A02: Dark Matter
Formations and clusterings of primordial black hole dark matter in the matter dominated Universe

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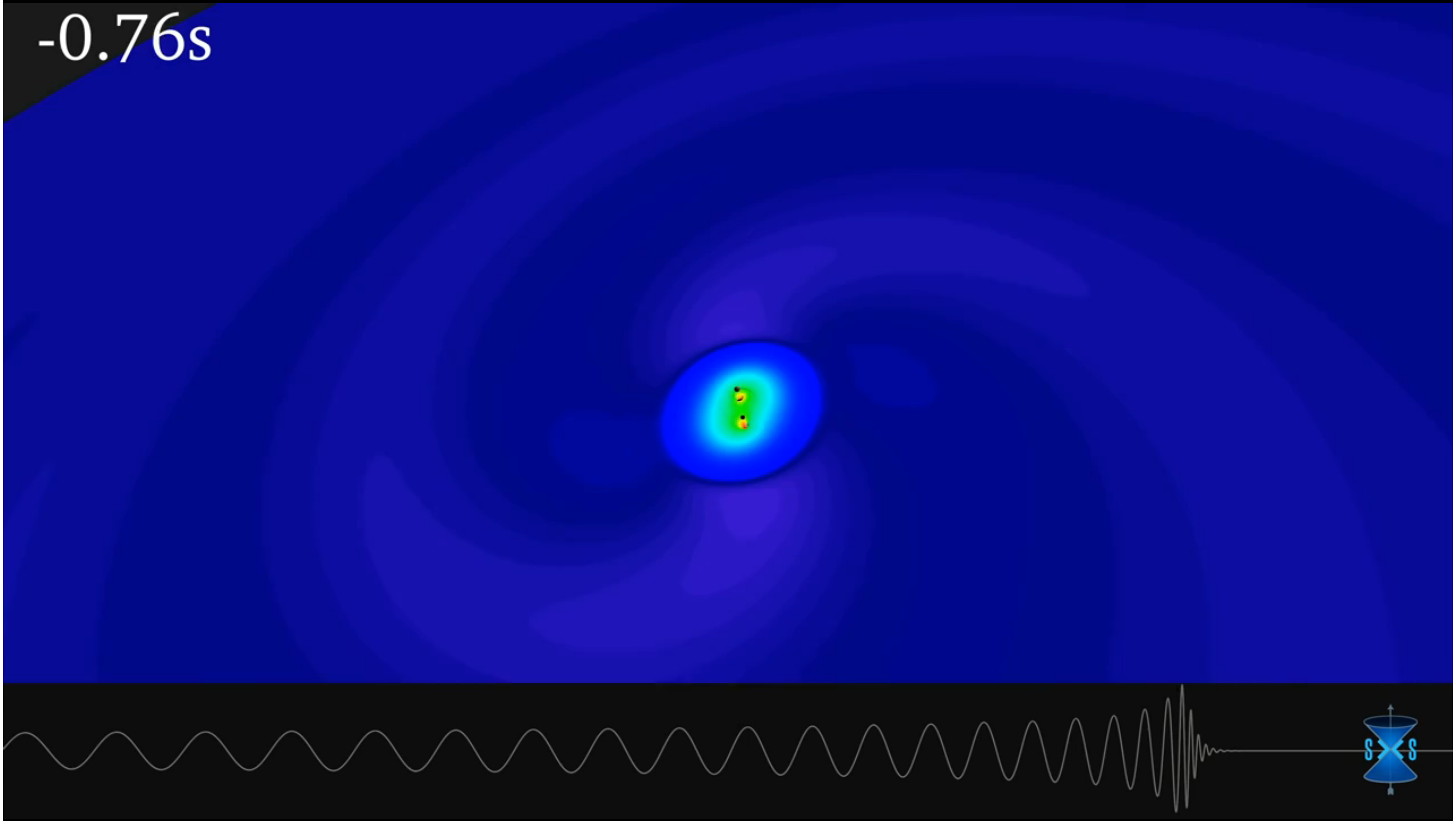
LIGO and Virgo have detected gravitational wave signals from Binary Black Holes

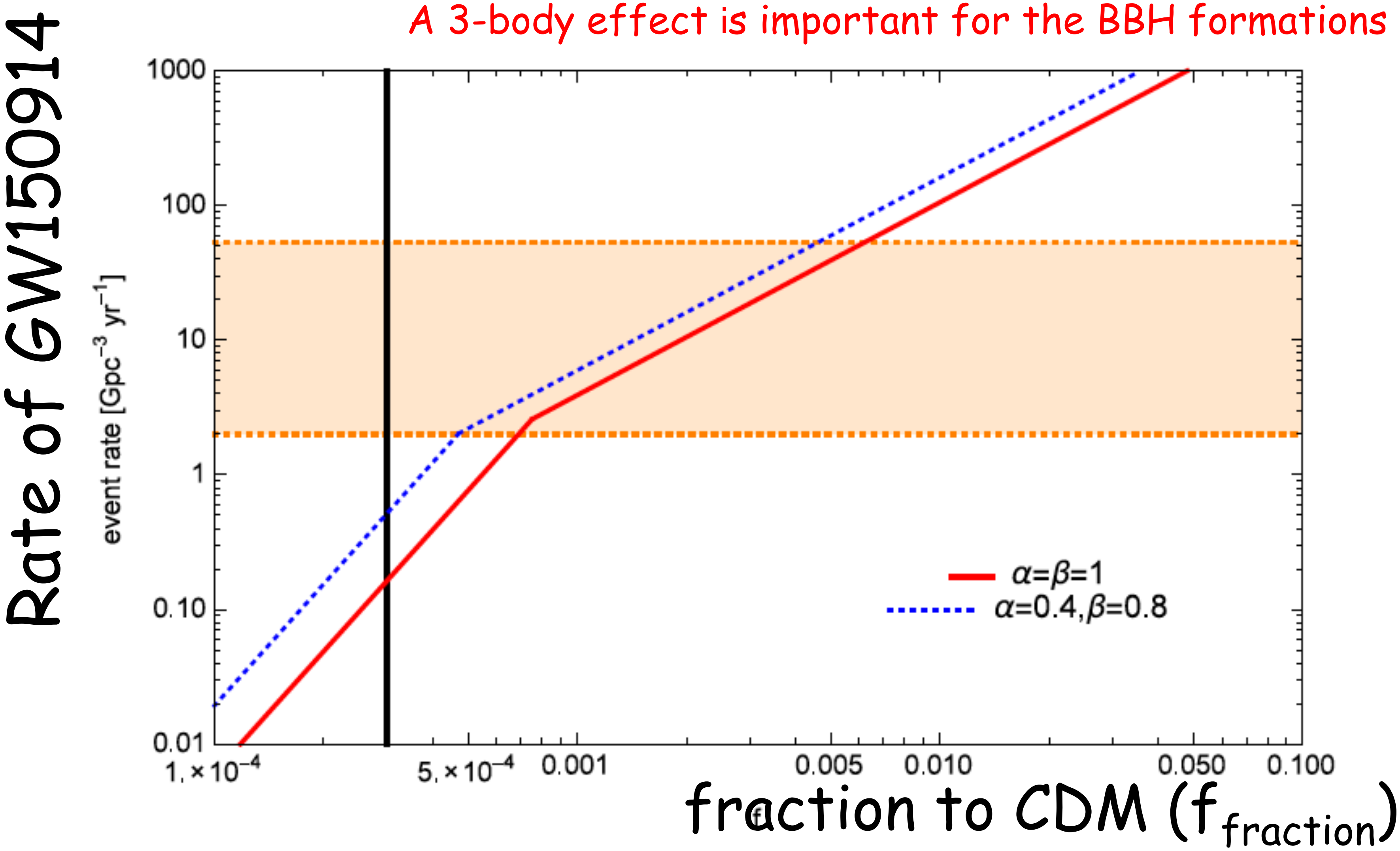
https://www.youtube.com/watch?v=1agm33iEAuo



GW150914 and its merger rates for 30 M_{solar} masses BBH

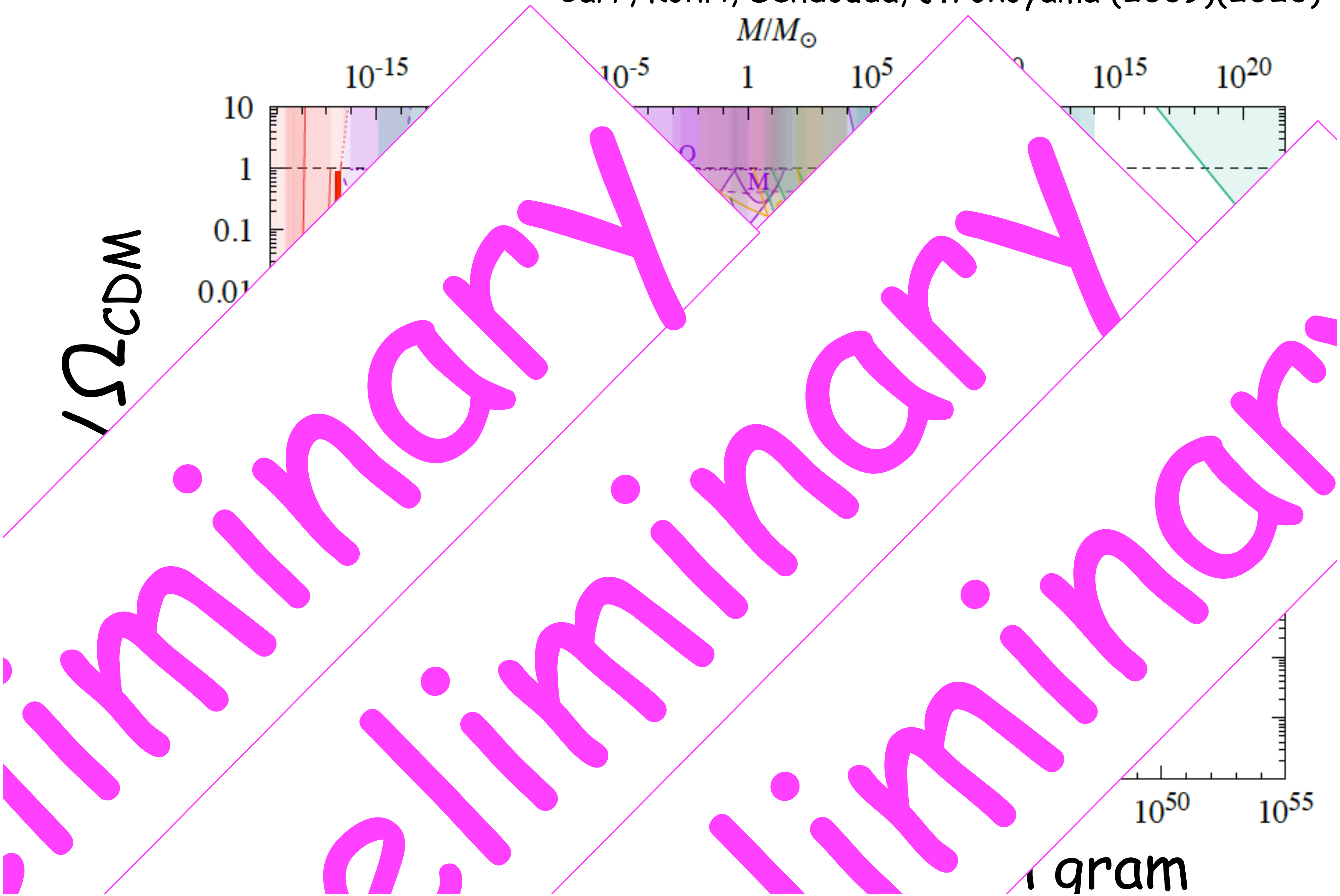
M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama (2016).

A 3-body effect is important for the BBH formations



Upper bounds on the fraction to CDM

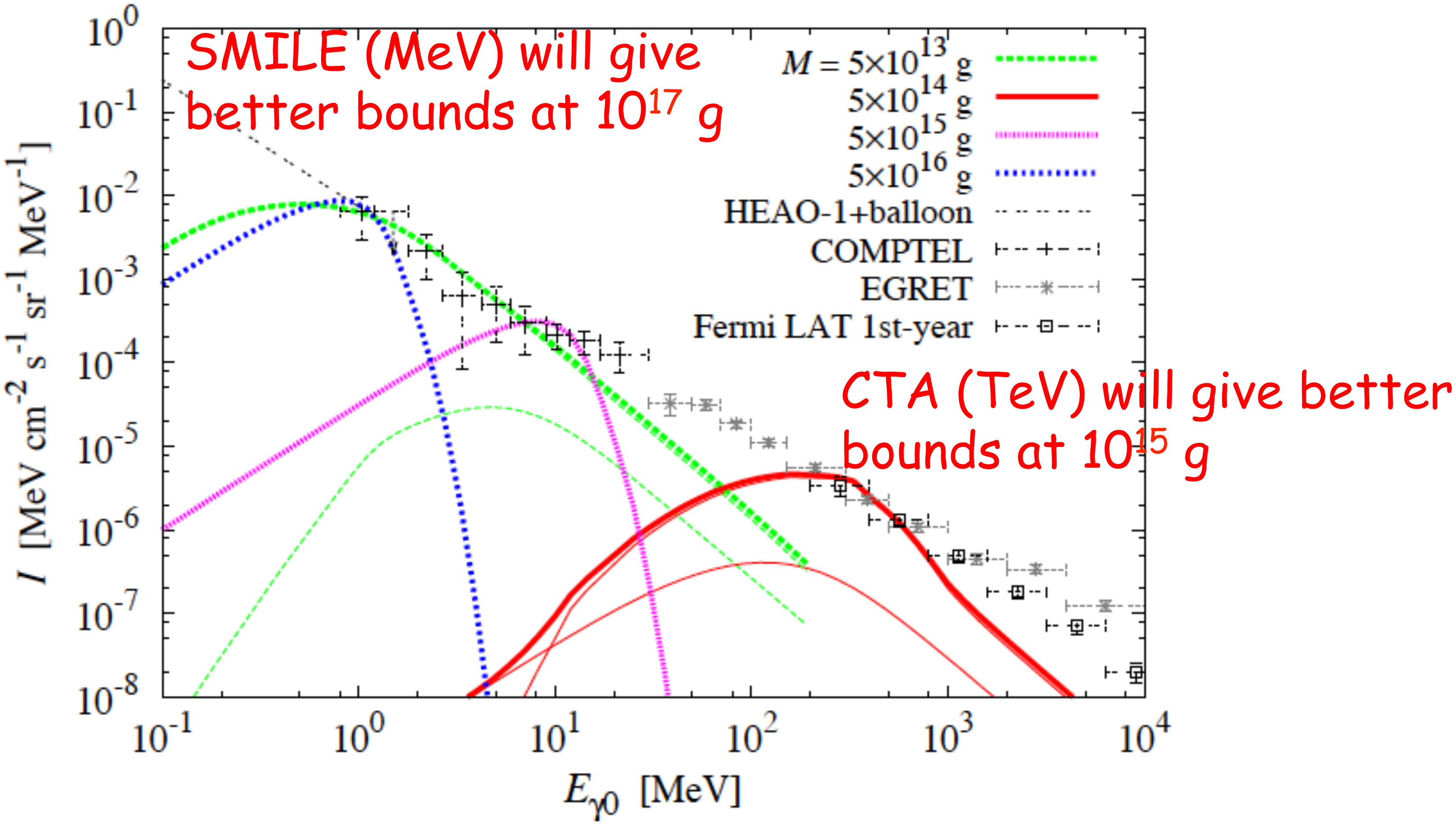
Carr, Kohri, Sendouda, J.Yokoyama (2009)(2020)



Evaporating PBHs through Hawking Process

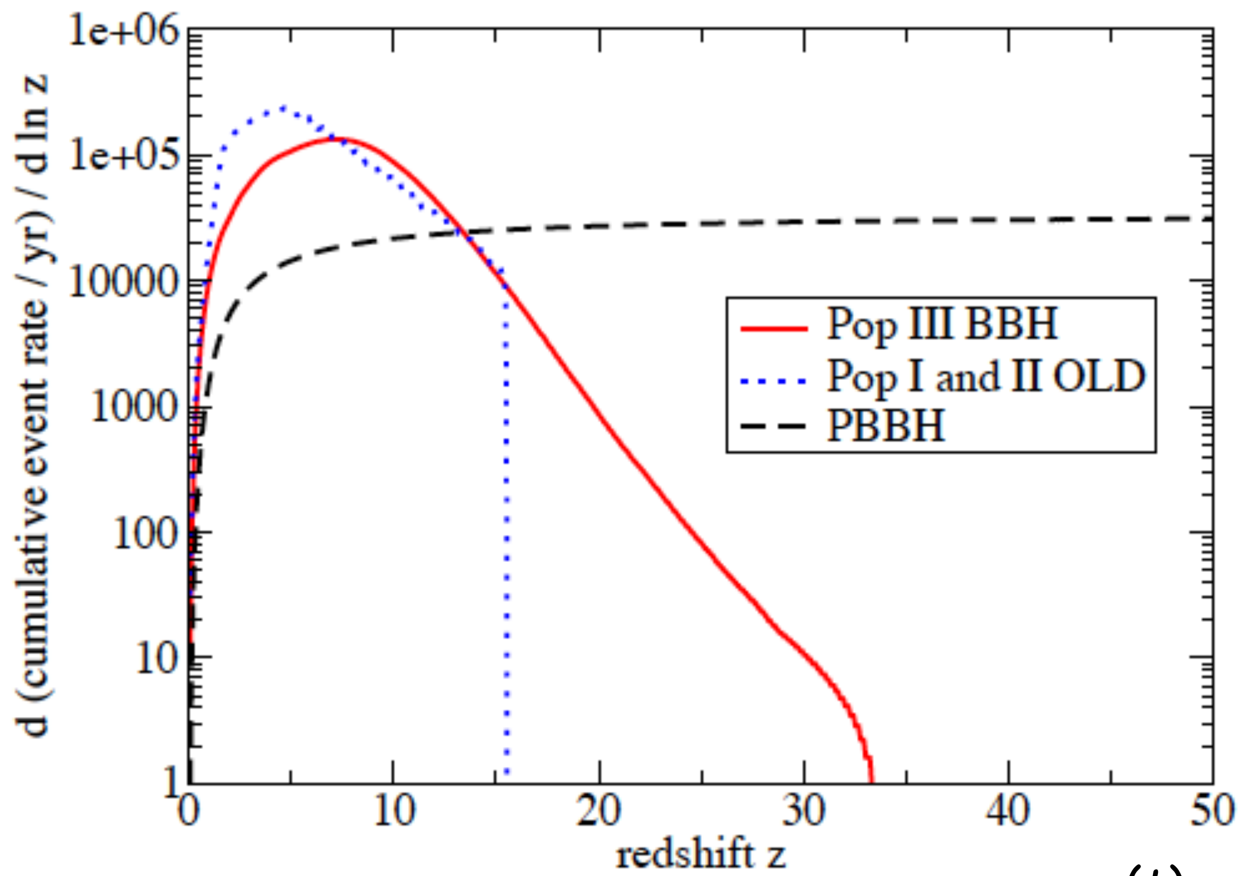
Carr, Kohri, Sendouda and Yokoyama (2010)

$$\mathrm{d}\dot{N}_s = \frac{\mathrm{d}E}{2\pi} \frac{\Gamma_s}{e^{E/T_{\mathrm{BH}}} - (-1)^{2s}}$$



DECIGO discriminates BPBHs from the normal BBHs

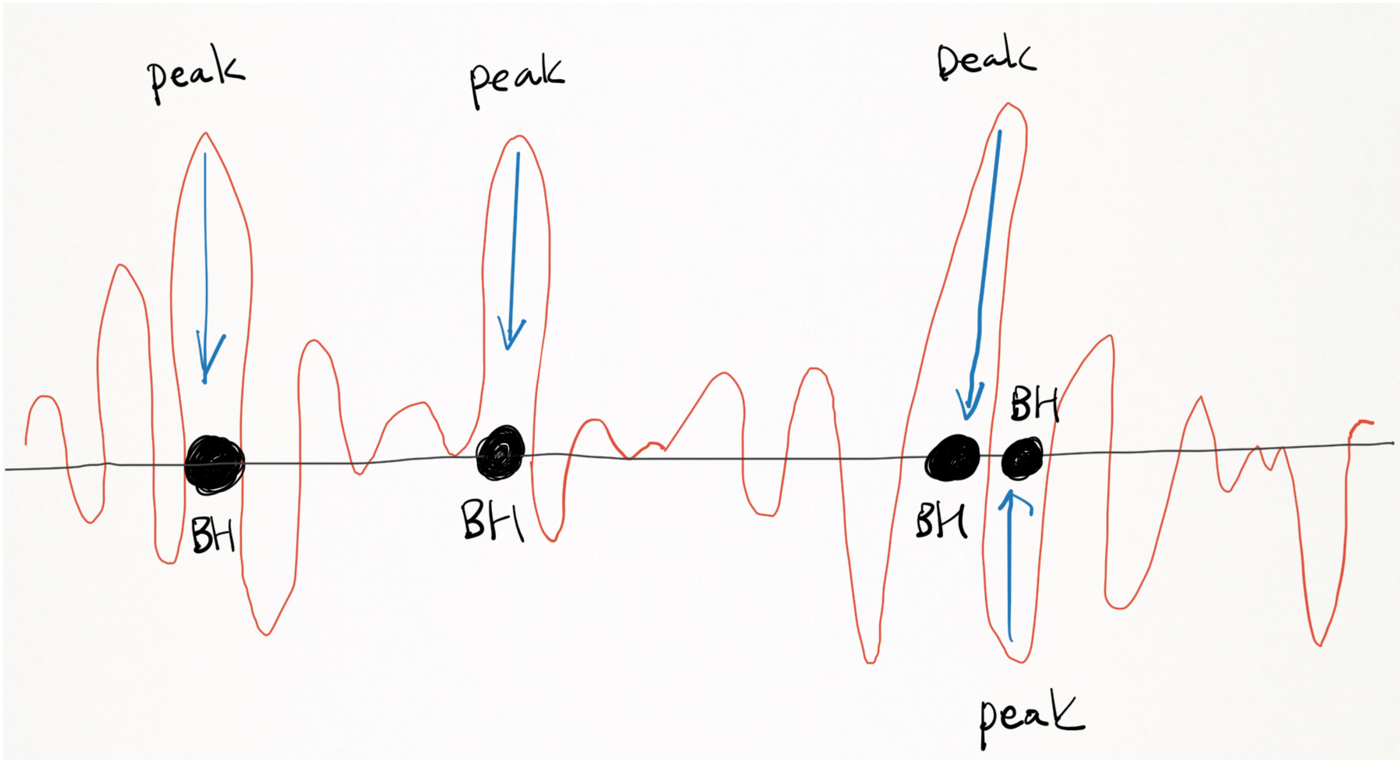
Takashi Nakamura et al, arXiv:1607.00897 [astro-ph.HE]



$$1/z \sim \frac{a(t)}{a(t_0)} \sim \left(t/10\mathrm{Gyr}\right)^{2/3}$$

Primordial Black Hole (PBH)

- Large perturbation at small scales was produced by Inflation at around $> 10^{-36}$ second



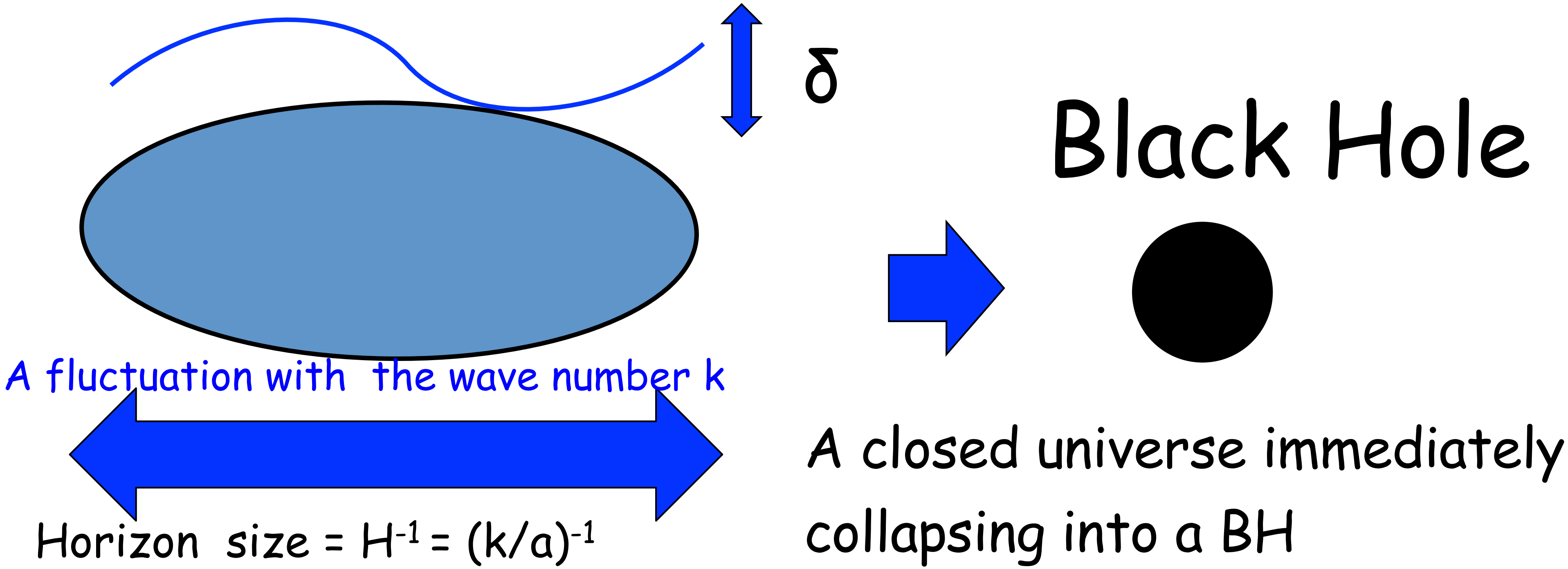
Conditions for a PBH formation in Radiation dominated (RD) Universe

Zel'dovich and Novikov (1967), Hawking (1971), Carr (1975)

Harada,Yoo and KK (2013)

- Gravity could be stronger than pressure

$$\delta > \delta_c \sim p/\rho \sim c_s^2 = w = 1/3$$



P_ζ (k) and PBH abundance β(M)

- Fraction of PBH to the total with Gaussian Statistics

For Peak Statistics,
e.g., see Yoo, Harada, Garriga, Kohri, 2018

$$\beta(M) \equiv \frac{\rho_{\rm PBH}(M)}{\rho_{\rm tot}} = 2\int_{\delta_{\rm th}}^{\infty} d\delta \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) = \mathrm{erfc}\left(\frac{\delta_{\rm th}}{\sqrt{2}\sigma}\right)$$

$\delta_{\rm th} \sim 1/3 - 0.5$

- Relation between β and fluctuation σ (or β and Ω)

$$\beta(M) \sim \mathrm{erfc}\left(\frac{\delta_{\rm th}}{\sqrt{2}\sigma}\right) \simeq \sqrt{\frac{2}{\pi}}\frac{\sigma}{\delta_{\rm th}} \exp\left(-\frac{\delta_{\rm th}^2}{2\sigma^2}\right)$$

$$= 1.5 \times 10^{-18} \left(\frac{m_{\rm PBH}}{10^{15}\,g}\right)^{1/2} \left(\frac{\Omega_{\rm PBH} h^2}{0.1}\right)$$

$\sigma^2 \sim P_\zeta$

Typical quantities of PBHs in RD

- Mass (horizon mass $=\rho(t_{form})\, H(t_{form})^{-3}$)

$$M_{PBH} \sim M_{pl}^2 t_{from} \sim \frac{M_{pl}^3}{T_{form}^2} \sim 10^{15} g \left(\frac{T_{form}}{3\times 10^8 GeV} \right)^{-2} \sim 30 M_\odot \left(\frac{T_{form}}{40 MeV} \right)^{-2}$$

- Lifetime

$$\tau_{PBH} \sim \frac{M_{PBH}^3}{M_{pl}^4} \sim 4\times 10^{17} \sec \left(\frac{M_{PBH}}{10^{15} g} \right)^3 \sim 3\times 10^{68} yrs \left(\frac{M_{PBH}}{30 M_\odot} \right)^3$$

- Hawking Temperature

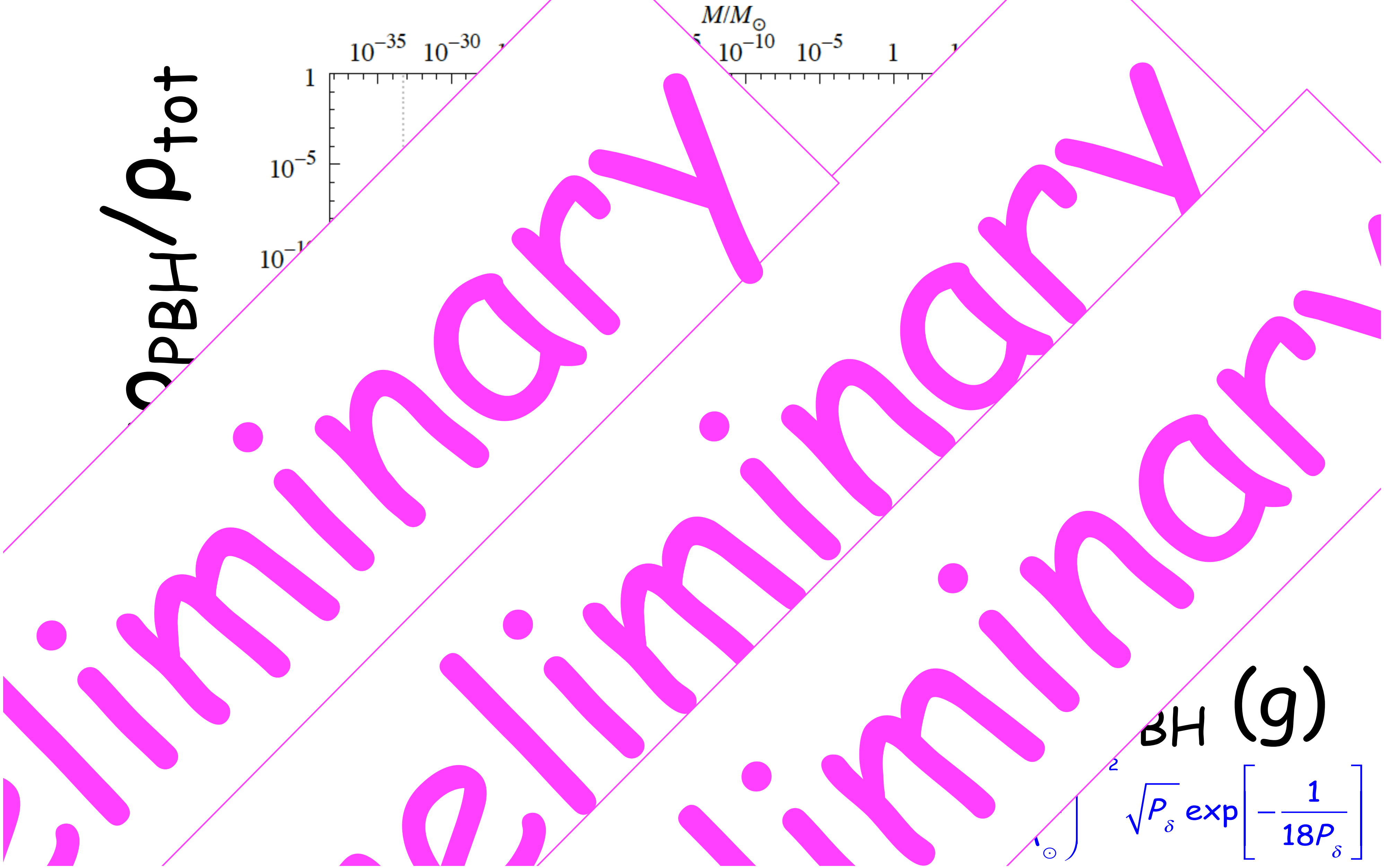
$$T_{PBH} \sim \frac{M_{pl}^2}{M_{PBH}} \sim 0.1 MeV \left(\frac{M_{PBH}}{10^{15} g} \right)^{-1} \sim 3\times 10^{-11} K \left(\frac{M_{PBH}}{30 M_\odot} \right)^{-1}$$

- Fraction to CDM

$$f_{fraction} \equiv \frac{\Omega_{PBH}}{\Omega_{CDM}} \sim \left(\frac{\beta}{10^{-18}} \right) \left(\frac{M_{PBH}}{10^{15} g} \right)^{-1/2} \sim \left(\frac{\beta}{10^{-8}} \right) \left(\frac{M_{PBH}}{30 M_\odot} \right)^{-1/2} \sim 10^8 \left(\frac{M_{PBH}}{30 M_\odot} \right)^{-1/2} \sqrt{P_\delta}\, \exp\left[-\frac{1}{18 P_\delta} \right]$$

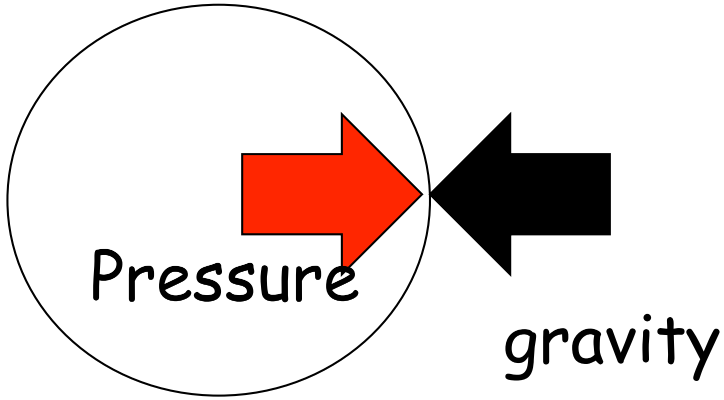
$\beta=\rho_{PBH}/\rho_{tot}$ vs M_{PBH}

Carr, Kohri, Sa[illegible]douda, J.Yokoyama (202[illegible]) in preparation



Features of PBH formations in RD

- Spherical due to radiation pressure



$(w \equiv p/\rho \sim 1/3)$

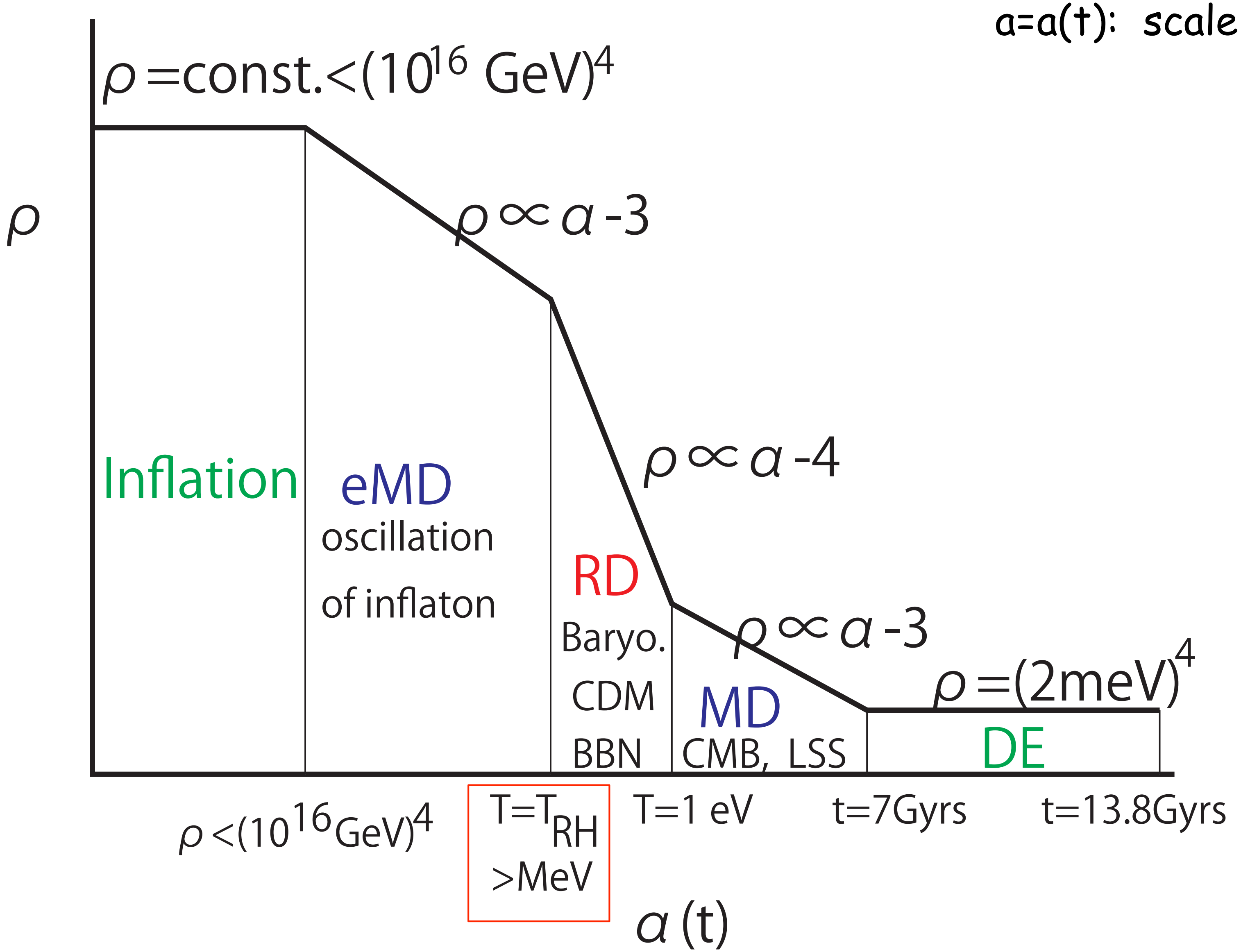
- Negligible evolutions of density perturbations

- Quite a small angular momentum

See, T.Chiba and S.Yokoyama, 2017
De Luca et al, 2019
Minxi He and Suyama, 2019

Cosmic history of energy density

a=a(t): scale factor



PBH formation at the (early) matter dominated (MD) Universe

Polnarev and Khlopov (1982)

Harada, Yoo, KK, Nakao, Jhingan (2016)

1. Pressure is negligible, which could induce an immediate collapse and producing more PBHs?

2. Density perturbations can evolve, which produces non-spherical objects and cannot be enclosed by the Horizon. That means less PBHs can be produced?

Matter Domination

- Three radius in Lagrangian coordinate q_i

Zel'dovich Approximation

$$r_1 = (a - \alpha b) q_1$$

$$r_2 = (a - \beta b) q_2$$

$$r_3 = (a - \gamma b) q_3$$

- Eccentricity $$e^2 = 1 - \left(\frac{r_2(t_c)}{r_3(t_c)}\right)^2 = 1 - \left(\frac{\alpha - \beta}{\alpha - \gamma}\right)^2$$
- Hoop with 2nd Elliptic funciton E(x)

$$\mathcal{C} = 16\left(1 - \frac{\gamma}{\alpha}\right) E\left(\sqrt{1 - \left(\frac{\alpha - \beta}{\alpha - \gamma}\right)^2}\right) r_f$$

- Hoop conjecture for PBH production

$$\mathcal{C} \lesssim 2\pi r_g.$$

Abundance of PBHs formed in MD

- Probability distribution by peak statistics (BBKS)

Doroshkevich (1970)

$$
\begin{aligned}
&w(\alpha,\beta,\gamma)d\alpha d\beta d\gamma \\
&= -\frac{27}{8\sqrt{5}\pi\sigma_3^6}\exp\left[-\frac{1}{10\sigma_3^2}(\alpha+\beta+\gamma)^2-\frac{1}{4\sigma_3^2}\{(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2\}\right] \\
&\cdot(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)d\alpha d\beta d\gamma.
\end{aligned}
$$

$$\sigma_H = \sqrt{5}\sigma_3$$

- Probability

$$\beta_0 = \int_0^\infty d\alpha \int_{-\infty}^{\alpha} d\beta \int_{-\infty}^{\beta} d\gamma\ \theta(1-h(\alpha,\beta,\gamma))w(\alpha,\beta,\gamma)$$

$$h(\alpha,\beta,\gamma) = \frac{2}{\pi}\frac{\alpha-\gamma}{\alpha^2}E\left(\sqrt{1-\left(\frac{\alpha-\beta}{\alpha-\gamma}\right)^2}\right)$$

$$h(\alpha,\beta,\gamma) := \mathcal{C}/(2\pi r_g)$$

Angular momentum produced by perturbations

Harada, Yoo, KK, nad Nakao (2017)

- Angular momentum

$$\mathbf{L}_c = \int_{a^3V} \rho \mathbf{r} \times \mathbf{v} d^3\mathbf{r} = \rho_0 a^4 \left(\int_V \mathbf{x} \times \mathbf{u} d^3\mathbf{x} + \int_V \mathbf{x}\delta \times \mathbf{u} d^3\mathbf{x} \right)$$

1st order effects   2nd order effects

- Density perturbation δ
- (Peculiar) Velocity perturbation

$$\mathbf{u} := a D\mathbf{x}/Dt$$

$$\mathbf{u}_1 = -\frac{t}{a}\nabla\psi_1$$

- Potential perturbation

$$\psi := \Psi - \Psi_0$$

Effects by finite angular momentum

Harada, Yoo, KK, Nakao (2017)

- Probability distribution

$$a_* := L/(GM^2/c)$$

$$f_{\mathrm{BH}(2)}(a_*)da_* \propto \frac{1}{a_*^{5/3}} \exp\left(-\frac{1}{2\sigma_H^{2/3}}\left(\frac{2}{5}\mathcal{I}\right)^{4/3}\frac{1}{a_*^{4/3}}\right) da_*$$

- Probability

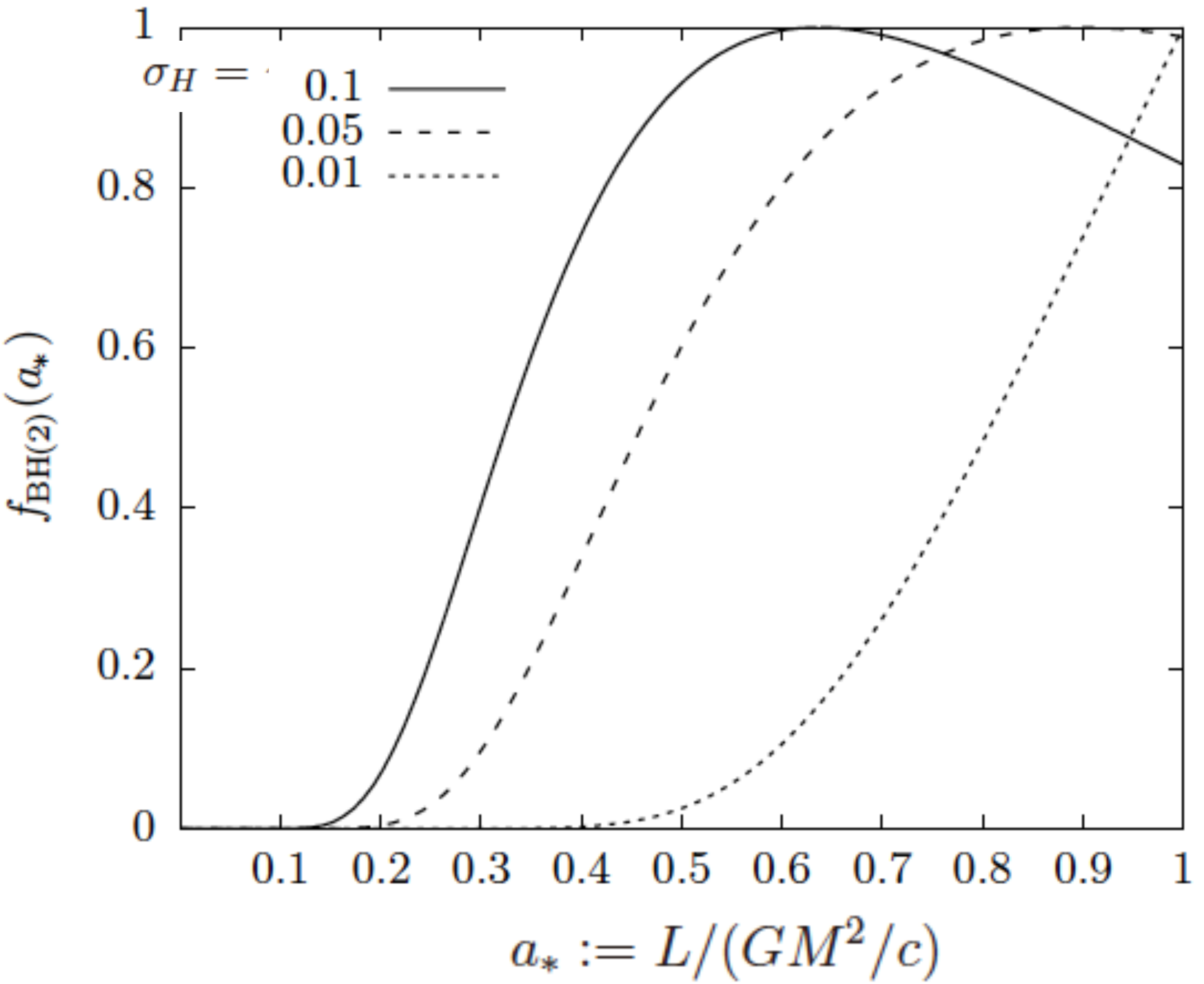
$$\beta_0 \simeq \int_0^\infty d\alpha \int_{-\infty}^{\alpha} d\beta \int_{-\infty}^{\beta} d\gamma \theta[\delta_H(\alpha,\beta,\gamma) - \delta_{\mathrm{th}}]\theta[1 - h(\alpha,\beta,\gamma)]w(\alpha,\beta,\gamma)$$

$$\delta_H(\alpha,\beta,\gamma) = \alpha+\beta+\gamma \qquad \delta_{\mathrm{th}} := \left(\frac{2}{5}\mathcal{I}\sigma_H\right)^{2/3}$$

Spin distribution

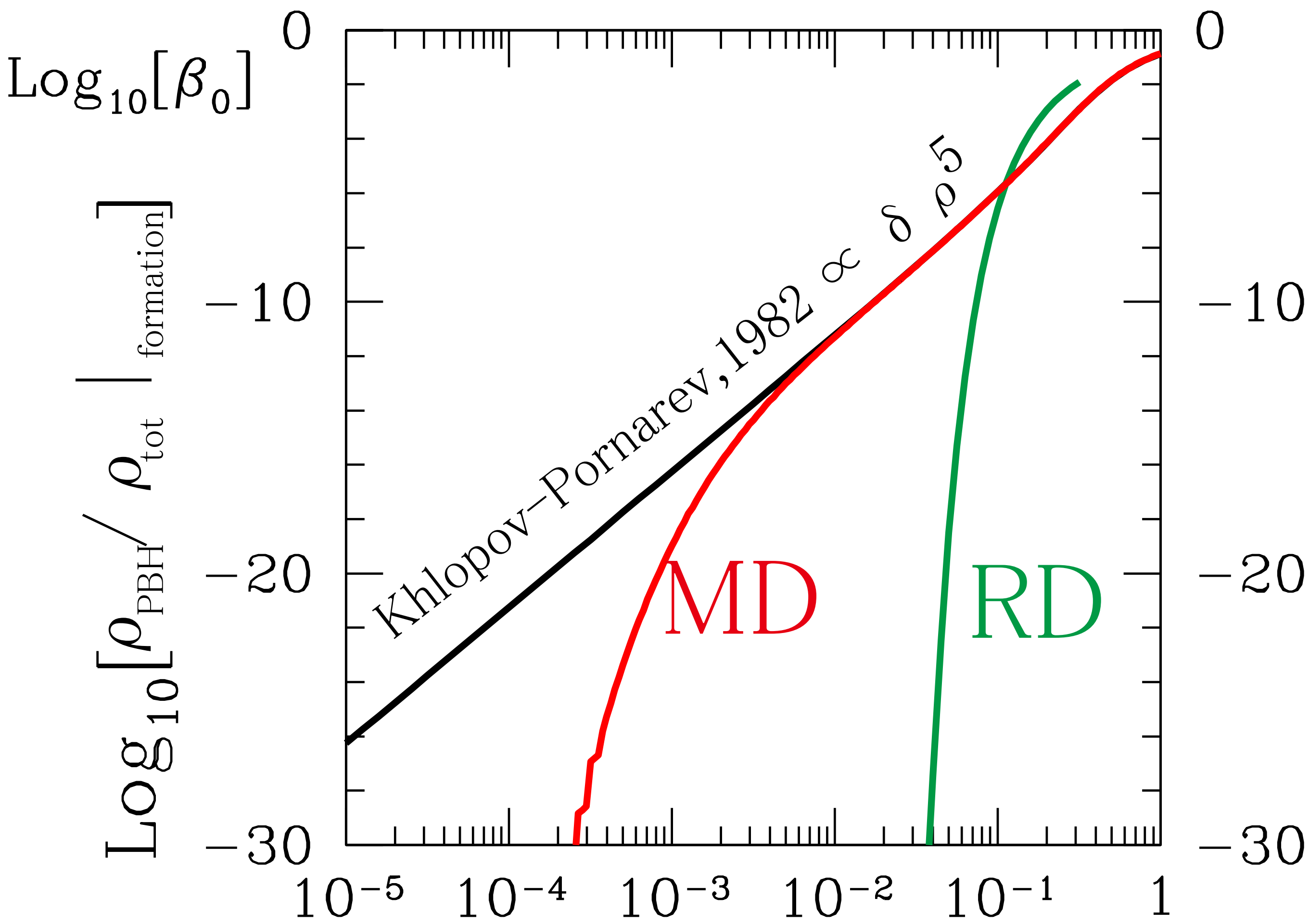
More highly-spinning halos cannot collapse into PBHs, which means that the PBHs produced tend to have high spins in MD

Harada, Yoo, KK, Nakao (2017)



Beta in matter-domination

Harada, Yoo, KK, Nakao (2017)



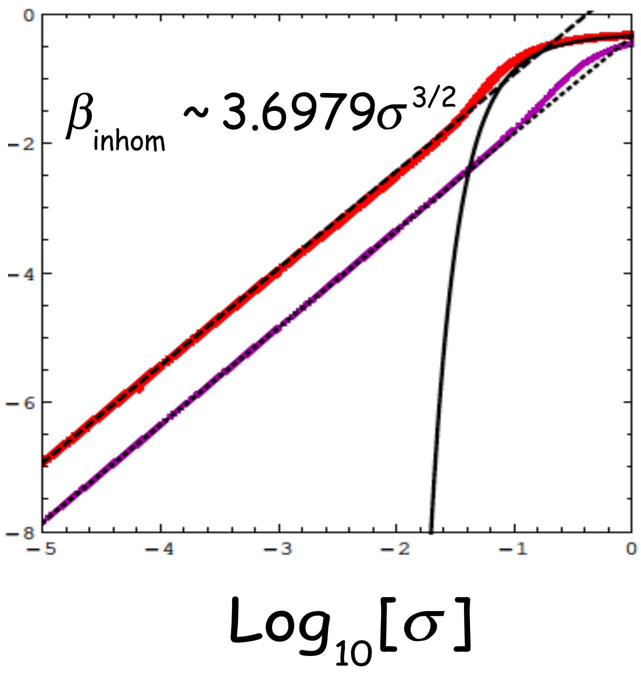
$\sigma_H = \sqrt{5}\sigma_3$ = $\delta\rho/\rho$

Effects of Inhomogeneity on PBH formations in Matter Domination

T.Kokubu, K.Kyutoku, K.Kohri, T.Harada, arXiv:1810.03490

Singularity should be enclosed by (apparent) horizon

$$\beta \equiv \frac{\rho_{PBH}}{\rho_{tot}}$$



$$\beta_{aniso} \simeq 0.05556\sigma^5$$

$$\beta_{inhom+aniso} \simeq \beta_{inhom} \times \beta_{aniso} = 0.2055\sigma^{13/2}$$

Inflation models

Type-III Hilltop inflation models

German, Ross, Sarkar (01)

- Potential in supergravity, e.g., KK, Lin and Lyth (07)

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 - \lambda\frac{\phi^p}{M_P^{p-4}} + \cdots$$

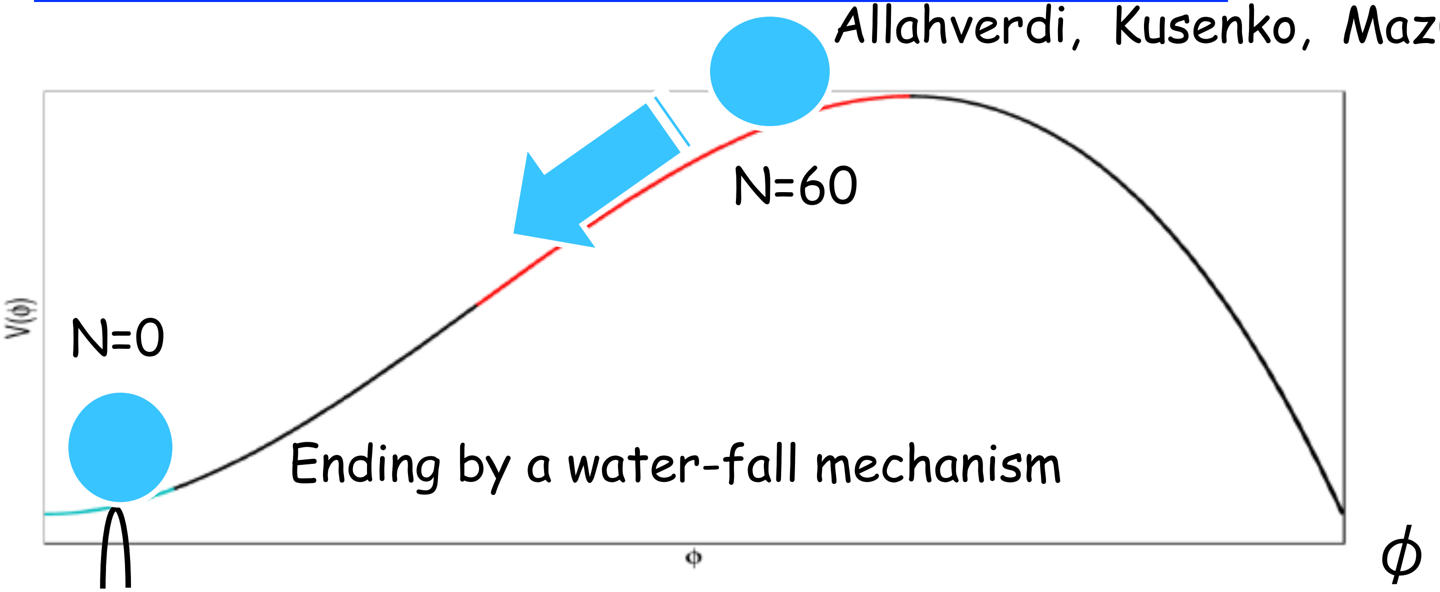
$$\equiv V_0\left(1 + \frac{1}{2}\eta_0\frac{\phi^2}{M_P^2}\right) - \lambda\frac{\phi^p}{M_P^{p-4}} + \cdots,$$

$$\eta_0 = \frac{\pm m^2 M_P^2}{V_0}$$

$\eta_0 > 0$ and $p > 2$

$$W = C\frac{\phi^p}{M_{pl}^{p-3}}, \quad \lambda \sim Cm_{3/2} / M_{pl} \text{ in SUGRA}$$

Allahverdi, Kusenko, Mazumdar (06)



Large running spectral index

- Curvature perturbation (scalar)

$$P_\zeta \sim \frac{V}{M_{pl}^4 \varepsilon} \sim \left(\delta T / T\right)^2$$

Only at large scales

- Higher order observables

Spectral index: $n_s - 1 = dP_\zeta / d\ln k = 2\eta - 6\varepsilon$

Running of spectral index: $\alpha_s = dn_s / d\ln k = -24\varepsilon^2 + 16\varepsilon\eta - \xi^{(2)}$

Running of running: $\beta_s = d\alpha_s / d\ln k = 192\varepsilon^3 + 192\varepsilon^2\eta - 32\varepsilon\eta^2 + (-24\varepsilon + 2\eta)\xi^{(2)} + 2\sigma^{(2)}$

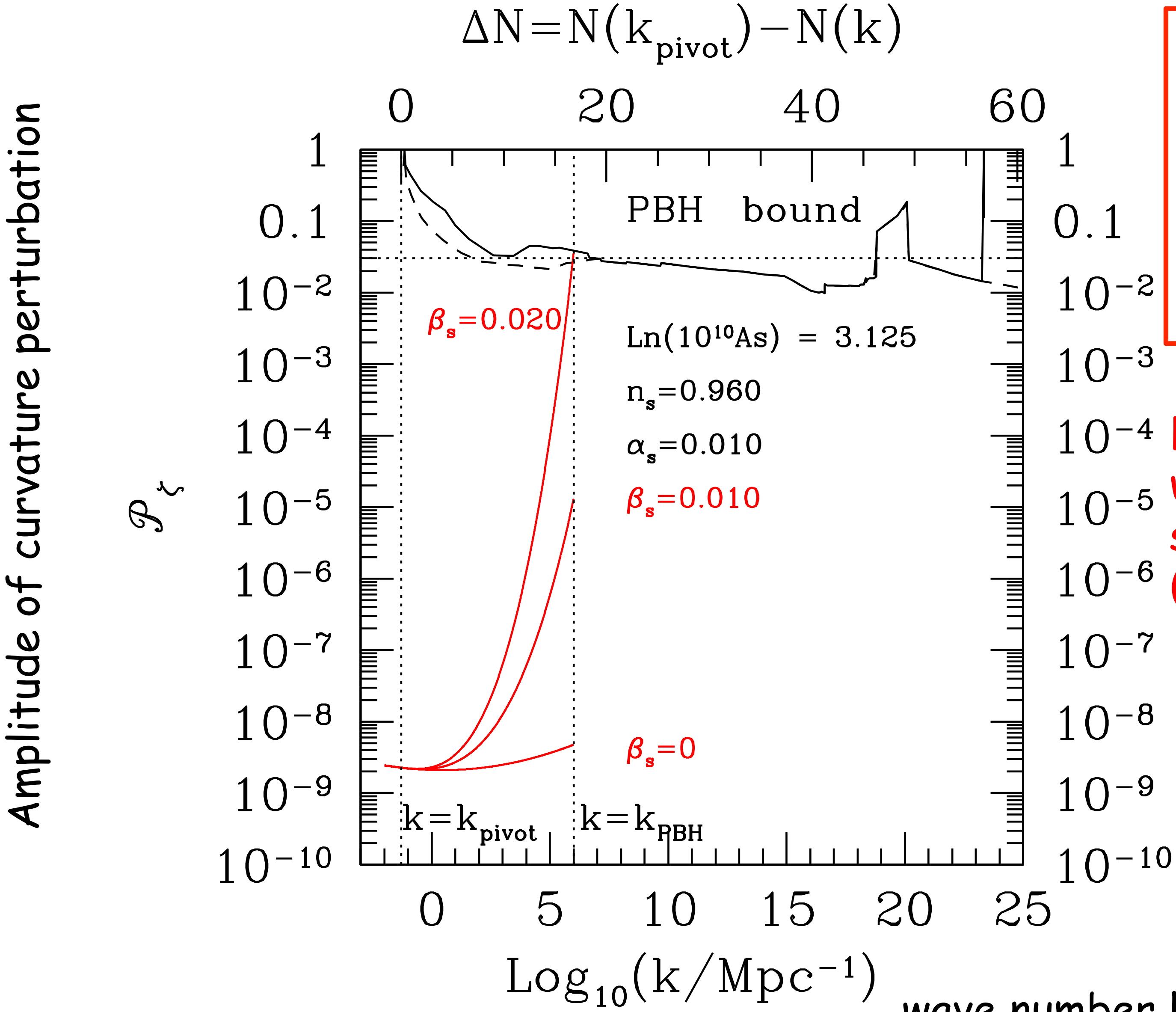
Simple parameterization of running of spectral indexes of curvature perturbation

KK and T.Terada, 2018

$$P_\zeta(k) = A_{\rm s}\left(\frac{k}{k_*}\right)^{n_{\rm s}-1+\frac{\alpha_{\rm s}}{2}\ln\left(\frac{k}{k_*}\right)+\frac{\beta_{\rm s}}{6}\left(\ln\left(\frac{k}{k_*}\right)\right)^2}$$

P_ζ vs k

KK and T.Terada, 2018



Planck (2015)

$n_s = 0.9586 \pm 0.0056,$

$\alpha_s = 0.009 \pm 0.010,$

$\beta_s = 0.025 \pm 0.013.$

at 68% C.L.

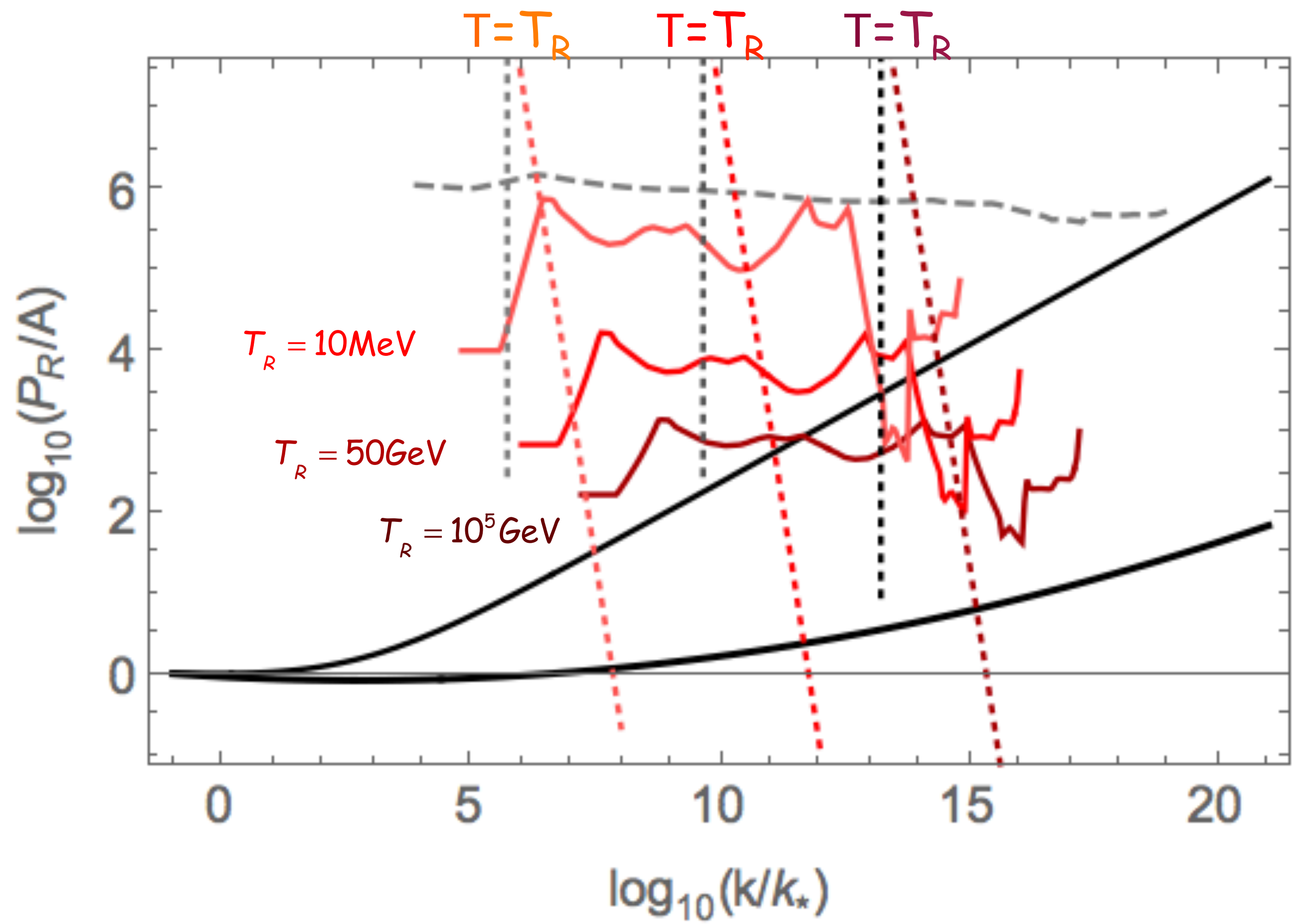
For inflation models with a big running, see Kohri, Lin Lyth (2008)

wave number k = p x a

Upper bounds on curvature perturbation in MD

Carr, Tenkanen and Vaskonen (2017)
KK and T.Terada, 2018

$\log_{10}[P_\zeta / P_{\zeta,\mathrm{observed}}]$



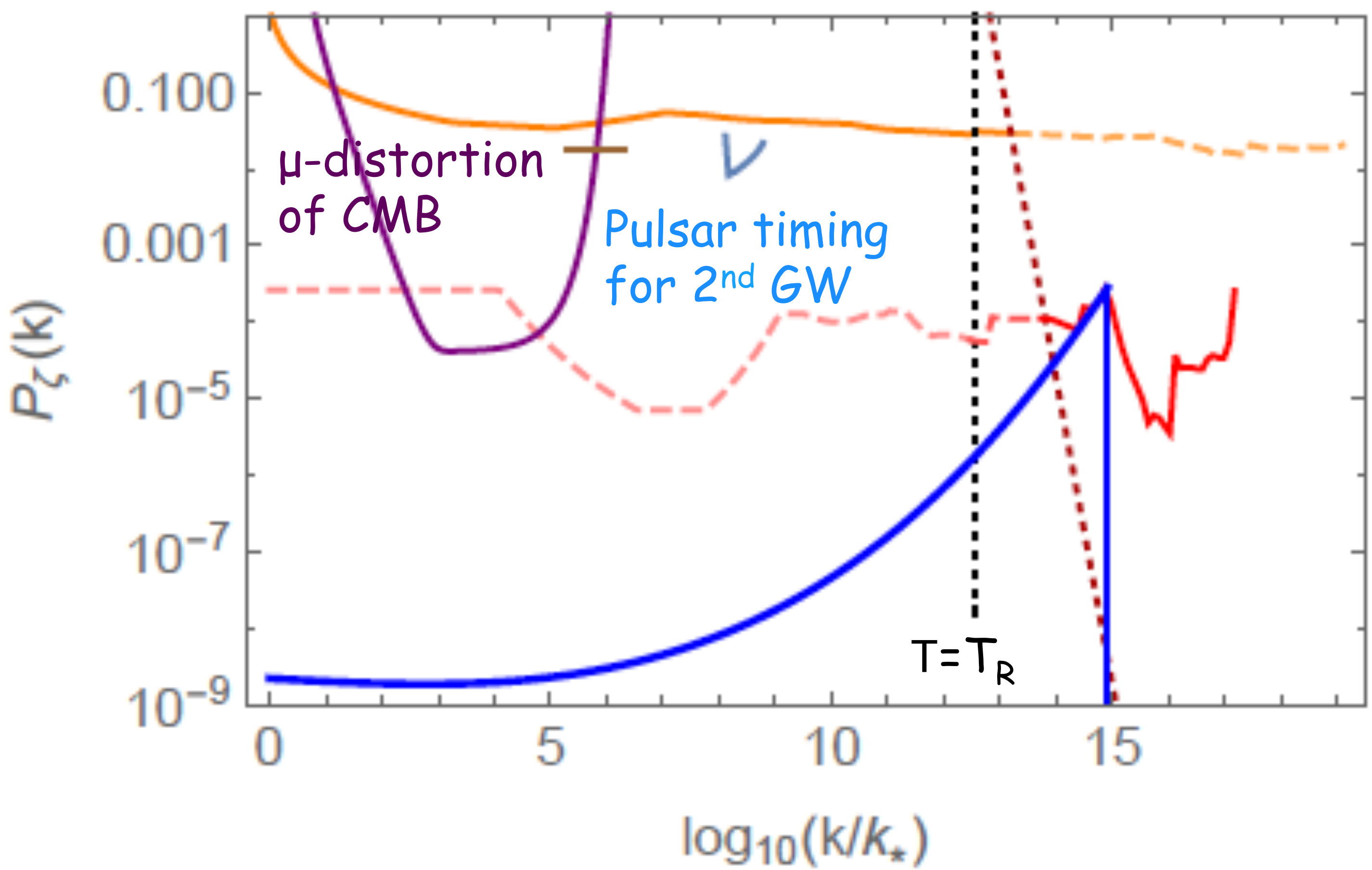
100% MD before reheating

100 % Dark Matter by PBHs with 10^{17} g masses

KK and T.Terada, 2018

$$T_{\mathrm{R}} = 10^4 \mathrm{GeV},$$

$$n_{\mathrm{s}} = 0.96, \alpha_{\mathrm{s}} = 0, \beta_{\mathrm{s}} = 0.0019485.$$

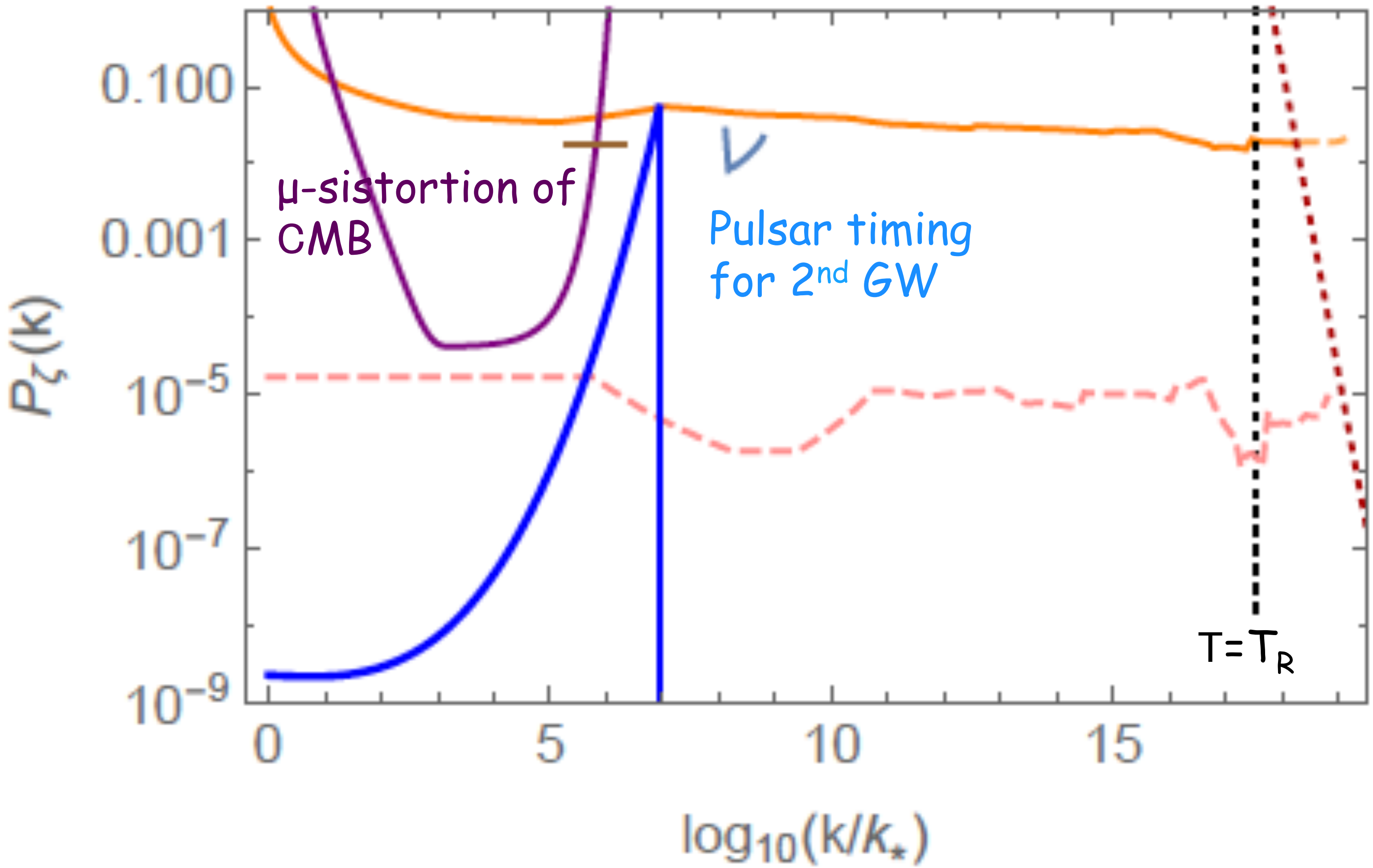


LIGO/VIRGO event with 30 Msolar

KK and T.Terada, 2018

$T_R = 10^9\mathrm{GeV}$

$n_s = 0.96, \alpha_s = 0, \beta_s = 0.026.$



2^{nd} order GWs enhanced at a sudden transition from MD to RD

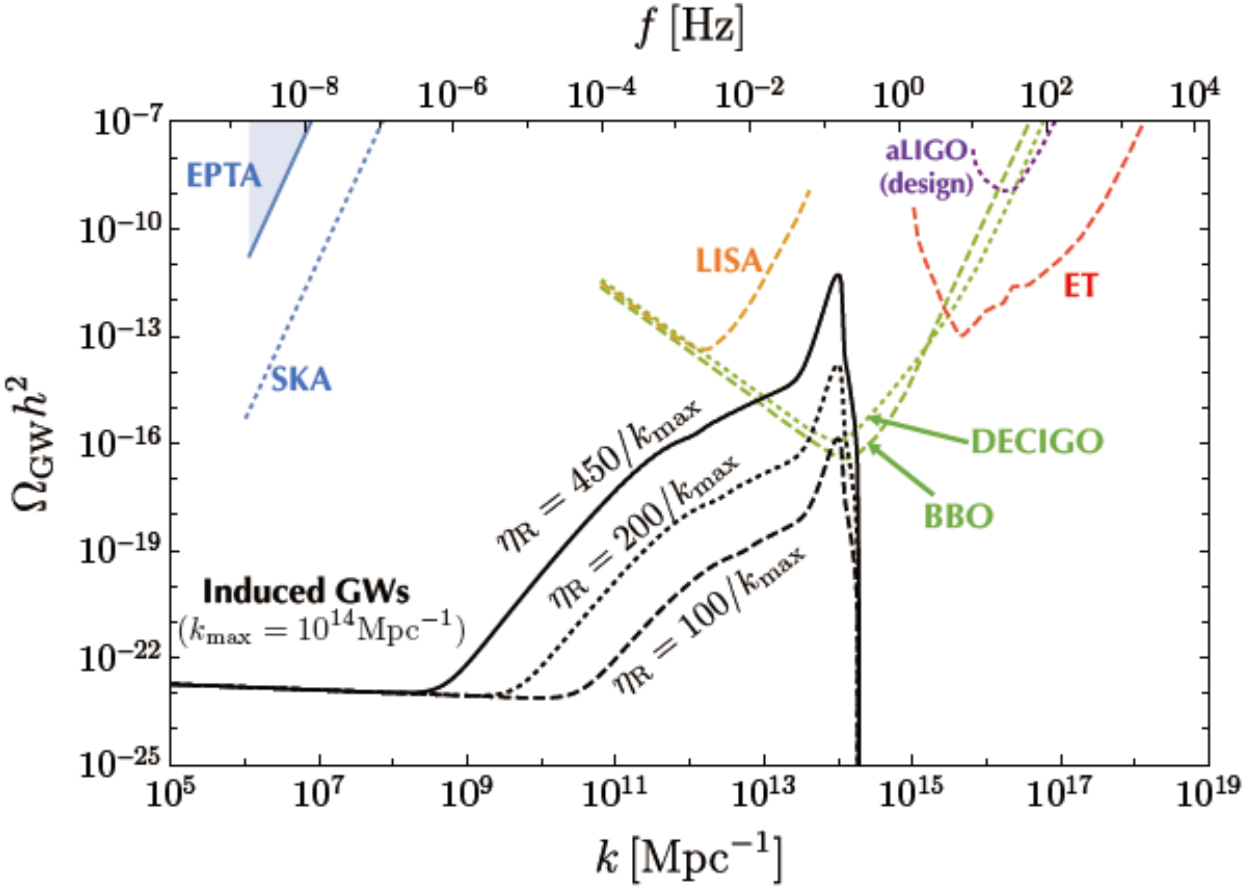
Inomata, Kohri, Nakama, Terada, 2019

See also, S. Kuroyanagi's talk in 2015

$\overline{\mathcal{P}_h(\eta,k)}$ ~ $\int\int f^2(u,v,x,x_R)$

$$f(u,v,\bar{x},x_{\rm R}) = \frac{3\left(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x}) + \Phi(u\bar{x})\Phi'(v\bar{x})) + 4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x})\right)}{25(1+w)}$$

This is big!



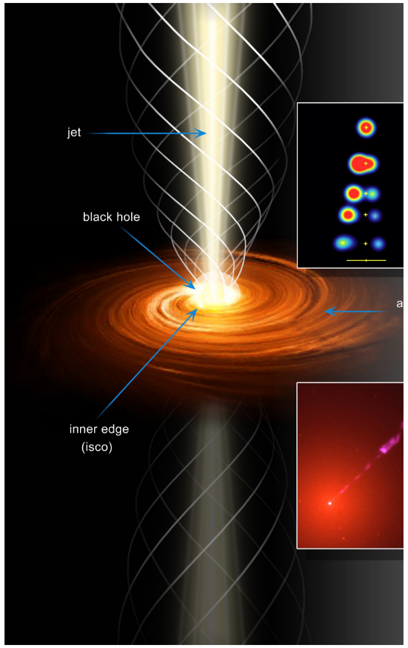
CMB bound on PBHs by COSMOLOGICAL disk-accretion in the late MD epoch

Poulin, Serpico, Calore, Clesse, KK (2017)

- A non-spherical accretion disk (ADAF(slim) + Standard disk) around a PBH caused by an angular momentum emits radiation

$$\dot{M}_{\rm HB} \equiv 4\pi\lambda\,\rho_\infty v_{\rm eff} r_{\rm HB}^2 \equiv 4\pi\lambda\,\rho_\infty \frac{(GM)^2}{v_{\rm eff}^3}$$

$$l \simeq \omega\, r_{\rm HB}^2 \simeq \left(\frac{\delta\rho}{\rho} + \frac{\delta v}{v_{\rm eff}}\right) v_{\rm eff} r_{\rm HB}$$



- CMB anisotropies are affected

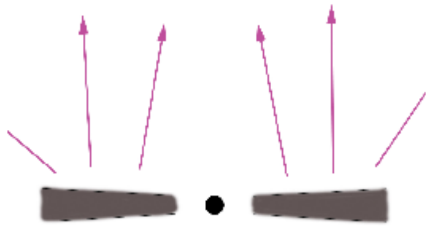
- From observations, we can constrain the number density of PBHs

An accretion disk around a black hole

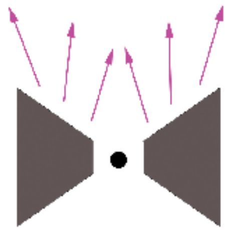
Kohri, Mineshige, 2002
Kohri, Narayan, Piran, 2005

Viscous heating process ⇔ Various cooling processes

i. Standard Accretion Disk (Standard Disk)
   - Radiative Cooling

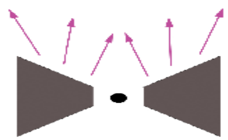


ii. Advection Dominated Accretion Flow (ADAF)
   - Advective cooling (entropy going into BH) gives RIAF (optically thin) or Slim Disk (optically-thick)



iii. Convection Dominated Accretion Flow (CDAF)
   - Convective cooling

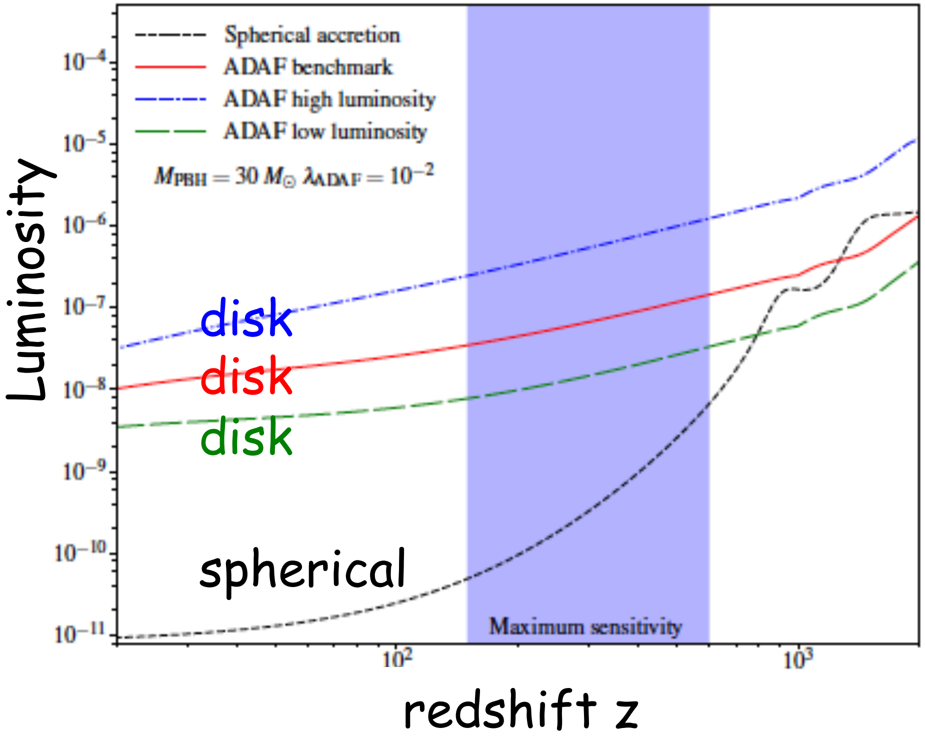
iv. Neutrino-Dominated Accretion Disk (NDAF)
   - Neutrino Cooling



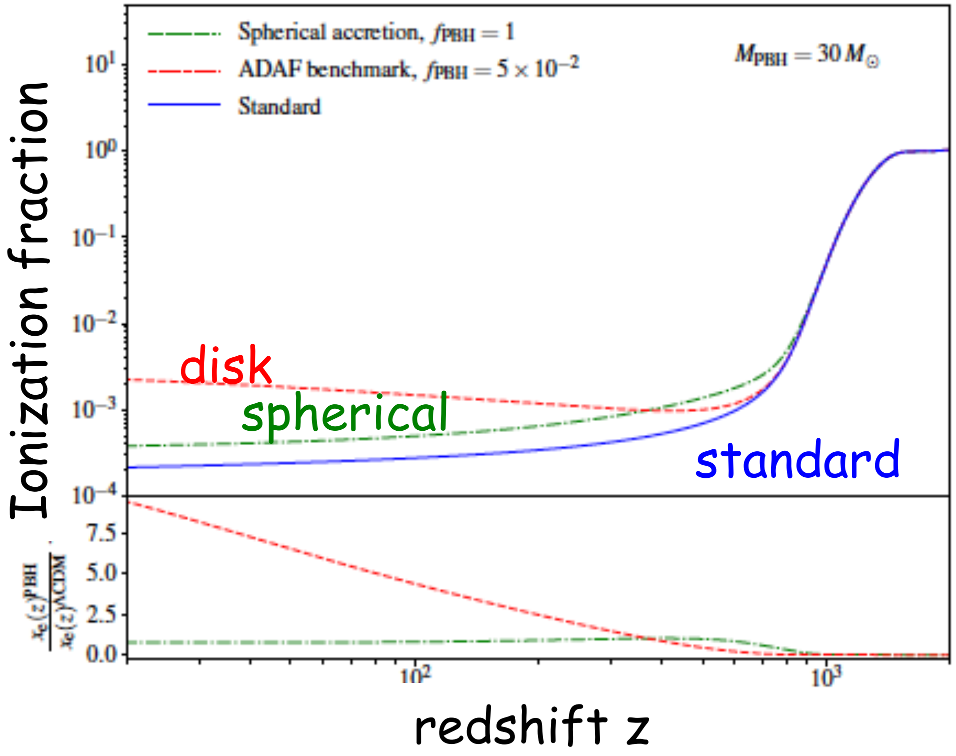
v. ...

Modified CMB anisotropy

Poulin, Serpico, Calore, Clesse, Kohri (2017)



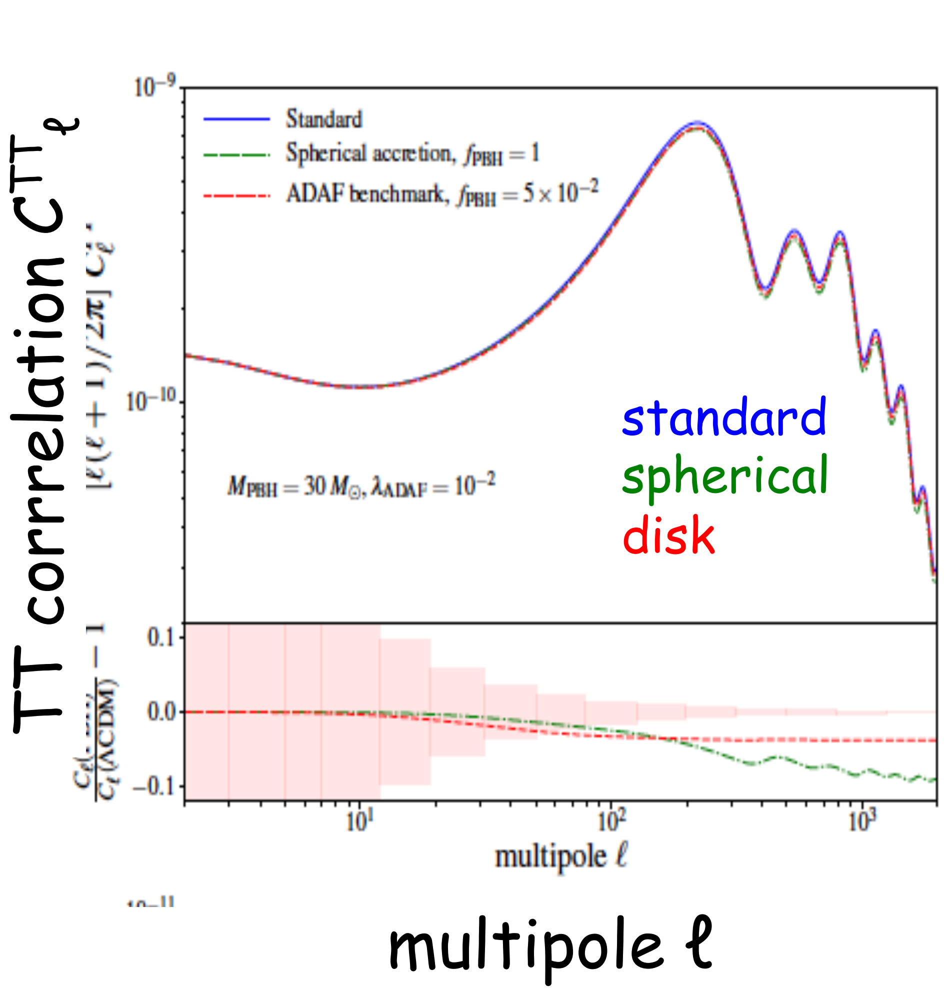
Luminosity

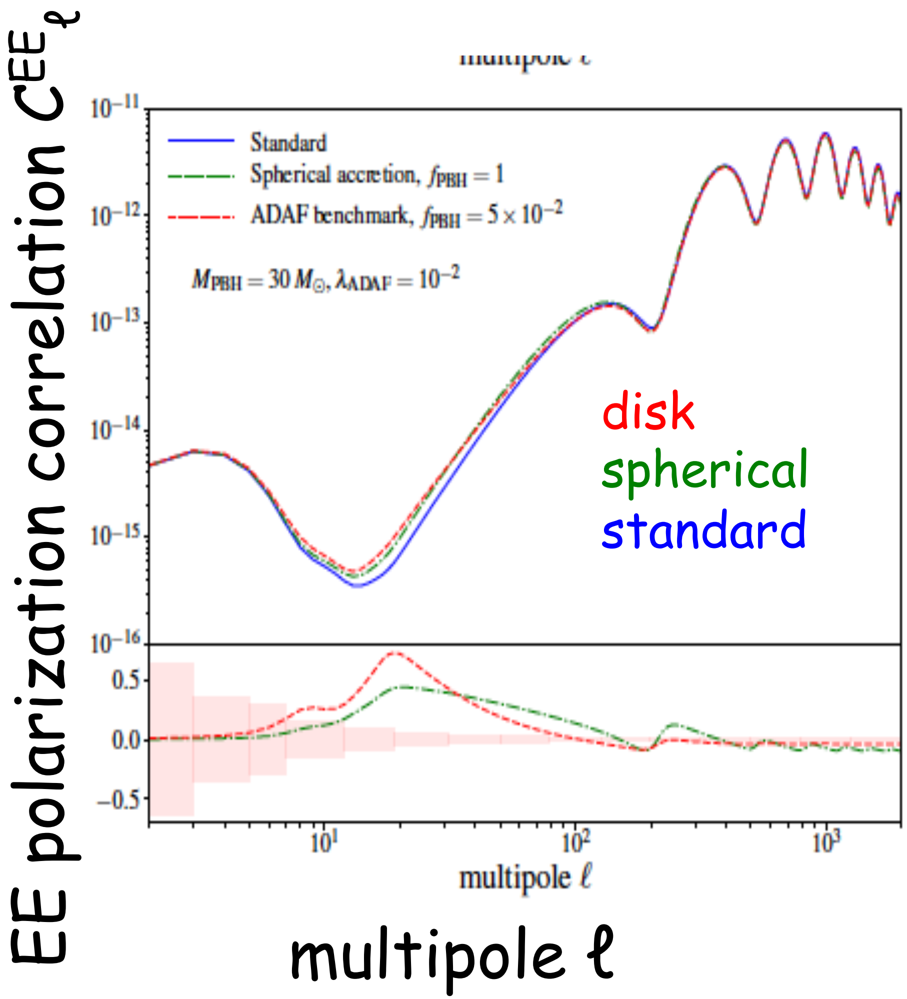


Ionization fraction

Modified CMB anisotropy

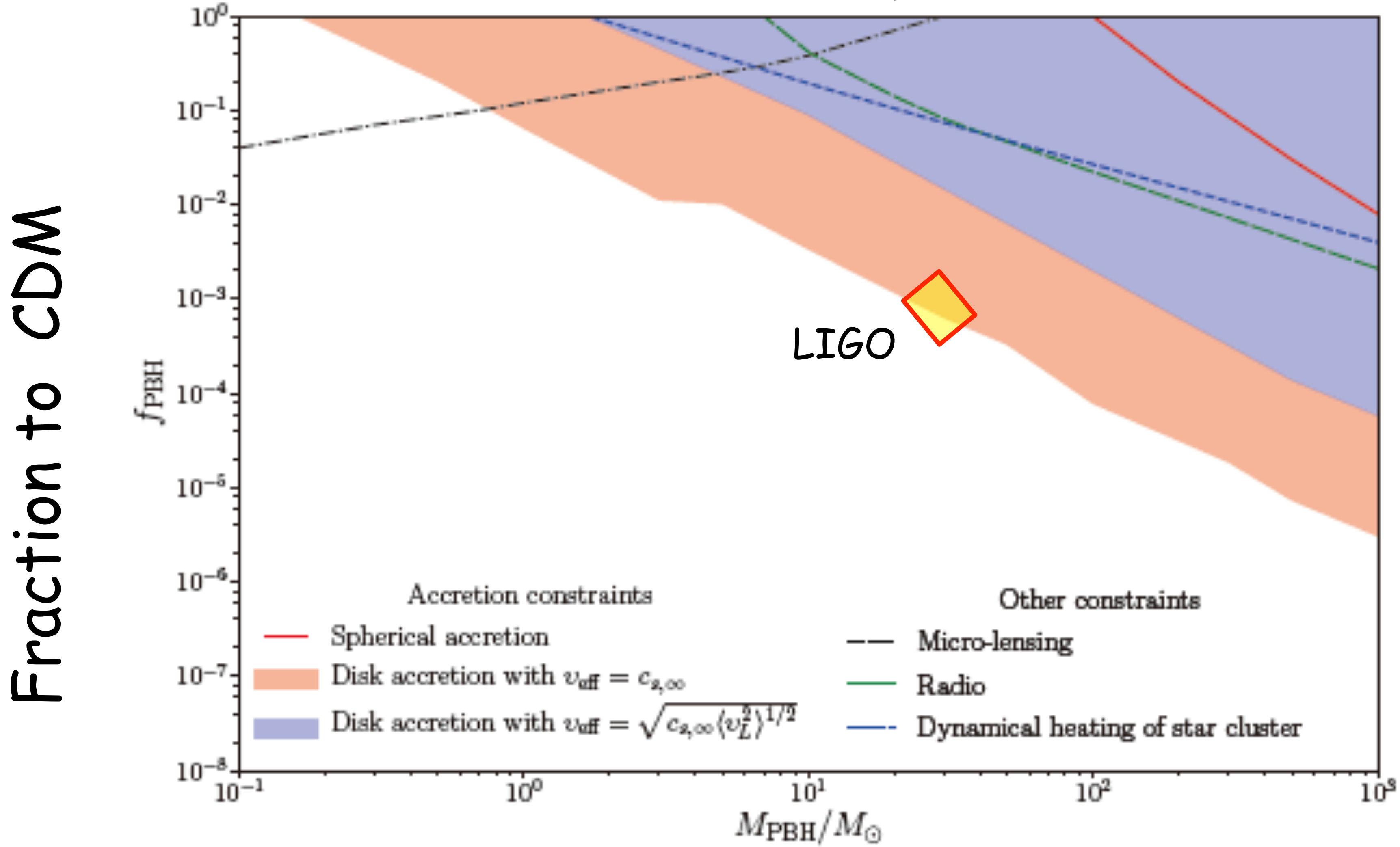
Poulin, Serpico, Calore, Clesse, Kohri (2017)





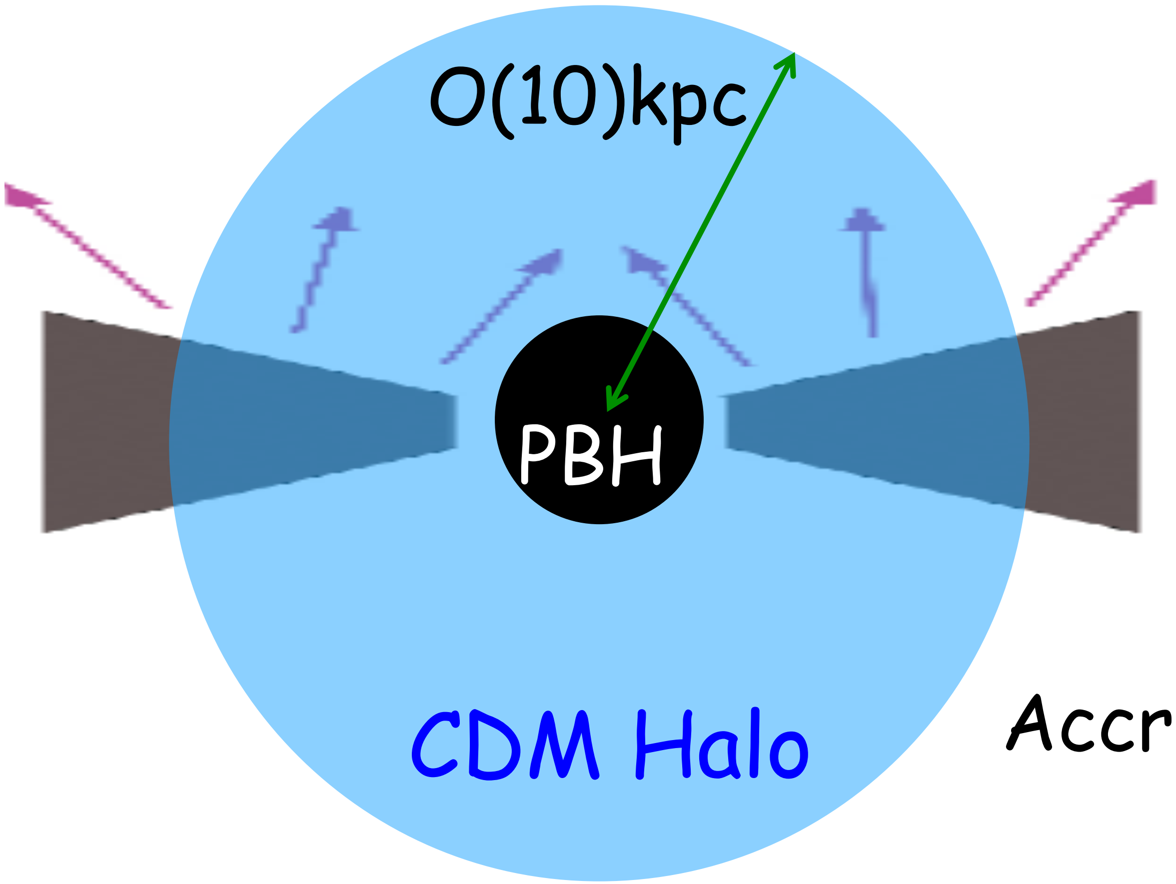
CMB bound by disk-accretion in the latest MD epoch

Poulin, Serpico, Calore, Clesse, KK (2017)



COSMOLOGICAL baryon accretion onto CDM halo with a PBH in the late MD epoch

Poulin, Serpico, Inman, Kohri, Hiroshima (2020)



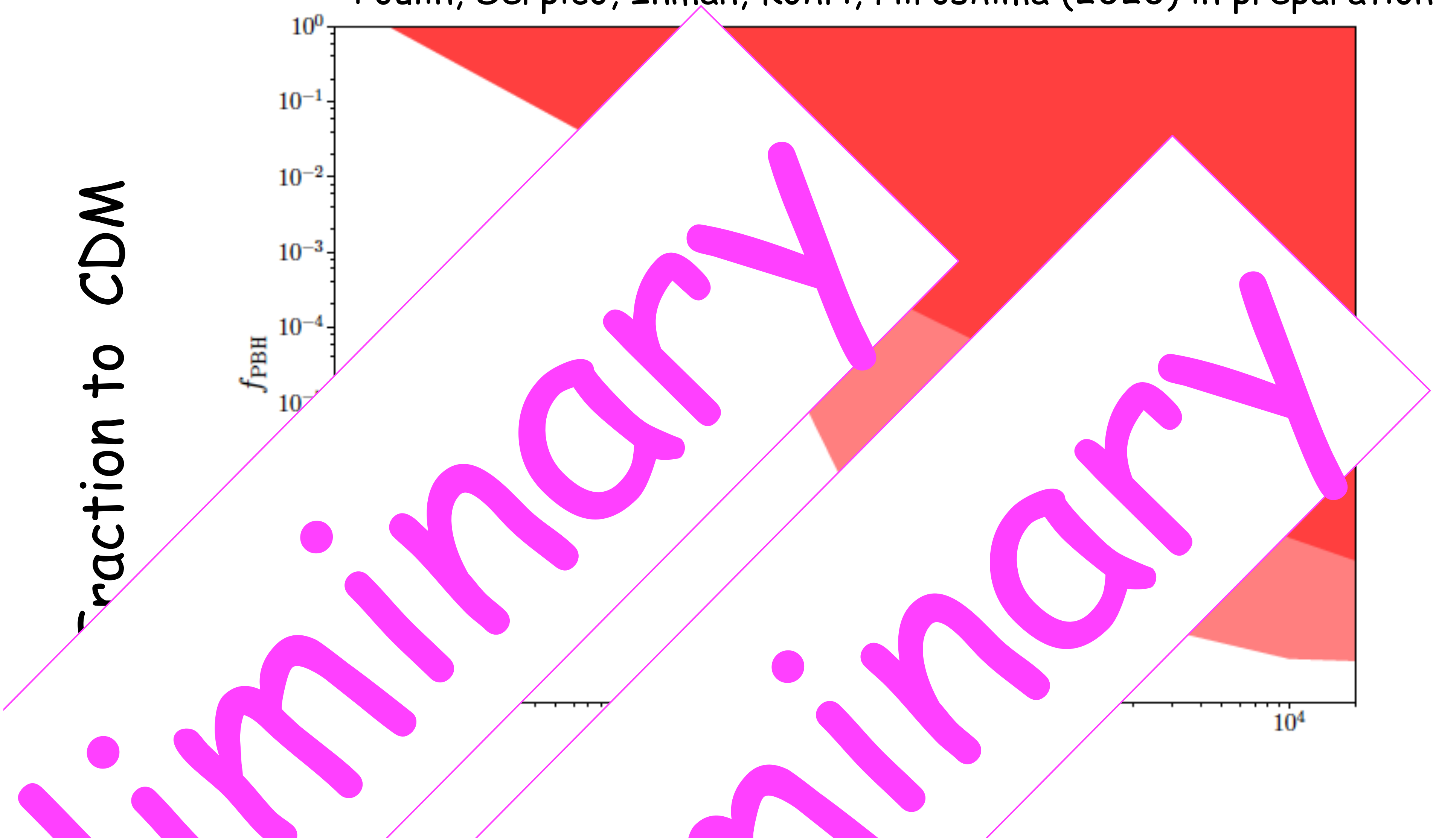
$\rho \propto r^{-2.25}$

Accretion onto CDM + PBH

z ~30 -- 100

CMB bound by disk-accretion in the latest MD epoch

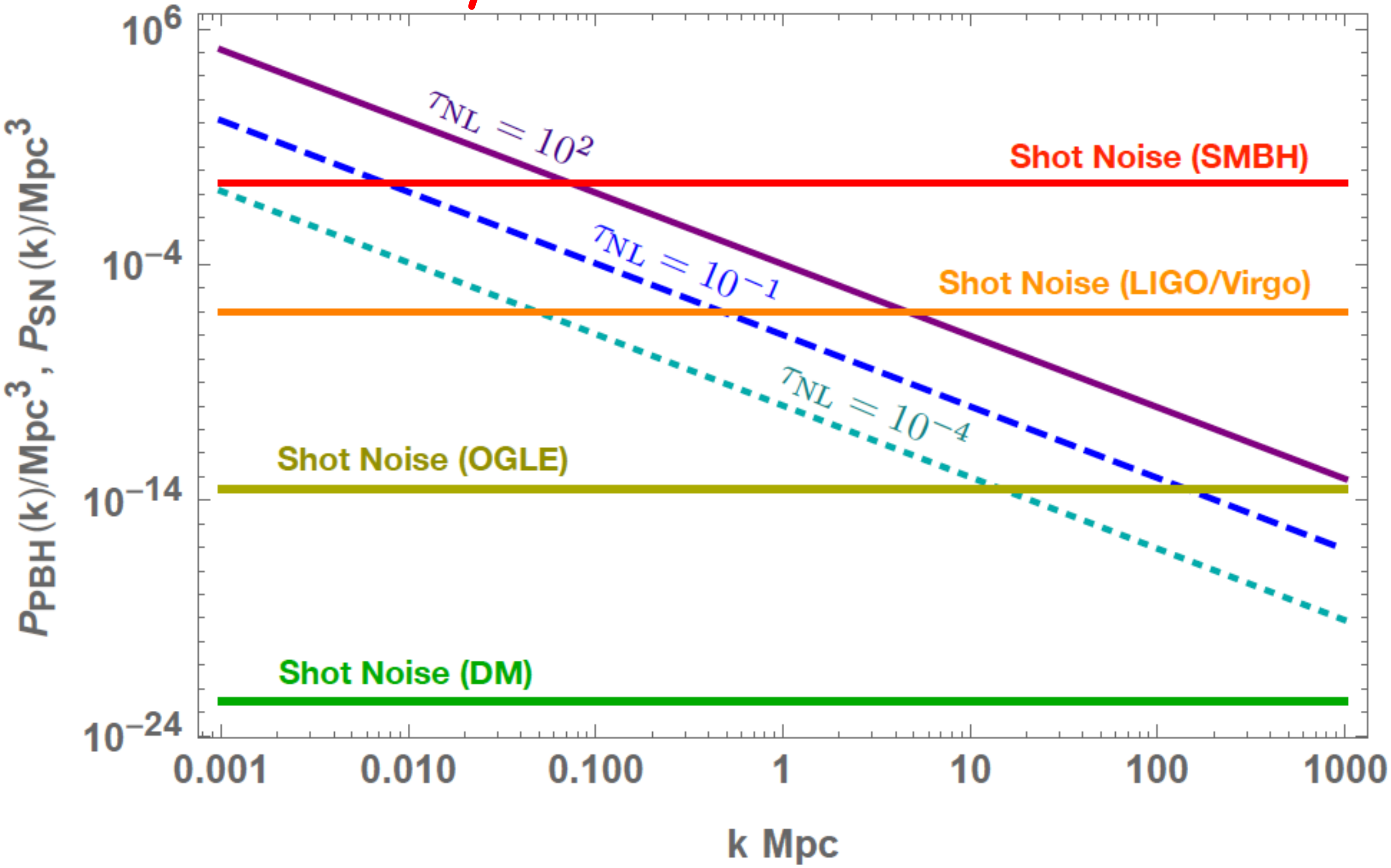
Poulin, Serpico, Inman, Kohri, Hiroshima (2020) in preparation



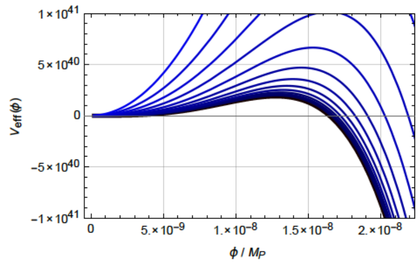
PBHs are clustering?

Matsubara, Terada, Kohri, S. Yokoyama, 1909.06048
See also, Suyama and S. Yokoyama (2019)
Tada and S.Yokoyama (2015)

See also S. Yokoyama's talk



Higgs stabilization due to evaporating PBHs?



Kohri and Matsui (2017)

- Potential with finite-temperature corrections

$$V_{\rm eff}(\phi) \simeq \frac{1}{2}\left(\lambda_{\rm eff} T_{\rm H}^2 + \kappa^2 T_{\rm H}^2\right)\phi^2 + \frac{\lambda_{\rm eff}}{4}\phi^4$$

$$\phi_{\rm max}^2 / T_{\rm H}^2 \approx \mathcal{O}(10)$$

- Probability to get over the potential

$$P(\phi > \phi_{\rm max}) \simeq \frac{\sqrt{2\langle\delta\phi^2\rangle_{\rm ren}}}{\pi\phi_{\rm max}} \exp\left(-\frac{\phi_{\rm max}^2}{2\langle\delta\phi^2\rangle_{\rm ren}}\right)$$

$$\langle\delta\phi^2\rangle_{\rm ren} / T_{\rm H}^2 \simeq \mathcal{O}(0.1)$$

$$\phi_{\rm max}^2 / \langle\delta\phi^2\rangle_{\rm rem} \sim 10^2$$

- This gives,

$$\mathcal{N}_{\rm PBH} \cdot P(\phi > \phi_{\rm max}) \lesssim 1$$

or

$$\beta \lesssim \mathcal{O}\left(10^{-21}\right)\left(\frac{m_{\rm PBH}}{10^9 {\rm g}}\right)^{3/2}$$

Summary

- PBH can be formed at small scales even in both radiation and matter dominated epochs
- More PBHs can be produced in MD for $\delta\rho/\rho \ll 1$
- We may detect gravitational wave signals secondarily-induced by large SCALAR fluctuations at small scales by e.g. DECIGO/BBO …
- We will be able to distinguish a model from others by using future small-scale probes such as PIXIE-like satellite (CMB μ-distortion), SKA/Ominiscope (21cm,Pulsar timing), CTA (gamma-ray) …