Walls for G-Hilb via Reid's recipe

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Main reference

- *B. Wormleighton; Walls for G-Hilb via Reid's recipe (2019)*

McKay Correspondence

- The n-dimensional McKay correspondence seeks to relate:

$$\begin{array}{c}\text{geometry of crepant resolutions}\\ \pi\colon Y \to X := \mathbb{C}^n/G\end{array} \quad \longleftrightarrow \quad \begin{array}{c}\text{representation theory}\\ \text{of } G\end{array}$$

  for finite $G \subseteq \mathrm{SL}_n(\mathbb{C})$.

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- Specifically, we will interpret walls in a stability space geometrically and combinatorially using representation theory.

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    - Homological Minimal Model Program (Wemyss)
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    - VGIT (Craw–Ishii)
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  - Homological Minimal Model Program (Wemyss)
  - quiver mutations (Nolla–Sekiya)
  - VGIT (Craw–Ishii)
  - ...
- We will emphasise the latter, though heavily inspired by the first two approaches.

Two dimensions

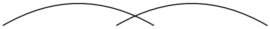
- Recall that in two dimensions the *G-Hilbert scheme*

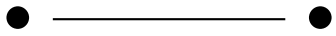
$$G\text{-Hilb}\,\mathbb{C}^2$$

is the unique crepant resolution of $\mathbb{C}^2/G$ when $G \subseteq \mathrm{SL}_2(\mathbb{C})$.

Example

- For the singularity of type A_2 the exceptional fibre in its crepant resolution and its dual graph are:





- G-Hilb can also be viewed as a moduli space of θ-stable quiver representations

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  Here Q^G is the McKay quiver for G, θ is a stability condition, and $\underline{\mathsf{d}}$ is a distinguished dimension vector.
- We denote the vertices of the McKay quiver by Q_0 and the arrows by Q_1.

- The stability condition θ lives in the *stability space*

$$\Theta_{\underline{\mathsf{d}}} := \{\eta \in \mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}^{Q_0}, \mathbb{R}) : \eta(\underline{\mathsf{d}}) = 0\}$$

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- Note that $\mathbb{Z}^{Q_0} = \mathrm{Rep}(G)$ and so we will often write elements of $\mathbb{Z}^{Q_0}$ as linear combinations of characters.

- The stability space $\Theta_{\underline{\mathsf{d}}}$ has a wall-and-chamber structure in which if θ, ϑ lie in the same open chamber $\mathfrak{C}$ then

$$\mathcal{M}_\theta(Q^G, \underline{\mathsf{d}}) \cong \mathcal{M}_\vartheta(Q^G, \underline{\mathsf{d}})$$

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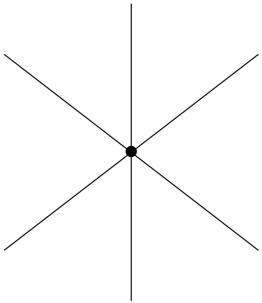
- We denote by $\mathcal{M}_{\mathfrak{C}}$ the moduli space for any generic $\theta \in \mathfrak{C}$.

Example

- In type A_2 the chambers in $\Theta_{\underline{d}}$ are:



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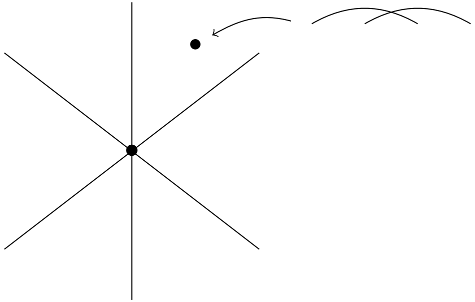

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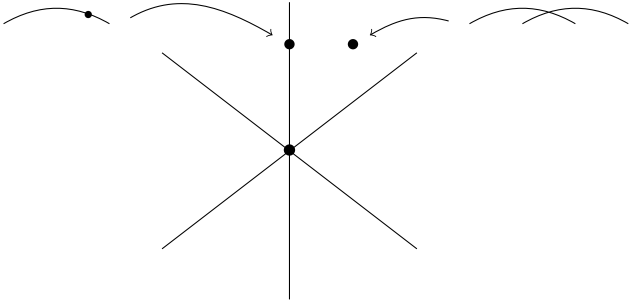

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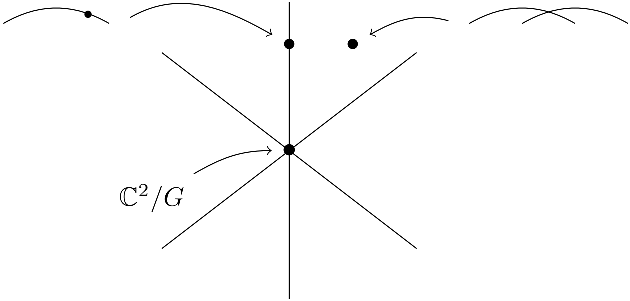
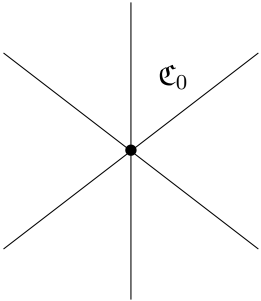


Example

- One chamber $\mathfrak{C}_0$ 'corresponds' to G-Hilb:



Theorem (Kronheimer)

Let $G \subseteq \mathrm{SL}_2(\mathbb{C})$ be a finite subgroup of type Γ. There is an identification of $\Theta_{\underline{\mathrm{d}}}$ with the Cartan subalgebra $\mathfrak{h}_\Gamma \subseteq \mathfrak{g}_\Gamma$ inside the simple Lie algebra of type Γ such that the walls in $\Theta_{\underline{\mathrm{d}}}$ correspond to the root hyperplanes in $\mathfrak{h}_\Gamma$.

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- it will be a 'local' description of the walls for $\mathfrak{C}_0$ only
- the geometry of wall-crossings is *much* richer in three dimensions

Three dimensions

- When $G \subseteq \mathrm{SL}_3(\mathbb{C})$ the same constructions

$$G\text{-Hilb}\,\mathbb{C}^3 \text{ and } \mathcal{M}_\theta(Q^G, \underline{\mathsf{d}})$$

yield crepant resolutions of $\mathbb{C}^3/G$ by BKR.

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- We denote the chamber for G-Hilb by $\mathfrak{C}_0$. This is the chamber containing

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- BKR also produces a triangulated equivalence

$$\Phi_{\mathfrak{C}} \colon D^b(\mathcal{M}_{\mathfrak{C}}) \to D^b_G(\mathbb{C}^3)$$

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- We denote by $\frac{1}{r}(a, b, c)$ the cyclic group of order r generated by

$$g = \begin{pmatrix} \varepsilon^a & & \\ & \varepsilon^b & \\ & & \varepsilon^c \end{pmatrix}$$

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  where ε is an rth root of unity, and $a + b + c \equiv 0 \bmod r$.
- In this situation $\mathcal{M}_{\mathfrak{C}}$ is a toric variety.

Theorem (Craw–Ishii)

For finite abelian $G \subseteq \mathrm{SL}_3(\mathbb{C})$, every (projective) crepant resolution of $\mathbb{C}^3/G$ occurs as $\mathcal{M}_{\mathfrak{C}}$ for some chamber $\mathfrak{C} \subseteq \Theta_{\underline{\mathsf{d}}}$.

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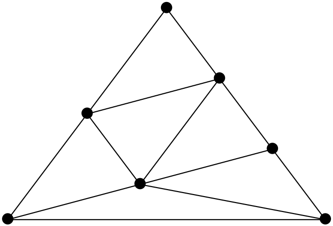
- Craw–Reid produce a fun algorithm for computing the triangulation for G-Hilb.

Example

This is G-Hilb for $G = \frac{1}{6}(1, 2, 3)$:



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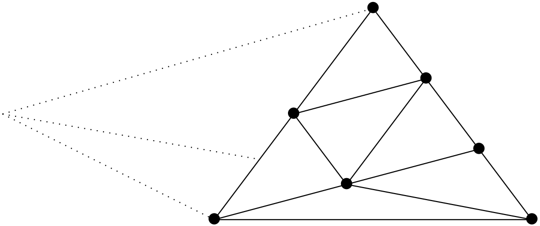

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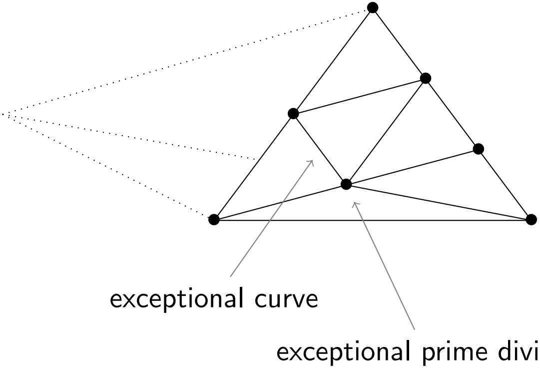

Reid's recipe

- Reid, Craw, Logvinenko, and Craw–Cautis–Logvinenko construct a labelling of the exceptional fibre of G-Hilb $\mathbb{C}^3$ by characters of G called *Reid's recipe*

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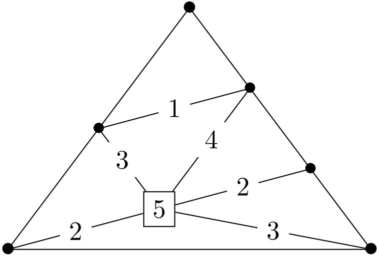
- Roughly, this labelling encodes where certain sheaves generating $D^b(G\text{-Hilb}\,\mathbb{C}^3)$ are supported.

Example

- Reid's recipe for $G = \frac{1}{6}(1,2,3)$ is:



- Here a denotes the character $\chi_a \colon g \mapsto \varepsilon^a$.

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More precisely,

- what are the equations of the walls?
- how can we describe the wall-crossing behaviour? (e.g. birational type, unstable locus, equivalences of derived categories,...)

Theorem (W. '19)

There is an algorithm – called the **unlocking procedure** *– to explicitly compute a set of inequalities defining* $\mathfrak{C}_0$ *from the data of Reid's recipe and the combinatorics of the exceptional fibre.*

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- *one can determine which of these inequalities are irredundant and so actually define walls of $\mathfrak{C}_0$.*
- *the birational type, unstable locus, and derived equivalence for the wall can be read from the wall equation.*

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- Each exceptional curve C produces a potentially redundant inequality defining $\mathfrak{C}_0$, and the unlocking procedure takes a curve to the collection of characters $\text{G-ig}(C)$ appearing in this inequality. The coefficients in these inequalities can also be calculated in a uniform way.

Let C be a curve marked with χ by Reid's recipe. Roughly, the unlocking procedure does the following:

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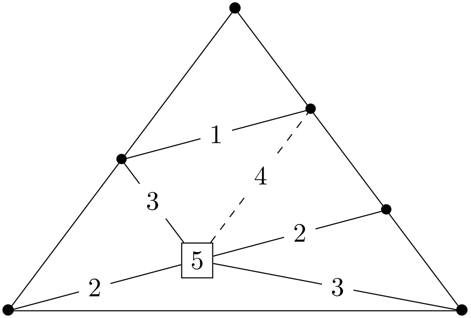

Unlocking for the 4-curve C_4 has $\text{G-ig}(C_4) = \{\chi_4\}$.

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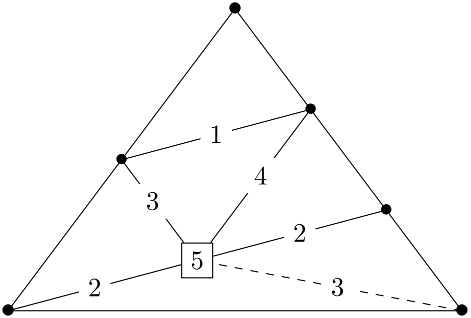

Unlocking for the dashed curve C_3 gives $\text{G-ig}(C_3) = \{\chi_3, \chi_4, \chi_5\}$.

Example (Unlocking for $G = \frac{1}{6}(1,2,3)$)

Using this, we compute the walls for $\mathfrak{C}_0$ to be

$$\begin{aligned}
\theta(\chi_1) &= 0 \\
\theta(\chi_2) + \theta(\chi_5) &= 0 \\
\theta(\chi_3) + \theta(\chi_5) &= 0 \\
\theta(\chi_4) &= 0 \\
\theta(\chi_5) &= 0 \\
\theta(\chi_2) + \theta(\chi_3) + \theta(\chi_4) + \theta(\chi_5) &= 0
\end{aligned}$$

The chamber $\mathfrak{C}_0$ is given by replacing $=$ by $>$ in these equations.

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- The curve C_3 from before produces the inequality

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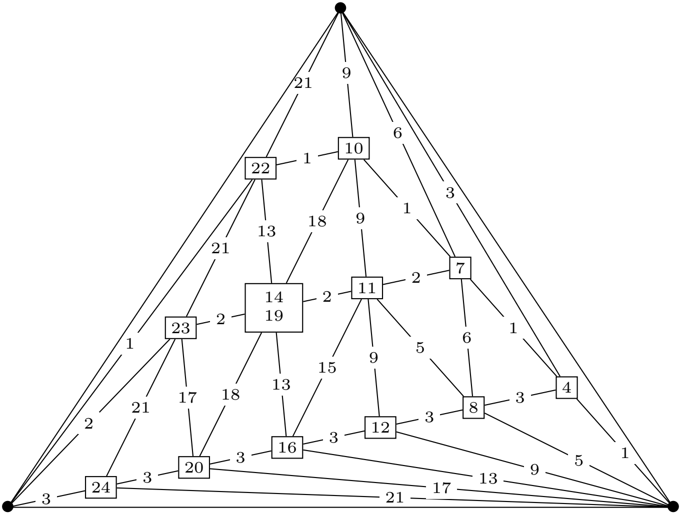
- Notice that this is the sum of two wall inequalities and so is redundant
- This comes from unlocking: the characters in the inequality come from the characters captured by the other 3-curve and from the unlocked 4-curve.

Example (Reid's recipe for $G = \frac{1}{25}(1, 3, 21)$)



Example (Unlocking for a 1-curve)

Let's consider the curve C marked with 1 that's dashed in the following picture.

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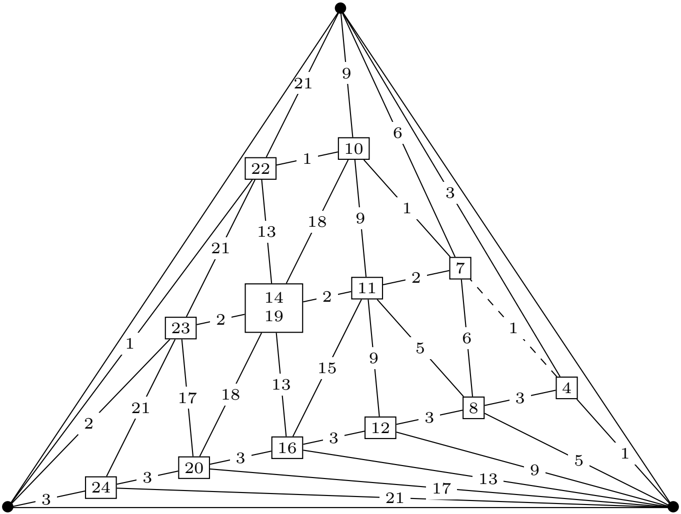

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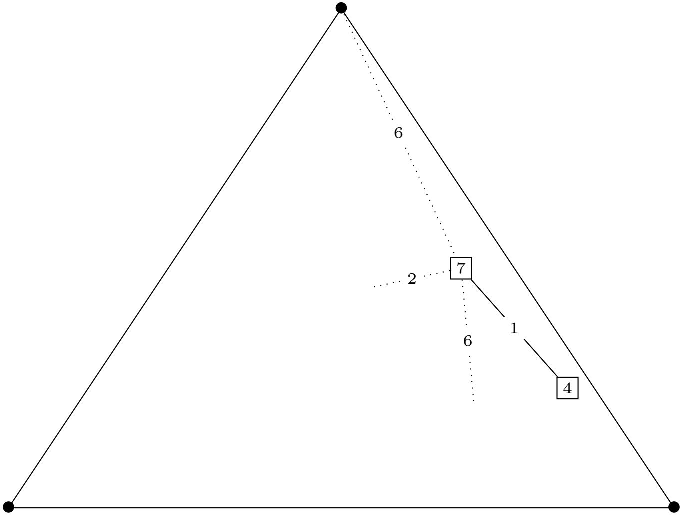

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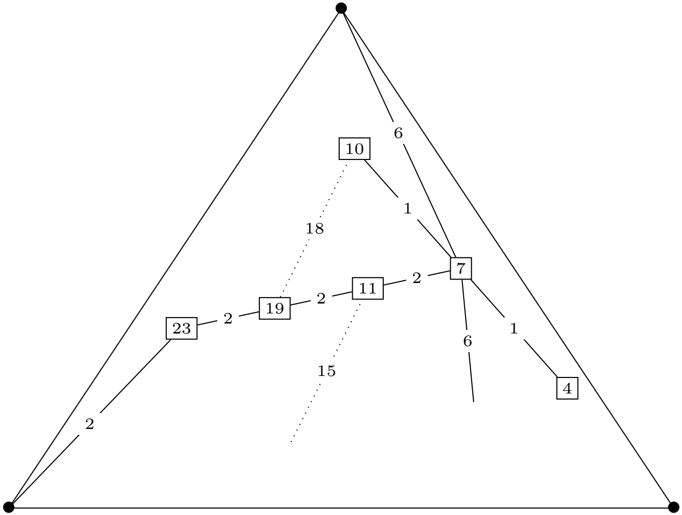

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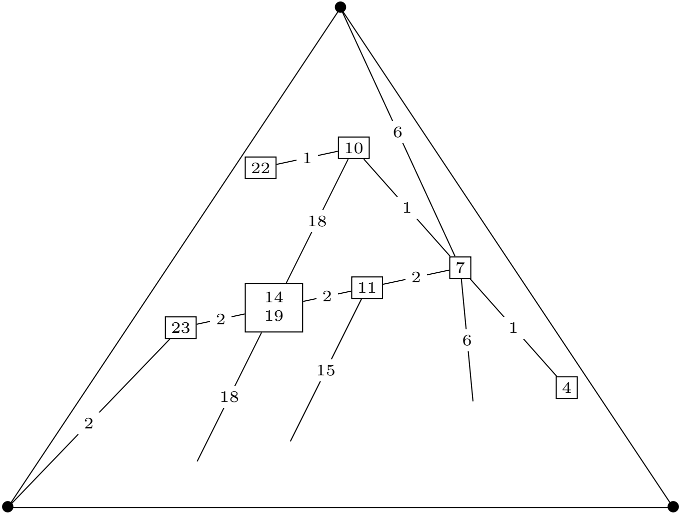

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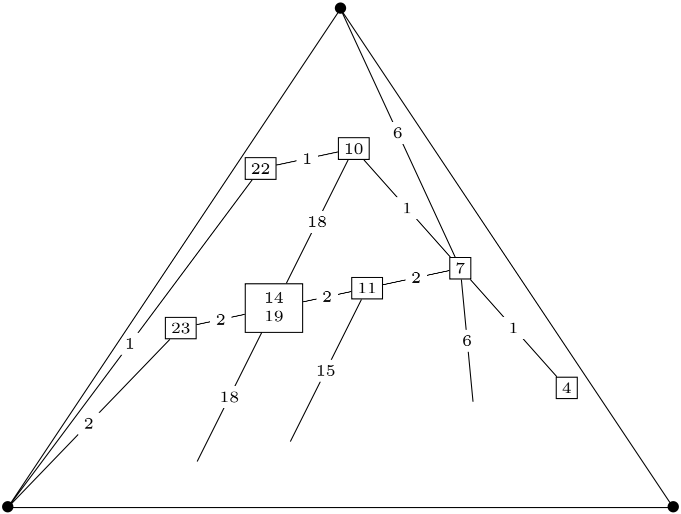

Example (Unlocking for a 1-curve)

We can conclude that the characters featuring in the inequality for C are

$$\{\chi_1, \chi_2, \chi_4, \chi_6, \chi_7, \chi_{10}, \chi_{11}, \chi_{14}, \chi_{15}, \chi_{18}, \chi_{19}, \chi_{22}, \chi_{23}\}$$

- We can also deduce some corollaries describing the structure of $\Theta_{\underline{d}}$ and the geometry of the wall-crossings.

- Given a wall $\mathfrak{w} \subseteq \mathfrak{C}$ there is a contraction morphism

$$\mathrm{cont}_{\mathfrak{w}} \colon \mathcal{M}_{\mathfrak{C}} \to \overline{\mathcal{M}}_{\theta}$$

for generic $\theta \in \mathfrak{w}$.

We say that $\mathfrak{w}$ is...

- Type 0 if $\mathrm{cont}_{\mathfrak{w}}$ is an isomorphism
- Type I if $\mathrm{cont}_{\mathfrak{w}}$ contracts a curve to a point
- Type II if $\mathrm{cont}_{\mathfrak{w}}$ contracts a divisor to a point
- Type III if $\mathrm{cont}_{\mathfrak{w}}$ contracts a divisor to a curve

Corollary (Craw–Ishii, W.)

The flop in each $(-1,-1)$-curve in $G\text{-Hilb}\,\mathbb{C}^3$ can be realised by a single wall-crossing from $\mathfrak{C}_0$. There are no walls of $\mathfrak{C}_0$ that contract a divisor to a point (Type II*).*

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Corollary (W.)

The unstable locus of each wall can be reconstructed combinatorially from the characters appearing in the wall equation.

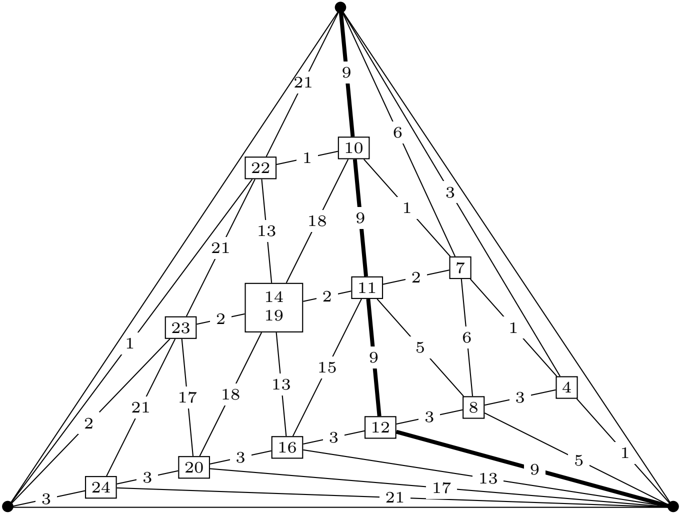
- Beyond assessing whether inequalities in a given example are redundant, the techniques of the theorem provide a general classification of the walls for $\mathfrak{C}_0$ in combinatorial terms.

Theorem (W. '19)

Suppose $G \subseteq \mathrm{SL}_3(\mathbb{C})$ *is a finite abelian subgroup. The walls of the chamber* $\mathfrak{C}_0$ *for* G*-Hilb and their types are as follows:*

- *a Type* I *wall for each exceptional* $(-1,-1)$*-curve,*
- *a Type* III *wall for each* generalised long side,
- *a Type* 0 *wall for each irreducible exceptional divisor,*
- *each remaining wall is of Type* 0 *and comes from a divisor parameterising a rigid quotient.*

Example (Reid's recipe for $G = \frac{1}{25}(1, 3, 21)$)



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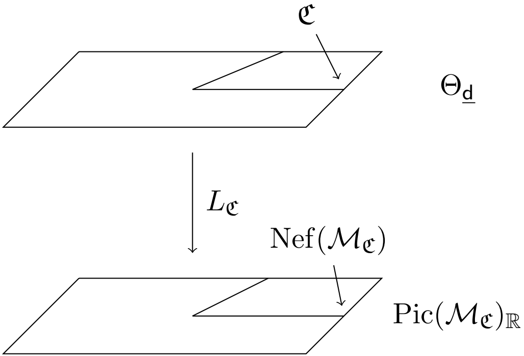

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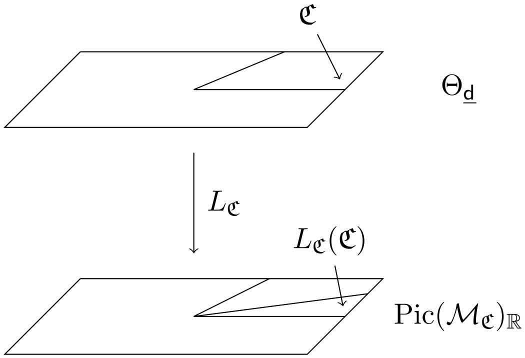

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- show that walls remember geometry
  - unlocking is reversible

Applications – in progress

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- One motivation for having explicit expressions for walls is to compare the position of chambers of different crepant resolutions of $\mathbb{C}^3/G$.

- Suppose $A \subseteq G$ is a normal subgroup with quotient $G/A = T$.

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- T acts on A-Hilb $\mathbb{C}^3$ and so one obtains the crepant resolution

$$T\text{-Hilb}\, A\text{-Hilb}\, \mathbb{C}^3 \to \mathbb{C}^3/G$$

Conjecture: Let $\mathfrak{C}_0$ denote the chamber for G-Hilb and let $\mathfrak{C}_1$ denote the/a chamber for T-Hilb A-Hilb.

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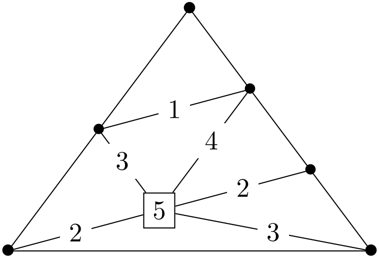
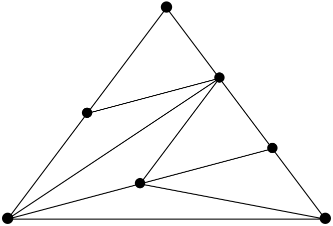
Example (Geometric evidence)

We return to $G = \frac{1}{6}(1,2,3)$ with G-Hilb $\mathbb{C}^3$ and Reid's recipe shown below.



Example (Geometric evidence)

G can be expressed as a direct product of $A = \frac{1}{3}(1,2,0)$ and $T = \frac{1}{2}(1,0,1)$. We show T-Hilb A-Hilb $\mathbb{A}^3$:



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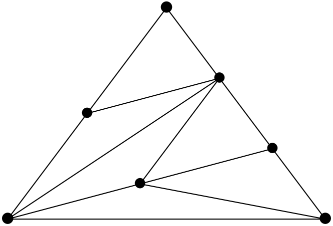

One obtains T-Hilb A-Hilb from flopping the $(-1,-1)$-curve labelled with χ_3, the only character lifted from the quotient T.

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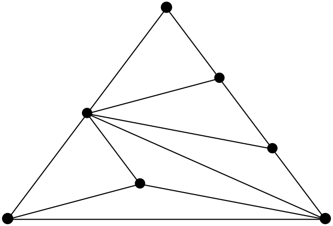
As G is a direct product we can also compute A-Hilb T-Hilb $\mathbb{A}^3$.



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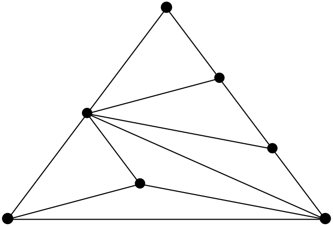

In this case two flops are required to reach A-Hilb T-Hilb, first in the curve marked with χ_4, then in the image of the curve marked with χ_2 in G-Hilb. These are exactly the characters lifted from A.

Algebraic evidence

- Ishii–Ito–Nolla de Celis construct a stability condition ϑ defining T-Hilb A-Hilb $\mathbb{C}^3$. It is not so hard to show that their stability condition satisfies the following:

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Lemma

Let χ be an irreducible representation of G. Then $\vartheta(\chi) < 0$ if and only if χ is lifted from T.

Algebraic evidence

- Recall that $\mathfrak{C}_0$ contains the locus where $\theta(\chi) > 0$ for all nontrivial irreducible representations χ.

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Algebraic evidence

- Recall that $\mathfrak{C}_0$ contains the locus where $\theta(\chi) > 0$ for all nontrivial irreducible representations χ.
- Hence, it is plausible that the negativity of $\vartheta(\chi)$ will contrast with the positivity in $\mathfrak{C}_0$.
- We hope to use the explicit expressions of walls of $\mathfrak{C}_0$ to codify this and offer further evidence towards the conjecture stated above.

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