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Revisiting slow-roll dynamics and the tensor tilt in general single-field inflation

Tsutomu Kobayashi

A01 連携研究者 + 公募研究

Rikkyo University

立教大学

Based on work with
Yosuke Mishima (Rikkyo)
arXiv:1911.02143,
Phys. Rev. D (to appear)

In cosmology textbooks…

Tensor modes from inflation: red tilt

$$n_t < 0$$

because

$$n_t = -2\epsilon, \quad \epsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}\frac{\rho + p}{\rho} \geq 0$$

Null energy condition (NEC)

🤔 ***Is this always true?***

Blue tensor from inflation

Radical scenarios…
Diffeomorphism-breaking inflation (solid inflation, massive gravity, …)

Endlich, et al. (2013); Cannone, et al. (2015); Fujita, et al. (2019); …

$$n_t \sim m_g^2/H_{\rm inf}^2 > 0$$

Kinetically driven G-inflation $H_{\rm inf} \sim$ constant kinetic energy

TK, Yamaguchi, Yokoyama (2010)

$$n_t = -2\epsilon > 0$$

Stable violation of NEC

Stay conservative…
Blue tensor *from single-field* ***slow-roll*** *inflation driven by* ***potential***
(breaking only time diffeomorphism invariance)?

Extended slow-roll models

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2} R + X - V(\phi) + \cdots, \quad X := -\frac{1}{2}(\partial\phi)^2$$

- Non-canonical kinetic term: $\kappa(\phi)X$
- "Galileon": $h(\phi)X\Box\phi$
- Non-minimal couplings: $f(\phi)R$

  $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

…… ……

$$\Rightarrow \quad 3M_{\rm eff}^2(\phi)H^2 \simeq V(\phi), \quad 3H\dot{\phi} + V' + \cdots \simeq 0$$

Inflation is driven by potential   Slow-roll dynamics is modified

Horndeski theory

Horndeski (1974)

Horndeski theory — *the most general scalar-tensor theory with 2nd-order field equations* — can be used for a comprehensive study of extended models of slow-roll inflation

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R$$
$$+ G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\phi\nabla_\nu\phi)^2\right]$$
$$+ G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3\right.$$
$$\left. - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

Deffayet et al. (2011); TK, Yamaguchi, Yokoyama (2011)

- Background dynamics
- Perturbations
- Reheating

Cf. EFT of inflation can incorporate only the perturbation dynamics on a given background

Unified description of slow-roll inflation

Kamada et al. (2012)

Taylor-expand the functions:

$$G_a(\phi, X) = g_a(\phi) + h_a(\phi)X + \mathcal{O}(X^2) \quad (a = 2, 3, 4, 5)$$

Assume "slow-roll" conditions:

$$-\frac{\dot{H}}{H^2} \ll 1, \quad \frac{\dot{g}_a}{Hg_a} \ll 1, \quad \frac{\dot{h}_a}{Hh_a} \ll 1$$

Under these assumptions, Kamada et al. argued that

All stable slow-roll models predict red tensor spectra

Counter-example: Gauss-Bonnet inflation

Satoh & Soda (2008); Satoh (2010); Koh, et al. (2018)

Tensor spectrum can be blue in stable slow-roll inflation with GB term,

$$f(\phi)\left(R^2-4R_{\mu\nu}R^{\mu\nu}+R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda}\right)$$

GB term leads to 2nd-order field equations, and hence is included within the Horndeski theory

→ *Something is wrong with Kamada et al. (2012)?*

Revisit (and improve) Kamada et al. (2012)!

GB term in Horndeski language

$$f(\phi)\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda}\right)$$

$$\Leftrightarrow \quad \begin{aligned} G_2 &= 8f''''X^2(3-\ln X), \quad & G_3 &= 4f'''X(7-3\ln X) \\ G_4 &= 4f''X(2-\ln X), \quad & G_5 &= -4f'\ln X \end{aligned}$$

TK, Yamaguchi, Yokoyama (2010)

$$\Rightarrow \quad G_2 \simeq G_3 \simeq G_4 \simeq 0, \quad G_5 = -4f'\ln X$$

at leading order in slow-roll

Taylor-expanded form in Kamada et al. (2012) fails to capture this ln *X* structure

$$G_a(\phi, X) = g_a(\phi) + h_a(\phi)X + \mathcal{O}(X^2) \quad (a = 2, 3, 4, 5)$$

Beyond Kamada et al.

$$G_a(\phi, X) = g_a(\phi) + \lambda_a(\phi) \ln X + h_a(\phi) X + \cdots$$

This includes GB inflation and other more general models

Friedmann equation

Singular terms ⇒ $\lambda_2 = \lambda_4 = 0$

$:= -g_2(\phi)$

$$6 g_4 H^2 \simeq V + \lambda_2 (2 - \ln X) - 6\lambda_4 H^2 \ln X + 6\lambda_3 H \dot{\phi} + 6\lambda_5 H^3 \dot{\phi}$$

Effective Planck mass

$g_4(\phi) > 0$

Assume *potential-dominance* conditions:

$$\delta_3 := \frac{\lambda_3 \dot{\phi}}{g_4 H} \lesssim \mathcal{O}(\epsilon), \quad \delta_5 := \frac{\lambda_5 H \dot{\phi}}{g_4} \lesssim \mathcal{O}(\epsilon)$$

Modified slow-roll dynamics

Einstein-Hilbert + canonical scalar

$$3M_{\rm Pl}^2 H^2 \simeq V$$

$$3H\dot{\phi} \simeq -V'$$

General slow-roll models in Horndeski

$$6g_4 H^2 \simeq V$$

$$3H\dot{\phi} \simeq -\frac{g_4^2}{\mathcal{I}}\left(\frac{V}{g_4^2}\right)'$$

$\mathcal{I}(\phi)$ $(\supset g_a(\phi),\ h_a(\phi),\ \lambda_a(\phi))$ effectively modifies the potential slope

$$\mathcal{I}(\phi) := \frac{U'}{2(U'+\varpi)}\left[u+\sqrt{u-4v(U'+\varpi)}\right],$$

$$U' := g_4^2\left(\frac{V}{g_4^2}\right)', \quad u := h_2 + \frac{h_4 V}{g_4}, \quad v := h_3 + \frac{h_5 V}{6g_4}, \quad \varpi := \frac{3V\lambda_3}{g_4} + \frac{\lambda_5}{6}\left(\frac{V}{g_4}\right)^2$$

Tensor tilt v.s. stability

Recall the standard story…

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2} R + X - V(\phi) \quad \text{or even} \quad \mathcal{L} = \frac{M_{\rm Pl}^2}{2} R + P(\phi, X)$$

Tensor tilt: $n_t = -2\epsilon$

> 0

Stability: $$\mathcal{L}_\zeta^{(2)} = \frac{a^3 M_{\rm Pl}^2 \epsilon}{c_s^2} \left[\dot{\zeta}^2 - a^{-2} c_s^2 (\partial \zeta)^2 \right]$$

Stability prohibits $n_t > 0$

Tensor tilt v.s. stability

Tensor power spectrum:

$$\mathcal{P}_{\rm GW} = \frac{H^2}{\pi^2 g_4}, \quad n_t = -2\epsilon - \frac{\dot{g}_4}{H g_4} = -\frac{\dot{\phi}^2 \mathcal{I}}{2 g_4 H^2}$$

$$\Rightarrow \quad n_t < 0 \;\Leftrightarrow\; \mathcal{I} > 0 \quad / \quad n_t > 0 \;\Leftrightarrow\; \mathcal{I} < 0$$

Stability:

$$\mathcal{L}_\zeta^{(2)} = \frac{a^3 \mathcal{F}}{c_s^2}\left[\dot{\zeta}^2 - a^{-2} c_s^2 (\partial\zeta)^2\right]$$

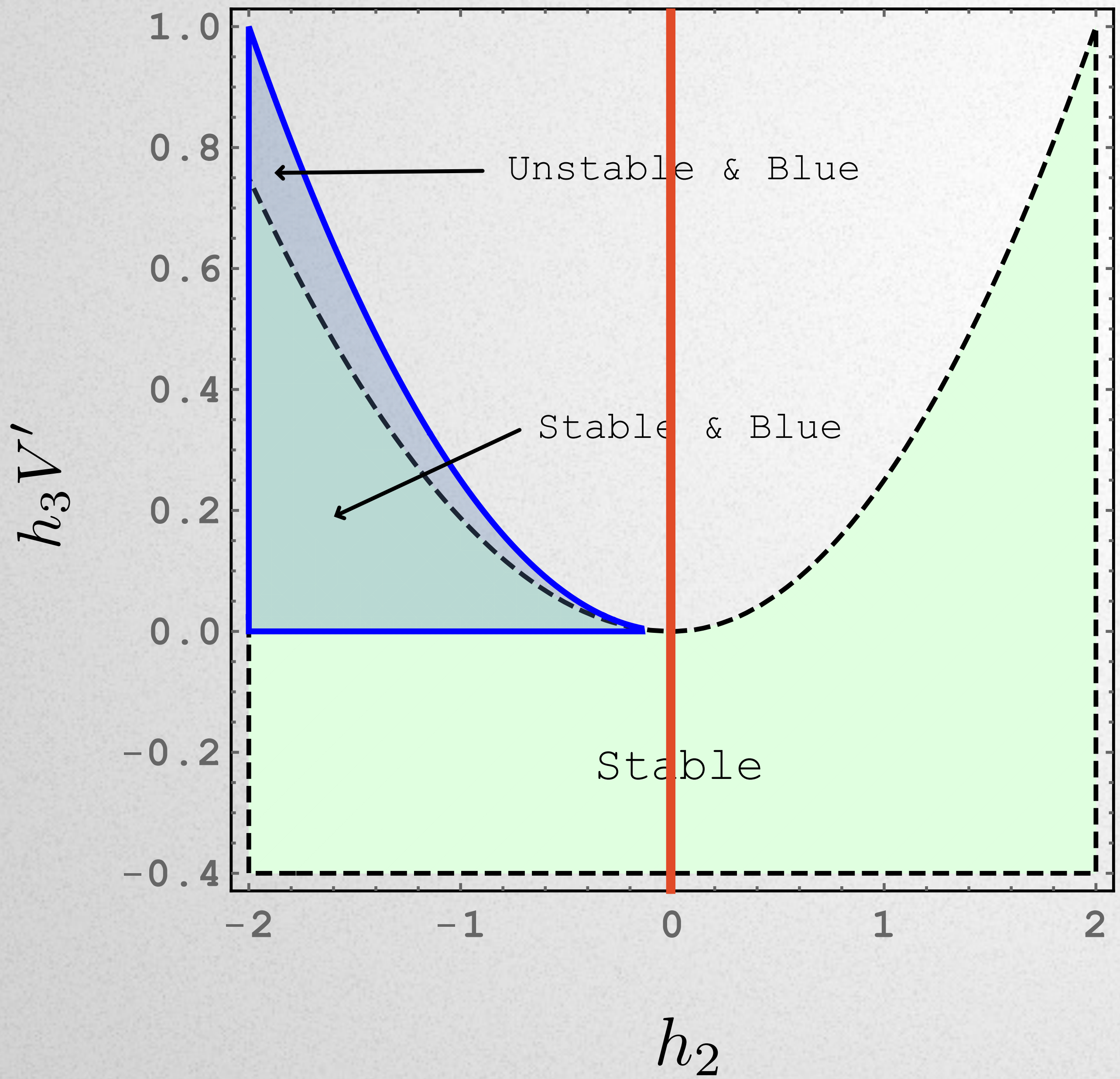
$\mathcal{F}$, c_s^2: > 0

Stability is NOT correlated with the sign of $\mathcal{I}$ in general

Case study 1

— Reanalysis of Kamada et al. (2012)

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}R + h_2(\phi)X - V(\phi) - h_3(\phi)X\Box\phi$$



Blue tensor even without $\ln X$

Kamada et al. unnecessarily assumed

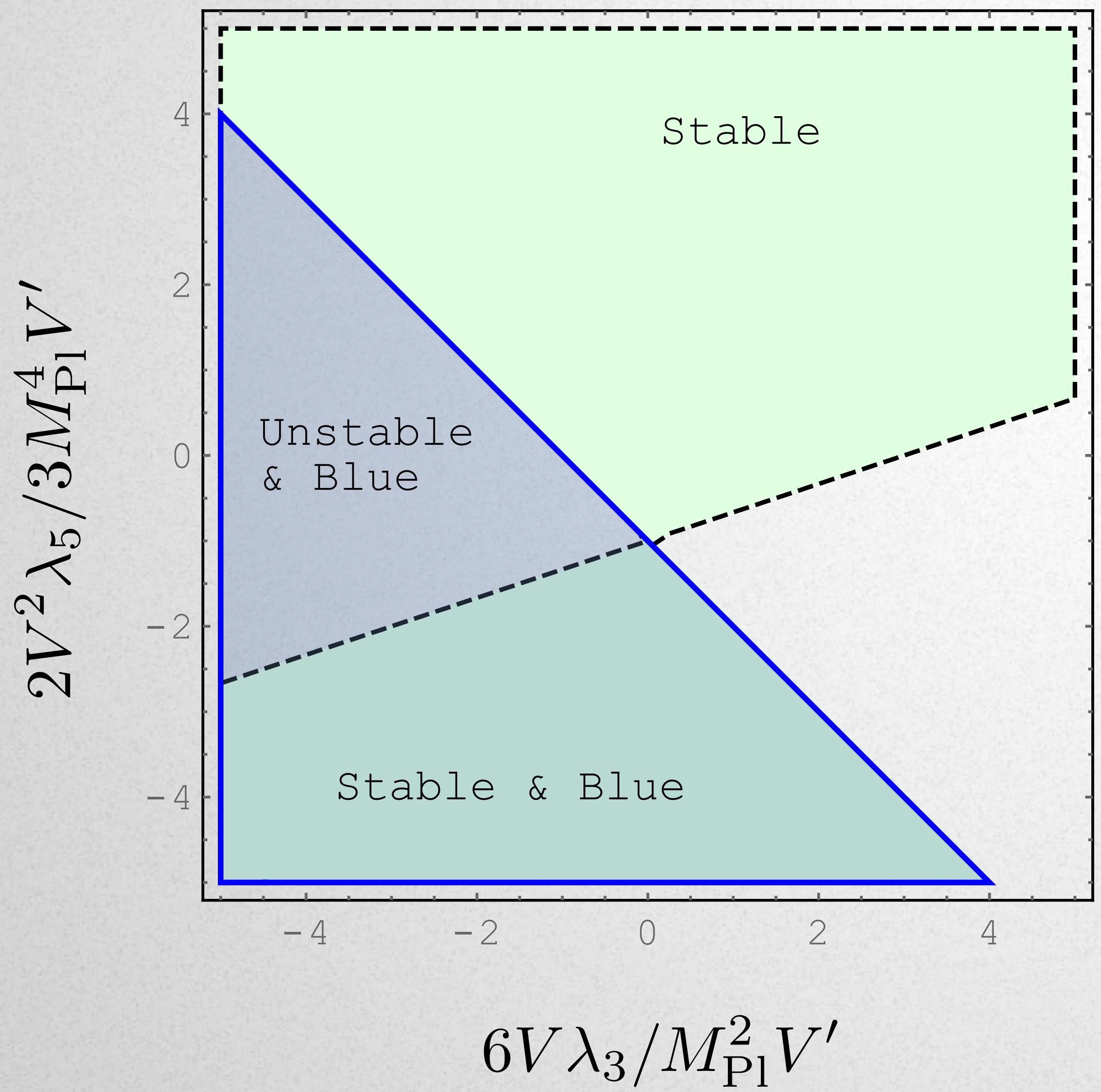
$$h_2(\phi) > 0$$

and simply overlooked the interesting model space

Case study 2

— Gauss-Bonnet inflation and further generalizations

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}R + X - V(\phi) - G_3(\phi, X)\Box\phi + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \cdots$$



$$G_3 = \lambda_3(\phi)\ln X$$
$$G_5 = \lambda_5(\phi)\ln X$$

For $\lambda_3 = 0$ this reproduces previous results on GB inflation

Summary

Yosuke Mishima & TK
arXiv:1911.02143, Phys. Rev. D (to appear)

- *Blue tensor spectra* *from* ***slow-roll*** *inflation driven by* ***potential***?
- **— YES!**
- We have improved the unified description of slow-roll inflation [Kamada et al. (2012)] in two ways:
  - **(1) reanalyzed the model space with blue tensor spectra which was carelessly overlooked;**
  - **(2) included ln X terms to accommodate more general models than previously considered.**