Primordial black holes

with primordial non-Gaussianitity

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公募研究(``koubo kenkyu”, open-solicited researches) 18H04356

「原始ブラックホール形成過程の精査とその観測的検証」

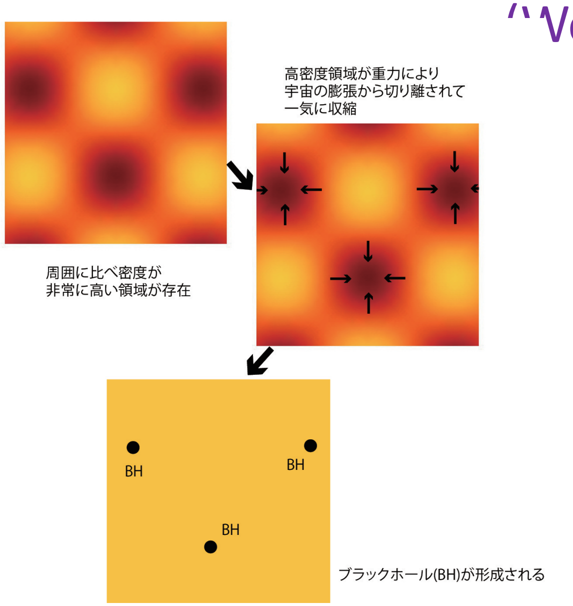
Brief intro. for PBH

Hawking (1971)
Carr and Hawking (1974), …

(also Zeldovich and Novikov (1967))

- Primordial Black Hole (PBH)
  - ✓ BHs formed in the early Universe (after inflation)
  - ✓ direct gravitational collapse of a overdense region
  - ✓ mass of formed BH ~ Hubble horizon mass at the formation

(We focus on the PBH formed in the radiation-dominated era)



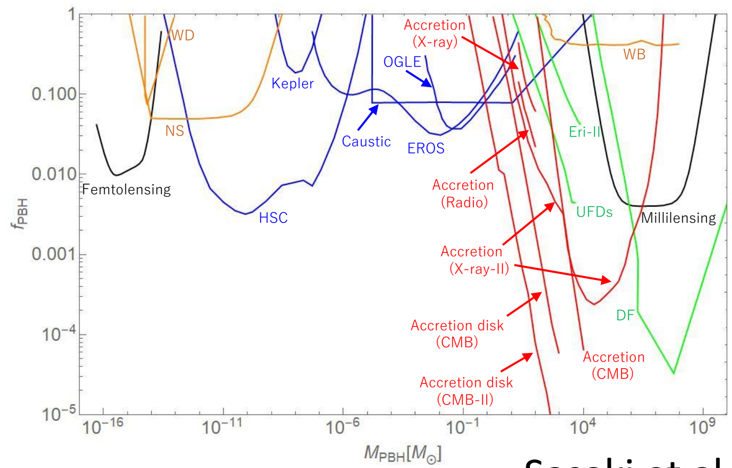
$$M = \gamma M_{\rm PH} = \frac{4\pi}{3}\gamma\rho H^{-3} \approx 2.03\times 10^5\gamma\left(\frac{t}{1\ {\rm s}}\right)M_\odot.$$

$$t \approx 0.738\left(\frac{g_*}{10.75}\right)^{-1/2}\left(\frac{T}{1\ {\rm MeV}}\right)^{-2}\ {\rm s},$$

Why PBHs?

✓a candidate of dark matter

$$M > 10^{15}\,\mathrm{g}(\sim 10^{-18}\,M_\odot)$$



Sasaki et al. (2018)

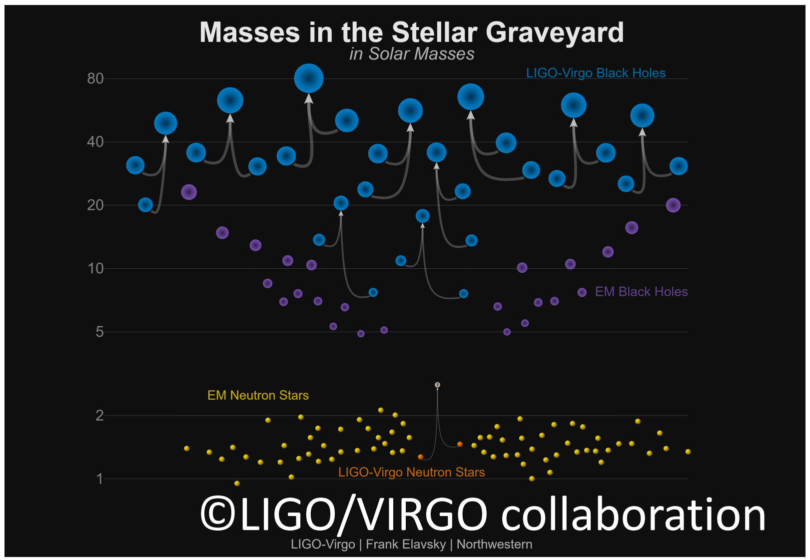
✓a "probe" of Hawking radiation

$$M < 10^{15}\,\mathrm{g}$$

✓a source of LIGO events

$$M \sim 10\,M_\odot$$

Nakamura et al.(1997),
Sasaki et al. (2016), Bird et al. (2016), …



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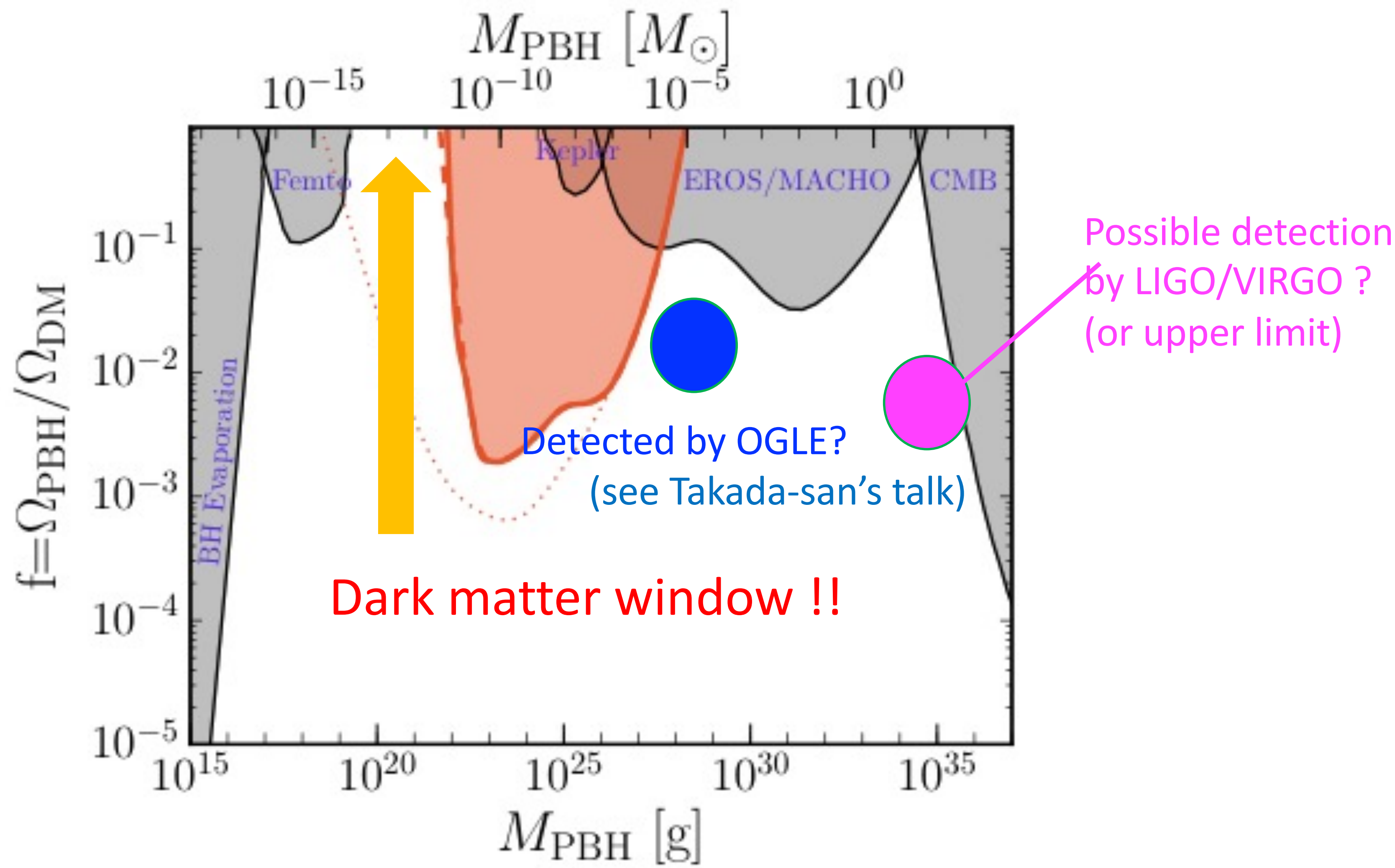
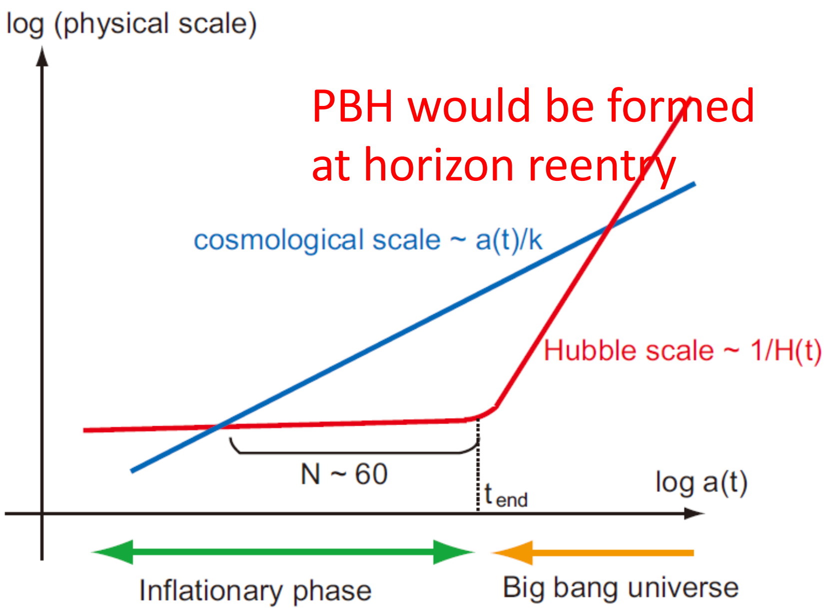
Current situation (conservative)

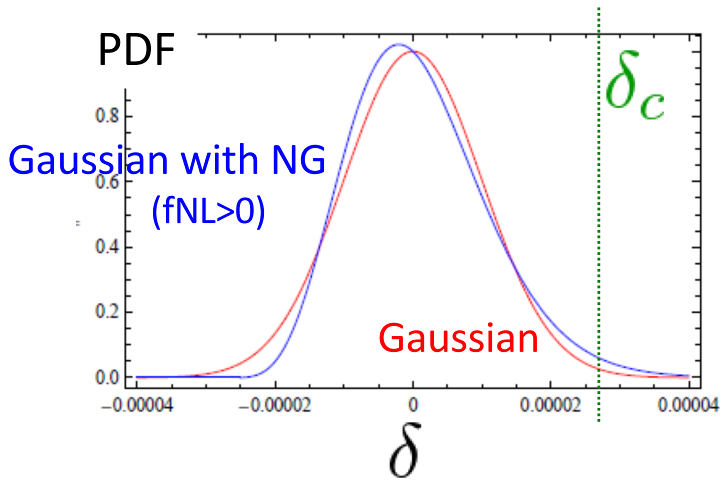


Niikura, Takada, SY+ 1901.07120, …

Picture of PBH formation

- Super-horizon, probability distribution func.



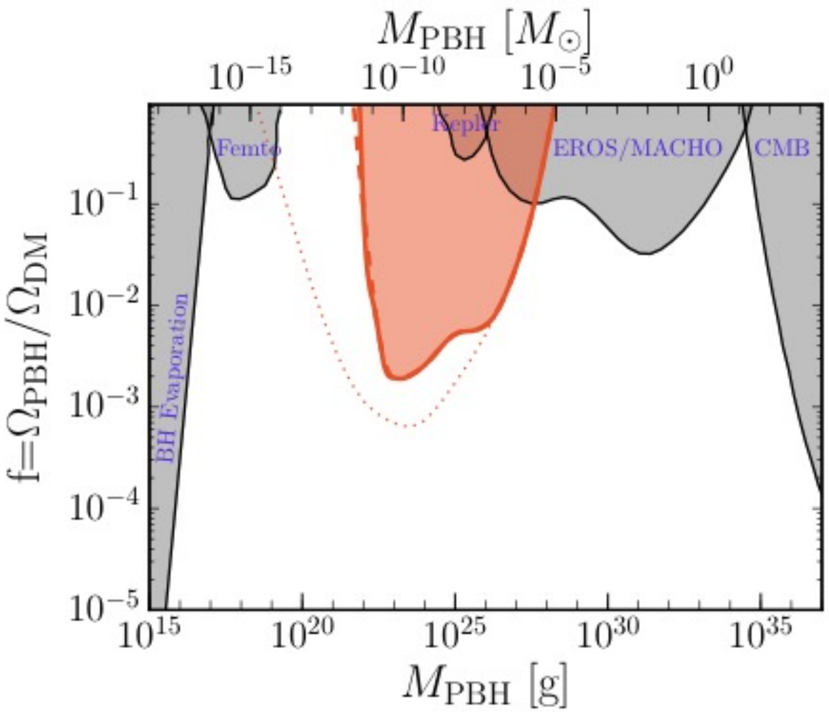


A region where $\delta > \delta_c$ would be BH.

Formation probability (abundance) is

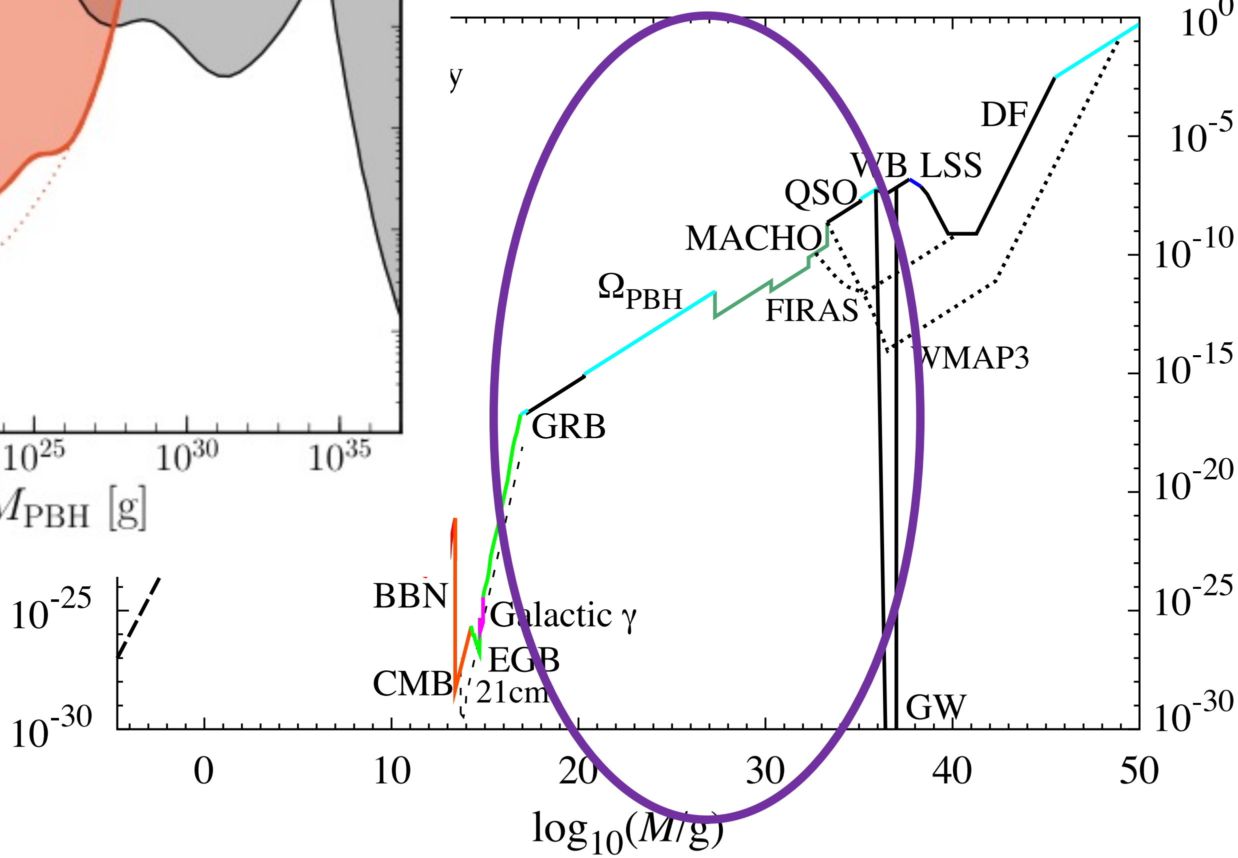
$$\beta(M) = \int_{\delta_c} P(\delta) d\delta$$

PBH is rare..



$$\beta(M) = \int_{\delta_c} P(\delta)d\delta$$

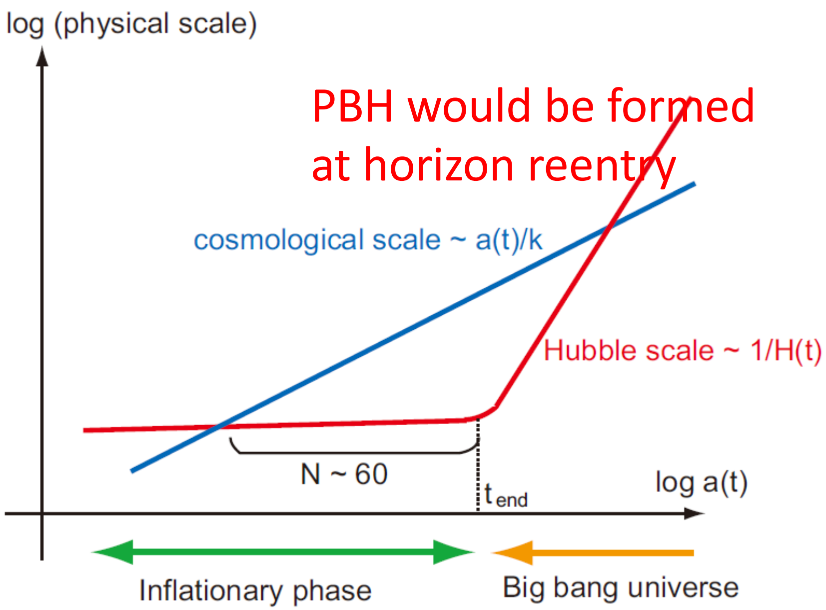
Sendouda et al. (2009)

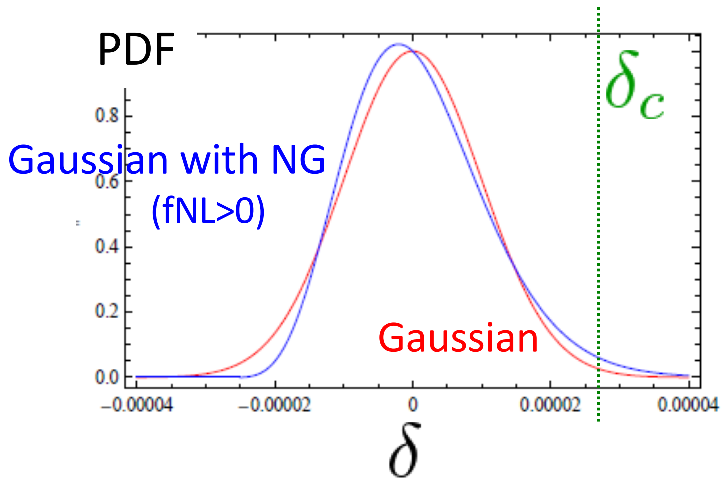


small beta ←→ rare object

Picture of PBH formation

- Super-horizon, probability distribution function (PDF)





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Formation probability (abundance) is

$$\beta(M) = \int_{\delta_c} P(\delta) d\delta$$

➔ non-Gaussian feature of primordial (super-horizon) fluctuations should be important !!

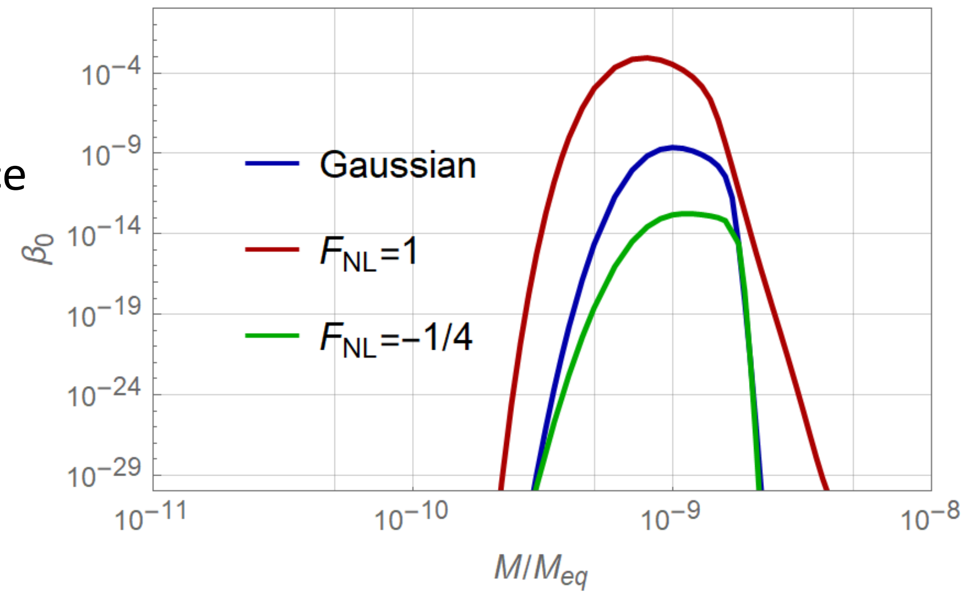
PBH abundance with nG

Yoo, Gong, SY, 1906.06790

Simple local type nG for initial curvature perturbations

$$\mathcal{R}_c(\boldsymbol{x}) = \mathcal{R}_{c,G}(\boldsymbol{x}) + \frac{3}{5} f_{\rm NL} \left(\mathcal{R}_{c,G}(\boldsymbol{x})^2 - \langle \mathcal{R}_{c,G}^2 \rangle \right)$$

abundance



Mass of formed PBH

Compared with Gaussian case,

Positive fNL
→ more efficient PBH formation

Negative fNL
→ suppressed PBH formation

N.B. non-linearity between curvature pert.
and density fluc. is included.

Any concrete example ?

PBH formation
with primordial magnetic fields (PMFs)

Saga, Tashiro, SY, 2002.01286

Energy momentum tensor for PMFs;

$$T^0_0 = -\rho_\gamma \Delta_B \,, \quad \text{; energy density}$$

$$T^i_j = p_\gamma \left(\Delta_B \delta^i_j + {\Pi_B}^i{}_j\right) \,, \quad \text{; isotropic pressure + anisotropic stress}$$

- Bardeen eq. for the scalar potential

$$\Phi'' + 3\mathcal{H}(1 + c_s^2)\Phi' + c_s^2 k^2 \Phi = \frac{3}{2} w \mathcal{H}^2 \left[\Gamma - \frac{2}{3}\Pi + 4\frac{\mathcal{H}'}{k^2}\Pi\right]$$

In radiation dominated era,

$$c_s^2 = w = \frac{1}{3}. \quad \mathcal{H} = \eta^{-1}$$

Γ ; entropy pert. = 0

$\Pi \neq 0$; anisotropic stress

- evolution eq. for tensor

$$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = 3w\mathcal{H}^2 \Pi_{ij}$$

Anisotropic stress of PMFs can induce both scalar and tensor on super-horizon
(if super-horizon PMFs exist.)

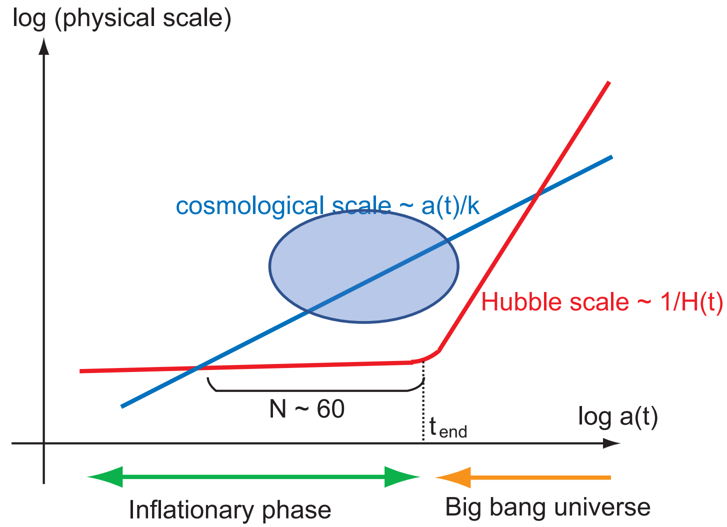
Solutions on super-horizon scales

see, e.g., Shaw and Lewis (2009)

✓ Curvature perturbations induced from PMFs anisotropic stress

(in comoving slice)

$$\mathcal{R}_c(\eta, \boldsymbol{k}) = \frac{1}{3}\xi(\eta)\Pi_B(\boldsymbol{k})$$



✓ Tensor perturbations

$$h_{ij}(\boldsymbol{k}) = \xi(\eta)\Pi_{Bij}(\boldsymbol{k})$$

$$\xi(\eta) \approx \ln\left(\frac{\eta}{\eta_B}\right) \quad \text{for} \quad \frac{\eta}{\eta_B} \gg 1$$

➔ almost constant (just log-dependence)

By using the above solutions, we can obtain (upper) limits on PMFs amplitude from already-obtained constraints on primordial scalar and tensor amplitudes !!

Note; vector mode in metric (vorticity) cannot grow even when PMFs exist.

Non-Gaussian property

Curvature perturbations induced from PMFs anisotropic stress;

$$\mathcal{R}_c(\eta, \boldsymbol{k}) = \frac{1}{3}\xi(\eta)\Pi_B(\boldsymbol{k})$$

and

$$\Pi_B(\boldsymbol{k}) = \left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right)\frac{9}{8\pi\rho_{\gamma,0}}\int\frac{\mathrm{d}^3k_1}{(2\pi)^3}\,B_i(\boldsymbol{k}_1)B_j(\boldsymbol{k}-\boldsymbol{k}_1)\ ,$$

Source term is given by a convolution of the magnetic fields.

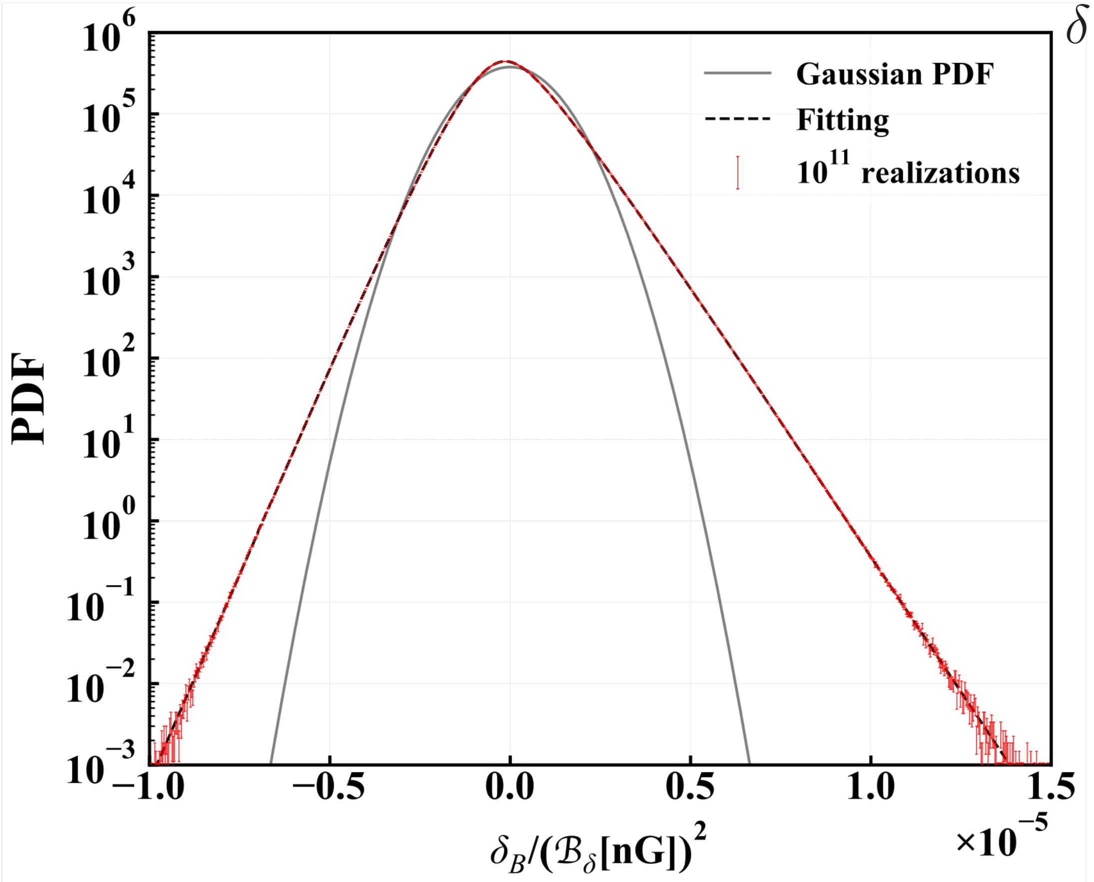
➔Even if ``B” is Gaussian,

induced curvature perturbations should be non-Gaussian!

PDF of density perturbations induced from PMFs

- Constructed by monte-carlo method

$$\delta(\boldsymbol{x}) = -\frac{4}{9}\frac{1}{a^2H^2}\triangle\mathcal{R}_c(\boldsymbol{x})$$



Saga, Tashiro, SY (2020)

see also Nakama and Suyama (2015,2016) (for 2nd order tensor)

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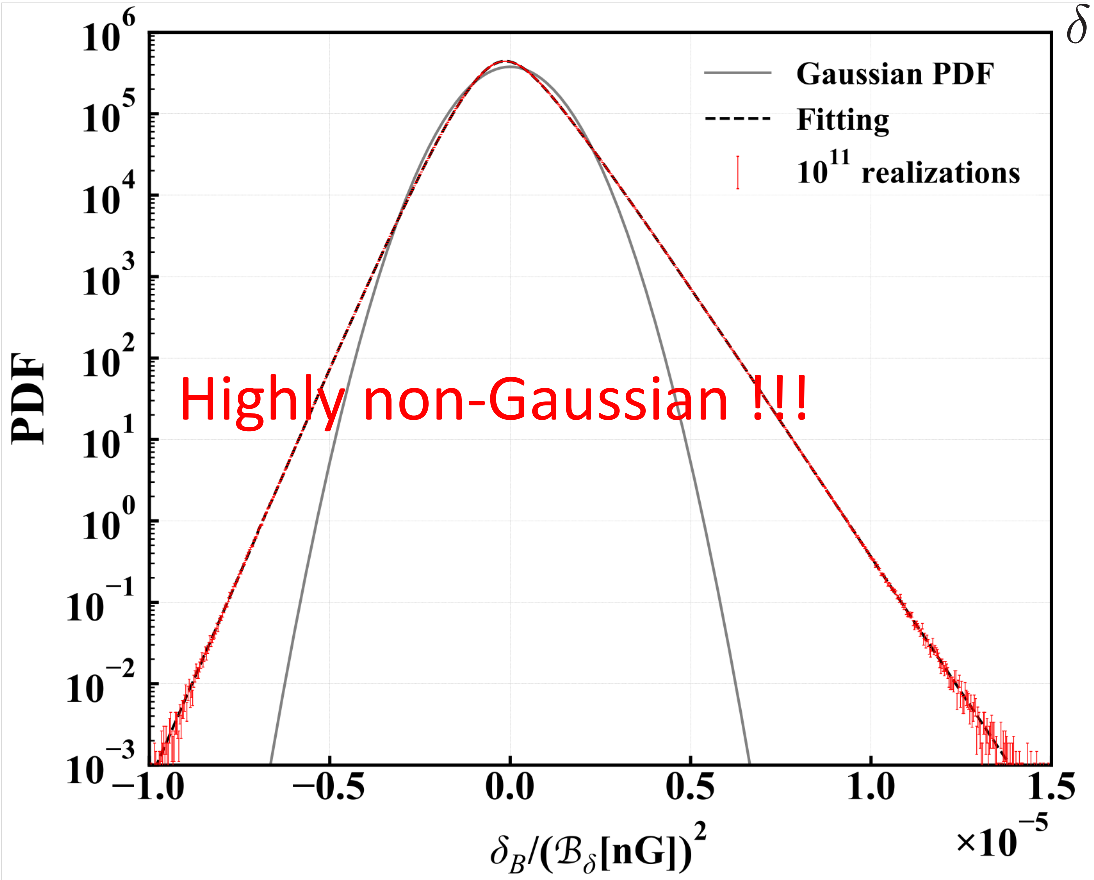

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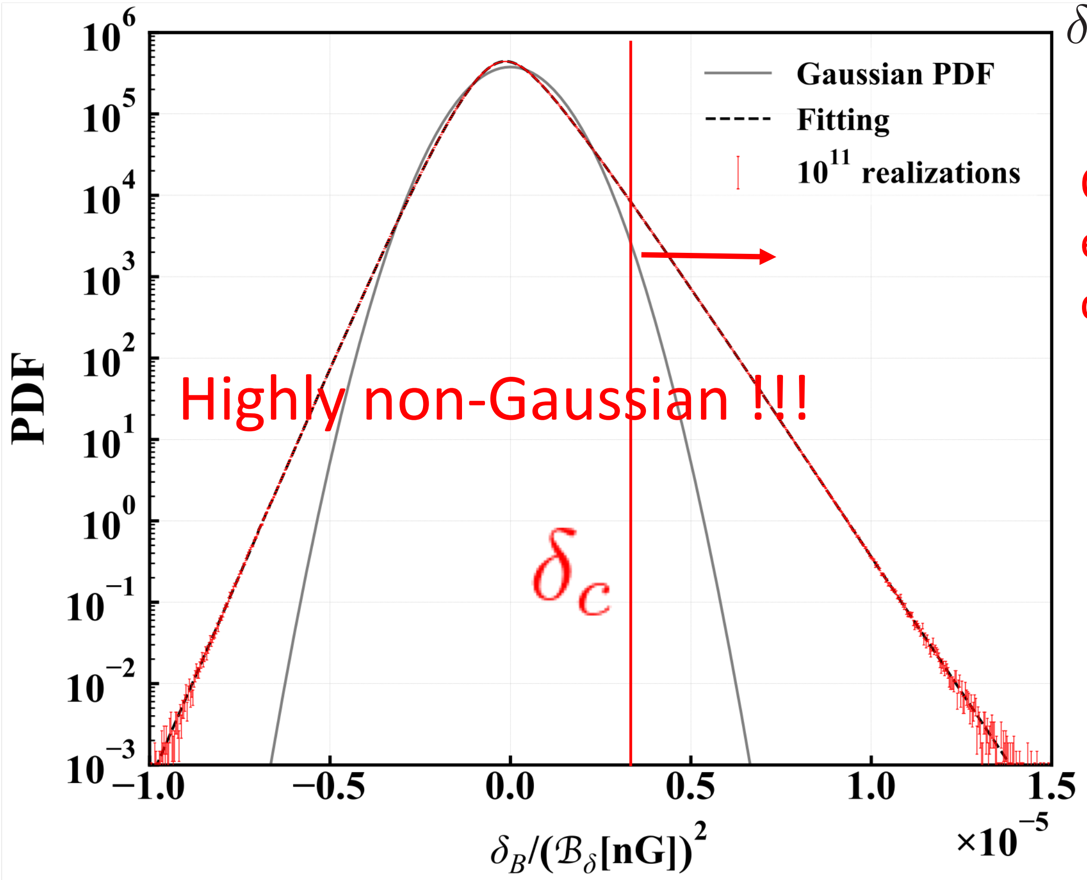


Obviously,
enhanced PBH formation!!
compared with Gaussian

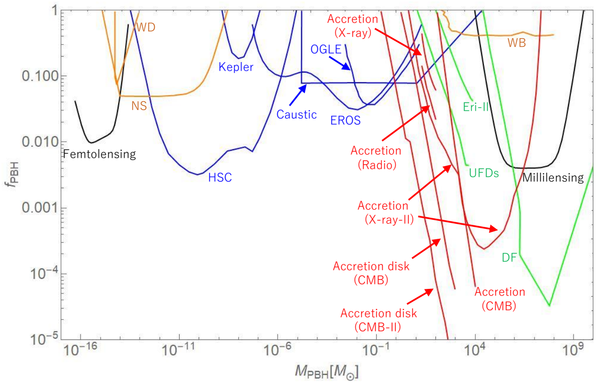
Saga, Tashiro, SY (2020)

see also Nakama and Suyama (2015,2016) (for 2nd order tensor)

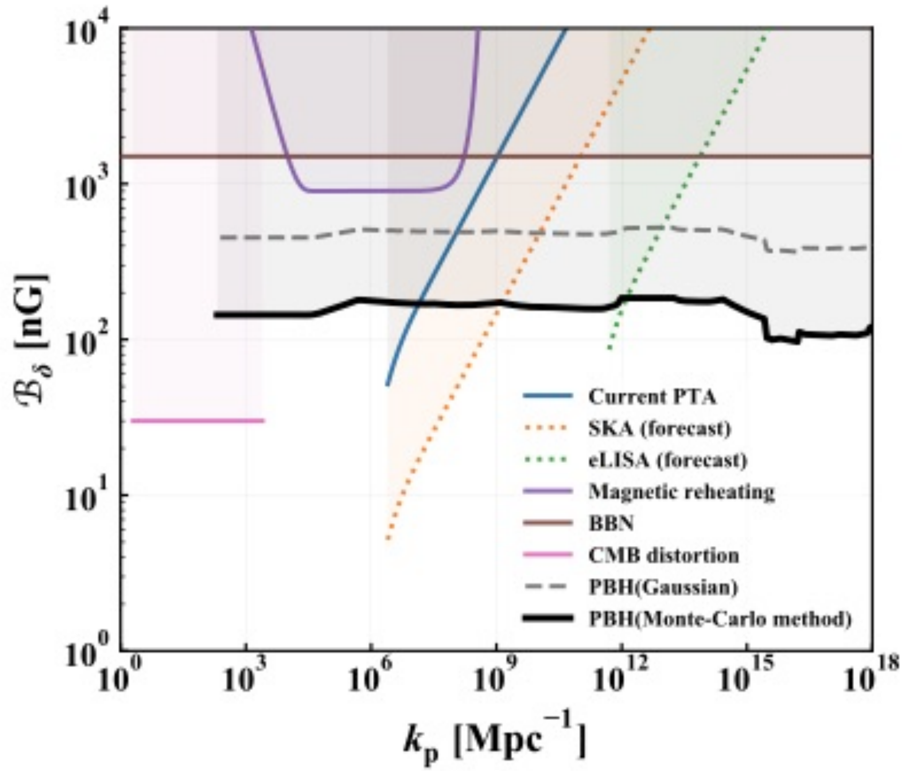
PBH formation
with primordial magnetic fields (PMFs)

Saga, Tashiro, SY, 2002.01286

Constraint on PBH abundance ➔ constraint on the amplitude PMFs



➔



About > 100 nano-Gauss

Corresponding to the mass of PBH

Other impacts of non-G?

Primordial 4-point NG
→ PBH clustering

Suyama, SY 1906.04958,
Matsubara, Terada, Kohri, SY 1909.04053…

Assuming simple local type non-Gaussianity,

Two-point correlation function;

$$\xi_{\rm PBH}(\boldsymbol{x}_1, \boldsymbol{x}_2) \coloneqq \nu^4\, \tau_{\rm NL}\, \xi_{\mathcal{R}_c}(\boldsymbol{x}_1, \boldsymbol{x}_2)$$

In Fourier space, $P_{\rm PBH}(k) \approx \nu^4\, \tau_{\rm NL}\, P_{\mathcal{R}_c}(k)$

$\nu := \delta_c/\sigma$

$$= 10^7 \times \left(\frac{\nu}{10}\right)^4 \left(\frac{\tau_{\rm NL}}{10^3}\right) P_{\mathcal{R}_c}(k)$$

$\tau_{\rm NL}$ is a non-linearity parameter in the trispectrum of curvature pert.

$$\begin{aligned}\langle \mathcal{R}_c(\boldsymbol{k}_1)\mathcal{R}_c(\boldsymbol{k}_2)\mathcal{R}_c(\boldsymbol{k}_3)\mathcal{R}_c(\boldsymbol{k}_4)\rangle &:= (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4) \\ &\times \left\{\frac{54}{25} g_{\rm NL}\left[P_{\mathcal{R}_c}(k_1)P_{\mathcal{R}_c}(k_2)P_{\mathcal{R}_c}(k_3) + 3\ \text{perms.}\right]\right. \\ &\left. + \underline{\tau_{\rm NL}}\left[P_{\mathcal{R}_c}(k_1)P_{\mathcal{R}_c}(k_2)P_{\mathcal{R}_c}(|\boldsymbol{k}_1+\boldsymbol{k}_3|) + 11\ \text{perms.}\right]\right\}\end{aligned}$$

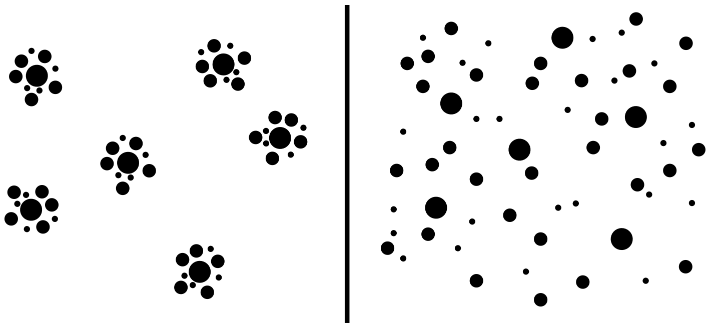
Clustering?

- "clustering"

= spatial distribution of PBHs

We focus on - at the formation
- during radiation dominated era

➔ Spatial distribution of PBHs on super-Hubble scales



Ali-Haimoud (2018)

✓ DM isocurvature fluctuations

Tada, SY (2015), Young, Byrnes (2015), …

✓ Event rate of PBH binary mergers

Raidal et al. (2017), Bringmann et al. (2018), …

✓ (additional adiabatic pert.??)

related to the Hawking radiation…

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$$\times\left\{\frac{54}{25}g_{\rm NL}\left[P_{\mathcal{R}_c}(k_1)P_{\mathcal{R}_c}(k_2)P_{\mathcal{R}_c}(k_3) + 3\text{ perms.}\right]\right.$$
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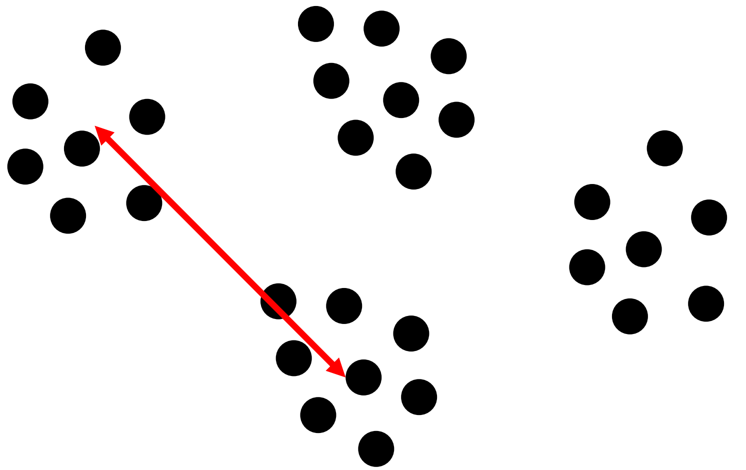
Why tau-NL ?



The spatial fluctuation of PBH number density is

$$\delta_{\rm PBH}(\boldsymbol{x}) = \frac{n_{\rm PBH}(\boldsymbol{x}) - \bar{n}}{\bar{n}}$$

$$\xi_{\rm PBH}(\boldsymbol{x}_1, \boldsymbol{x}_2) := \langle \delta_{\rm PBH}(\boldsymbol{x}_1)\delta_{\rm PBH}(\boldsymbol{x}_2)\rangle$$

Naively, we can expand this in terms of density fluc. as

$$\delta_{\rm PBH}(\boldsymbol{x}) \sim A\,\delta(\boldsymbol{x}) + B\,\delta(\boldsymbol{x})^2 + \cdots$$

density fluc. ←→ curvature pert.

$$\delta(\boldsymbol{x}) = -\frac{4}{9}\frac{1}{a^2H^2}\triangle\mathcal{R}_c(\boldsymbol{x})$$

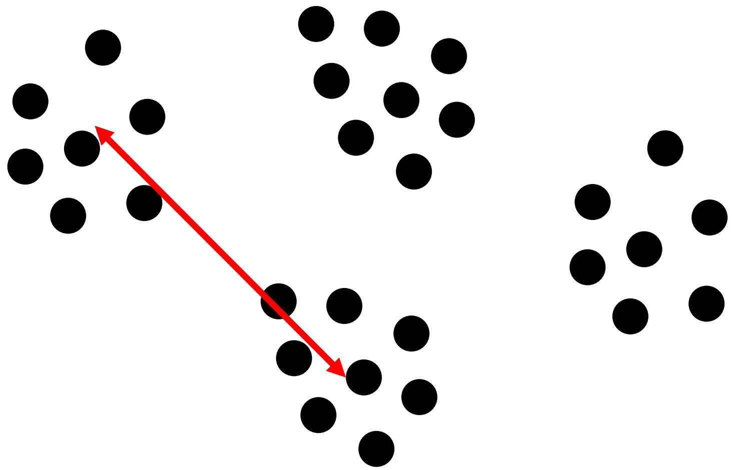
Correlation function is

$$\xi_{\rm PBH}(\boldsymbol{x}, \boldsymbol{y}) = \langle \delta_{\rm PBH}(\boldsymbol{x})\delta_{\rm PBH}(\boldsymbol{y})\rangle$$

$$\approx A^2\langle\delta(\boldsymbol{x})\delta(\boldsymbol{y})\rangle + AB\langle\delta(\boldsymbol{x})\delta(\boldsymbol{y})^2\rangle + B^2\langle\delta(\boldsymbol{x})^2\delta(\boldsymbol{y})^2\rangle + \cdots$$

$$+(\boldsymbol{x} \leftrightarrow \boldsymbol{y})$$

Why tau-NL ?



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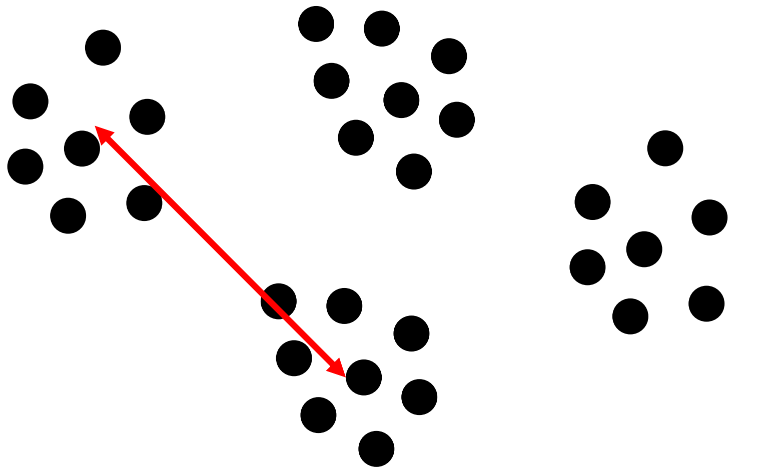
??

On super-horizon scales, $|x - \boldsymbol{y}| \gg (aH)^{-1}$, these should be zero..

Why tau-NL ?



$$\xi_{\rm PBH}(\boldsymbol{x}_1, \boldsymbol{x}_2) := \langle \delta_{\rm PBH}(\boldsymbol{x}_1)\delta_{\rm PBH}(\boldsymbol{x}_2)\rangle$$

If Gaussian,

$$\frac{\langle \delta^2(\boldsymbol{x})\delta^2(\boldsymbol{y})\rangle}{\langle \delta^2(\boldsymbol{x})\rangle^2} - 1 = 2\langle \delta(\boldsymbol{x})\delta(\boldsymbol{y})\rangle^2$$

On super-horizon scales, $|\boldsymbol{x}-\boldsymbol{y}| \gg (aH)^{-1}$, this should be zero..

density fluc. ←→ curvature pert.

$$\delta(\boldsymbol{x}) = -\frac{4}{9}\frac{1}{a^2H^2}\triangle\mathcal{R}_c(\boldsymbol{x})$$

On the other hand, if we assume

$$\mathcal{R}_c(\boldsymbol{x}) = (1+\underline{\alpha}\,\underline{\chi(\boldsymbol{x})})\underline{\phi(\boldsymbol{x})}$$

dominant source of PBH formation (small scales)

constant parameter

has super-Hubble scale correlation

then, $$\delta(\boldsymbol{x}) = -(1+\alpha\chi(\boldsymbol{x}))\frac{4}{9}\frac{1}{a^2H^2}\triangle\phi(\boldsymbol{x})$$

➔ $$\frac{\langle \delta^2(\boldsymbol{x})\delta^2(\boldsymbol{y})\rangle}{\langle \delta^2(\boldsymbol{x})\rangle^2} - 1 \approx 4\alpha^2\langle \chi(\boldsymbol{x})\chi(\boldsymbol{y})\rangle$$: super-Hubble correlation in the correlation of the local variance !!

Primordial 4-point NG
→ PBH clustering

Assuming simple local type non-Gaussianity,

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Suyama, SY (2019)
Matsubara, Terada, Kohri, SY (2019), …

If this form can be applied to the CMB scale, this should behave as a DM isocurvature, and it is highly suppressed ! ➔ PBH DM scenario has to be strongly constrained !!!

Tada, SY (2015)

Implication ?

- From CMB observations,

  DM isocurvature perturbation is tightly constrained.
  (less than 1% compared with the adiabatic curvature perturbations!)

$$P_{\rm iso}/P_{\rm adi} \lesssim O(0.01)$$

Please check Planck 2018

- If we assume PBH is a dominant component of DM,

$$P_{\rm iso} = P_{\rm PBH} = 0.01 \times \left(\frac{\nu}{10}\right)^4 \left(\frac{\tau_{\rm NL}}{10^{-6}}\right) P_{\rm adi}$$

should be so small !!

- If we ``believe" Maldacena's consistency relation,

$$\tau_{\rm NL} \sim f_{\rm NL}^2 \sim (1-n_s)^2 \sim O(10^{-4})$$

Crisis on PBH-DM scenario?

$$P_{\rm iso} = P_{\rm PBH} = 0.01 \times \left(\frac{\nu}{10}\right)^4 \left(\frac{\tau_{\rm NL}}{10^{-6}}\right) P_{\rm adi}$$

$$\Updownarrow$$

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PBH can not be DM?

Crisis on PBH-DM scenario?

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$$\Updownarrow$$

$$\tau_{\rm NL} \sim f_{\rm NL}^2 \sim (1-n_s)^2 \sim O(10^{-4})$$

PBH can not be DM?

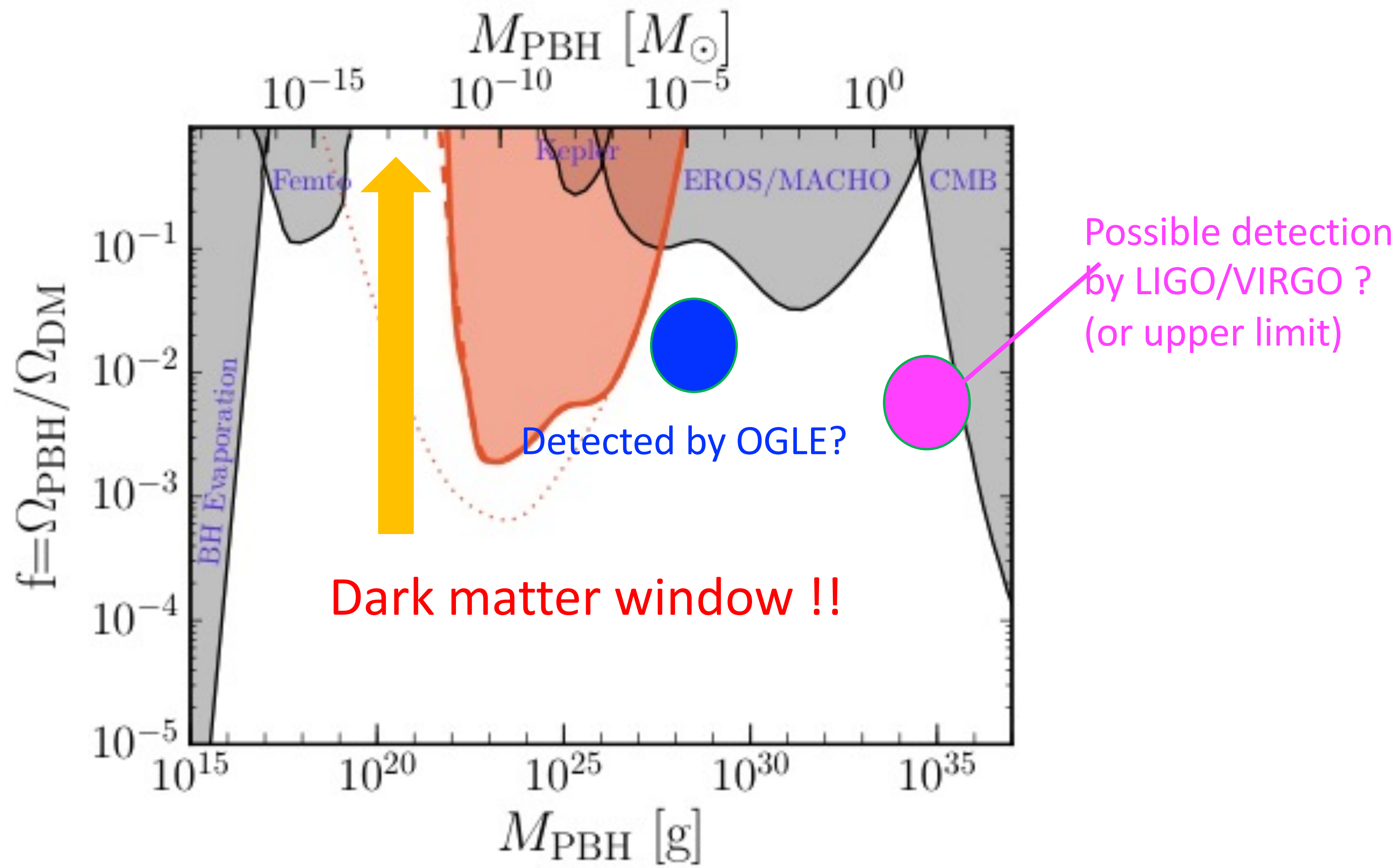
Maybe Yes, PBH can be DM.

Maldacena's consistency relation would be invalid.

1. In inflationary models associated with PBH formation, slow-roll violation might be essential.
2. Due to so-called ``local observer effect'', for single field inflation case, non-linearity parameter fNL and tauNL should be zero.

We need to investigate this issue more carefully.

Still PBH is interesting !!



Niikura, Takada, SY+ (2019), …

Summary

- PBH is an interesting target as a DM and LIGO BH (also exoplanet?).

- In formation process, non-Gaussianity becomes important. ➔ abundance, clustering

(➔ secondary GWs! (in Sasaki-san's talk))

Future issues

- ➢ Non-Gaussianity could induce spinning PBH?
- ➢ Evaluate the clustering feature for concrete models
- ➢ Extended ``Maldacena's consistency relation" ? …