The McKay correspondence, mutation and related topics

Report of Contributions

https://indico.ipmu.jp/e/314

Contribution ID: **1** Type: **not specified**

8/14 [Karin Baur] Cluster categories of Grassmannians

The category of maximal CM modules over a quotient of a preprojective algebra is an additive categorification of Scott's cluster structure on the coordinate ring of the Grassmannian (Jensen-King-Su). We study this category and associated root combinatorics.
This is joint work with D. Bogdanic, A. Garcia Elsener and with J. Li

Contribution ID: **2** Type: **not specified**

7/24 [Michael Wemyss] Tits Cone Intersections and Applications

I will explain some new structures that can be obtained from ADE Dynkin diagrams, which visually are very beautiful, and have some surprising applications to both two-dimensional and three-dimensional algebraic geometry. Most of the talk will explain how to construct these new objects, and will explain some of the combinatorial results that we can prove about them. I will then highlight briefly some (new!) applications to Kleinian singularities, and also some applications to 3-fold flopping contractions through mutation and stability conditions. This is joint work with Yuki Hirano, and with Osamu Iyama.

Contribution ID: **3** Type: **not specified**

7/31 [Timothy Logvinenko] McKay correspondence and perverse schobers

A classical result by Seidel and Thomas shows that there is a faithful categorical action of the braid group Br_n on the derived category D(Y) of the minimal resolution Y of the Kleinian singularity C^2/G of A_{n-1}-type. The generators of Br_n act by spherical twists around the exceptional curves of Y. Recall that the classifying space of Br_n is the big open stratum (h/W)*0 of h/W stratified by positive roots, where h is a Cartan subalgebra of the corresponding Lie algebra sl*{n} and W is the Weil group. We can therefore view a categorical action of Br_n on D^b(Y) as a local system of triangulated categories on (h/W)_0. In this talk, I will discuss a joint ongoing work with my PhD student Chris Seaman to extend this local system to a perverse schober on the whole of h/W. The idea is to use well-known interpretation of h/W as the theta-stability parameter space for the GIT problem which constructs Y as a moduli space of G-constellations. We then aim to construct a "window-shift" shober for this GIT problem similar to that recently constructed by Spenko and van den Bergh in the Halpern-Leistner and Sam setup of a quasi-symmetric linear action of a reductive group. In our case, the action isn't quasi-symmetric, and numerous complications arise.

Contribution ID: **4** Type: **not specified**

7/17 [Ayako Kubota] On the G-Hilbert scheme of the closure of the regular nilpotent orbit of type A

Adjoint orbits of nilpotent elements in a semisimple Lie algebra are called nilpotent orbits, and their closures are known to have symplectic singularities. In this talk, we consider nilpotent orbits of type A, and we discuss resolutions of singularities of the closure of the regular nilpotent orbit by means of the G-Hilbert scheme associated with the Cox realization of the singularity.

Contribution ID: **5** Type: **not specified**

7/17 [Liana Heuberger] Combinatorial Reid's Recipe for Dimer Models

Reid's recipe marks interior line segments and lattice points in the fan of G-Hilb with certain nontrivial irreducible representations of G. Our goal is to generalise this by marking the toric fan of a crepant resolution of any affine Gorenstein singularity, in a way that is compatible with both the G-Hilb case and its categorical counterpart known as Derived Reid's Recipe. The result is a combinatorial version of the algorithm via consistent dimer models, whose key ingredient is adapting Nakamura's jigsaw transformations for G-Hilb to our context. This is joint work with Alastair Craw and Jesus Tapia Amador.

Contribution ID: **6** Type: **not specified**

7/17 [Alastair Craw] Punctual Hilbert schemes of Kleinian singularities

It is well known that the Hilbert scheme of n points on the minimal resolution of a Kleinian singularity is a Nakajima quiver variety, but what about the Hilbert scheme of n points on the Kleinian singularity itself? I'll describe joint work with Gammelgaard, Gyenge and Szendroi in which we construct these Hilbert schemes as quiver varieties for the framed McKay quiver using multigraded linear series.

Contribution ID: **7** Type: **not specified**

7/17 [Takehiko Yasuda] The motivic McKay correspondence in arbitrary characteristics

I will speak about a recent result of mine, which has been the main conjecture in the filed for years. In characteristic zero, the McKay correspondence in terms of motivic invariants is formulated as the equality of the stringy motive of the quotient variety in question and some finite sum of powers of L in a version of the complete Grothendieck ring of varieties which is taken over conjugacy classes of the given finite group. In generalization to arbitrary characteristic, the finite sum is replaced with a motivic integral over the moduli space of G-torsors over the punctured formal disk. I also talk about an application to singularities.

Contribution ID: **9** Type: **not specified**

7/24 [Eleonore Faber] Complex reflection groups, their McKay quivers, and the McKay correspondence

Finite complex reflection groups were classified by Shepherd and Todd: up to finitely many exceptions they are the groups G(r,p,n) or the Symmetric groups. This talk is about a combinatorial description of the McKay quivers of the groups G(r,p,n).
Furthermore, I will comment on a McKay correspondence for complex reflection groups. This is joint work with R.-O. Buchweitz, C. Ingalls, and M. Lewis.

Contribution ID: **10** Type: **not specified**

8/14 [Xiuping Su] Categorification and the quantum Grassmannian

Let CM(A) be the category of Cohen-Macaulay modules of a certain Gorenstein order. Equivalently,

CM(A) is a category of equivariant Cohen-Macaulay modules for the plane curve singularity $x^k = y^{n-k}$

This category provides an (additive) categorification for the Grassmannian cluster algebra $\mathbb{C}[\mathrm{Gr}(k,n)]$.

In this talk, I will define an invariant $\kappa(M,N)$ for $M, N \in \mathrm{CM}(A)$ and discuss its properties. I will then explain

how to use this invariant to construct quantum seed data and its link to Newton-Okounkov bodies constructed by Rietsch-Williams.
The quantum seed is compatible with mutations and it determines a quantum cluster algebra, which is isomorphic to the quantum Grassmannian.
This talk is based on joint work with B T Jensen and A King.

Contribution ID: **11** Type: **not specified**

8/ 7 [Miles Reid] Trihedral groups: old and new ideas and methods, (with many problems still to solve)

Section 1. What groups are we talking about? G = A x) T with A diag in SL(3) and T the 3-cycle (x,y,z)
Section 2. Affine pieces of G-Hilb corr. to combinatorics (a) Leng partitions (b) trihedral boats
Section 3. (a) and (b) by computer algebra: Running my Magma code is a fun, easy do-it-yourself game:
follow the instructions on https:/homepages.warwick.ac.uk/~masda/McKay/tri
Section 4. Compute some affine pieces in the baby case A = V4 = mu2 + mu2 by generators and relations
(or as quiver moduli, somewhat implicitly).
My title says ideas, methods and open problems, not results or theorems.

Contribution ID: **12** Type: **not specified**

7/24 [Jenny August] Stability and McKay

Through the 3-dimensional McKay Correspondence, we may associate a finite-dimensional algebra, known as a contraction algebra, to each minimal model of certain 3-fold singularities. By sitting at the intersection of the worlds of finite-dimensional algebras and geometry, contraction algebras have some remarkable properties. In this talk, I'll describe how these properties allow us to easily determine the stability manifolds of these algebras and moreover, I'll try to describe why the corresponding story for surfaces is not so simple.

Contribution ID: **13** Type: **not specified**

7/24 [Sibylle Schroll] Grassmannian categories of infinite rank from hypersurface singularities

We construct Grassmannian categories of infinite rank as graded maximal Cohen-Macauley modules over a hypersurface singularity, providing an infinite analogue of the Grassmannian cluster categories introduced by Jensen, King, and Su. We show that the generically free rank one modules in a Grassmannian category of infinite rank are in a structure preserving bijection with the Plücker coordinates in a Grassmannian cluster algebra of infinite rank. This follows from a combinatorial dimension formula for the extensions between generically free rank one modules. This is joint work with Jenny August, Man-Wai Cheung, Eleonore Faber and Sira Gratz.

Contribution ID: **14** Type: **not specified**

7/31 [Rina Anno] Nil Hecke bimodule categories

Suppose we have a triangulated category with a DG-enhanceable braid group action, such as the derived category of coherent sheaves on the minimal resolution of a Kleinian singularity. Then we can use the generators of the braid group action to cook up a new triangulated category with the same objects using a construction that is similar to that of the nil Hecke algebra, a network of other triangulated categories corresponding to its "block subalgebras", and functors between them corresponding to certain diagrams. In particular, this network includes the categories and functors (for each generator) that Ed Segal used in 2016 to prove that every derived autoequivalence is a spherical twist. This is joint work in progress with Timothy Logvinenko.

Contribution ID: **15** Type: **not specified**

7/31 [Will Donovan] Windows on the Pfaffian-Grassmannian correspondence

The Pfaffian-Grassmannian correspondence relates certain pairs of non-birational Calabi-Yau threefolds which can be proved to be derived equivalent. I construct a family of derived equivalences using mutations of an exceptional collection on the relevant Grassmannian, and explain a mirror symmetry interpretation. This follows a physical analysis of Eager, Hori, Knapp, and Romo, and builds on work with Addington and Segal.

Contribution ID: **16** Type: **not specified**

7/31 [Amihay Hanany] Quivers for Hypersurface Symplectic Singularities

Symplectic singularities which are hypersurfaces are very rare. There is a conjectured list of all such singularities, where the most studied are of course the Klein (Du Val) singularities. Nakajima quivers are known for these and it is natural to ask for quivers of all the other hypersurfaces. This talk focuses on such quivers and includes less studied quivers like orthosymplectic and non simply laced quivers.

Contribution ID: **17** Type: **not specified**

7/31 [Ryo Yamagishi] Crepant resolutions and moduli of G-constellations for abelian groups

For a finite subgroup G of SL(n,C), a moduli space of G-constellations is a generalized notion of the G-Hilbert scheme, and it is expected that every (projective) crepant resolution X of C^n/G is obtained as such a moduli space. In the talk I will construct an explicit morphism from the resolution X to a moduli space for abelian G and discuss when it becomes an isomorphism.

Contribution ID: **18** Type: **not specified**

8/ 7 [Karin Schaller] Mirror symmetry constructions for quasi-smooth Calabi-Yau hypersurfaces in weighted projective spaces

We consider a general combinatorial framework for constructing mirrors of quasi-smooth Calabi-Yau hypersurfaces defined by weighted homogeneous polynomials. Our mirror construction shows how to obtain mirrors being Calabi-Yau compactifications of non-degenerate affine hypersurfaces associated to certain Newton polytopes. This talk is based on joint work with Victor Batyrev.

Contribution ID: **19** Type: **not specified**

8/ 7 [Johannes Hofscheier] Cohomology rings of toric bundles

The celebrated BKK theorem expresses the number of roots of a system of generic Laurent polynomials in terms of the mixed volume of the corresponding system of Newton polytopes. Pukhlikov and Khovanskii noticed that the cohomology ring of smooth projective toric varieties can be computed via this theorem. In this talk, I will report on joint work with Khovanskii and Monin where we extend this description to toric bundles. Our approach relies on a generalisation of the BKK theorem and a description of graded-commutative algebras which satisfy Poincaré duality. We conclude the presentation with some computations of cohomology rings.

Contribution ID: **20** Type: **not specified**

8/ 7 [Alvaro Nolla de Celis] On Reid's Recipe for non Abelian groups

I will show some examples of Reid's Recipe on G-HilbC^3 when G is a non Abelian finite subgroup of SL(3,C). For some cases I will comment on possible approaches towards the recipe for the general case.

Contribution ID: **21** Type: **not specified**

8/ 7 [Ben Wormleighton] Walls for G-Hilb via Reid's recipe

Many crepant resolutions of Gorenstein quotient singularities can be realised as moduli of quiver representations, which depends on a stability condition. The space of stability conditions has a wall-and-chamber structure that captures much, and in some cases all, of the birational geometry of the singularity. We study this chamber decomposition for abelian subgroups of SL(3) and give an explicit combinatorial / representation-theoretic description of the walls for the chamber corresponding to the G-Hilbert scheme. This description includes the geometry of each wall-crossing, and makes heavy use of Reid's recipe. We outline some work in progress using these results to compare the G-Hilbert scheme with another crepant resolution 'Hilb of Hilb' introduced by Ishii—Ito—Nolla de Celis.

Contribution ID: **22** Type: **not specified**

8/ 7 [Kohei Sato] On the crepant Fujiki-Oka resolutions

We show a necessary and sufficient condition for Fujiki-Oka resolutions of Gorenstein abelian quotient singularities to be crepant in all dimensions. Moreover, we prove that all three dimensional Gorenstein abelian quotient singularities possess a crepant Fujiki-Oka resolution as a corollary.

Contribution ID: **23** Type: **not specified**

8/14 [Akihiro Higashitani] Newton-Okounkov bodies of flag varieties and combinatorial mutations

Combinatorial mutations of polytopes ware introduced by Akhtar-Coates-Galkin-Kasprzyk in the context of mirror symmetry for Fano varieties. In this talk, the details of combinatorial mutations will be explained. As an application to the theory of Newton-Okounkov bodies of flag varieties, it will be also explaind that specific Newton-Okounkov of flag varieties, including string polytopes, Nakashima-Zelevinsky polytopes and FFLV polytopes, can be connected by iterated combinatorial mutations. This talk is based on the joint work with Naoki Fujita.

Contribution ID: **24** Type: **not specified**

8/14 [Yusuke Nakajima] Combinatorial mutations of polygons via dimer models

In this talk, I consider a dimer model on the real two-torus T, which is a bipartite graph described on T.
For a dimer model, we can assign the lattice polygon, and a dimer model enjoys rich information regarding toric geometry associated to such a polygon.
On the other hand, there is the operation called the combinatorial mutation of a polytope, which makes a given polytope another one.
This mutation is important to understand mirror partners for Fano manifolds.
Under these backgrounds, I expect that there is a certain operation for a dimer model that induces the combinatorial mutation of the associated polygon.
In my talk, I will introduce the operation which I call the deformation of a dimer model, and show that this operation realizes my expectation.
This talk is based on a joint work with A. Higashitani.

Contribution ID: **25** Type: **not specified**

8/14 [Louis-Philippe Thibault] Tilting objects in singularity categories and levelled mutations

In 1989, Reiten and Van den Bergh showed that for every finite subgroup G of SL(2,k), the skew-group algebra k[x,y]#G is Morita equivalent to the preprojective algebra over the extended Coxeter-Dynkin quiver associated to G via the McKay correspondence, thus providing another bridge between Kleinian singularities and representation theory. In the context of Iyama's higher Auslander-Reiten theory, it is natural to ask whether the same holds true for finite subgroups of SL(n,k) and higher preprojective algebras. In the first part of this talk, I will give a class of subgroups for which the skew-group algebra is not Morita equivalent to a higher preprojective algebra.

We will then move on to study the graded singularity category over the invariant ring k[x_1,…,x_n]^G. When the skew-group algebra is endowed with a grading giving it the structure of a preprojective algebra, Amiot, Iyama and Reiten showed that this category admits a tilting object. In the second part of this talk, we will be motivated by the case where the skew-group algebra does not admit such grading structure. We will explain that, in certain situations, one can use levelled mutations to obtain tilting objects in the graded singularity category.

Contribution ID: **26** Type: **not specified**

7/31 [Victor Batyrev] On the stringy E-functions of minimal models

The stringy E-function of a singular algebraic variety was invented for testing mirror symmetry in case when a singular Calabi-Yau variety does not admit a crepant resolution. In my talk I will explain how to apply the stringy E-functions in minimal model program and how to compute them for minimal models of non-degenerate hypersurfaces in toric varieties.

Contribution ID: **27** Type: **not specified**

7/17 [Mahito Tanno] The wild McKay correspondence for Z/p^n Z and its application

Poster

Contribution ID: **28** Type: **not specified**

7/24 [Okke an Garderen] Donaldson-Thomas invariants and flopping contractions

Poster

Contribution ID: **29** Type: **not specified**

7/24 [Toshiya Yurikusa] Dense g-vector fans for tame algebras

Poster

Contribution ID: **31** Type: **not specified**

8/14 [Yasuaki Gyoda] Compatibility degree of cluster complexes

(Poster) I will introduce a new function on the set of pairs of cluster variables, which we call it the compatibility degree (of cluster complexes). The compatibility degree which I deal with in this talk is a generalization of the "classical" compatibility degree introduced by Fomin and Zelevinsky. The classical one defines the generalized associahedra, and it is used to give the classification of cluster algebras of finite type. Cao and Li generalized this degree to that of cluster complexes by using d-vectors. We give another generalization by using f-vectors on the basis of their studies. This is joint work with Changjian Fu.

Contribution ID: **32** Type: **not specified**

8/14 [Haibo Jin] Simple-minded systems in cluster categories and singularity categories

Poster

Contribution ID: **33** Type: **not specified**

8/14 [Norihiro Hanihara] Cluster categories from Calabi-Yau algebras

Poster

Contribution ID: **34** Type: **not specified**

8/ 7 [Yusuke Sato] Crepant resolutions for two-parameter Cyclic Quotient Singularities

Poster