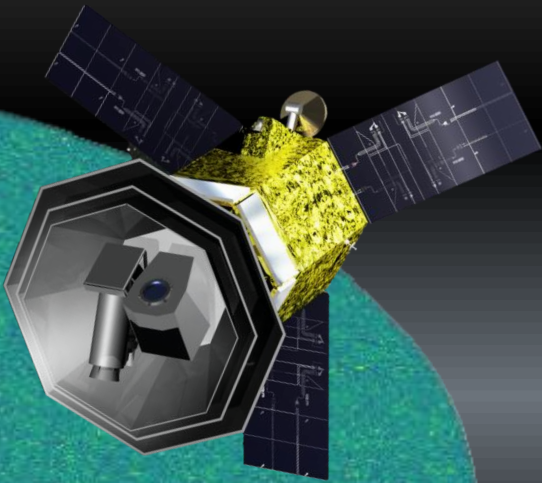
Simultaneous determination of the cosmic birefringence and miscalibrated polarisation angles from LiteBIRD



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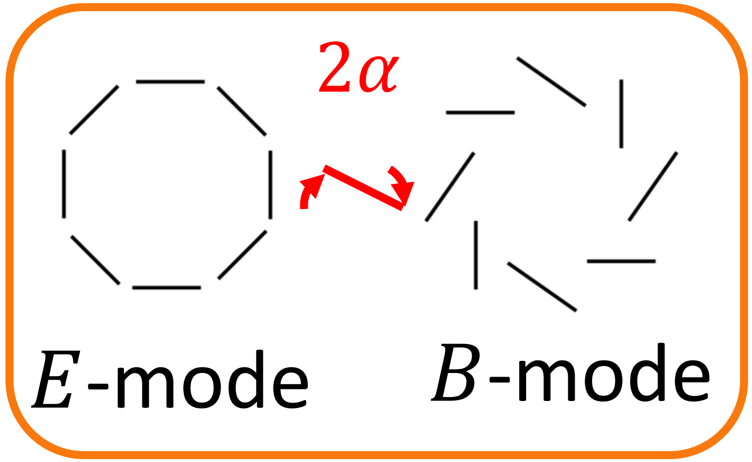
Introduction:



- Miscalibration of detector rotation angle (α) creates spurious B−mode from E−mode

$$\underbrace{C_\ell^{BB,o}}_{\text{observed}} = \underbrace{C_\ell^{EE}}_{\text{before detectors}}\sin(2\alpha) + \underbrace{C_\ell^{BB}}cos(2\alpha) \quad \cdots (1)$$

We need to determine α to calibrate rotation angle

- In past experiments, this α was calculated assuming that EB correlation of CMB is zero:

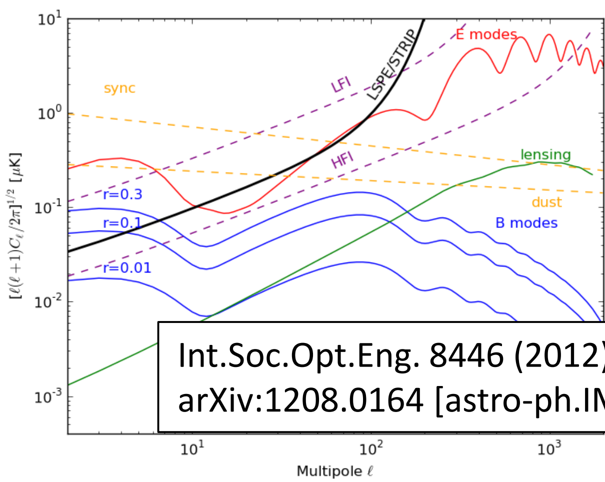
$$C_\ell^{EB,o} = \frac{1}{2}\left(\underbrace{C_\ell^{EE,CMB} - C_\ell^{BB,CMB}}_{\text{From theory}}\right)\sin(4\alpha) \quad \cdots (2)$$

“Self-calibration” by Brian G. Keating et al.（2013）

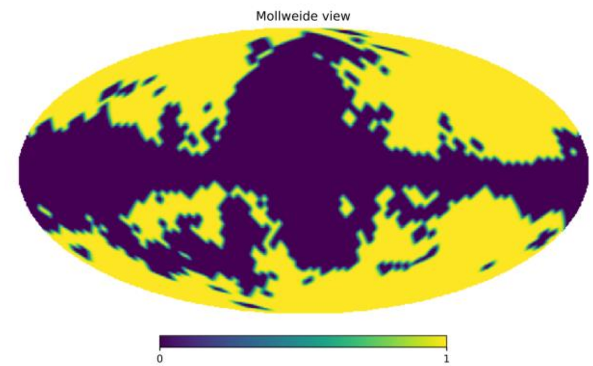
Some issues in "self-calibration"

Foreground emissions should be small

- Because foreground signals also have α,
  - We need to know the foreground model
  - We need to mask the Galactic plane



Int.Soc.Opt.Eng. 8446 (2012) 84467C
arXiv:1208.0164 [astro-ph.IM]



Cosmological EB correlation should be zero

- We lose sensitivity to cosmic birefringence

To solve these issues

We relate observed *E*- and *B*- modes to the intrinsic ones as

$$E^{o}_{\ell,m} = E_{\ell,m}\cos(2\alpha) - B_{\ell,m}\sin(2\alpha)$$
$$B^{o}_{\ell,m} = E_{\ell,m}\sin(2\alpha) + B_{\ell,m}\cos(2\alpha) \quad \cdots(3)$$

From these equations, we find

$$C_{\ell}^{EB,o} = \frac{1}{2}\left(C_{\ell}^{EE,o} - C_{\ell}^{BB,o}\right)\tan(4\alpha) + \frac{C_{\ell}^{EB}}{\cos(4\alpha)} \quad \cdots(4)$$

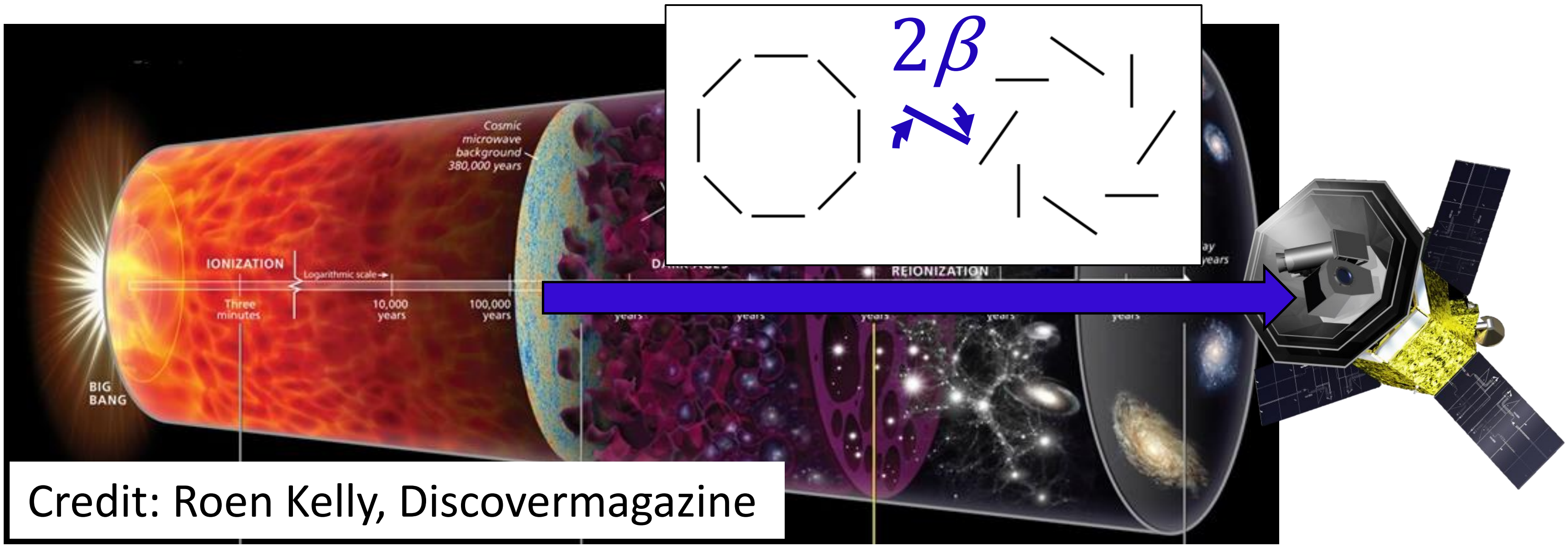
G. B. Zhao et al.（2015）

Our work

- We can estimate α with only observed data
- If we assume theory CMB power spectra, we can estimate an additional angle!

Cosmic birefringence

During the long travel from the recombination era to Earth, CMB can be rotated by some physics (e.g. axionic fields)



Credit: Roen Kelly, Discovermagazine

In this case,

- Foreground term: rotated only by α
- CMB term: rotated by $\alpha+\beta$

Equations including birefringence rotation:

The coefficients become

$$E^{\rm o}_{\ell,m} = E^{\rm fg}_{\ell,m}\cos(2\alpha) - B^{\rm fg}_{\ell,m}\sin(2\alpha) + E^{\rm CMB}_{\ell,m}\cos(2\alpha+2\beta) - B^{\rm CMB}_{\ell,m}\sin(2\alpha+2\beta) + E^{\rm N}_{\ell,m}$$

$$B^{\rm o}_{\ell,m} = E^{\rm fg}_{\ell,m}\sin(2\alpha) + B^{\rm fg}_{\ell,m}\cos(2\alpha) + E^{\rm CMB}_{\ell,m}\sin(2\alpha+2\beta) + B^{\rm CMB}_{\ell,m}\cos(2\alpha+2\beta) + B^{\rm N}_{\ell,m}$$

$\cdots (5)$

From them, we derived

$$\begin{aligned}
&\left(C_\ell^{EB,\rm o} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\rm o} - C_\ell^{BB,\rm o}\right)\right)\cos(4\alpha) = \\
&\quad (C_\ell^{EE,\rm CMB} - C_\ell^{BB,\rm CMB})\sin(4\beta)/2 \\
&\quad - (C_\ell^{EE,\rm N} - C_\ell^{BB,\rm N})\sin(4\alpha)/2 \\
&\quad + C_\ell^{EB,\rm fg} + C_\ell^{EB,\rm N}\cos(4\alpha) + C_\ell^{EB,\rm CMB}\cos(4\beta) \\
&\quad + (C_\ell^{E^{\rm fg}B^{\rm CMB}} + C_\ell^{E^{\rm CMB}B^{\rm fg}})\cos(2\beta) + (C_\ell^{E^{\rm fg}E^{\rm CMB}} - C_\ell^{B^{\rm fg}B^{\rm CMB}})\sin(2\beta) \\
&\quad + (C_\ell^{E^{\rm fg}B^{\rm N}} + C_\ell^{B^{\rm fg}E^{\rm N}})\cos(2\alpha) - (C_\ell^{E^{\rm fg}E^{\rm N}} - C_\ell^{B^{\rm fg}B^{\rm N}})\sin(2\alpha) \\
&\quad + (C_\ell^{E^{\rm CMB}B^{\rm N}} + C_\ell^{B^{\rm CMB}E^{\rm N}})\cos(2\alpha-2\beta) \\
&\quad - (C_\ell^{E^{\rm CMB}E^{\rm N}} - C_\ell^{B^{\rm CMB}B^{\rm N}})\sin(2\alpha-2\beta)\,.
\end{aligned} \qquad \cdots (6)$$

Equations including birefringence rotation:

If we take an ensemble average

$$\langle C_\ell^{EB,\mathrm{o}}\rangle = \frac{\tan(4\alpha)}{2}\left(\langle C_\ell^{EE,\mathrm{o}}\rangle - \langle C_\ell^{BB,\mathrm{o}}\rangle\right) + \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(\langle C_\ell^{EE,\mathrm{CMB}}\rangle - \langle C_\ell^{BB,\mathrm{CMB}}\rangle\right) \quad \cdots (7)$$

$$\boxed{+ \frac{1}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{fg}}\rangle + \frac{\cos(4\beta)}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{CMB}}\rangle .}$$

Assume these to be zero

Therefore, we can determine both miscalibration and birefringence-rotation angles simultaneously!

Construct a likelihood for determination of α and β

If we take an ensemble average

$$\langle C_\ell^{EB,\mathrm{o}}\rangle = \frac{\tan(4\alpha)}{2}\left(\langle C_\ell^{EE,\mathrm{o}}\rangle - \langle C_\ell^{BB,\mathrm{o}}\rangle\right) + \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(\langle C_\ell^{EE,\mathrm{CMB}}\rangle - \langle C_\ell^{BB,\mathrm{CMB}}\rangle\right) \quad \cdots (7)$$

$$\boxed{+\frac{1}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{fg}}\rangle + \frac{\cos(4\beta)}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{CMB}}\rangle .}$$

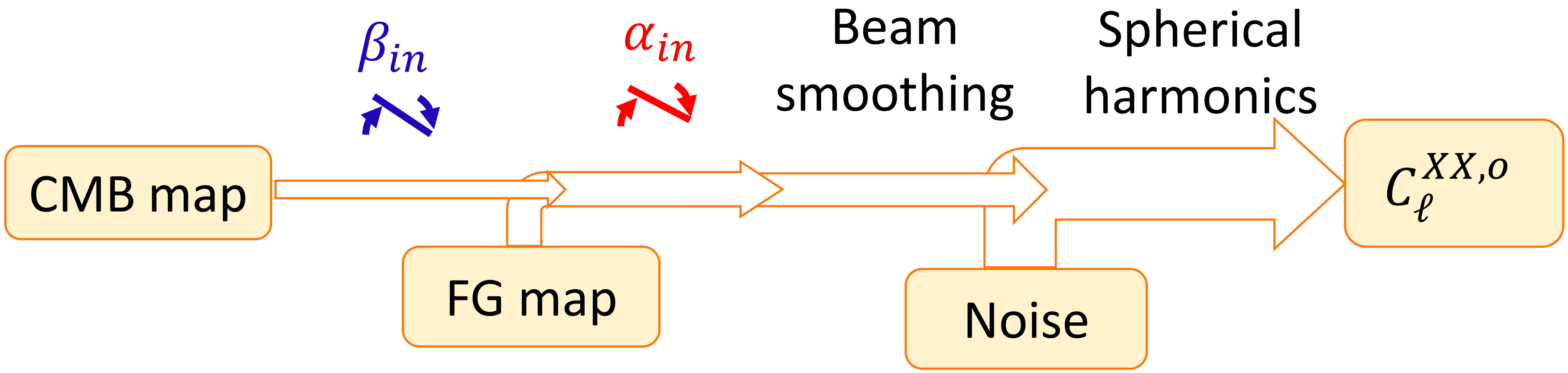
Assume these to be zero

$$-2\ln\mathcal{L} = \sum_{\ell=2}^{\ell_{\max}} \frac{\left[C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right) - \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(C_\ell^{EE,\mathrm{CMB}} - C_\ell^{BB,\mathrm{CMB}}\right)\right]^2}{\mathrm{Var}\left(C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right)\right)} \quad \cdots (8)$$

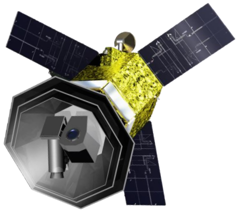
Minimise $-2\ln\mathcal{L}$ to determine α and β

Sky simulation setup for the validation

- Components
  - Thermal dust: modified black body
  - Synchrotron: simple power law
  - CMB: tensor-to-scalar ratio r = 0
  - Noise: white noise with LiteBIRD polarisation sensitivity
- N_{side} is 512 and l_{max} is 1024
- Compute EB power spectra from full-sky maps



Sky simulation setup for the validation: LiteBIRD



We extract representative 4 frequencies to show how the method works

	Frequency	polarisation sensitivity (uK’)	Beam size in FWHM (arcmin)
Synchrotron	50	24.0	48
CMB	119	7.6	25
Dust + CMB	195	5.8	20
Dust	235	7.7	19

LiteBIRD parameters extracted from M. Hazumi et al., J. Low Temp. Phys. 194, 443 (2019).

Before the simultaneous determination: α only case

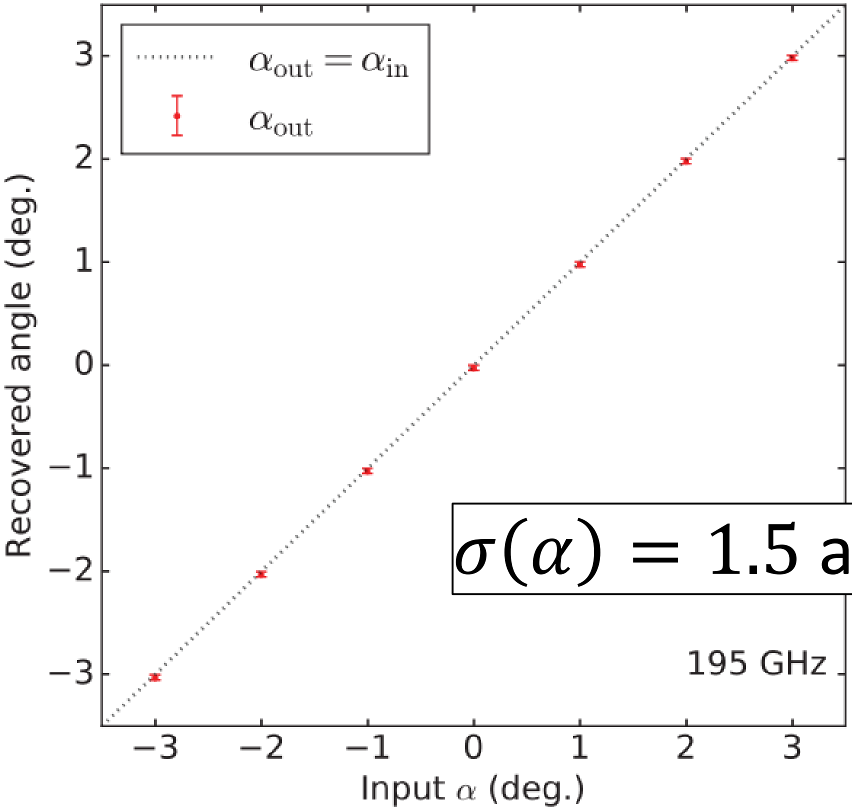
If we assume $\beta_{in}=0$, we can set $\beta=0$ in the Likelihood as,

$$-2\ln\mathcal{L} = \frac{\left[C_\ell^{EB,o} - \frac{1}{2}\left(C_\ell^{EE,o} - C_\ell^{BB,o}\right)\tan(4\alpha)\right]^2}{\mathrm{Var}\left(C_\ell^{EB,o} - \frac{1}{2}\left(C_\ell^{EE,o} - C_\ell^{BB,o}\right)\tan(4\alpha)\right)}. \qquad \cdots(10)$$

With this likelihood, we can determine α.

α only estimation at 195 GHz

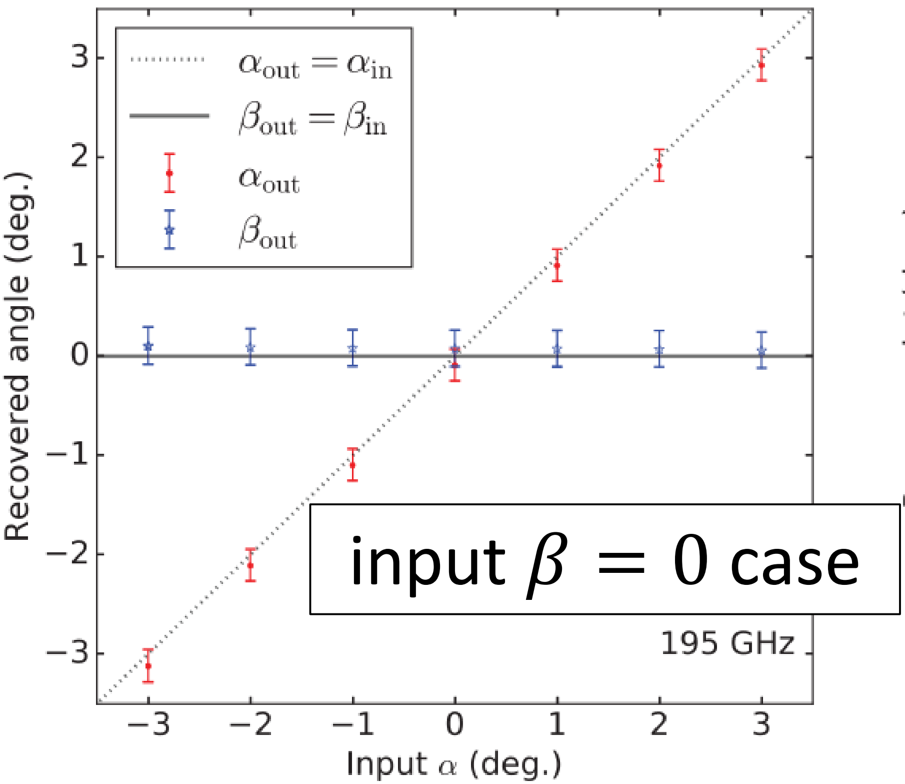
We set β_{in}=0 and try whether we can determine α_{in}

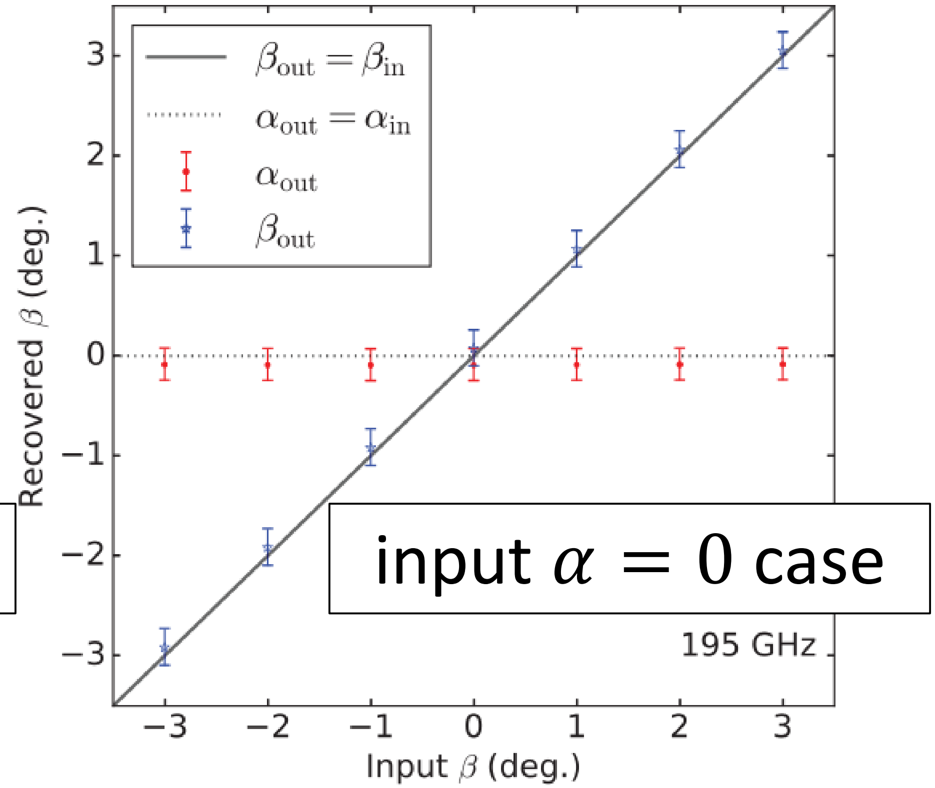


$\sigma(\alpha) = 1.5$ arcmin

We can recover the correct α without theoretical power spectra

Simultaneous determination at 195 GHz





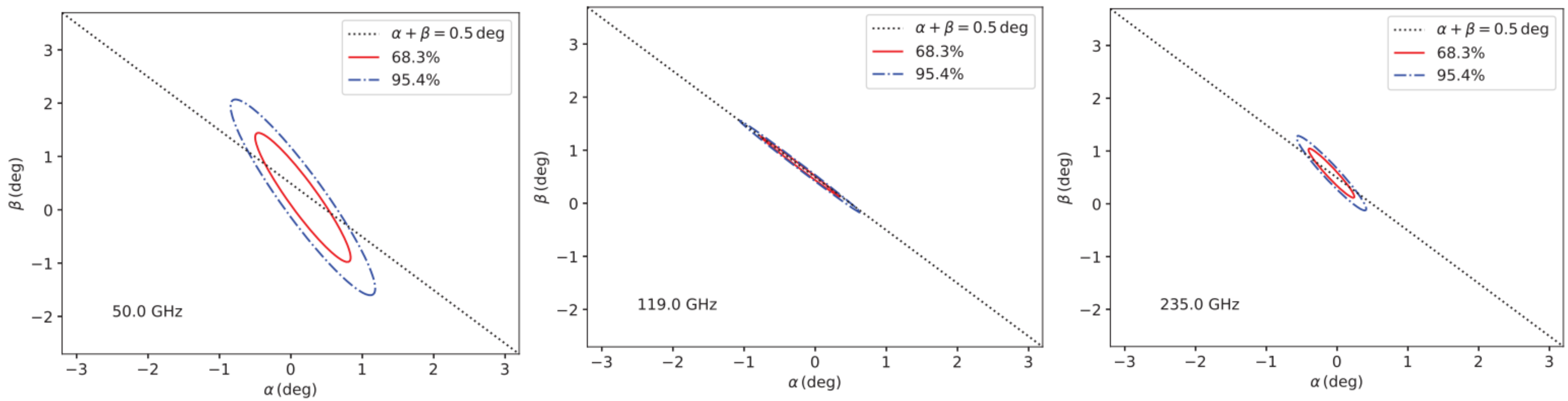
$\sigma(\alpha) = 9.6$ arcmin and $\sigma(\beta) = 11$ arcmin

(current 1σ upper bound for β is 30 arcmin)

We can recover the correct α and β simultaneously

Correlation between α and β

$$E^{\mathrm{o}}_{\ell,m} = E^{\mathrm{fg}}_{\ell,m}\cos(2\alpha) - B^{\mathrm{fg}}_{\ell,m}\sin(2\alpha) + E^{\mathrm{CMB}}_{\ell,m}\cos(2\alpha+2\beta) - B^{\mathrm{CMB}}_{\ell,m}\sin(2\alpha+2\beta)$$



Synchrotron channel (50 GHz)

CMB channel (119 GHz)

Dust channel (235 GHz)

- CMB has a power to determine α+β
- FG has a power to determine α

Achievements in “Cosmic Acceleration” Innovative Area

This is an interdisciplinary study of B01 and D01 group

Publication:

- “Simultaneous determination of the cosmic birefringence and miscalibrated polarisation angles from CMB experiments”, PTEP, Volume 2019, Issue 8, August 2019, 083E02, https://doi.org/10.1093/ptep/ptz079

Presentations and posters:

- Poster: COSMO19, RWTH Aachen University, Germany
- Presentation: B-mode from Space, MPA

Lead to the future developments:

- “Determination of miscalibrated polarization angles from observed CMB and foreground EB power spectra: Application to partial-sky observation”, arXiv:2002.03572

Summary

There was a consensus in the CMB community that the measurement of the cosmic birefringence and the polarization angle calibration cannot be done simultaneously

We have shown that this is not the case.

We can determine the birefringence angle of order 10 arcmin with LiteBIRD

This is a great news!

We made interdisciplinary studies and give a seed to the future developments

Backups

Variance

- With full-sky power spectra (not cut-sky pseudo power spectra), we can calculate variance exactly as

$$
\begin{aligned}
&\mathrm{Var}\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right] \\
&= \left\langle\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right]^2\right\rangle - \langle C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\rangle^2 \\
&= \frac{1}{2\ell+1}\langle C_\ell^{EE}\rangle\langle C_\ell^{BB}\rangle + \frac{\tan^2(4\alpha)}{4}\frac{2}{2\ell+1}\left(\langle C_\ell^{EE}\rangle^2 + \langle C_\ell^{BB}\rangle^2\right) \\
&\quad - \tan(4\alpha)\frac{2}{2\ell+1}\langle C_\ell^{EB}\rangle\left(\langle C_\ell^{EE}\rangle - \langle C_\ell^{BB}\rangle\right) + \underbrace{\boxed{\frac{1}{2\ell+1}\left(1-\tan^2(4\alpha)\right)\langle C_\ell^{EB}\rangle^2.}}_{=\ 0}
\end{aligned}
\qquad \cdots (9)
$$

- We approximate $\langle C_\ell^{XY}\rangle \approx C_\ell^{XY,o}$
- We ignore $\langle C_\ell^{EB}\rangle^2$ term because it's small and yields bias
  - Even if $\langle C_\ell^{EB}\rangle \approx 0$, $C_\ell^{EB,o}$ has a small non-zero value with fluctuation, and ${C_\ell^{EB,o}}^2$ yields bias

Future prospect: Possibility to determine foreground *EB*

- In general, *EB* is related to *EE* and *BB* as

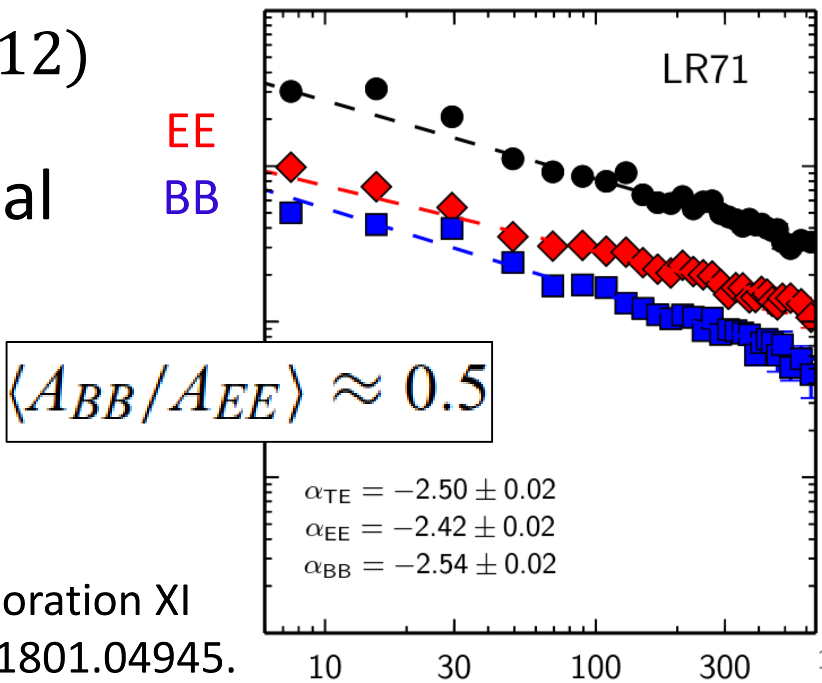
$$C_\ell^{EB} = f_c \sqrt{\langle C_\ell^{EE} \rangle \langle C_\ell^{BB} \rangle} \quad \cdots (11)$$

where f_c is a correlation coefficient.

- We assume *BB* is proportional to *EE* as

$$\langle C_\ell^{BB,fg} \rangle = \xi \langle C_\ell^{EE,fg} \rangle \quad \cdots (12)$$

  - This seems valid for thermal dust observed by Planck



Planck Collaboration XI (2018), arXiv:1801.04945.

Relate correlation parameters to rotation angle

- If *EB* correlation is small enough to meet

$$0 \le \frac{2 f_c \sqrt{\xi}}{1-\xi} \le 1, \qquad \cdots (13)$$

we can put f_c and ξ into the rotation angle γ as,

$$\frac{\sin(4\gamma)}{2} = \frac{f_c \sqrt{\xi}}{1-\xi}. \qquad \cdots (14)$$

- Therefore, we can determine foreground EB correlation, if we give up measuring β

Table 1: Polarisation sensitivity and beam size of the LiteBIRD telescopes [15]

Frequency (GHz)	Polarization Sensitivity ($\mu K'$)	Beam Size in FWHM (arcmin)
40	37.5	69
50	24.0	56
60	19.9	48
68	16.2	43
78	13.5	39
89	11.7	35
100	9.2	29
119	7.6	25
140	5.9	23
166	6.5	21
195	5.8	20
235	7.7	19
280	13.2	24
337	19.5	20
402	37.5	17

Foreground

$$\langle C_\ell^{EB,\mathrm{fg}}\rangle = \frac{f_c\sqrt{\xi}}{1-\xi}\left(\langle C_\ell^{EE,\mathrm{fg}}\rangle - \langle C_\ell^{BB,\mathrm{fg}}\rangle\right). \longrightarrow \frac{\sin(4\gamma)}{2}\left(\left\langle C_\ell^{EE,fg}\right\rangle - \left\langle C_\ell^{BB,fg}\right\rangle\right)$$

$$E_{\ell,m}^{\mathrm{o,fg}} = E_{\ell,m}^{\mathrm{fg}}\cos(2\gamma) - B_{\ell,m}^{\mathrm{fg}}\sin(2\gamma),$$
$$B_{\ell,m}^{\mathrm{o,fg}} = E_{\ell,m}^{\mathrm{fg}}\sin(2\gamma) + B_{\ell,m}^{\mathrm{fg}}\cos(2\gamma),$$

Replace α-> α+γ
β->-γ
in birefringence estimation

$$\begin{aligned}
&\left(C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right)\right)\cos(4\alpha+4\gamma) = \\
&\quad (C_\ell^{EE,\mathrm{CMB}} - C_\ell^{BB,\mathrm{CMB}})\sin(-4\gamma)/2 \\
&\quad + C_\ell^{EB,\mathrm{fg}} + C_\ell^{EB,\mathrm{N}}\cos(4\alpha+4\gamma) + C_\ell^{EB,\mathrm{CMB}}\cos(-4\gamma) \\
&\quad - (C_\ell^{EE,\mathrm{N}} - C_\ell^{BB,\mathrm{N}})\sin(4\alpha+4\gamma)/2 \\
&\quad + (C_\ell^{E^{\mathrm{fg}}B^{\mathrm{CMB}}} + C_\ell^{E^{\mathrm{CMB}}B^{\mathrm{fg}}})\cos(-2\gamma) + (C_\ell^{E^{\mathrm{fg}}E^{\mathrm{CMB}}} - C_\ell^{B^{\mathrm{fg}}B^{\mathrm{CMB}}})\sin(-2\gamma) \\
&\quad + (C_\ell^{E^{\mathrm{fg}}B^{\mathrm{N}}} + C_\ell^{B^{\mathrm{fg}}E^{\mathrm{N}}})\cos(2\alpha+2\gamma) - (C_\ell^{E^{\mathrm{fg}}E^{\mathrm{N}}} - C_\ell^{B^{\mathrm{fg}}B^{\mathrm{N}}})\sin(2\alpha+2\gamma) \\
&\quad + (C_\ell^{E^{\mathrm{CMB}}B^{\mathrm{N}}} + C_\ell^{B^{\mathrm{CMB}}E^{\mathrm{N}}})\cos(2\alpha+4\gamma) \\
&\quad - (C_\ell^{E^{\mathrm{CMB}}E^{\mathrm{N}}} - C_\ell^{B^{\mathrm{CMB}}B^{\mathrm{N}}})\sin(2\alpha+4\gamma)
\end{aligned}$$

Equations including birefringence rotation – carefully-

We should not forget noise and other correlations!

$$E^{\rm o}_{\ell,m} = E^{\rm fg}_{\ell,m}\cos(2\alpha) - B^{\rm fg}_{\ell,m}\sin(2\alpha) + E^{\rm CMB}_{\ell,m}\cos(2\alpha+2\beta) - B^{\rm CMB}_{\ell,m}\sin(2\alpha+2\beta) + E^{\rm N}_{\ell,m},$$

$$B^{\rm o}_{\ell,m} = E^{\rm fg}_{\ell,m}\sin(2\alpha) + B^{\rm fg}_{\ell,m}\cos(2\alpha) + E^{\rm CMB}_{\ell,m}\sin(2\alpha+2\beta) + B^{\rm CMB}_{\ell,m}\cos(2\alpha+2\beta) + B^{\rm N}_{\ell,m}$$

Noise

$$\left(C_\ell^{EB,{\rm o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,{\rm o}} - C_\ell^{BB,{\rm o}}\right)\right)\cos(4\alpha) =$$

$$(C_\ell^{EE,{\rm CMB}} - C_\ell^{BB,{\rm CMB}})\sin(4\beta)/2$$

$$+ C_\ell^{EB,{\rm fg}} + C_\ell^{EB,{\rm N}}\cos(4\alpha) + C_\ell^{EB,{\rm CMB}}\cos(4\beta)$$

$$- (C_\ell^{EE,{\rm N}} - C_\ell^{BB,{\rm N}})\sin(4\alpha)/2$$

$$+ (C_\ell^{E^{\rm fg}B^{\rm CMB}} + C_\ell^{E^{\rm CMB}B^{\rm fg}})\cos(2\beta) + (C_\ell^{E^{\rm fg}E^{\rm CMB}} - C_\ell^{B^{\rm fg}B^{\rm CMB}})\sin(2\beta)$$

$$+ (C_\ell^{E^{\rm fg}B^{\rm N}} + C_\ell^{B^{\rm fg}E^{\rm N}})\cos(2\alpha) - (C_\ell^{E^{\rm fg}E^{\rm N}} - C_\ell^{B^{\rm fg}B^{\rm N}})\sin(2\alpha)$$

$$+ (C_\ell^{E^{\rm CMB}B^{\rm N}} + C_\ell^{B^{\rm CMB}E^{\rm N}})\cos(2\alpha - 2\beta)$$

$$- (C_\ell^{E^{\rm CMB}E^{\rm N}} - C_\ell^{B^{\rm CMB}B^{\rm N}})\sin(2\alpha - 2\beta)$$

Variance

$$\begin{aligned}
&\mathrm{Var}\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right] \\
&= \langle\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right]^2\rangle - \langle C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\rangle^2 \\
&= \frac{1}{2\ell+1}\langle C_\ell^{EE}\rangle\langle C_\ell^{BB}\rangle + \frac{\tan^2(4\alpha)}{4}\frac{2}{2\ell+1}\left(\langle C_\ell^{EE}\rangle^2 + \langle C_\ell^{BB}\rangle^2\right) \\
&\quad - \tan(4\alpha)\frac{2}{2\ell+1}\langle C_\ell^{EB}\rangle\left(\langle C_\ell^{EE}\rangle - \langle C_\ell^{BB}\rangle\right) + \frac{1}{2\ell+1}\left(1-\tan^2(4\alpha)\right)\langle C_\ell^{EB}\rangle^2. \qquad \text{(A1)}
\end{aligned}$$

$$\begin{aligned}
&\mathrm{Var}\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right] \\
&\approx \frac{1}{2\ell+1}C_\ell^{EE,\mathrm{o}}C_\ell^{BB,\mathrm{o}} + \frac{\tan^2(4\alpha)}{4}\frac{2}{2\ell+1}\left[(C_\ell^{EE,\mathrm{o}})^2 + (C_\ell^{BB,\mathrm{o}})^2\right] \\
&\quad - \tan(4\alpha)\frac{2}{2\ell+1}C_\ell^{EB,\mathrm{o}}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right).
\end{aligned}$$