Tokyo Tech

Computations of quantities characterizing PBHs formed from primordial perturbations

-report of my research activities-

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Who am I?

I am a member of A01 group (Misao's group).

I receive a grant of public research (2018-2020).
"Theoretical studies to test stellar mass PBHs"

Achievements during this innovative area (- July 2015)

Totally 24 papers in 9 journals(PRL, PRD, JCAP, PTEP, CQG, JHEP, JPSJ, PASJ, ApJ)

16 papers by collaborations with members in this area

8 papers on PBHs (triggered by the LIGO events)

Proposal of the PBH hypothesis as an explanation of the LIGO events

$\boldsymbol{f_{PBH} \sim 10^{-3}}$ → Strong constraint on PBH dark matter scenario

Mass distribution of the merger rate

Spin, clustering of PBHs, PBH mass function

etc.

IOP Publishing Classical and Quantum Gravity

Class. Quantum Grav. **35** (2018) 063001 (76pp) https://doi.org/10.1088/1361-6382/aaa7b4

Topical Review

arXiv:1801.05235

Primordial black holes—perspectives in gravitational wave astronomy

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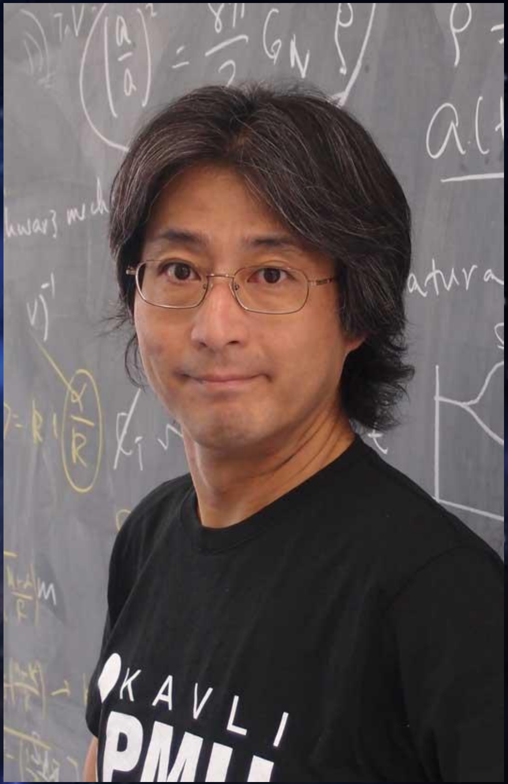
Abstract

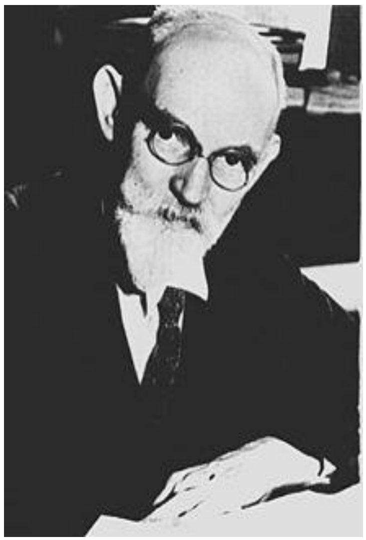
This article reviews current understanding of primordial black holes (PBHs), with particular focus on those massive examples ($\gtrsim 10^{15}$ g) which remain at

第1回村山杯



加速クイズ

加速膨張と言えば、ド・ジッターであるが、ド・ジッターの写真はどれ？



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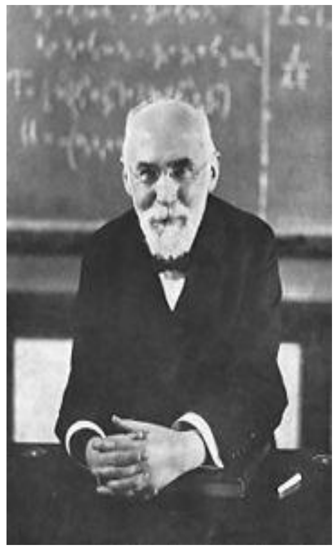

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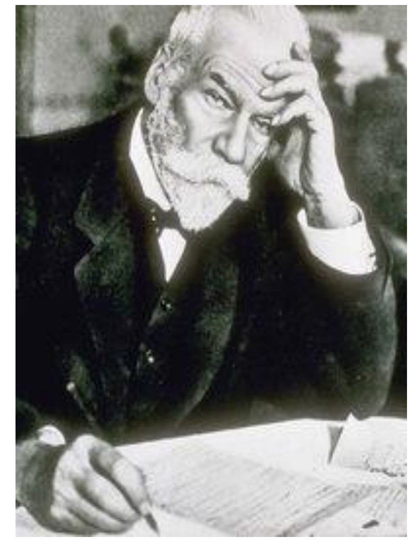

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Achievements during this fiscal year (-April 2019)

Totally 5 papers

- Quantum Ostrogradsky theorem,
  H.Motohashi, TS, 2001.02483
- A novel formulation of the PBH mass function,
  TS, S.Yokoyama, 1912.04687, to appear in PTEP
- Formation threshold of rotating primordial black holes,
  M.He, TS, 1906.10987, PRD 100, 063520 (2019)
- Clustering of primordial black holes with non-Gaussian initial fluctuations,
  TS, S.Yokoyama, 1906.04958, PTEP 103E02 (2019)
- A large mass hierarchy from a small nonminimal coupling,
  C.Ringeval, TS, M.Yamaguchi, 1903.03544, PRD 99, 123524(2019)

ICTS | INTERNATIONAL CENTRE *for* THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Summer School on Gravitational Wave Astronomy

15 July 2019 to 26 July 2019

Primordial black holes and GW astronomy

(lecture: 90min×5, exercise : 90min×5)

Papers on PBHs

- Clustering of primordial black holes with non-Gaussian initial fluctuations,
  TS, S.Yokoyama, 1906.04958
  clustering

- Formation threshold of rotating primordial black holes,
  M.He, TS, 1906.10987
  spin

- A novel formulation of the PBH mass function,
  TS, S.Yokoyama, 1912.04687
  mass function

Computations of PBH-related quantities from the primordial density perturbations

1) “Clustering of primordial black holes with non-Gaussian initial fluctuations”

TS, S.Yokoyama

This was already addressed by Shuichiro’s talk.

Motivation of the study

There has been confusion in the literature as to super-Hubble clustering of PBHs.

Result

We showed that PBHs cluster on super-Hubble scale if the seed perturbation has local-type trispectrum parametrized by τ_{NL}.

$$\xi^{(2)}_{PBH}(r) = \boldsymbol{\tau_{NL}} \left(\frac{4\delta_{th}}{9\sigma} \right)^4 \xi^{(2)}_{\mathcal{R}}(r)$$

correlation function of PBHs ($\xi^{(2)}_{PBH}$)

correlation function of the primordial perturbation ($\xi^{(2)}_{\mathcal{R}}$)

2) "Formation threshold of rotating primordial black holes" M.He, TS

Motivation of the study

Spin distribution of BHs may be used to test a hypothesis that LIGO BHs are PBHs.

Formal expression of the spin distribution of PBHs at time t

$$W(J,t) = \int dJ' Q(J,J',t) \int_{\delta_{\rm th}(J')} P(\delta_M, J') d\delta_M .$$

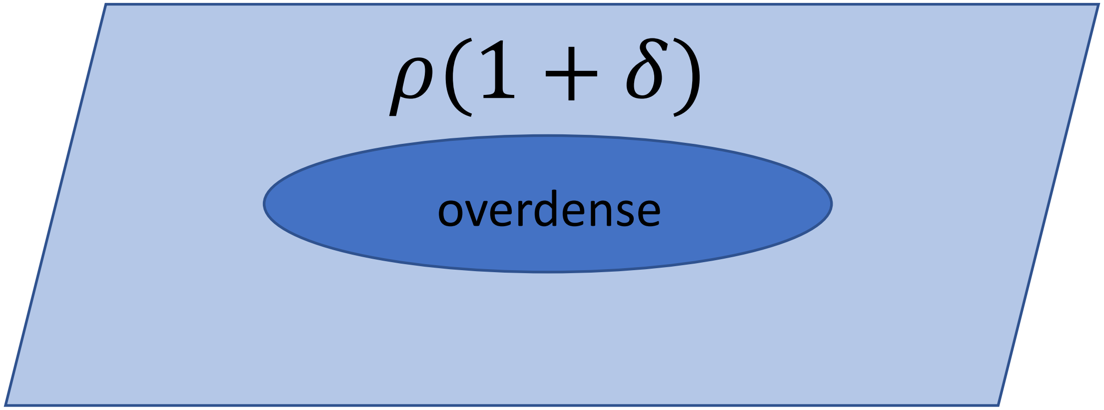
$P(\delta_M, J)$: probability that region collapsing to a BH has (δ_M, J) (given in the literature)

$\delta_{th}(J)$: Formation threshold of PBH (derived in my paper)

$Q(J, J', t)$: spin evolution from J' to J (given in the literature)

Carr's formula - analytic formula for $\delta_{\mathrm{th}}(J=0)$-

Carr 1975

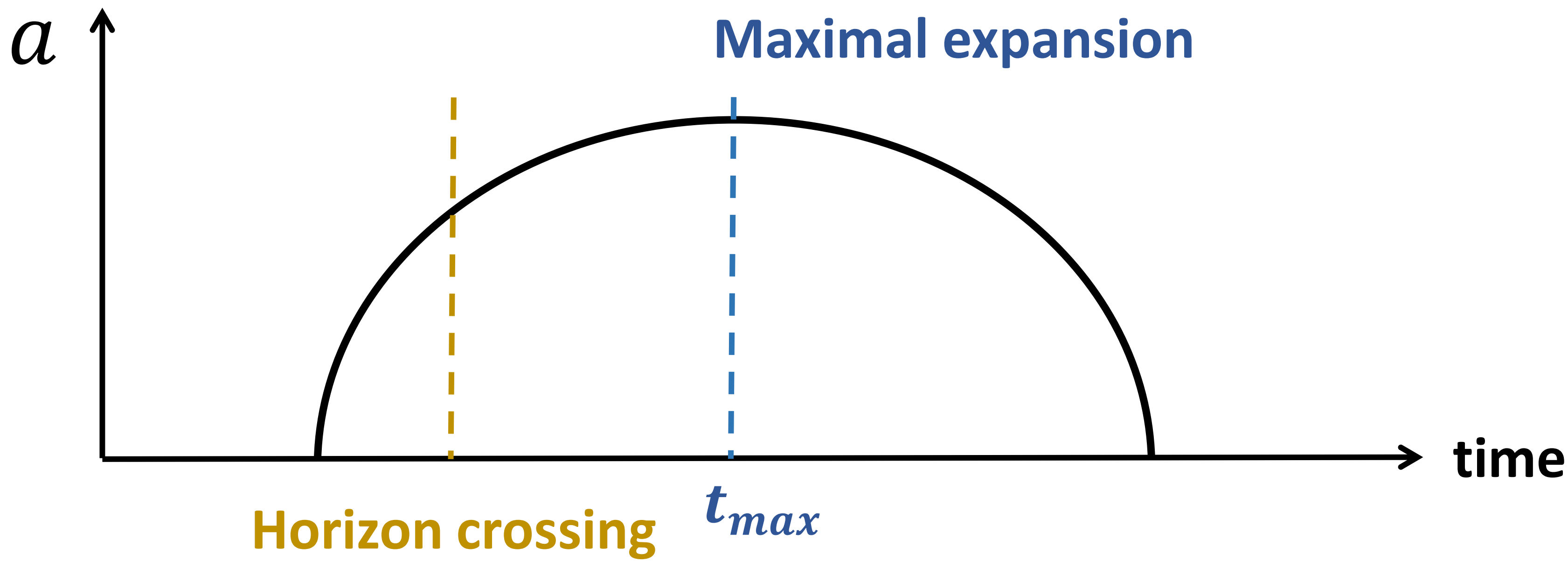


Assumption

Overdense region is uniform

$$H^2 = \frac{8\pi G}{3}\bar{\rho}(1+\delta) - \frac{1}{a^2}$$

Regard the overdense region as a closed FLRW universe.



Carr's criterion

Size of the region > Jeans length at $t = t_{max}$

$$\frac{a_{max}}{a_{hc}}\frac{1}{H_{hc}} > R_J = \frac{c_s}{\sqrt{G\rho_{max}}} = c_s a_{max}$$

$$\delta_{\mathrm{th}} = c_s^2$$ ← Carr's formula

Inclusion of the angular momentum

$$\omega^2 = c_s^2 k^2 + 4\Omega^2 - 4\pi G\rho$$

Ω: angular velocity

Use $\frac{c_s}{\sqrt{G\rho - \Omega^2/\pi}}$ as the Jeans length

$$\delta_{\rm th}(J) = c_s^2 + \frac{25 c_s^4}{6(1+c_s^2)^2} a_K^2$$

a_K: Kerr parameter

Formation threshold increases in proportional to J^2, which indicates that highly spinning PBHs are unlikely.

3) "A novel formulation of the PBH mass function" TS, S.Yokoyama

PBH mass function

$f(M)d\ln M$: fraction of PBHs in $(M, M + d\ln M)$

Primordial density perturbations $\Rightarrow$ $f(M)$

Existing methods to compute the PBH mass function

i) Press-Schechter formalism

ii) Peak theory

Motivation of the study

These two methods have issues at the conceptual level.

e.g. $\beta = f(M)$

Basic variable for the PBH formation

$\theta(\vec{x}, R)$: density contrast smoothed on scale R

Change of R $\Longleftrightarrow$ Change of the density contrast at different horizon crossing time

Criterion of the PBH formation

$$\vec{\nabla}\theta = 0, \partial_R \theta = 0$$

Main result

$$f(M) = \frac{M}{n_{\rm PBH}} \int dR \, \langle J\delta(M - m(R, \theta_R))\Theta(\theta_R - \theta_{\rm th}) \prod_{a=1}^{4} \delta(\theta_{R,a})\Theta(-\lambda_a)\rangle$$

- Automatically derived from the new criterion
- Applicable to any type of primordial perturbations

Similar works

- Germani, Sheth, 1912.07072
- Luca, Franciolini, Riotto, 2001.04372
- Young, Musco, 2001.06469

Summary

- Clustering of primordial black holes with non-Gaussian initial fluctuations,
  TS, S.Yokoyama, 1906.04958
  clustering

- Formation threshold of rotating primordial black holes,
  M.He, TS, 1906.10987
  spin

- A novel formulation of the PBH mass function,
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