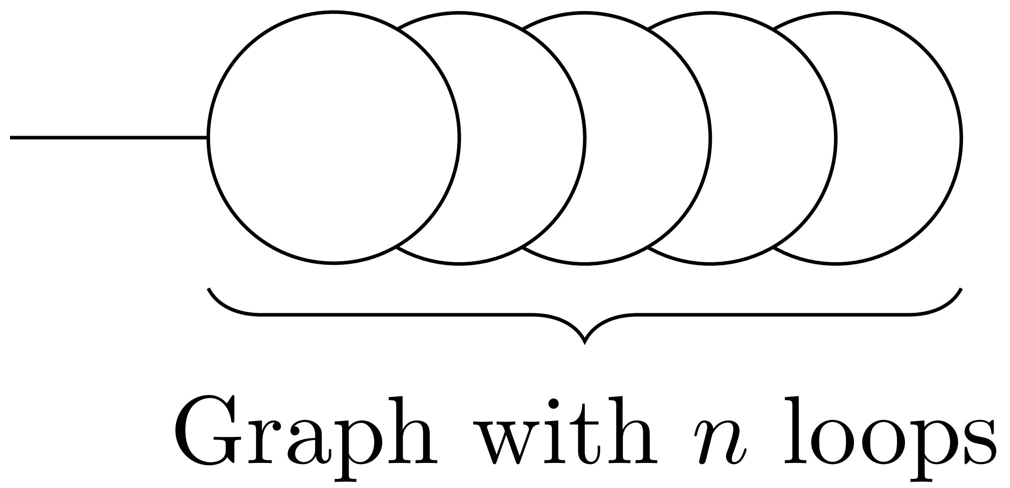
Quivers for hyper surface symplectic singularities

Slices in Sp(n)

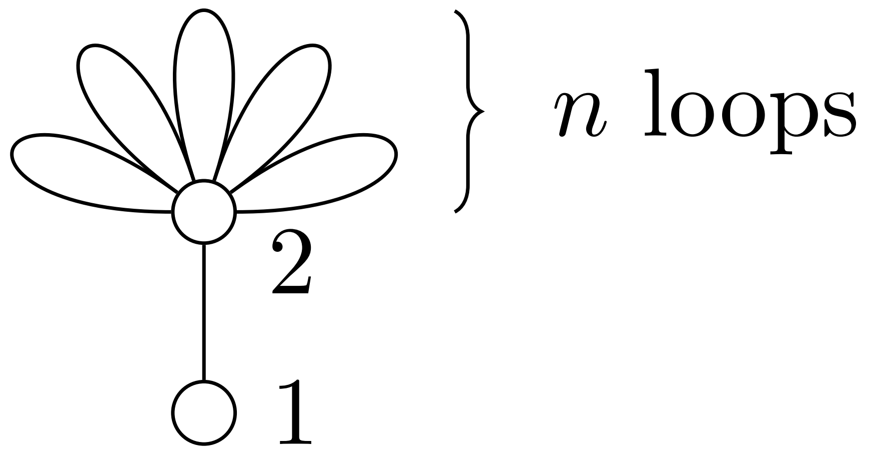
X_n

$\dim_{\mathbb{H}} = 2$

$\mathcal{H}\Big($



$\Big) = \mathcal{C}\Big($

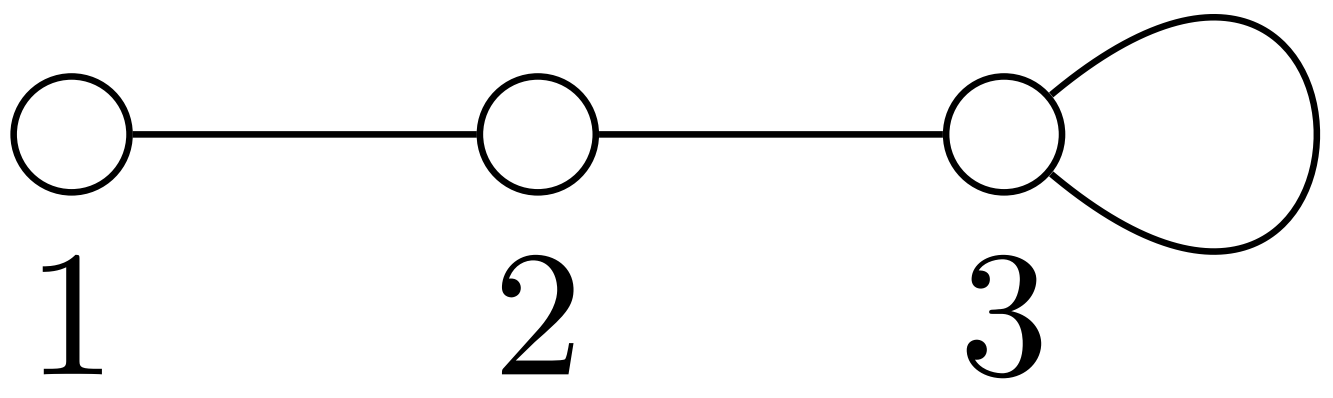


$\Big)$

Quivers for hyper surface symplectic singularities

Slice in G2

$$\hat{X}$$
$$\dim_{\mathbb{H}} = 3$$



Thank you