# On the crepant Fujiki-Oka resolutions

( Joint work with Yusuke Sato )

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# 1. The Gorenstein Quotient Singularities in dimension 2

1. $(\mathbb{C}^2/G, 0)$: a cyclic quotient singularity.

$\Rightarrow$ $^\exists$**The correspondence w.r.t. self-intersection numbers.**

{The coefficients of the Hirzebruch-Jung continued fraction}
$\longleftrightarrow$ {The self-intersection numbers of the excep. div. of the min. resol. of $\mathbb{C}^2/G$}

2. $(\mathbb{C}^2/G, 0)$: a Gorenstein quotient singularity.

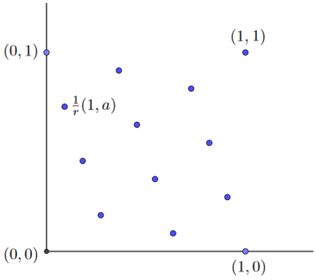
$\Rightarrow$ $^\exists$**The Mckay correspondence.**

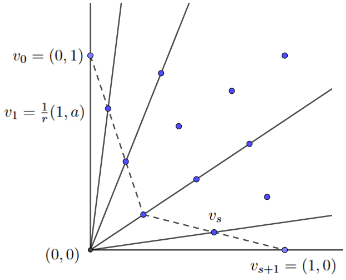
{The non-trivial irreducible representations of $G$}
$\longleftrightarrow$ {The excep. div. of the min. resol. of $\mathbb{C}^2/G$}

# Example 1.

$\mathbb{C}^2/G$: a C.Q.S. of $\frac{1}{r}(1,a)$-type where $1 \le a \le r-1$.
$(N', \sigma)$: a toric model of $\mathbb{C}^2/G$ where $N' := \mathbb{Z}^3 + \frac{1}{r}(1,a)\mathbb{Z}$, $\sigma := \mathbb{R}_{\ge 0}(1,0) + \mathbb{R}_{\ge 0}(0,1)$.

$$\frac{r}{a} = x_1 - \frac{1}{x_2 - \frac{1}{x_3 - \cdots \frac{1}{x_s}}} = [x_1, \ldots, x_s] \text{ where } x_1, \ldots, x_s \in \mathbb{Z}_{>0}.$$



# 2. The Gorenstein Quotient Singularities in higher dimension

2.1 A generalization of Hirzebruch-Jung continued fractions by Tadashi Ashikaga[1]

**Definition 1.**

$n \in \mathbb{Z}_{\geq 1}$, $r \in \mathbb{N}$, $\mathbf{a} = (a_1, \ldots, a_n) \in \mathbb{Z}^n$ s.t. $0 \leq a_i \leq r-1$ $(1 \leq i \leq n)$.
We call the symbol

$$\frac{\mathbf{a}}{r} = \frac{(a_1, \ldots, a_n)}{r}$$

an $n$-**dimensional proper fraction**.

[1] T. Ashikaga, *Multidimensional continued fractions for cyclic quotient singularities and Dedekind sums*, Kyoto J. Math. Vol. **59** (2019), no.4, 993–1039.

## Definition 2.

A proper fraction s.t. at least one component of $\mathbf{a}$ is $1$ is called **semi-unimodular**.

## Setting 1.

A semi-unimodular proper fraction forms as the following.

$$\frac{\mathbf{a}}{r} = \frac{(1, a_2, \ldots, a_n)}{r}$$

## Definition 3.

The **age** of an $n$-dimensional proper fraction $\frac{\mathbf{a}}{r} = \frac{(a_1,\ldots,a_n)}{r}$ is defined as

$$\mathrm{age}\left(\frac{\mathbf{a}}{r}\right) = \frac{1}{r}\sum_{i=1}^{n} a_i.$$

## Notation 1.

$\mathbb{Q}_n^{prop}$: the set of $n$-dimensional proper fractions.

$\overline{\mathbb{Q}_n^{prop}} := \mathbb{Q}_n^{prop} \cup \{\infty\}$.

$\overline{\mathbb{Z}^n} := \mathbb{Z}^n \cup \{\infty\}$.

## Definition 4.

$\frac{\mathbf{a}}{r}$: an $n$-dimensional semi-unimodular proper fraction.

$(i)$ For $2 \leq i \leq n$, the $i$-**th round down map** $Z_i : \overline{\mathbb{Q}_n^{prop}} \to \overline{\mathbb{Z}^n}$ is defined by

$$Z_i\left(\frac{(1, a_2, \ldots, a_n)}{r}\right) = \begin{cases} \left(\lfloor \frac{1}{a_i} \rfloor, \lfloor \frac{a_2}{a_i} \rfloor, \ldots, \lfloor \frac{a_{i-1}}{a_i} \rfloor, \lfloor \frac{-r}{a_i} \rfloor, \lfloor \frac{a_{i+1}}{a_i} \rfloor, \ldots, \lfloor \frac{a_n}{a_i} \rfloor\right) & \text{if } a_i \neq 0 \\ \infty & \text{if } a_i = 0 \end{cases}$$

and $Z_i(\infty) = \infty$ where $\lfloor x \rfloor$ is the greatest integer not exceeding $x$.

$(ii)$ For $2 \leq i \leq n$, the $i$-**th remainder map** $R_i : \overline{\mathbb{Q}_n^{prop}} \to \overline{\mathbb{Q}_n^{prop}}$ is defined by

$$R_i\left(\frac{(1, a_2, \ldots, a_n)}{r}\right) = \begin{cases} \left(\frac{\overline{1}^{a_i}, \overline{a_2}^{a_i}, \ldots, \overline{a_{i-1}}^{a_i}, \overline{-r}^{a_i}, \overline{a_{i+1}}^{a_i}, \ldots, \overline{a_n}^{a_i}}{a_i}\right) & \text{if } a_i \neq 0 \\ \infty & \text{if } a_i = 0 \end{cases}$$

and $R_i(\infty) = \infty$ where $\overline{a_j}^{a_i}$ is an integer satisfying $0 \leq \overline{a_j}^{a_i} < a_i$ and $\overline{a_j}^{a_i} \equiv a_j$ modulo $a_i$.

## Example 2.

$$v = \frac{(1,2,5)}{8}$$

$$\begin{aligned} Z_2(v) &= (0,-4,2), \\ Z_3(v) &= (0,0,-2), \\ R_2(v) &= \frac{(1,0,1)}{2}, \\ R_3(v) &= \frac{(1,2,2)}{5}. \end{aligned}$$

## Definition 5.

$\frac{\mathbf{a}}{r}$: an $n$-dimensional semi-unimodular proper fraction.

(i) The **remainder polynomial** $\mathcal{R}_*\left(\frac{\mathbf{a}}{r}\right) \in \overline{\mathbb{Q}_n^{prop}}[x_2, \ldots, x_n]$ is defined by

$$\mathcal{R}_*\left(\frac{\mathbf{a}}{r}\right) = \frac{\mathbf{a}}{r} + \sum_{(i_1, i_2, \ldots, i_l) \in \mathbf{I}^l,\, l \geq 1} (R_{i_l} \cdots R_{i_2} R_{i_1})\left(\frac{\mathbf{a}}{r}\right) \cdot x_{i_1} x_{i_2} \cdots x_{i_l}$$

where we exclude terms with coefficients $\infty$ or $\frac{(0,0,\ldots,0)}{1}$.

(ii) The **round down polynomial** $Z_*\left(\frac{\mathbf{a}}{r}\right) \in \overline{\mathbb{Z}^n}[x_2, \ldots, x_n]$ is defined by

$$\mathcal{Z}_*\left(\frac{\mathbf{a}}{r}\right) = \sum_{j=2}^{n} Z_j\left(\frac{\mathbf{a}}{r}\right) x_j + \sum_{j=2}^{n} \sum_{(i_1, i_2, \ldots, i_l) \in \mathbf{I}^l,\, l \geq 1} (Z_j R_{i_l} \cdots R_{i_2} R_{i_1})\left(\frac{\mathbf{a}}{r}\right) \cdot x_{i_1} x_{i_2} \cdots x_{i_l} x_j$$

where $\mathbf{I} = \{2, \ldots, n\}$ signifies the index set of the variables.

## Example 3.

Let $v = \frac{(1,2,8)}{11}$, then the remainder polynomial is

$$\begin{aligned}\mathcal{R}_*\left(\frac{(1,2,8)}{11}\right) = \frac{1}{11}(1,2,8) &+ \frac{1}{2}(1,1,0)x_2 + \frac{1}{8}(1,2,5)x_3 \\ &+ \frac{1}{2}(1,0,1)x_3x_2 + \frac{1}{5}(1,2,2)x_3x_3 \\ &+ \frac{1}{2}(1,1,0)x_3x_3x_2 + \frac{1}{2}(1,0,1)x_3x_3x_3.\end{aligned}$$

The round down polynomial is

$$\begin{aligned}\mathcal{Z}_*\left(\frac{(1,2,8)}{11}\right) &= (0,-6,4)x_2 + (0,0,-2)x_3 \\ &+ (1,-4,2)x_3x_2 + (0,0,-2)x_3x_3 \\ &+ (0,-3,1)x_3x_3x_2 + (0,1,-3)x_3x_3x_3.\end{aligned}$$

$$\mathcal{R}_*\left(\frac{(1,2,8)}{11}\right) = \frac{1}{11}(1,2,8) \quad + \quad \frac{1}{2}(1,1,0)x_2 + \frac{1}{8}(1,2,5)x_3$$
$$+ \quad \frac{1}{2}(1,0,1)x_3x_2 + \frac{1}{5}(1,2,2)x_3x_3$$
$$+ \quad \frac{1}{2}(1,1,0)x_3x_3x_2 + \frac{1}{2}(1,0,1)x_3x_3x_3.$$

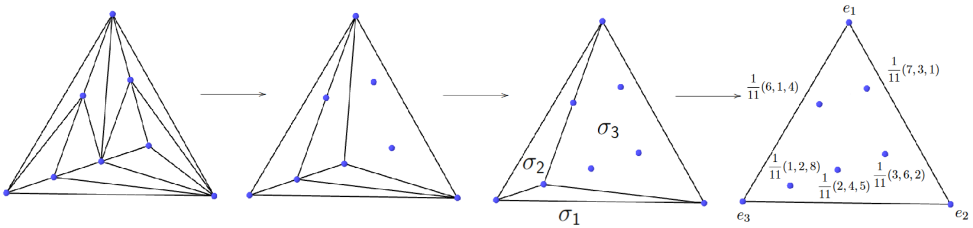



Figure: The basic triangulation of $\mathfrak{s}_G$ by **Fujiki-Oka resolution**

**Fact 1.**

For a **semi-isolated quotientsingularity** (i.e. a C.Q.S. of $\frac{1}{r}(1, a_2, \ldots, a_n)$-type), every Fujiki-Oka resolution is always smooth and have a relation with a **multi-dimensional continued fraction** (i.e. a pair of a remainder polynomial and a round down polynomial).

**Question 1.**

When does the McKay correspondence on the Fujiki-Oka resolutions hold?

**Question 2.**

When is a Fujiki-Oka resolution crepant?

## 2.2 Necessary and sufficient condition for the Fujiki-Oka resolutions to be crepant

**Theorem 1. (K.S, Y.Sato)**

For a C.Q.S. of $\frac{1}{r}(1, a_2, \ldots, a_n)$-type, the corresponding Fujiki-Oka resolution is crepant if and only if the ages of all the coefficients of $\mathcal{R}_* \left( \frac{(1, a_2, \ldots, a_n)}{r} \right)$ are $1$.

**Outline of the proof.**
1. $G := \left\langle \frac{1}{r}(1, a_2, \ldots, a_n) \right\rangle$ s.t. $1 + a_2 + \cdots + a_n \geq 2r \Rightarrow \mathbb{C}^n/G$ has no toric crepant resolutions.

2. Assume $G := \left\langle \frac{1}{r}(1, a_2, \ldots, a_n) \right\rangle$ s.t. $1 + a_2 + \cdots + a_n = r$.
The Fujiki-Oka resolution of $\mathbb{C}^n/G$ is crepant $\Leftrightarrow$ all the $\mathbb{C}^n/G_i$ have a toric crepant resol.
where $\mathbb{C}^n/G_i$ is the C.Q.S. of $\frac{1}{a_i}(1, \overline{a_2}^{a_i}, \ldots, \overline{a_{i-1}}^{a_i}, \overline{-r}^{a_i}, \overline{a_{i+1}}^{a_i}, \ldots, \overline{a_n}^{a_i})$-type.

3. For $\boldsymbol{x} = (x_1, \ldots, x_n) \in N_{i\mathbb{R}}$, the map $\phi_i : N_{i\mathbb{R}} \hookrightarrow N'_{\mathbb{R}}$ is defined as follows:
$\phi_i(\boldsymbol{x}) = \left(x_1 + \frac{1}{r}x_i, x_2 + \frac{a_2}{r}x_i, \ldots, x_{i-1} + \frac{a_{i-1}}{r}x_i, \frac{a_i}{r}x_i, x_{i+1} + \frac{a_{i+1}}{r}x_i, \ldots, x_n + \frac{a_n}{r}x_i\right)$
where $X(N_i, \sigma) \cong \mathbb{C}^n/G_i$ and $N_i = \mathbb{Z}^n + \frac{1}{a_i}(1, \overline{a_2}^{a_i}, \ldots, \overline{a_{i-1}}^{a_i}, \overline{-r}^{a_i}, \overline{a_{i+1}}^{a_i}, \ldots, \overline{a_n}^{a_i})\mathbb{Z}$.

□

## Corollary 1.

For all three dimensional semi-isolated Gorenstein quotient singularities, the Fujiki-Oka resolutions are crepant.

**Outline of the proof.**

1. $1 + a_2 + a_3 + \cdots + a_n = r \Rightarrow \mathrm{age}\left(\mathcal{R}_i\left(\frac{(1, a_2, \ldots, a_n)}{r}\right)\right) \in \mathbb{Z}$.
2. $1 + \overline{-r}^a + \overline{b}^a < 2a$
3. By Theorem 1.

□

**Question 3.**

Does there exist a crepant fujiki-Oka resolution if a semi-isolated singularity $\mathbb{C}^n/G$ has a crepant resolution? $\to$ No.

**Example 4.**

The C.Q.S. of $\frac{1}{40}(1,3,9,27)$-type has a toric crepant resolution by [2]. However, the Fujiki-Oka resolution is not crepant.

[2] D. I. Dais, M. Henk, and G. M. Ziegler, *On the existence of crepant resolutions of Gorenstein Abelian quotient singularities in dimensions*$\geq 4$, Contemp. Math. **423**, Amer. Math. Soc., Providence, RI, 2006.

$$R_*\left(\frac{(1,3,9,27)}{40}\right) = \frac{(1,3,9,27)}{40} + \frac{(1,2,0,0)}{3}x_2 + \frac{(1,3,5,0)}{9}x_3 + \frac{(1,3,9,14)}{27}x_4$$

$$+ \frac{(1,1,0,0)}{2}x_2x_2 + \frac{(1,0,2,0)}{3}x_3x_2 + \frac{(1,3,1,0)}{5}x_3x_3 + \frac{(1,0,0,2)}{3}x_4x_2$$

$$+ \frac{(1,3,0,5)}{9}x_4x_3 + \frac{(1,3,9,1)}{14}x_4x_4 + \frac{(1,0,1,0)}{2}x_3x_2x_3 + \frac{(1,1,1,0)}{3}x_3x_3x_3$$

$$+ \frac{(1,0,0,1)}{2}x_4x_2x_4 + \frac{(1,0,0,2)}{3}x_4x_3x_2 + \frac{(1,3,0,1)}{5}x_4x_3x_4 + \frac{(1,1,0,1)}{3}x_4x_4x_2$$

$$+ \frac{(1,3,4,1)}{9}x_4x_4x_3 + \frac{(1,0,0,1)}{2}x_4x_3x_2x_4 + \frac{(1,1,0,1)}{3}x_4x_3x_4x_2$$

$$+ \frac{(1,0,1,1)}{3}x_4x_4x_3x_2 + \boxed{\frac{(1,3,3,1)}{4}x_4x_4x_3x_3} + \cdots$$

The age is 2.

## 2.3 The Fujiki-Oka resolutions in abelian case

$G \subset SL(n, \mathbb{C})$: a finite abelian subgroup.
Assume the ages of all the generators of $G$ are $1$.

There exist a **basic generating system** of $G$ as follows:

$$\left\{ \frac{1}{r_1}(a_{11}, a_{12}, \ldots, a_{1n}), \frac{1}{r_2}(0, a_{22}, \ldots, a_{2n}), \ldots, \frac{1}{r_{n-1}}(0, \ldots, 0, a_{n-1\ n-1}, a_{n-1\ n}) \right\}$$

where $r_i, a_{ij}$ $(1 \leq i \leq n-1,\ i \leq j \leq n)$ are positive integers satisfying $\mathrm{LCM}(r_1, \ldots, r_{n-1}) = |G|$ and the following conditions:

(i) $a_{ii} = 0 \Rightarrow a_{ij} = 0$ for $i \leq j \leq n$,

(ii) $a_{ii} \neq 0 \Rightarrow a_{ii} = 1$ and $\displaystyle\sum_{j=i}^{n} a_{ij} = r_i$.

$G$ can be decomposed to the cyclic components as follows:

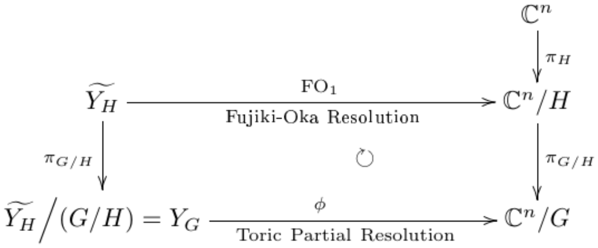
$$G \cong \left\langle \frac{1}{r_1}(a_{11}, a_{12}, a_{13}) \right\rangle \times \cdots \times \left\langle \frac{1}{r_{n-1}}(0, \ldots, 0, a_{n-1\ n-1}, a_{n-1\ n}) \right\rangle .$$

**Note 1.**

Clearly, every cyclic component can be decomposed to the product of $p$-Sylow subgroups.

$G \subset SL(n, \mathbb{C})$: a finite abelian subgroup.
$H$: a component of the above decomposition by cyclic subgroups of $G$.

$\mathbb{C}^n/H$: semi-isolated $\Rightarrow$ $^\exists(\widetilde{Y_H}, \mathrm{FO}_1)$: Fujiki-Oka resolution, $^\exists(Y_G, \phi)$: the toric partial resolution satisfying the following diagram:



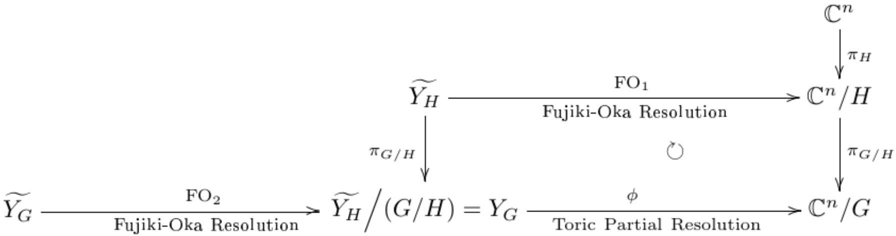

where $\pi_H$ (resp. $\pi_{G/H}$) is the quotient map by $H$ (resp. $G/H$).

All the singularities in $Y_G$ are semi-isolated $\Rightarrow$ $^{\exists}(\widetilde{Y_G}, \mathrm{FO}_2)$: a Fujiki-Oka resolutions for the quotient singularities in $Y_G$.

$$\widetilde{Y_G} \xrightarrow[\text{Fujiki-Oka Resolution}]{\mathrm{FO}_2} \widetilde{Y_H}\Big/(G/H) = Y_G$$

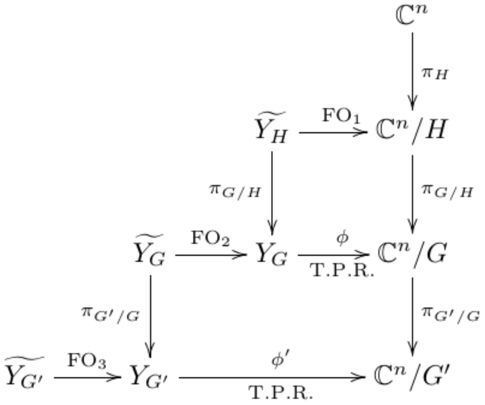
## Note 2.

Every singularity in $Y_G$ is at worst Gorenstein cyclic quotient singularity which is canonical but not terminal because of the construction.




## Definition 6.

We call the resolution $(\widetilde{Y_G}, \mathrm{FO}_2 \circ \phi)$ in the above diagrams an **iterated Fujiki-Oka resolution** of $\mathbb{C}^n/G$.




As $(\widetilde{Y_{G'}}, \mathrm{FO}_3 \circ \phi')$ in the above, iterated Fujiki-Oka resolutions can be extended under the suitable conditions. We also call these resolutions and the ordinary Fujiki-Oka resolutions **iterated Fujiki-Oka resolutions**.

## Lemma 1.

$G \subset SL(n, \mathbb{C})$: a finite abelian subgroup.
There exists at least one iterated Fujiki-Oka resolution for $\mathbb{C}^n/G$.

**Outline of the proof.**

$\left\{\frac{1}{r_1}(a_{11}, a_{12}, \ldots, a_{1n}), \ldots, \frac{1}{r_{n-1}}(0, \ldots, 0, a_{n-1\ n-1}, a_{n-1\ n})\right\}$: a basic generating system of $G$.

1. $H_1 = \left\langle \frac{1}{r_{n-1}}(0, \ldots, 0, a_{n-1\ n-1}, a_{n\ n}) \right\rangle$.
2. We have the Fujiki-Oka resolution $X(N_1, \Sigma_1)$ of the singularity $\mathbb{C}^n/H_1$.
3. $H_2 = \left\langle \frac{1}{r_{n-1}}(0, \ldots, 0, a_{n-2\ n-2}, a_{n-2\ n-1}, a_{n-2\ n}) \right\rangle \times \left\langle \frac{1}{r_{n-1}}(0, \ldots, 0, a_{n-1\ n-1}, a_{n-1\ n}) \right\rangle$.

4. We have the quotient map $\pi_{H_2/H_1} : \mathbb{C}^n/H_1 \to \mathbb{C}^n/H_2 = X(N_2, \Sigma_1)$.

Every maximal cone in $\Sigma_1$ is semi-unimodular, and we have an iterated Fujiki-Oka resolution $X(N_2, \Sigma_2)$.

5. By repeating similar operation to the above for the subgroup sequence:

$$H_1 \subset H_2 \subset \cdots \subset H_{n-1} = G,$$

we have the sequence of iterated Fujiki-Oka resolutions:

$$\widetilde{Y_{H_1}} = X(N_1, \Sigma_1), \widetilde{Y_{H_2}} = X(N_2, \Sigma_2), \ldots, \widetilde{Y_G} = X(N_{n-1}, \Sigma_{n-1}).$$

□

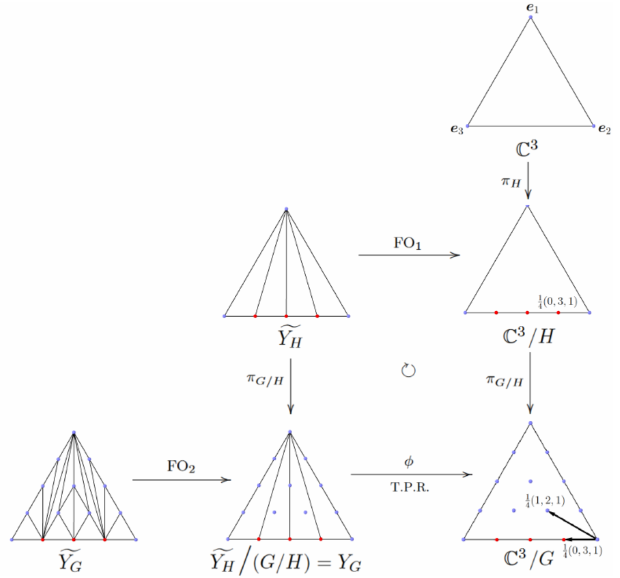
**Theorem 2. (K.S, Y.Sato)**

$\mathbb{C}^n/G$: an $n$-dimensional Gorenstein abelian quotient singularity.
$\widetilde{Y_{H_1}}, \widetilde{Y_{H_2}}, \ldots, \widetilde{Y_{H_k}} = \widetilde{Y_G}$: the above sequence of iterated Fujiki-Oka resolutions for $\mathbb{C}^n/G$.
If the ages of all the coefficients in the remainder polynomials associated with every $\widetilde{Y_{H_i}}$ $(i = 1, \ldots, k)$ are $1$, then the corresponding iterated Fujiki-Oka resolution $\widetilde{Y_G}$ for $\mathbb{C}^n/G$ is crepant.

**Corollary 2.**

Assume that $G$ is a finite abelian subgroup of $SL(3, \mathbb{C})$. Then a crepant iterated Fujiki-Oka resolution exists for $\mathbb{C}^3/G$.

**Example 5.** $G := \left\langle \frac{1}{4}(1,3,0), \frac{1}{4}(0,1,3) \right\rangle$.



# 3. Related topics

1. $G := \langle \frac{1}{r}(1, 1, r-2) \rangle \subset SL(3, \mathbb{C}) \Rightarrow$ $^{\exists}$A unique projective crepant Fujiki-Oka resolution of $\mathbb{C}^3/G$ which coincides with $\mathrm{A}\text{-}\mathrm{Hilb}(\mathbb{C}^3)$.

2. $\mathbb{C}^3/G$: an isolated Gorenstein quotient singularity $\Rightarrow$ The Fujiki-Oka resolutions can be obtained from three ways by changing generator of $G$.

$\rightarrow$ It can be shown that the Fujiki-Oka resolution is isomorphic to $\mathrm{A}\text{-}\mathrm{Hilb}(\mathbb{C}^3)$ if and only if these three Fujiki-Oka resolutions are isomorphic to each other as toric varieties.

**Conjecture 1.**

Let $\mathbb{C}^3/G$ be a semi-isolated Gorenstein quotient singularity. If a Fujiki-Oka resolution is isomorphic to $\mathrm{A}\text{-}\mathrm{Hilb}(\mathbb{C}^3)$, then the projective toric crepant resolution of $\mathbb{C}^3/G$ exists uniquely up to isomorphism as toric varieties.

In the case that $G = \left\langle \frac{1}{r}(1, a, r-a) \right\rangle$, $X = \mathbb{C}^3/G$ has an economic resolution.

**Definition 7.**

Let $G = \left\langle \frac{1}{r}(1, a, r-a) \right\rangle$ and $N' = \mathbb{Z}^3 + \frac{1}{r}(1, a, r-a)\mathbb{Z}$. Let $\boldsymbol{v}_i = \frac{1}{r}(i, \overline{ai}^r, \overline{r-ai}^r) \in N'$ for each integer $1 \leq i \leq r-1$. The **economic resolution** of $\mathbb{C}^3/G$ is obtained by the consecutive weighted blow-ups at $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{r-1}$ from $\mathbb{C}^3/G$.

Since the weighted blow-up with $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_{r-1}$ coincides with the Fujiki-Oka resolution, the Fujiki-Oka resolution is an economic resolution. As S.J.Jung showed, economic resolutions can be expressed in some moduli spaces. Thus, Fujiki-Oka resolutions can be written as a moduli space in the case of $G = \left\langle \frac{1}{r}(1, a, r-a) \right\rangle$.

### Theorem 3. [3]

The economic resolution $Y$ of a three fold terminal quotient singularity $X = \mathbb{C}^3/G$ is isomorphic to the birational component $Y_\theta$ of the moduli space $\mathcal{M}_\theta$ of $\theta$-stable $G$-constellations for a suitable parameter $\theta$.

Kedzierski has shown that $\mathrm{A\text{-}Hilb}(\mathbb{C}^3)$ is an economic resolution in some special cases.

### Theorem 4. [4]

Let $G \subset GL(3, \mathbb{C})$ be the finite subgroups generated by $\frac{1}{r}(1, a, r-a)$ with $a = 1$ or $r-1$. Then $\mathrm{A\text{-}Hilb}(\mathbb{C}^3)$ is isomorphic to the economic resolution of the quotient variety $\mathbb{C}^3/G$.

[3] S. J. Jung, *Terminal Quotient Singularities in Dimension Three via Variation of GIT*, Jour. of Algebra **468** (2016) 354–394.
[4] O. Kedzierski, *Cohomology of the $G$-Hilbert scheme for $\frac{1}{r}(1,1,r-1)$*, Serdica Math. J. **30** (2004), no.2-3, 293–302.

**Thank you for listening!**

For the details, see **arXiv:2004.03522 [math.AG]**.