CROSS-CORRELATION OF THE ASTROPHYSICAL GW BACKGROUND WITH GALAXY CLUSTERING

VALERI VARDANYAN

(Based on arXiv:1910.08353, G. Cañas-Herrera, O. Contigiani, V.V.)

KAVLI-IPMU, FEBRUARY, 2020

LEIDEN AT THE SUNSET

Anisotropic background from astro sources

- Gravitational waves from distinct astrophysical mergers are already measured.

- This is an important channel of gravitational wave detection. However, one can also try to measure the superimposed signal from all the astrophysical mergers in the universe known as the stochastic gravitational wave background of astrophysical origin.

- This channel of gravitational wave detection might in fact contain a complementary information with respect to the standard channel mentioned above.

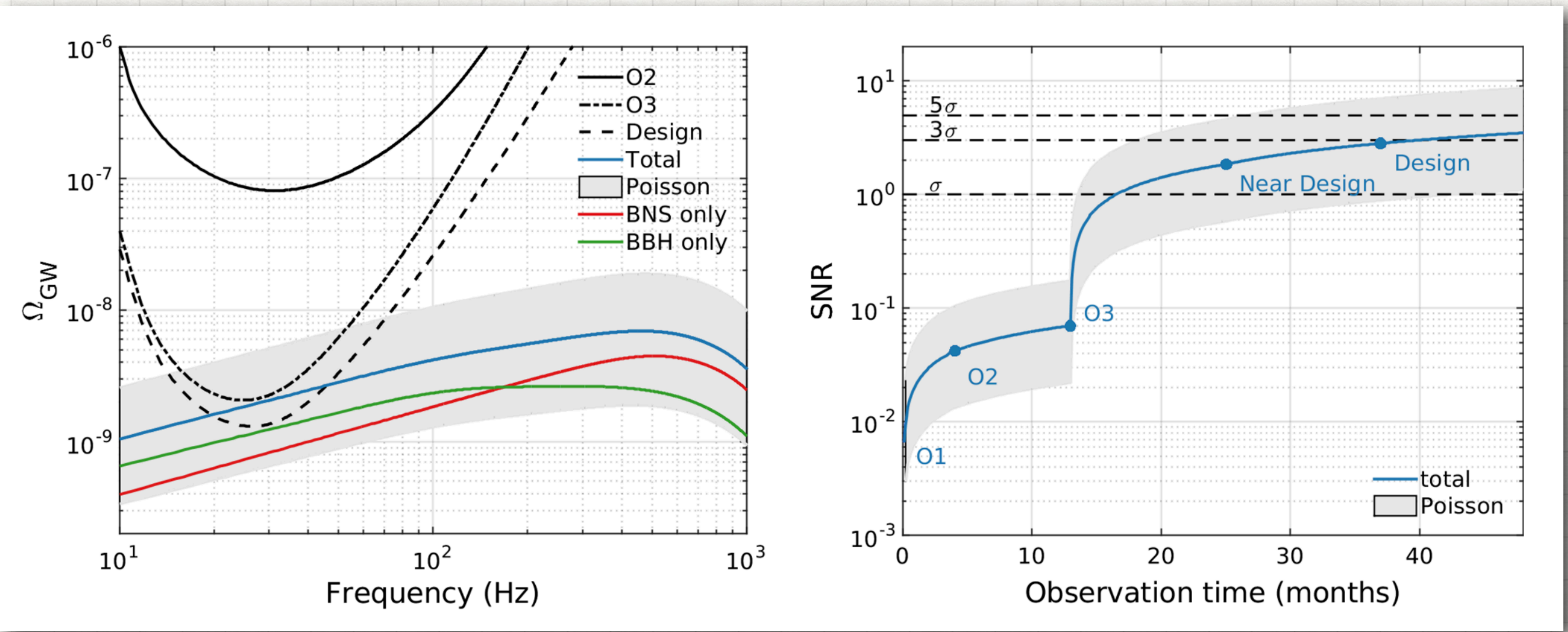
- One merit of this signal is that it also includes contributions from weak, individually unresolvable sources.

- Another important merit is the potential sensitivity to source redshift distribution.

- We explored the potential of the anisotropies of the astrophysical gravitational wave background to constrain the cosmological parameters and various ingredients of the source physics.

Anisotropic background from astro sources

$$\Omega_{\rm GW}(f,\theta) = \frac{f}{\rho_c H_0}\int_0^{z_{\rm max}} dz \frac{R_m(z;\theta) dE_{\rm GW}(f_s;\theta)/df_s}{(1+z)E(\Omega_M,\Omega_\Lambda,z)}.$$



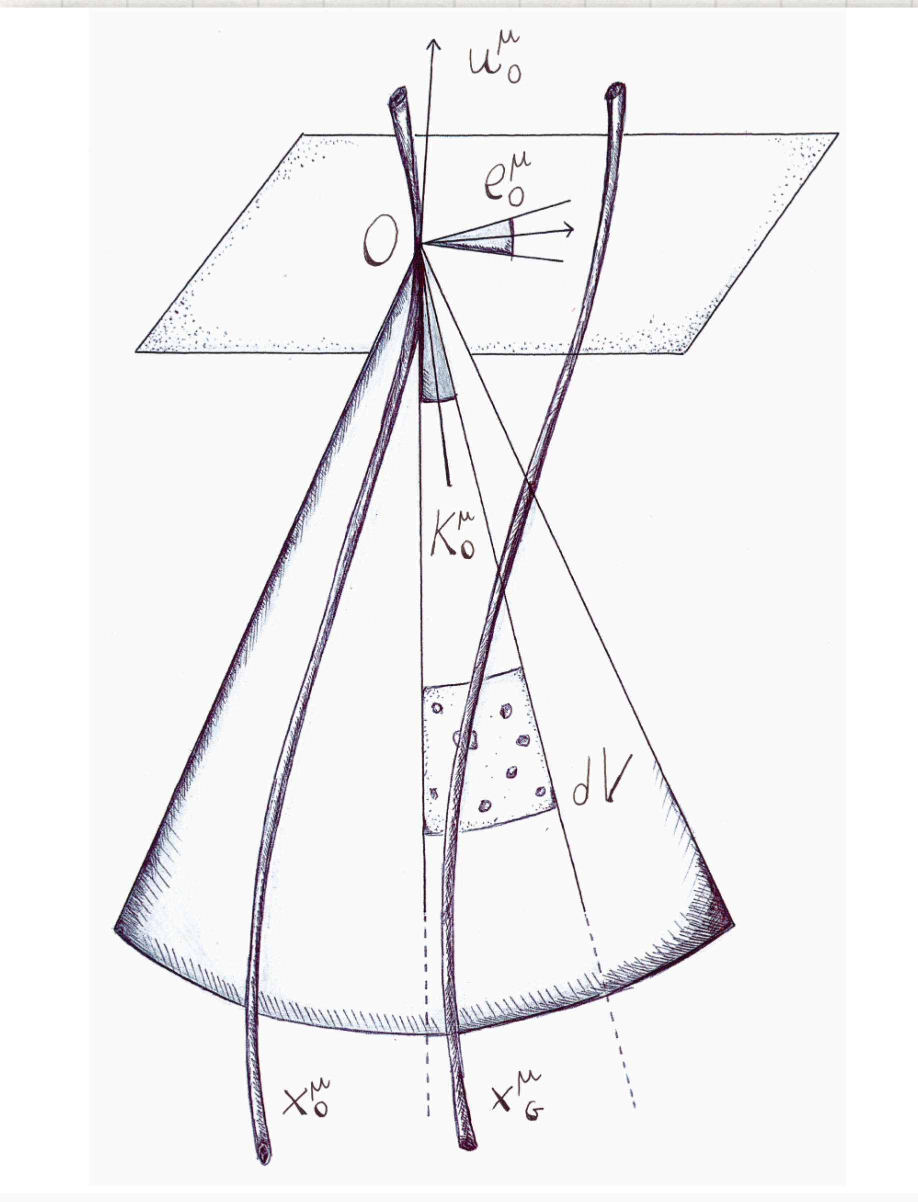
LIGO/VIRGO Collaboration: "GW170817: Implications for the Stochastic Gravitational-Wave Background from Compact Binary Coalescences"

Anisotropic background from astro sources

- **GW-s travel on perturbed backgrounds**
- **Wave trajectory parametrized by an affine parameter λ**
- **Tangent vector $k^\mu = \mathrm{d}x^\mu/\mathrm{d}\lambda$**
- **Satisfies the geodesic equation**

$$k^\mu \nabla_\mu k^\nu = 0$$

- **Integrate over past light-cone**



$$\frac{\mathrm{d}^3\rho_{\mathrm{GW}}}{\mathrm{d}\nu_{\mathrm{O}}\mathrm{d}^2\Omega_{\mathrm{O}}}(\nu_{\mathrm{O}}, e_{\mathrm{O}}) = \int \mathrm{d}\lambda \int \mathrm{d}\theta_{\mathrm{G}}\, \Phi\left[x^\mu(\lambda), \nu_{\mathrm{O}}, \theta_{\mathrm{G}}\right] \frac{\mathrm{d}^3\mathcal{N}_{\mathrm{G}}}{\mathrm{d}\lambda\, \mathrm{d}^2\Omega_{\mathrm{O}}}\left[x^\mu(\lambda), \theta_{\mathrm{G}}\right]$$

$$\mathrm{d}^3\mathcal{N}_{\mathrm{G}}\left[x^\mu(\lambda), \theta_{\mathrm{G}}\right] \equiv n_{\mathrm{G}}\left[x^\mu(\lambda), \theta_{\mathrm{G}}\right] \mathrm{d}^3V\left[x^\mu(\lambda)\right]$$

Cusin, Pitrou, Uzan, 2018

Anisotropic background from astro sources

$$\frac{\mathrm{d}^3\rho_{\rm GW}}{\mathrm{d}\nu_{\rm O}\mathrm{d}^2\Omega_{\rm O}}(\nu_{\rm O},e_{\rm O}) = \int \mathrm{d}\lambda \int \mathrm{d}\theta_{\rm G}\, \Phi\left[x^\mu(\lambda),\nu_{\rm O},\theta_{\rm G}\right]\, \frac{\mathrm{d}^3\mathcal{N}_{\rm G}}{\mathrm{d}\lambda\,\mathrm{d}^2\Omega_{\rm O}}\left[x^\mu(\lambda),\theta_{\rm G}\right]$$

- **Study perturbatively**

$$C_\ell(\nu_{\rm O}) = \frac{2}{\pi}\int \mathrm{d}k\, k^2 |\mathcal{E}_\ell(k,\nu_{\rm O})|^2$$

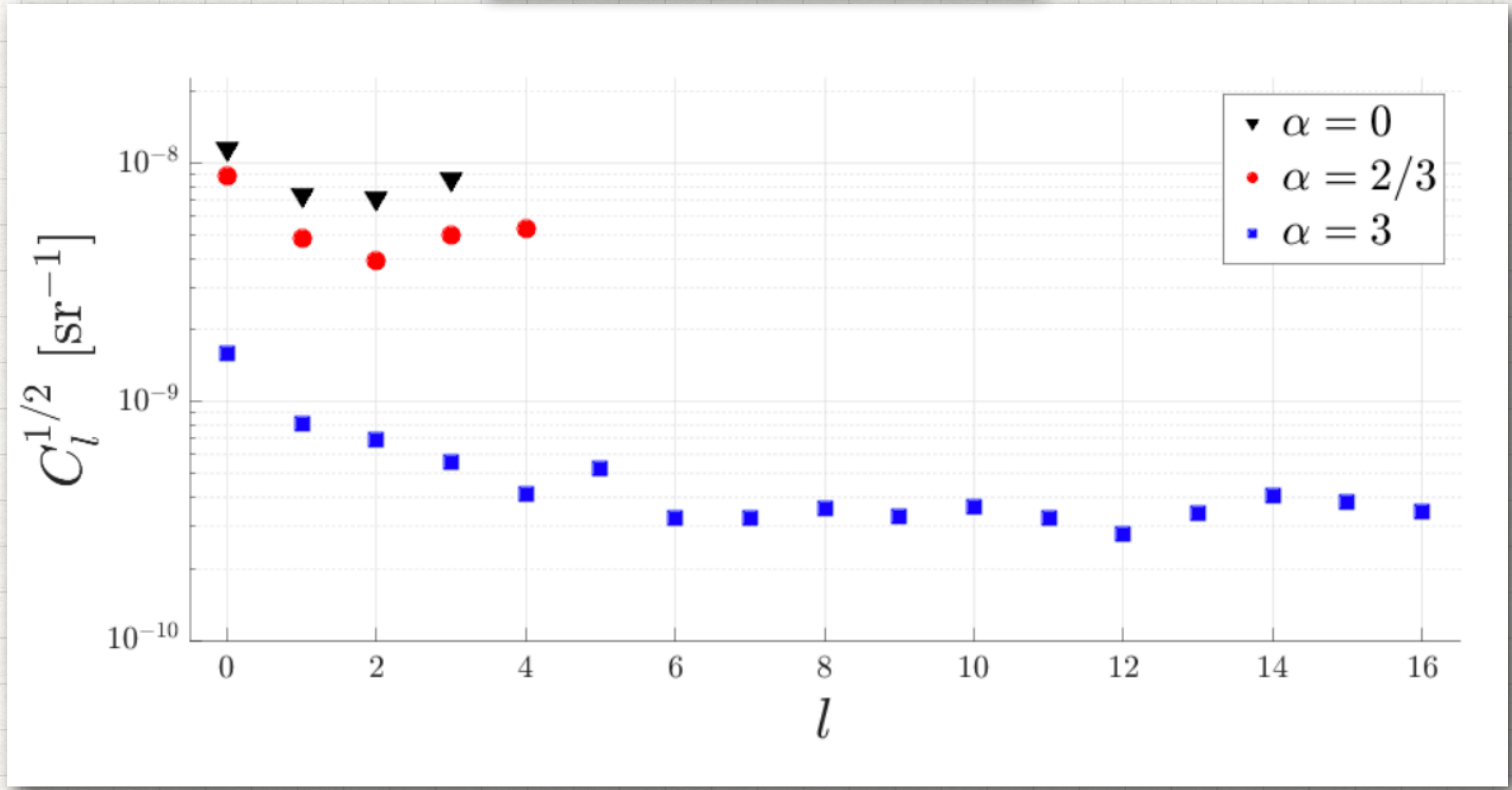
$$\begin{aligned}\hat{\mathcal{E}}_\ell(k,\nu_{\rm O}) &= \frac{1}{4\pi}\int \mathrm{d}\eta\, a^4 \int \mathrm{d}\theta_{\rm G}\, \mathcal{L}_{\rm G}(\bar\nu_{\rm G},\theta_{\rm G})\,\bar n_{\rm G}(\eta,\theta_{\rm G}) \times \\ &\times \Bigg\{\left[4\hat\Psi_k(\eta)+4\hat\Pi_k(\eta)+b\hat\delta_k(\eta)\right] j_\ell(k\Delta\eta) - 2k\hat v_k(\eta) j_\ell'(k\Delta\eta) - 6\int_{\eta_{\rm O}}^{\eta}\mathrm{d}\eta'\,\dot{\hat\Psi}_k(\eta') j_\ell(k\Delta\eta') + \\ &+ \frac{1}{\mathcal{L}_{\rm G}}\frac{\partial\mathcal{L}_{\rm G}}{\partial\nu_{\rm G}}\bigg|_{\bar\nu_{\rm G}}\frac{\nu_{\rm O}}{a}\left[-\hat\Psi_k(\eta)j_\ell(k\Delta\eta) - \hat\Pi_k(\eta)j_\ell(k\Delta\eta) + k\hat v_k(\eta)j_\ell'(k\Delta\eta) + 2\int_{\eta_{\rm O}}^{\eta}\mathrm{d}\eta'\,\dot{\hat\Psi}_k(\eta')j_\ell(k\Delta\eta')\right]\Bigg\}\end{aligned}$$

Cusin, Pitrou, Uzan, 2018

Anisotropic background from astro sources

$$\Omega_{\rm gw}(f,\Theta) = \frac{2\pi^2}{3H_0^2} f^3 H(f)\mathcal{P}(\Theta)$$

$$H(f) = (f/f_{\rm ref})^{\alpha-3}$$



LIGO/VIRGO Collaboration: "Directional limits on persistent gravitational waves using data from Advanced LIGO's first two observing runs"

Anisotropic background from astro sources

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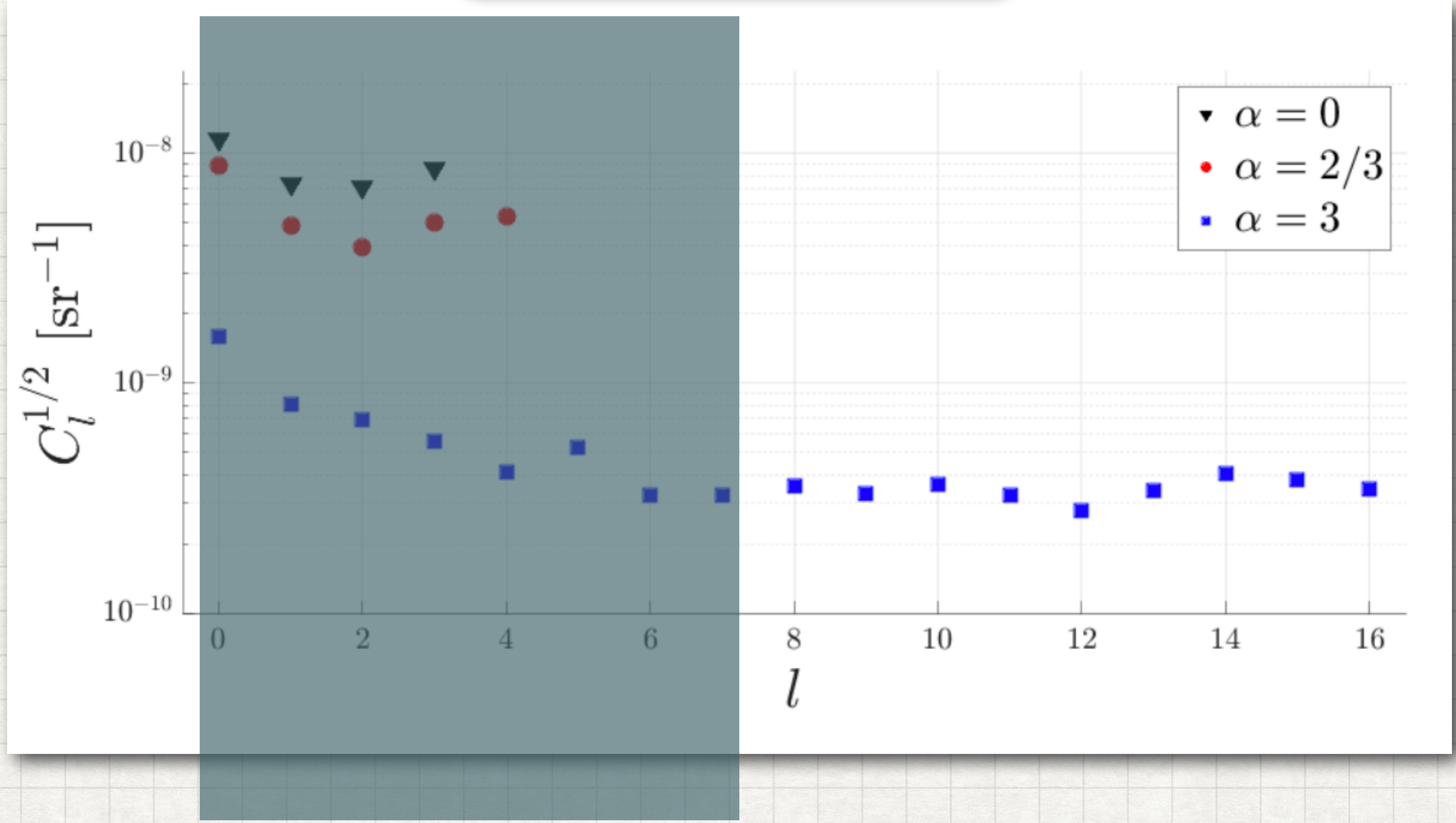


Cosmic-variance-dominated

Modelling the signal

The map on the sky

$$\Omega_{\rm GW}(\nu_0,\hat{\mathbf{r}}) = \frac{\nu_0}{\rho_{\rm c}}\frac{{\rm d}\rho_{\rm GW}(\nu_0,\hat{\mathbf{r}})}{{\rm d}\nu_0{\rm d}^2\hat{\mathbf{r}}}$$

Let's model this as

$$\Omega_{\rm GW}(\hat{\mathbf{r}}) \equiv \int dr\, r^2 \mathcal{K}(r) n(\vec{\mathbf{r}})$$

In terms of matter overdensity

$$\delta_{\rm g}(\vec{\mathbf{r}}) \equiv n(\vec{\mathbf{r}})/\bar{n}(r) - 1$$

$$\bar{n}(r) \equiv \int {\rm d}^2\hat{\mathbf{r}}\, n(\vec{\mathbf{r}})/4\pi$$

$$\Omega_{\rm GW}(\hat{\mathbf{r}}) = \int dr\, r^2 \boxed{\mathcal{K}(r)} \bar{n}(r) \left(\delta_{\rm g}(\vec{\mathbf{r}}) + 1\right)$$

Astrophysical kernel

Modelling the signal

$$\Omega_{\rm GW}(\hat{\mathbf{r}}) = \int dr\, r^2 \boxed{\mathcal{K}(r)}\, \bar{n}(r)\, (\delta_{\rm g}(\vec{\mathbf{r}}) + 1)$$

Astrophysical kernel

From real space to angular space

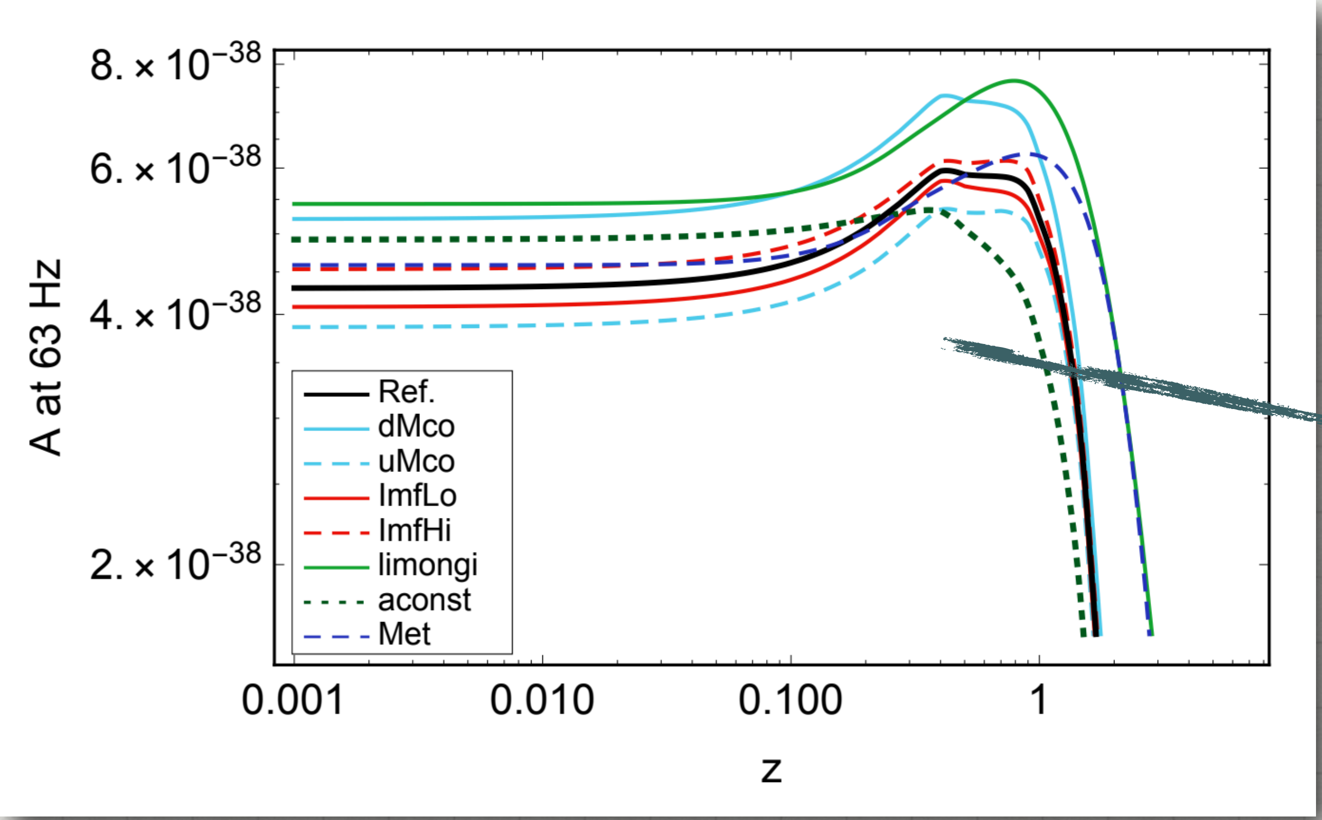
$$C_\ell^{\rm GW} = 4\pi \int_{k_{\rm min}}^{k_{\rm max}} \frac{dk}{k} |\delta\Omega_\ell|^2 \mathcal{P}(k) + B_\ell^{\rm GW}$$

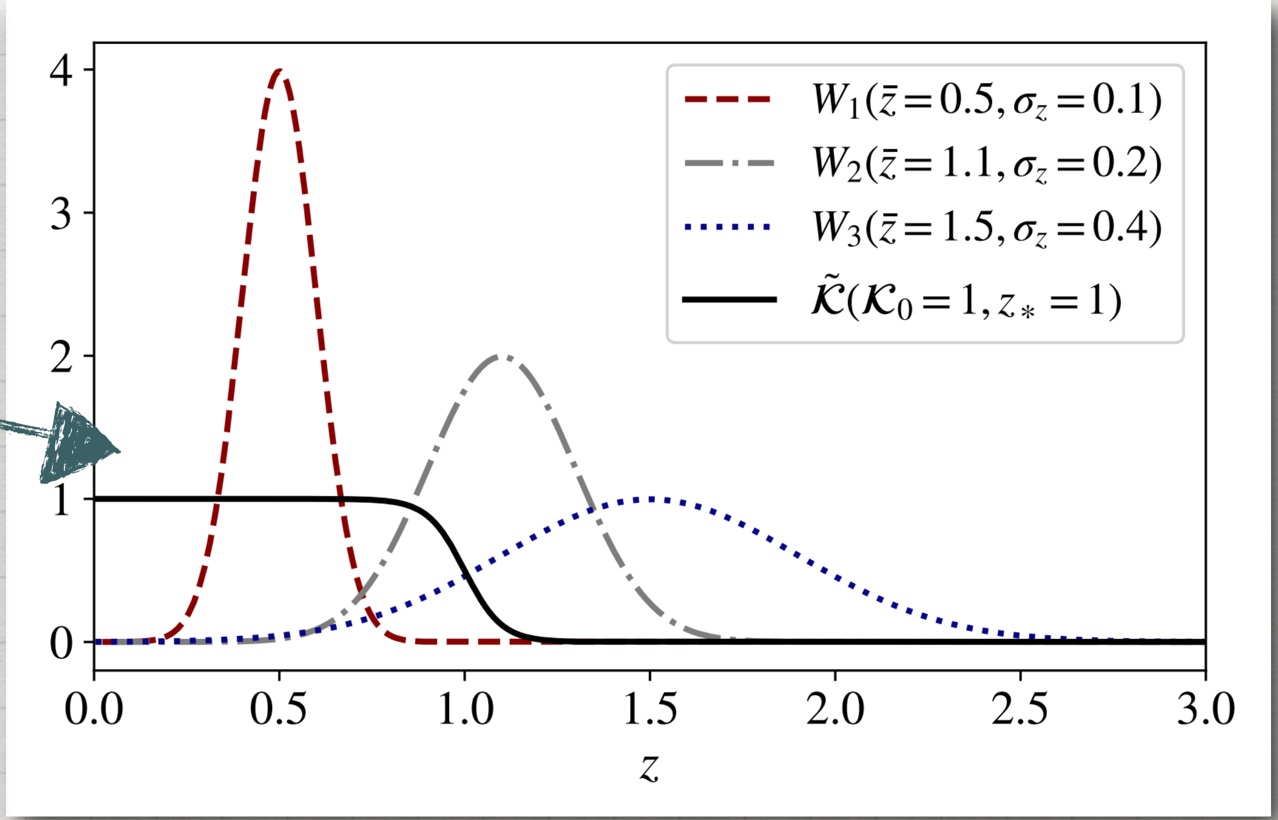
- **Primordial power spectrum** $\mathcal{P}(k) \ = \ A_{\rm s}\,(k/k_*)^{n_{\rm s}-1}$
- **Integrands** $\delta\Omega_\ell(k) = \int dr\, r^2 \mathcal{K}(r)\bar{n}(r) T_g(k,r) j_\ell(kr)$
- **Shot-noise contribution** $B_\ell^{\rm GW} = \int dr\, \mathcal{K}^2(r)\bar{n}(r) r^2 \left[1 + \frac{1+z(r)}{R(r)T_O}\right]$

Modelling the signal

Astrophysical models indicate that the shape of the kernel function is step-like

$$\tilde{\mathcal{K}}(r) \equiv \mathcal{K}(r)\bar{n}(r)r^2,$$

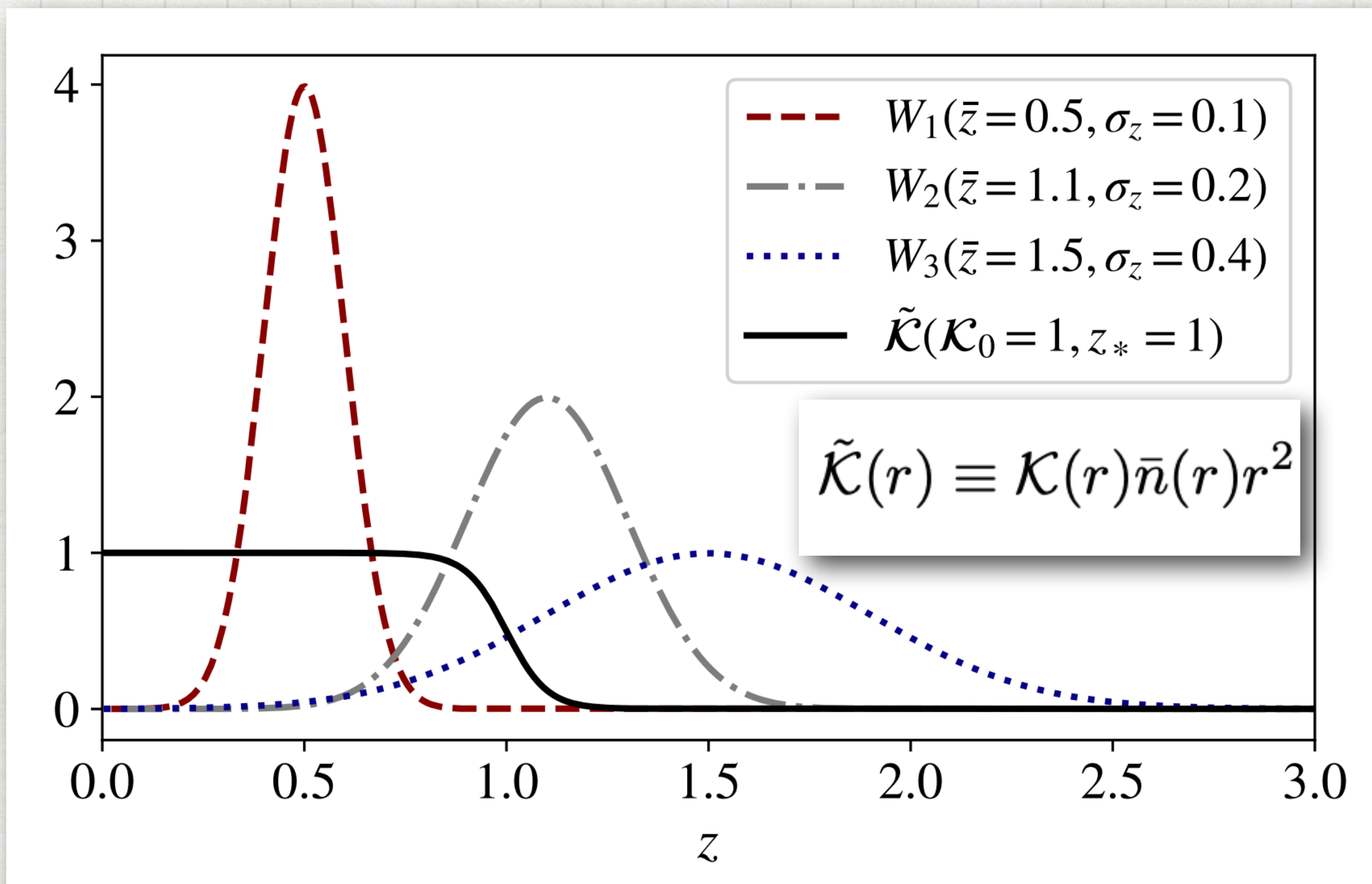




Cusin, Dvorkin, Pitrou, Uzan, 2019

$$\mathcal{K}(r) = \frac{\mathcal{K}_0}{2\bar{n}(r)r^2}\left[\tanh\left(10(z_* - z)\right) + 1\right]$$

Shortcomings of the autocorrelation



$$\mathcal{K}(r) = \frac{\mathcal{K}_0}{2\bar{n}(r)r^2}\left[\tanh\left(10(z_* - z)\right) + 1\right]$$

$$C_\ell^{\mathrm{GW}} = 4\pi \int_{k_{\min}}^{k_{\max}} \frac{dk}{k} |\delta\Omega_\ell|^2 \mathcal{P}(k)$$

$$\delta\Omega_\ell(k) = \int dr\, r^2 \mathcal{K}(r)\bar{n}(r) T_g(k,r) j_\ell(kr)$$

Limber approximation

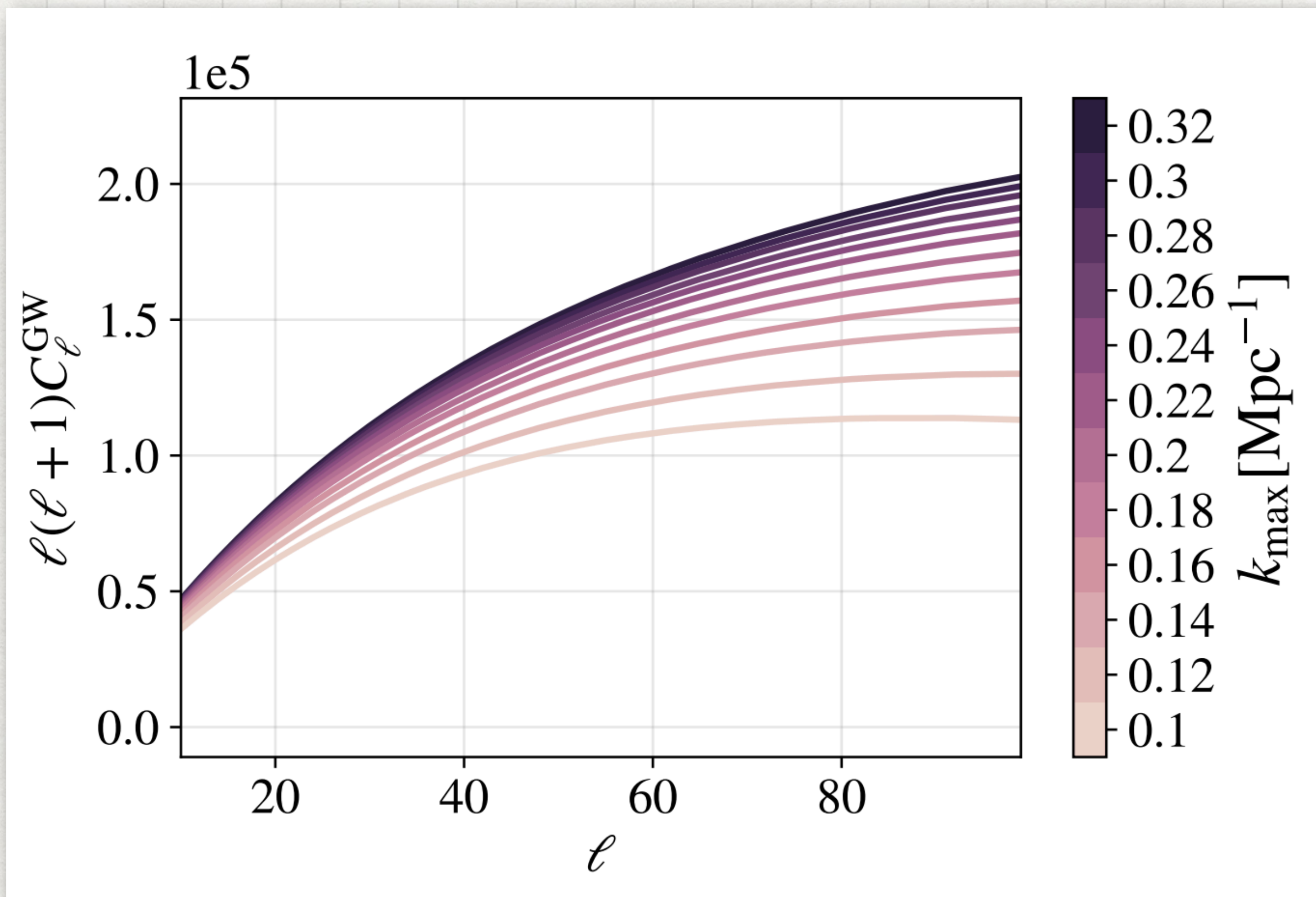
$$j_\ell(x) \to \sqrt{\frac{\pi}{2\alpha}}\delta_{\mathrm{D}}(\alpha - x)$$

$$C_\ell^{\mathrm{GW}} \approx \frac{2\pi^2}{\alpha} \int_{k_{\min}}^{k_{\max}} \frac{\mathrm{d}k}{k^3} \boxed{\tilde{\mathcal{K}}^2\left(\frac{\alpha}{k}\right)} \mathcal{S}^2\left(k, \frac{\alpha}{k}\right)$$

Serves as a filter in k-space

$$\tilde{\mathcal{K}}(r) \equiv \mathcal{K}(r)\bar{n}(r)r^2, \quad \mathcal{S}(k,r) \equiv T_g(k,r)\mathcal{P}(k)^{1/2}.$$

Shortcomings of the autocorrelation



High-k modes severely contribute.

$$C_\ell^{\rm GW} = 4\pi \int_{k_{\rm min}}^{k_{\rm max}} \frac{dk}{k} |\delta\Omega_\ell|^2 \mathcal{P}(k)$$

$$\delta\Omega_\ell(k) = \int dr\, r^2 \mathcal{K}(r)\bar{n}(r) T_g(k,r) j_\ell(kr)$$

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GW-GC cross-correlation

$$\Omega_{\mathrm{GW}}(\hat{\mathbf{r}}) = \int dr\ r^2 \boxed{\mathcal{K}(r)} \bar{n}(r)\left(\delta_{\mathrm{g}}(\vec{\mathbf{r}}) + 1\right)$$

$$\Delta(\hat{\mathbf{r}}) = \int dr\ \boxed{W_i(r)} \delta_g(\vec{\mathbf{r}})$$

From real space to angular space

$$C_\ell^{\mathrm{GW}} = 4\pi \int_{k_{\min}}^{k_{\max}} \frac{dk}{k} |\delta\Omega_\ell|^2 \mathcal{P}(k) + B_\ell^{\mathrm{GW}}$$

$$\delta\Omega_\ell(k) = \int dr\ r^2 \mathcal{K}(r)\bar{n}(r) T_g(k,r) j_\ell(kr)$$

$$B_\ell^{\mathrm{GW}} = \int dr\ \mathcal{K}^2(r)\bar{n}(r) r^2 \left[1 + \frac{1+z(r)}{R(r)T_O}\right]$$

$$C_\ell^{\mathrm{GC}} = 4\pi \int \frac{dk}{k} |\Delta_\ell(k)|^2 \mathcal{P}(k) + \frac{1}{n_i}$$

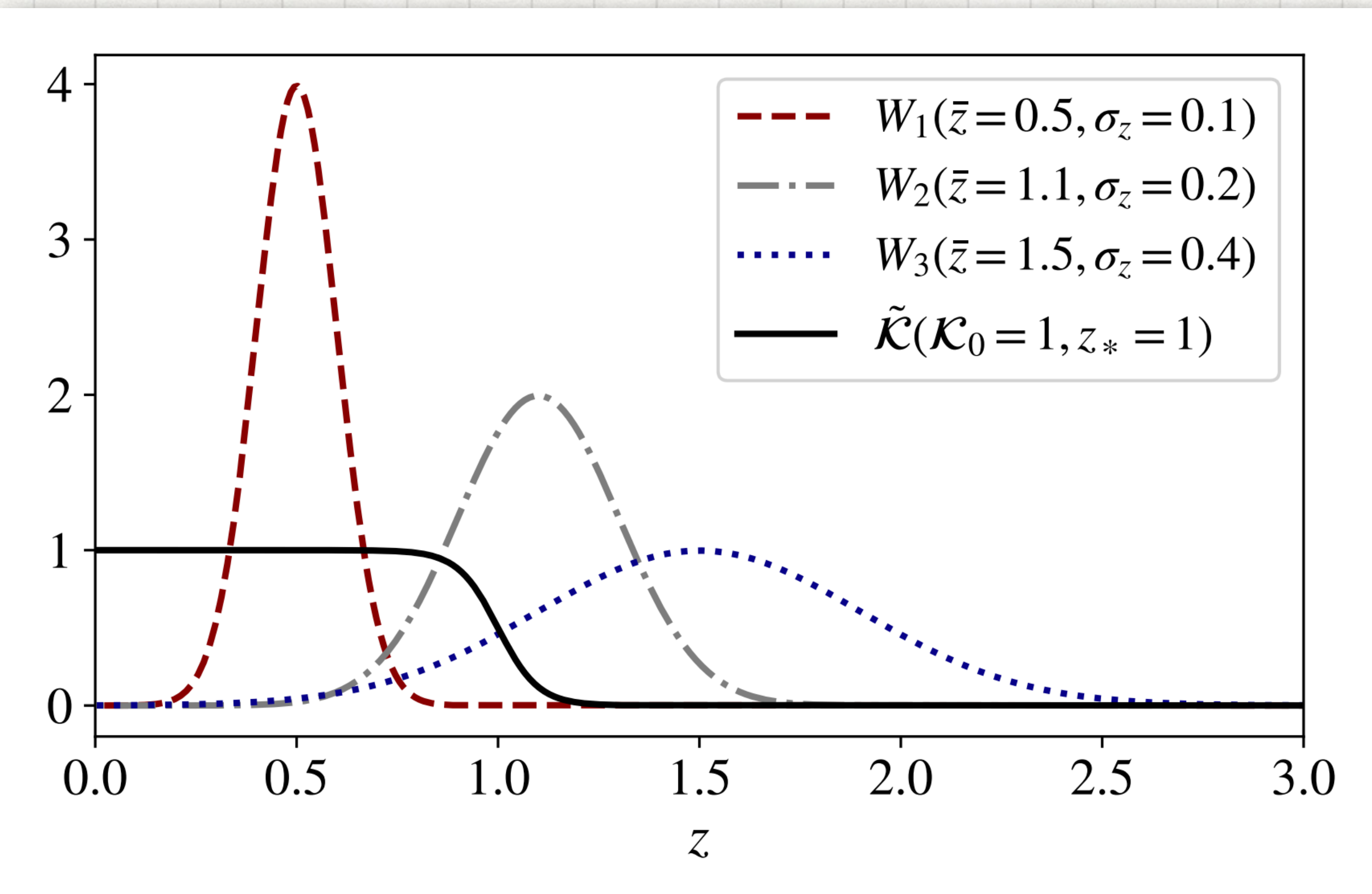
$$\Delta_\ell(k) = \int dr\ W_i(r) T_g(k,r) j_\ell(kr)$$

GW-GC cross-correlation

$$C_\ell^{GW\times GC} = 4\pi \int \frac{dk}{k}\, \delta\Omega_\ell^*(k)\Delta_\ell(k)\mathcal{P}(k) + B_\ell$$

$$B_\ell = \int dr\, W_i(r)\mathcal{K}(r)$$

GW-GC cross-correlation



$$C_\ell^{GW\times GC} = 4\pi \int \frac{dk}{k}\, \delta\Omega_\ell^*(k)\Delta_\ell(k)\mathcal{P}(k) + B_\ell$$

$$\delta\Omega_\ell(k) = \int dr\, r^2\mathcal{K}(r)\bar{n}(r)T_g(k,r)j_\ell\,(kr)$$

$$\Delta_\ell(k) = \int dr\, W_i(r)T_g(k,r)j_\ell(kr)$$

Limber approximation

$$j_\ell(x) \to \sqrt{\frac{\pi}{2\alpha}}\delta_{\rm D}\,(\alpha - x)$$

$$\tilde{\mathcal{K}}(r) \equiv \mathcal{K}(r)\bar{n}(r)r^2, \quad \mathcal{S}(k,r) \equiv T_g(k,r)\mathcal{P}(k)^{1/2}$$

$$C_\ell^{\rm GC\times GW} \approx \frac{2\pi^2}{\alpha}\int_{k_{\rm min}}^{k_{\rm max}} \frac{{\rm d}k}{k^3} \boxed{W_i\left(\frac{\alpha}{k}\right)\tilde{\mathcal{K}}\left(\frac{\alpha}{k}\right)}\mathcal{S}^2\left(k, \frac{\alpha}{k}\right)$$

Serves as a filter in k-space

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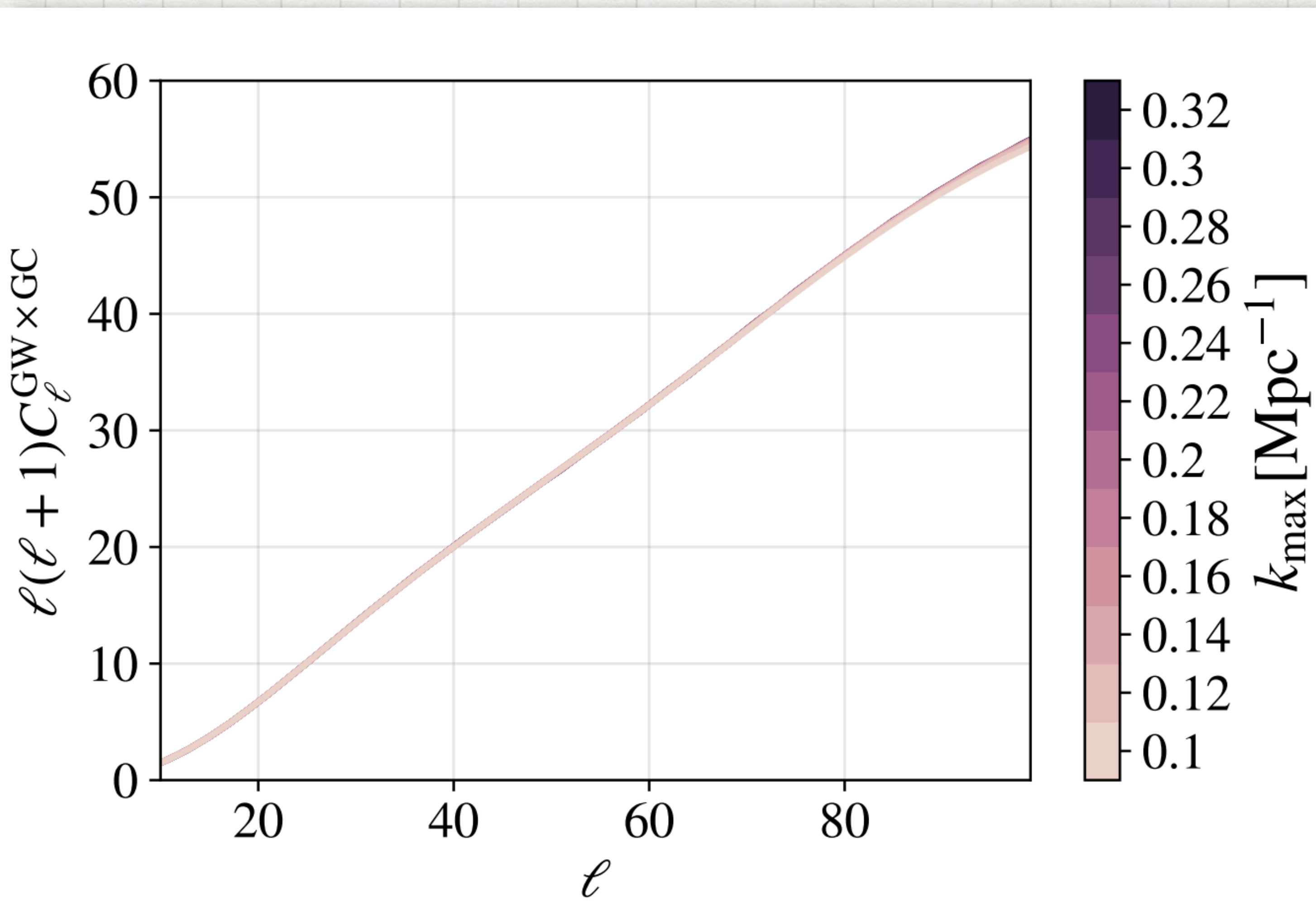

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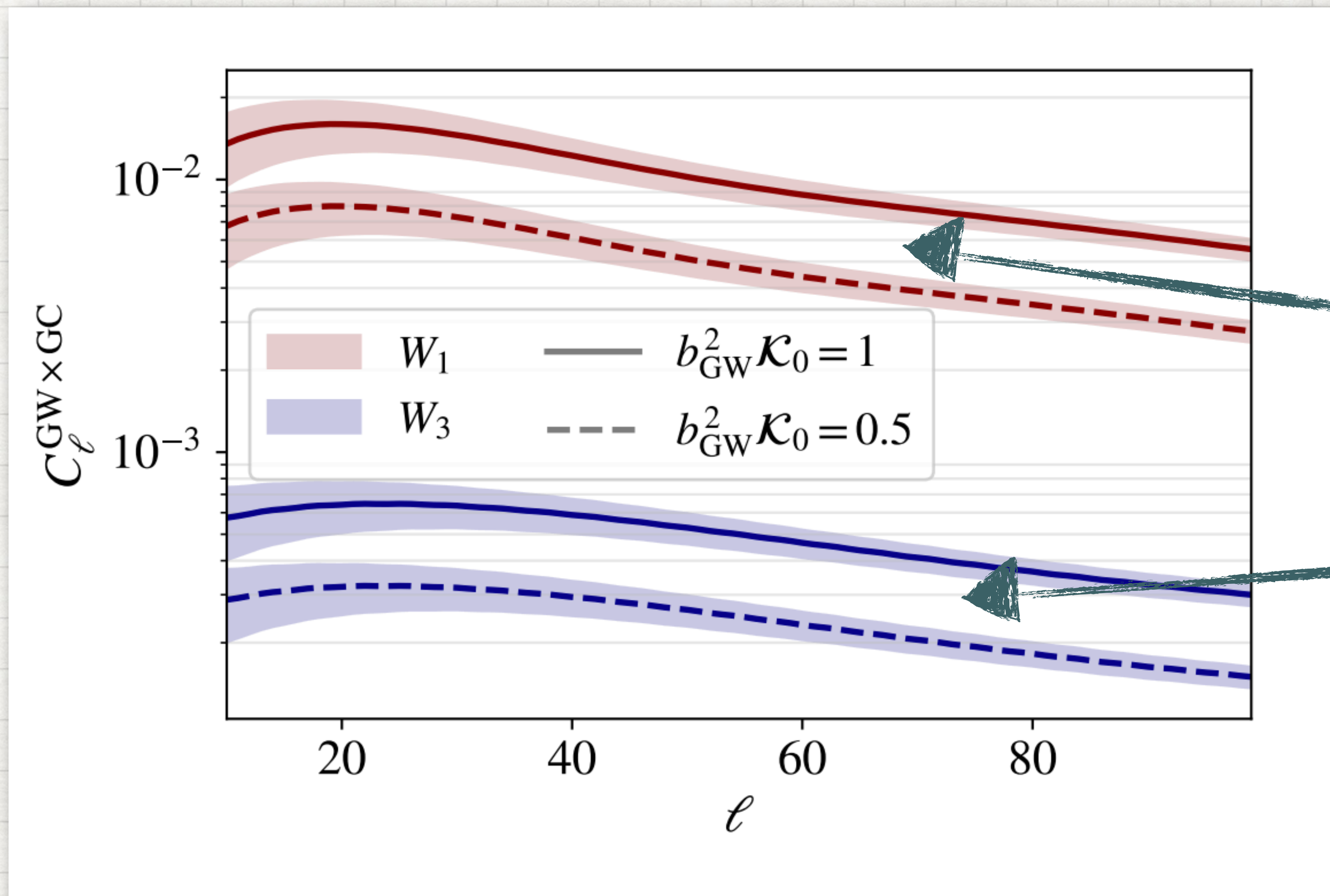
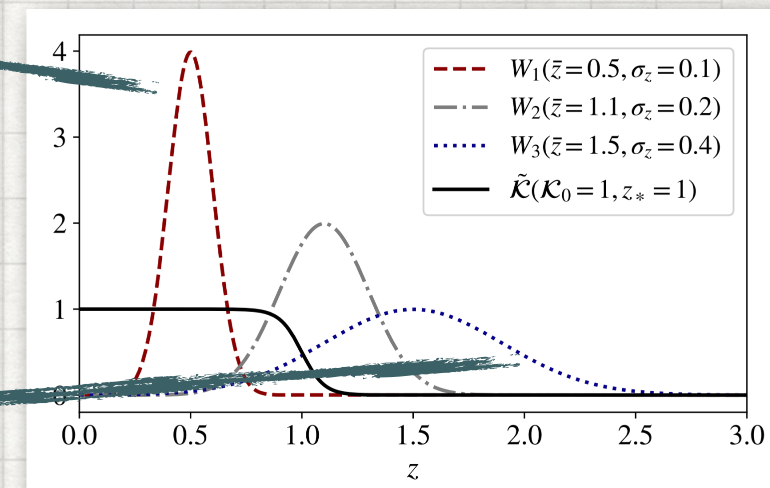
$$C_\ell^{\mathrm{GC}\times\mathrm{GW}} \approx \frac{2\pi^2}{\alpha}\int_{k_{\min}}^{k_{\max}} \frac{\mathrm{d}k}{k^3} \boxed{W_i\left(\frac{\alpha}{k}\right)\tilde{\mathcal{K}}\left(\frac{\alpha}{k}\right)} \mathcal{S}^2\left(k, \frac{\alpha}{k}\right)$$

Serves as a filter in k-space

GW-GC cross-correlation

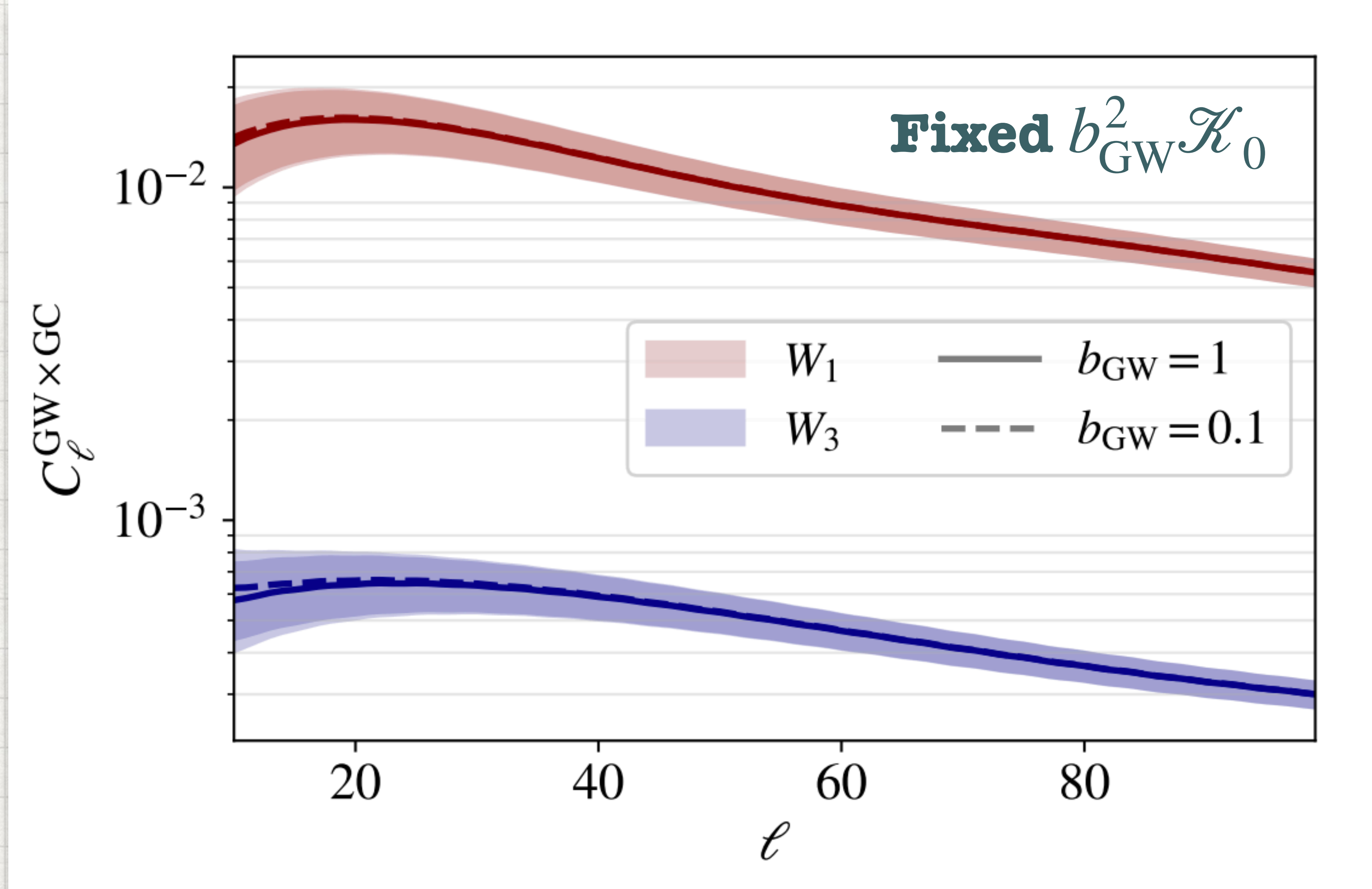


$$\left(\frac{\Delta C_\ell}{C_\ell}\right)^2 = \frac{2}{2\ell+1}$$



Directly sensitive to changes in the combination of the kernel amplitude and galaxy bias.

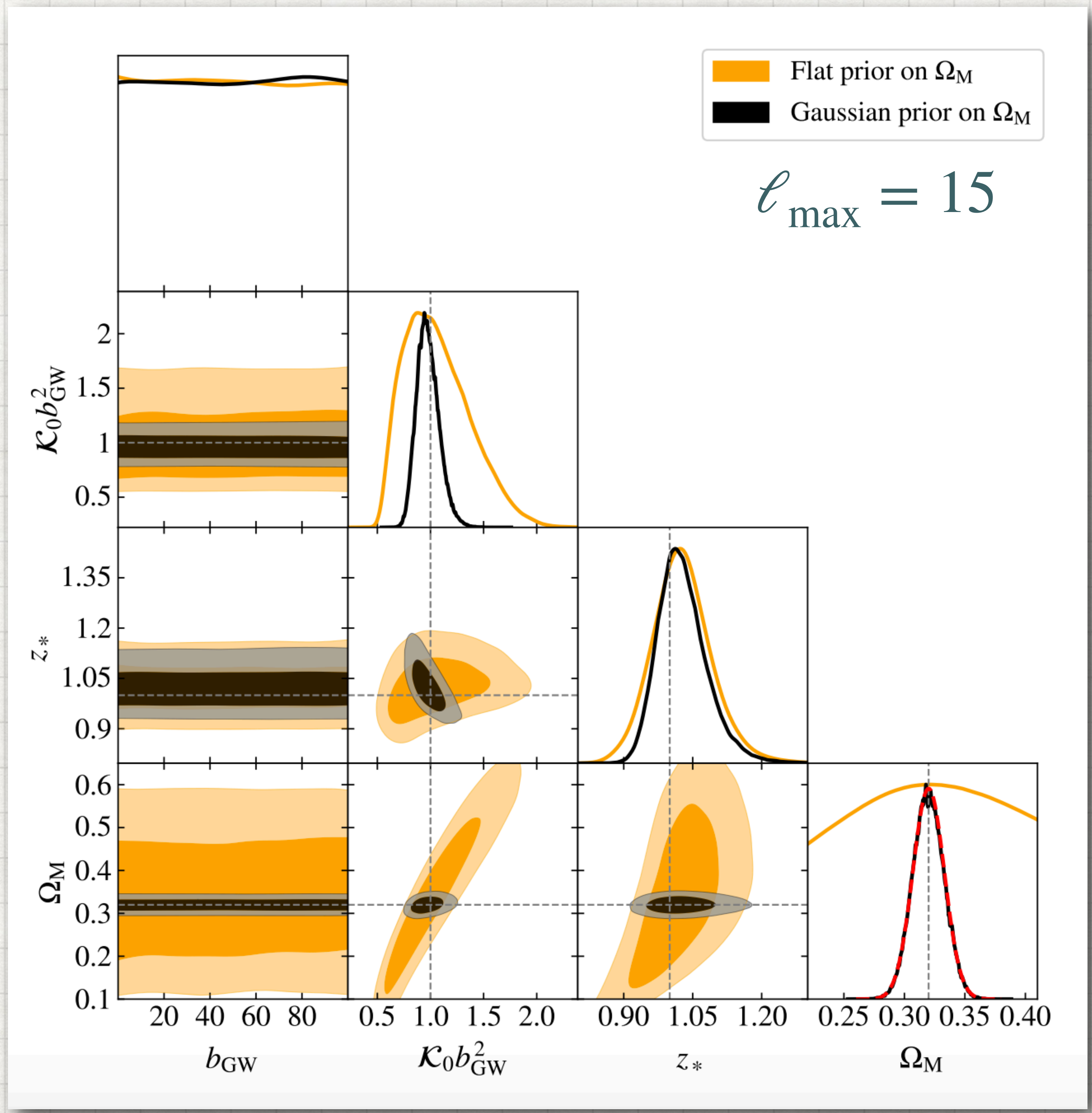
GW-GC cross-correlation



Note that the bias and the amplitude are completely degenerate.

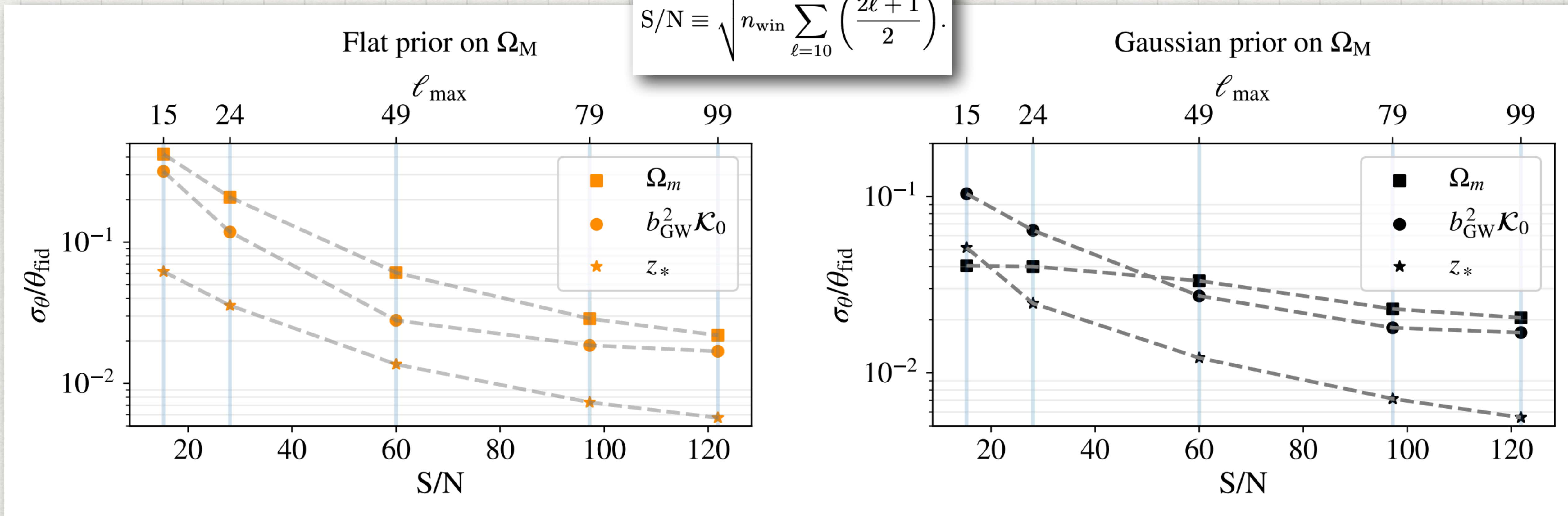
Using the autocorrelation is essential in breaking this degeneracy.

Dependence on Ω_M prior



What to expect?

$$S/N \equiv \sqrt{n_{\rm win} \sum_{\ell=10}^{\ell_{\rm max}} \left(\frac{2\ell+1}{2}\right)}.$$

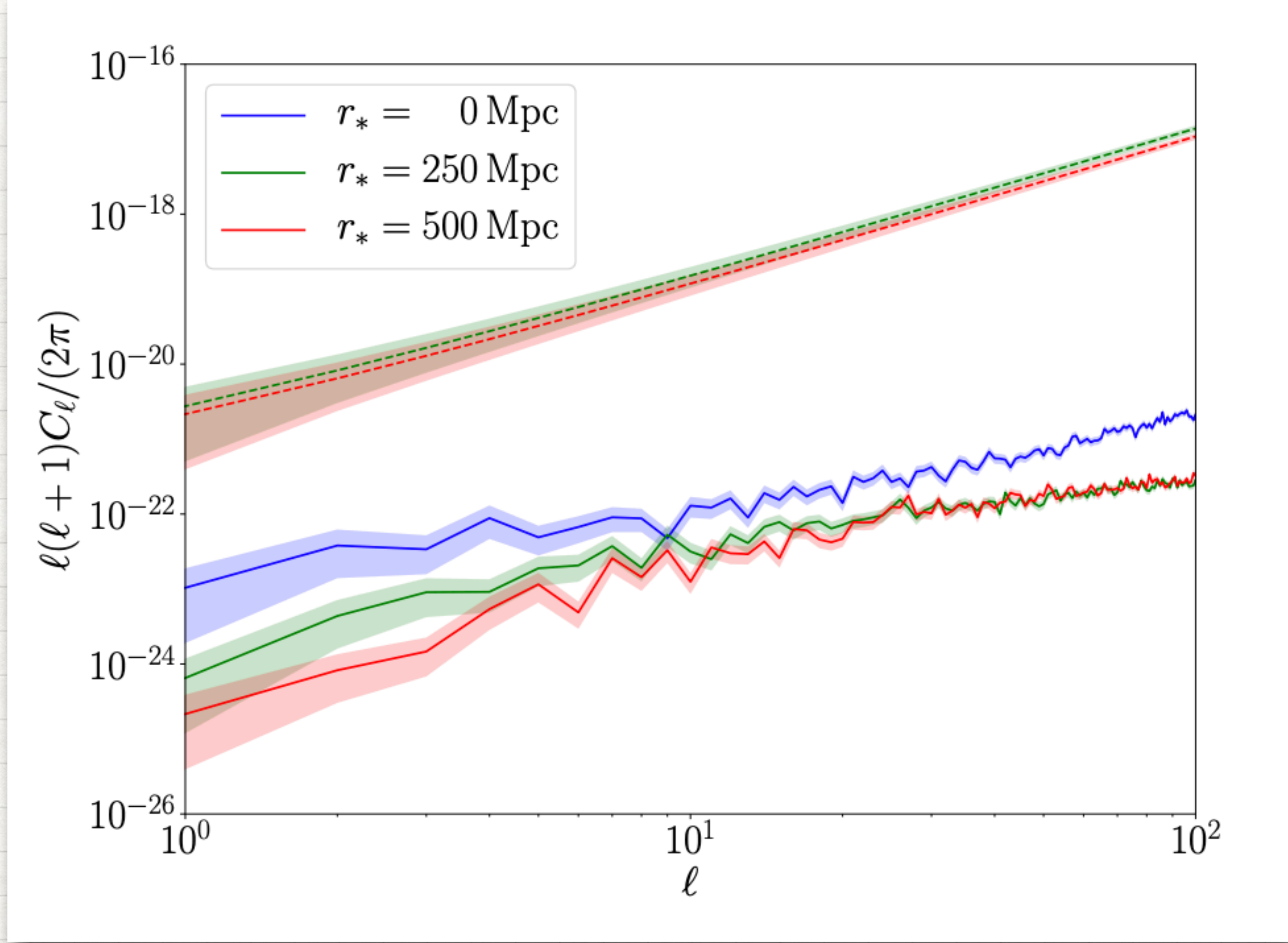


- **The anisotropies are likely to first be detected via the cross-correlation**
- **Very sensitive to the features in the astrophysical kernel** $\mathscr{K}(z)$
- **Useful for cosmology if** $\ell_{\rm max} \sim 100$ **is detected**

Shot noise: autocorrelation

$$\mathcal{B} = \frac{\pi^2}{9}(t_H \nu_\mathrm{o})^6 \int_0^{r_\mathrm{p}} \mathrm{d}r \left(\frac{1+z}{r_H r}\right)^2 \int \mathrm{d}\zeta\, \bar{n}\left[(1+z)\frac{R}{T_\mathrm{o}} + R^2\right]\mathcal{S}^2.$$

Shot noise: autocorrelation



Jenkins, Sakellariadou, 2019

Shot noise: cross-correlation

$$C_\ell^{GW\times GC} = 4\pi \int \frac{dk}{k}\, \delta\Omega_\ell^*(k)\Delta_\ell(k)\mathcal{P}(k) + B_\ell$$

$$B_\ell = \int dr\, W_i(r)\mathcal{K}(r)$$

$$B_\ell = \int d^2\hat{\mathbf{r}} P_\ell(\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}')\mathrm{Cov}[\Omega(\hat{\mathbf{r}}),\Delta(\hat{\mathbf{r}}')]_{\mathrm{SN}}$$

$$\mathrm{Cov}[\Omega(\hat{\mathbf{r}}),\Delta(\hat{\mathbf{r}}')]_{\mathrm{SN}} = \int dr \int dr' \frac{r^2}{\bar{n}} \times \mathrm{Cov}[\mathcal{K}(r)n(\vec{\mathbf{r}}), W_i(r')n(\vec{\mathbf{r}}')]_{\mathrm{SN}}$$

Number of events in δV_i

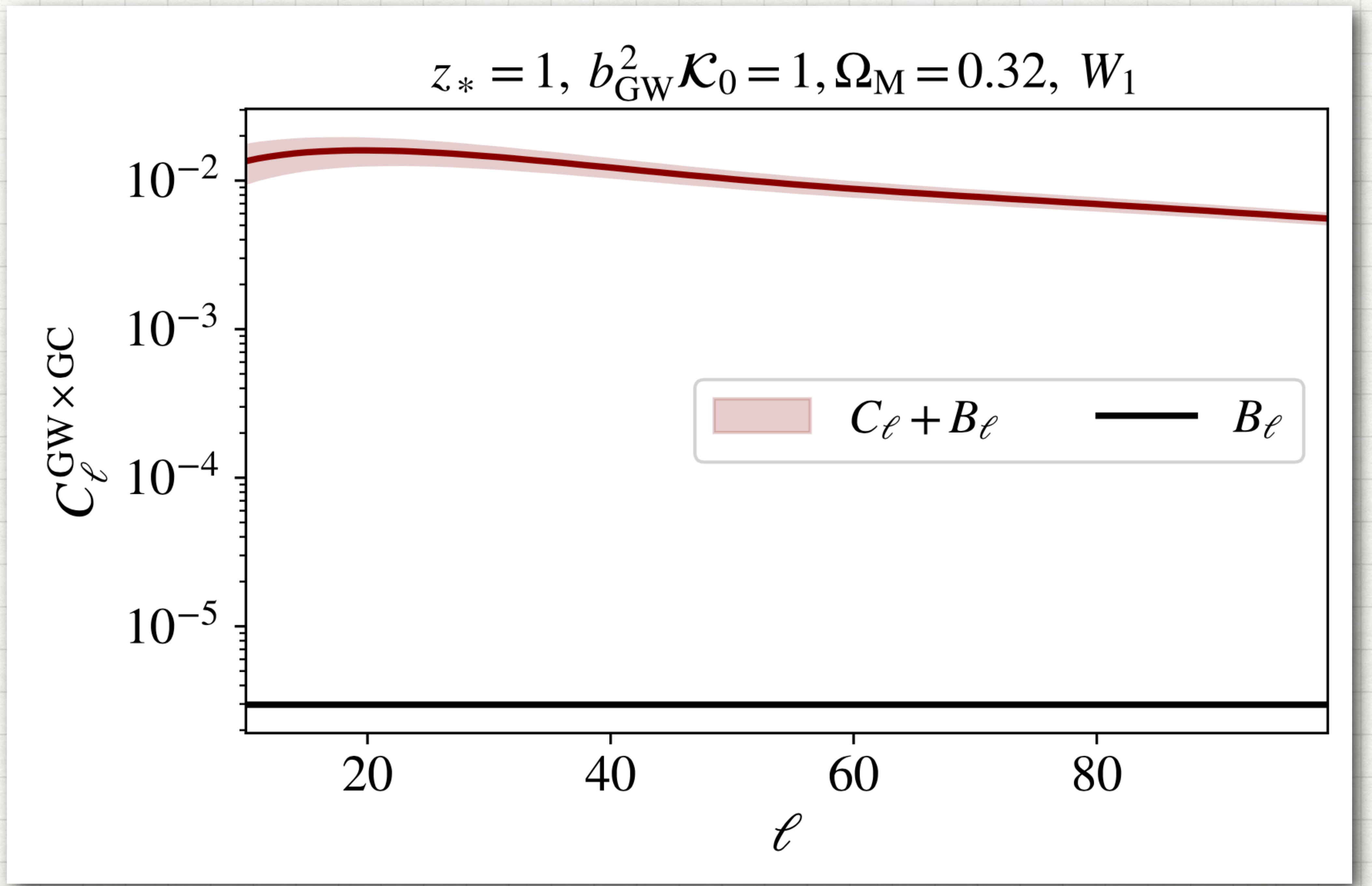
$$\Lambda_i = \sum_k^{N_i} \lambda_k$$

δV_i

$$\mathrm{Var}[\Lambda_i] = \langle\Lambda_i^2\rangle - \langle\Lambda_i\rangle^2 = \langle N_i\rangle\left(\langle\lambda\rangle + \langle\lambda\rangle^2\right)$$

$$\mathrm{Cov}[\mathcal{K}(r)n(\vec{\mathbf{r}}), W_i(r')n(\vec{\mathbf{r}}')]_{\mathrm{SN}} = \bar{n}(r)W_i(r)\mathcal{K}(r)\delta^3(\vec{\mathbf{r}}-\vec{\mathbf{r}}').$$

Shot noise: cross-correlation



Some ideas

- **Astrophysical GWB and its anisotropies: cross-correlation with galaxy catalogues is a promising observable.**
  - **Will LIGO+Euclid be able to detect the cross correlation?**
  - **Useful for GW lensing?**
  - **Frequency dependence detectable?**
  - **Anisotropic GWB from exotic sources**
    - **Primordial Black Hole mergers**
    - **Cosmic strings**
    - **Sensitivity to dark matter models**