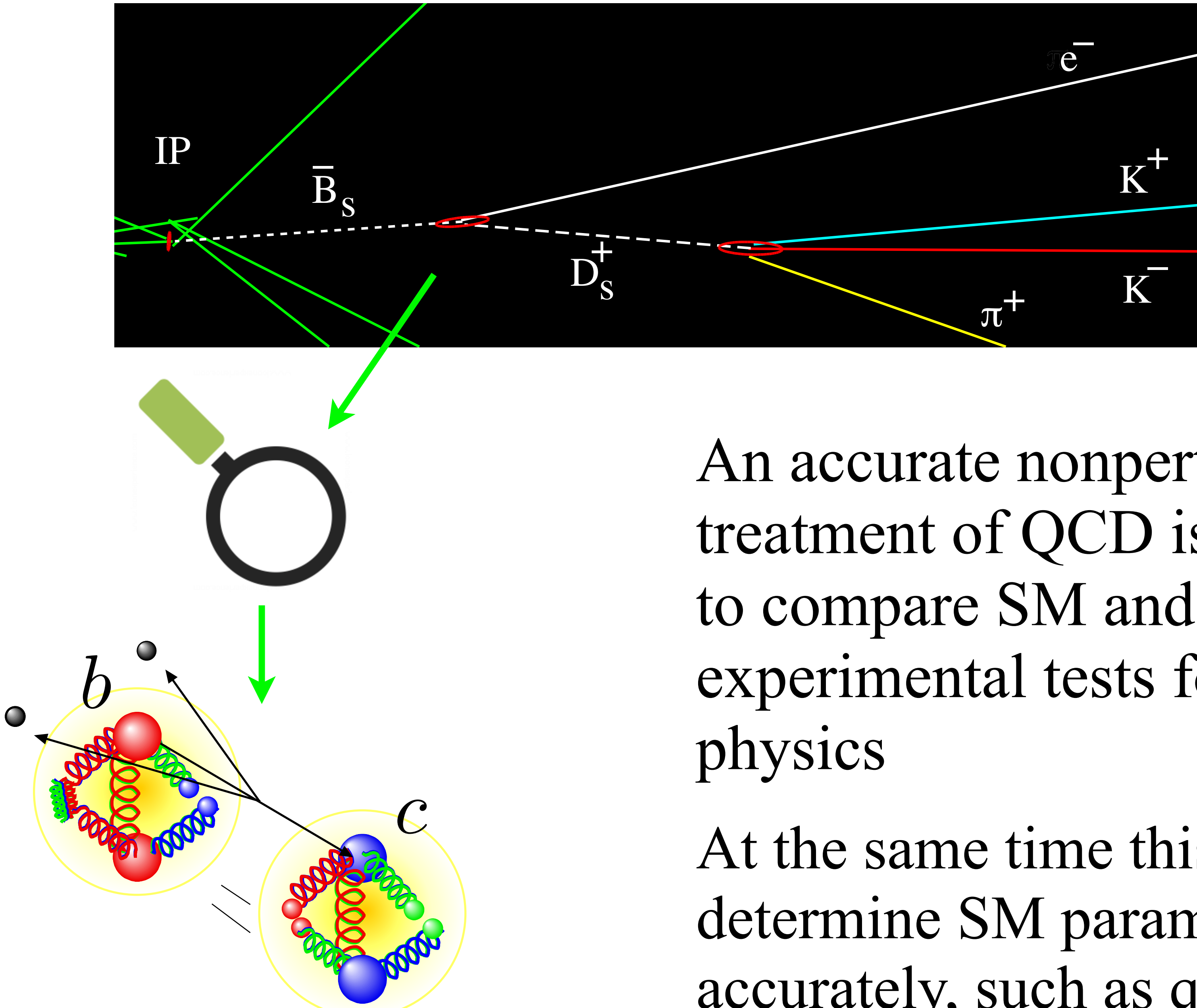
Lattice QCD: m_b, m_c and b and c form factors

Christine Davies
University of Glasgow
HPQCD collaboration

Beauty2020
Sept. 2020

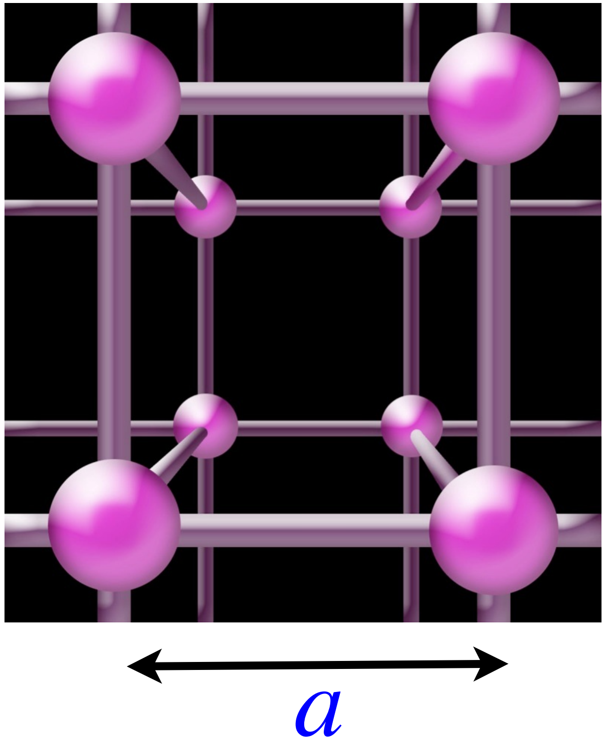
Quark low-energy strong interactions are a major complication in testing the Standard Model



ALEPH@LEP

An accurate nonperturbative treatment of QCD is necessary to compare SM and low-energy experimental tests for new physics

At the same time this allows us to determine SM parameters accurately, such as quark masses

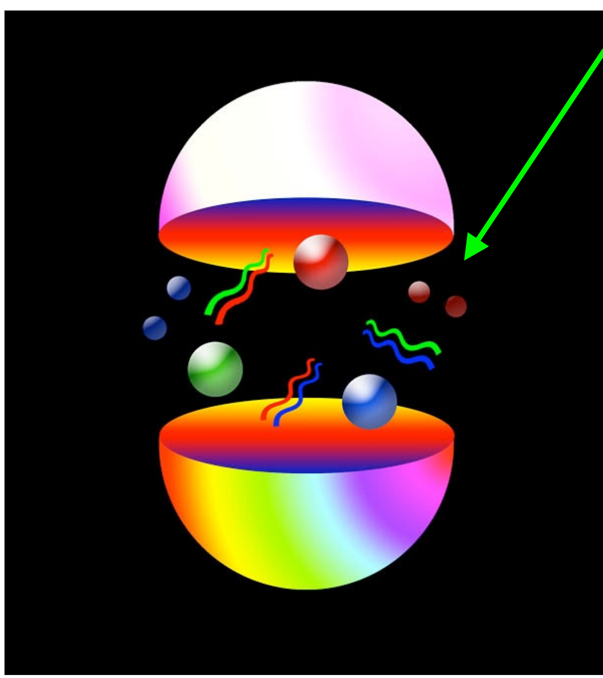


Lattice QCD provides such an approach.

Must be able to determine results in physical continuum limit.

Final accuracy depends on :

- control of lattice spacing dependence
- tuning of quark masses
- normalisation of operators (for matrix elements)



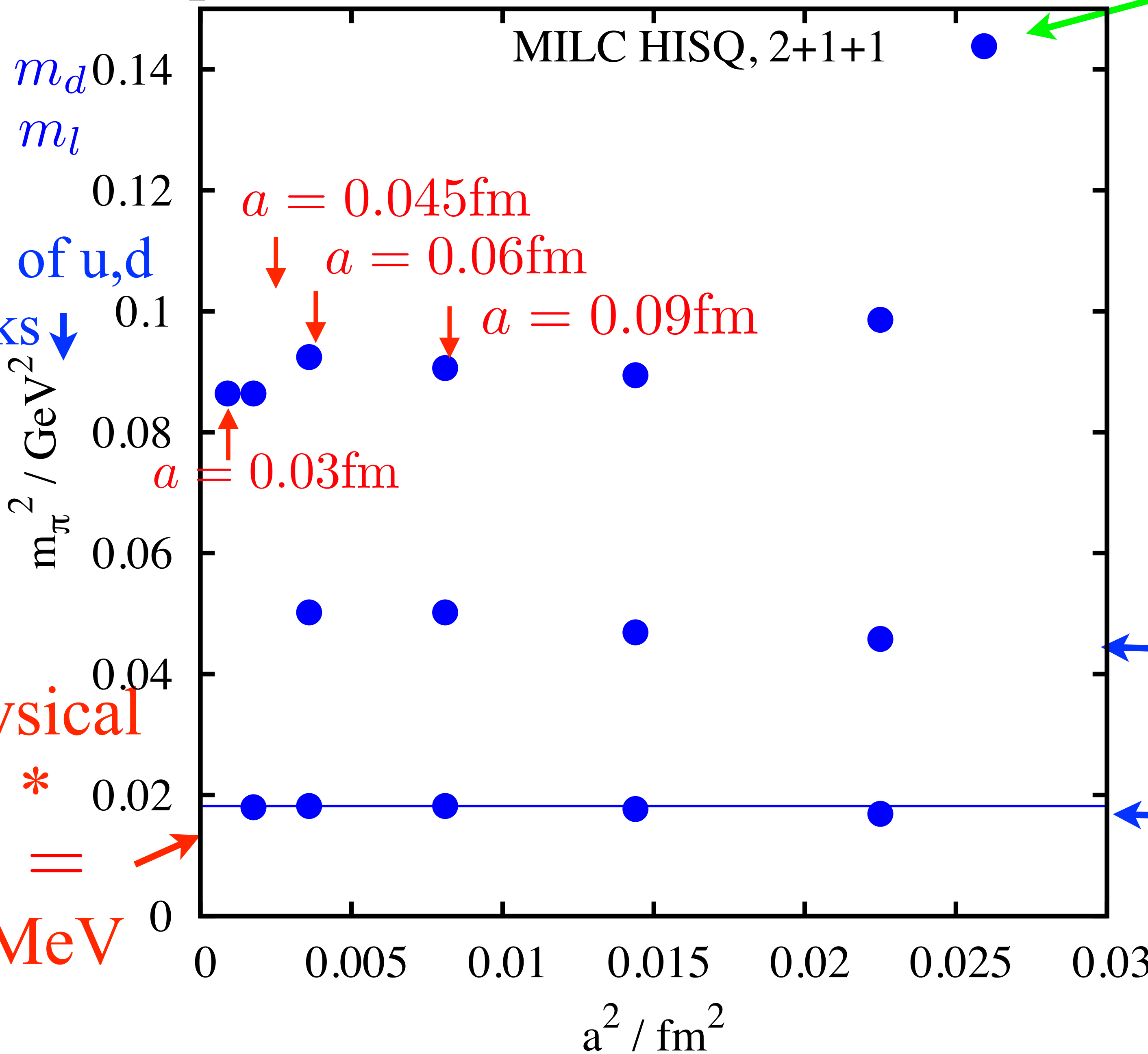
HPQCD's Highly Improved Staggered Quark (HISQ) action particularly good here since it has small discretisation errors, is numerically efficient, fully nonperturbative operator norm. is usually possible.

E.Follana, et al, HPQCD, hep-lat/0610092.

Parameters for gluon field configurations generated with HISQ sea quarks

$m_u = m_d = m_l$

mass of u,d quarks ↓



*physical $m_{u/d}$ *

$m_{\pi^0} =$ 135 MeV

"2nd generation" lattices inc. c quarks in sea

HISQ = Highly improved staggered quarks -very accurate discretisation

E.Follana, et al, HPQCD, hep-lat/0610092.

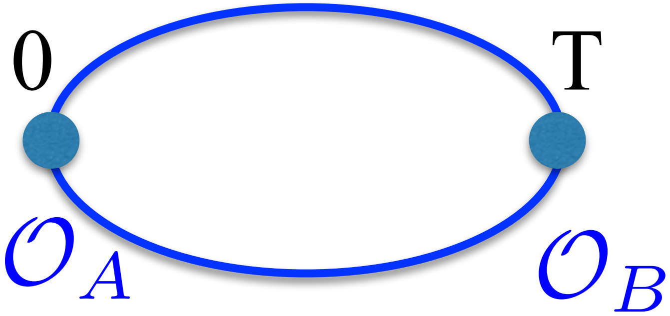
$m_{u,d} \approx m_s/10$

$m_{u,d} \approx m_s/27$

Volume: $m_\pi L > 3$

n_f =1+1+1+1 configs also generated

Meson Correlation functions are constructed from valence quark propagators



2-point function

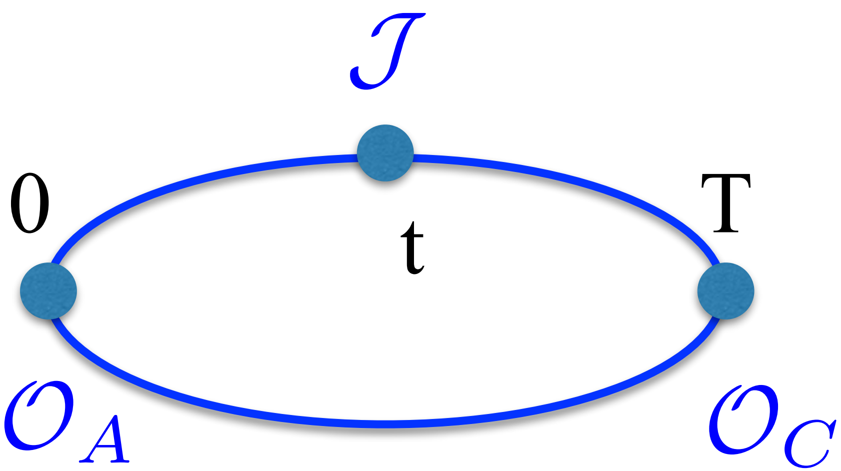
Multiple states need to be inc. in fit

$$C_2 = \sum_n A_n B_n e^{-M_n T}$$

$\langle 0|\mathcal{O}_A|n\rangle$

decay constant, if O normalised

Meson mass - Ground-state mass can be very accurate. Use to tune lattice quark mass



3-point function

$$C_3 = \sum_{m,n} A_n J_{nm} C_m \, e^{-M_n t} e^{-M_m (T-t)}$$

$\langle n|\mathcal{J}|m\rangle$ → form factor, if J normalised

Determining c, b quark masses from lattice QCD

Lattice quark masses can be tuned very accurately from ground-state meson masses, e.g. J/ψ for c.

The issue is accurate conversion to the $\overline{\mathrm{MS}}$ scheme

MULTIPLE DIFFERENT methods for doing this now - can compare results at 1% level.

NEW - HPQCD:2005.01845 includes QED effects.

$$S_{\mathrm{Feynman}} \propto k^2 |A_\mu^{\mathrm{QED}}(k)|^2$$

'Quenched' QED: Choose A from Gaussian dist. in mom. space, set zero-modes to zero: $(\mathrm{QED})_{\mathrm{L}}$

Quark electric charge ↓

$$U_\mu^{\mathrm{QED}} = \exp(-ie_q a A_\mu^{\mathrm{QED}}(x + a\hat{\mu}/2))$$

Feed into Dirac eq. solver along with gluons

Meson mass changes with QED - must retune quark masses - and conversion to $\overline{\mathrm{MS}}$ changes.

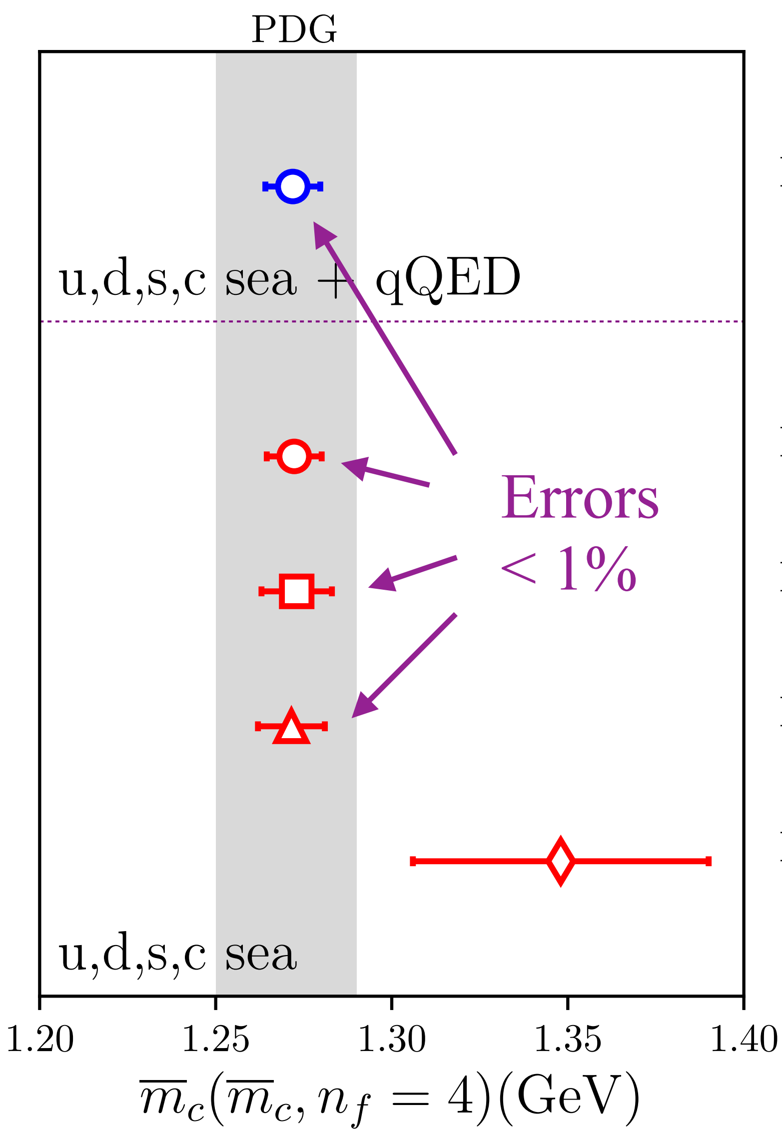
Effects ~0.1% but visible

Determining c quark mass from lattice QCD

Update on m_c :

`HPQCD : D. Hatton et al, 2005.01845`

Tune m_c from J/ψ, use SMOM scheme to get to $\overline{\mathrm{MS}}$ inc. analysis of nonpert. condensate effects



HPQCD HISQ RI-SMOM 2005.01845 $\overline{m}_c(\overline{m}_c) = 1.2719(78)\mathrm{GeV}$

HPQCD HISQ RI-SMOM 2005.01845

FNAL/MILC/TUM HISQ MRS 1802.04248

Tune m_c from Ds, minimal renormalon-sub. (MRS) scheme connects to $\overline{\mathrm{MS}}$

HPQCD HISQ JJc 1408.4169

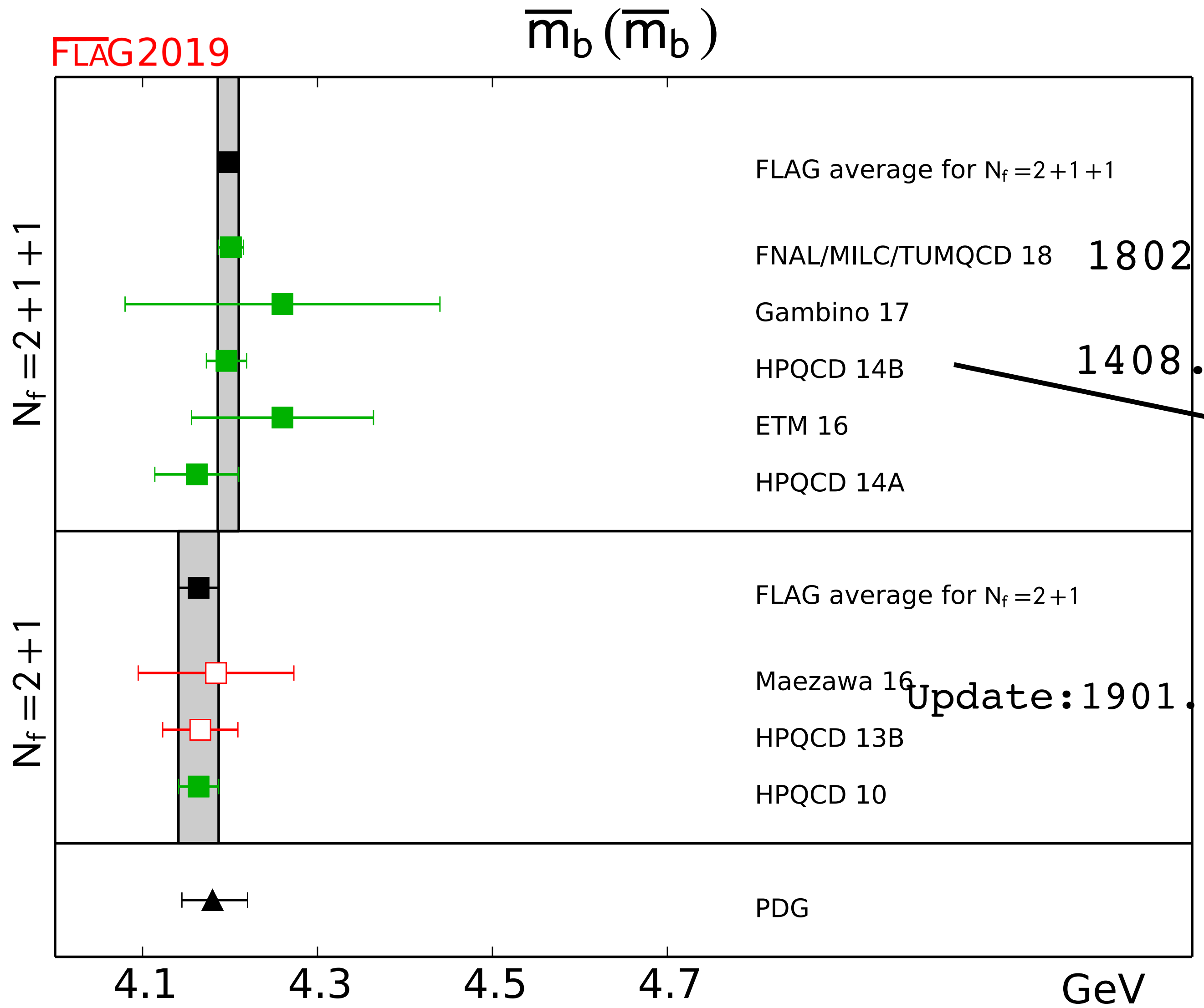
Tune using η_c. Time-moments of correlators allow match to $\overline{\mathrm{MS}}$

ETMC twisted mass RI-MOM 1403.4504

Impact of including QED:
Lowers $\overline{m}_c(3\ \mathrm{GeV})$ by 0.18(2)% ;
Almost no effect on $\overline{m}_c(\overline{m}_c)$

PDG: take note of uncertainties

Determining b quark mass from lattice QCD



FLAG:1902.08191 $\overline{m}_b(\overline{m}_b)$ GeV

1802.04248

Tune m_b from Bs, minimal renormalon-sub. (MRS) scheme connects to $\overline{\text{MS}}$. HISQ b.

4.195(14) GeV

1408.5768

Vector JJ correlator Time-moments + cont. pert. theory. NRQCD b.

Update:1901.06424

4.196(23) GeV

<0.5% uncertainty. Include estimate of effects of QED.

Determining c, b quark masses from lattice QCD

Update on m_b/m_c :

$$\left.\frac{m_1^{\rm latt}}{m_2^{\rm latt}}\right|_{a=0} = \frac{\overline{m}_1(\mu)}{\overline{m}_2(\mu)}$$

In pure QCD. Determine lattice mass ratio fully non-perturbatively by mapping m_h/m_c vs meson mass.

$$\frac{m_b}{m_c} = 4.570(10)$$ ← 0.2%

preliminary

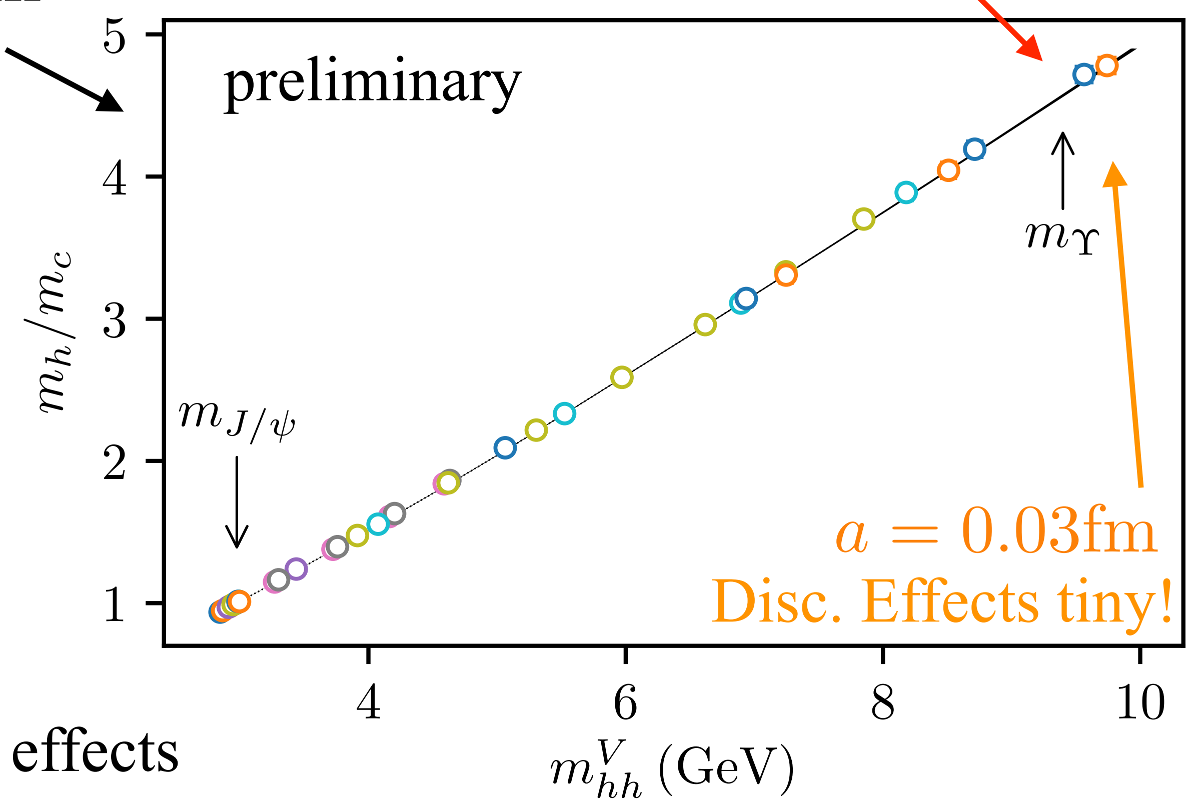
`HPQCD:preliminary, HISQ,` tune from bottom/charmonium

Inclusion of QED underway (ratio then μ-dependent)

`cf. FNAL/MILC/ TUMQCD:1802.04248,`HISQ Ds/Bs tuning and MRS.

$$\frac{m_b}{m_c} = 4.577(8)(12)$$

Missing QED self-energy effects

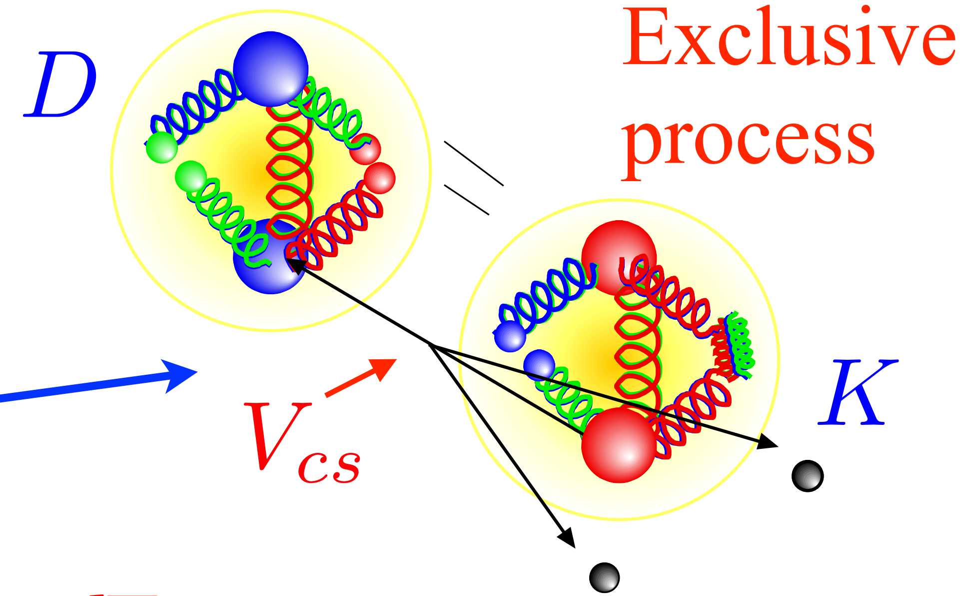


Weak decays probe hadron structure and quark couplings.

Hadronic matrix elements of weak current calculable in lattice QCD

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to l\nu & K \to l\nu & B \to l\nu \\ & K \to \pi l\nu & B \to \pi l\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \pi l\nu & D \to K l\nu & B \to D^{(*)} l\nu \\ D \to l\nu & D_s \to l\nu & \Lambda_b \to \Lambda_c l\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle & \end{pmatrix}$$

CKM matrix



$$\frac{d\Gamma}{dq^2} \propto V_{ab}^2 |f(q^2)|^2$$

Expt = CKM x theory(QCD)

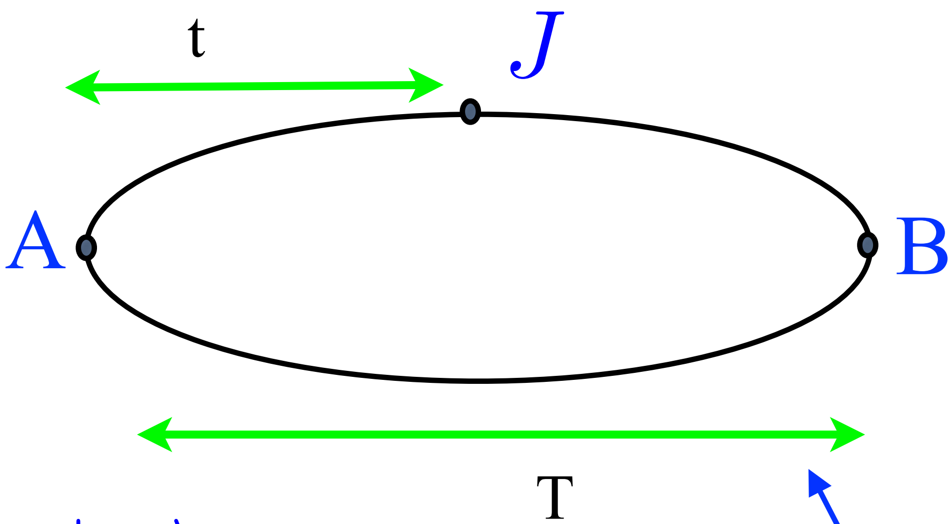
Need precision lattice QCD to get accurate CKM elements to test Standard Model.

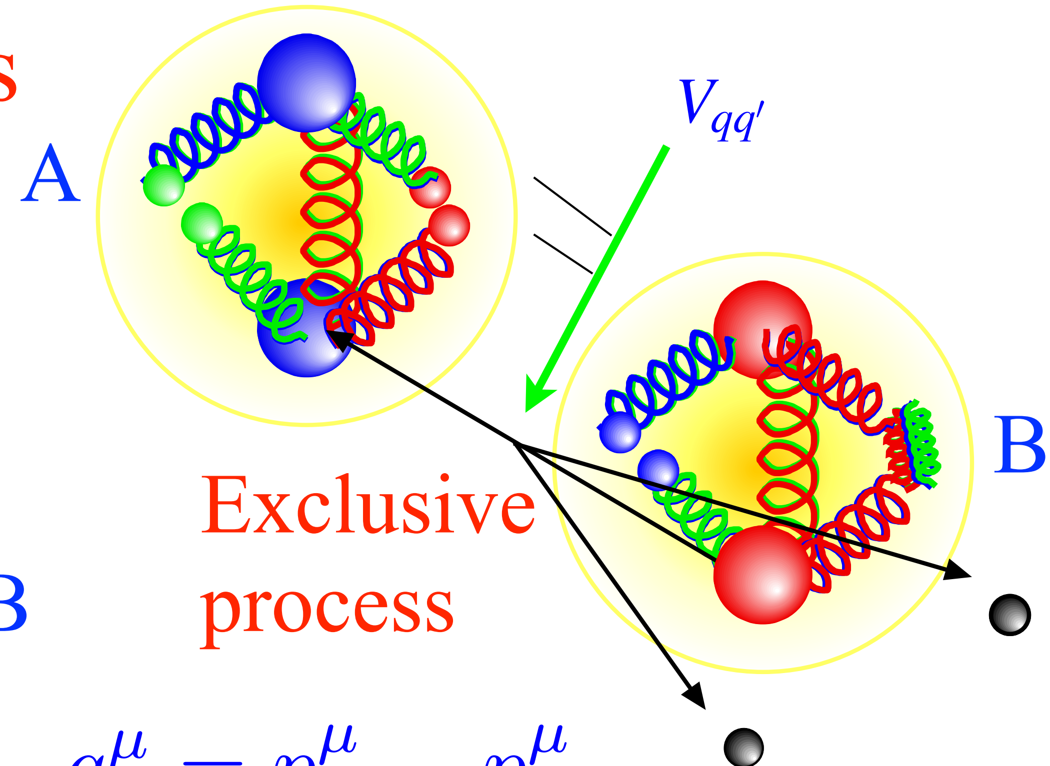
Aim : sub-1% errors

For semileptonic processes, can also map out q^2/angular behaviour for tests of SM.

Semileptonic form factors

QCD info. encoded in form factors, functions of q^2





$$q^\mu = p_A^\mu - p_B^\mu$$

$q^2_{max} = (M_A - M_B)^2$, zero recoil

$\langle A|J|B\rangle$

determined from fits to 3-point correlators, gives form factors, $f(q^2)$

PS to PS case: f_+, f_0, with $f_+(0)=f_0(0)$

PS to V case: V, A_0, A_1, A_2 ← Zero recoil

Tensor form factors can also be calculated

Issues are: discretisation errors, normalisation of current J and, for b case, coverage of large q^2 range

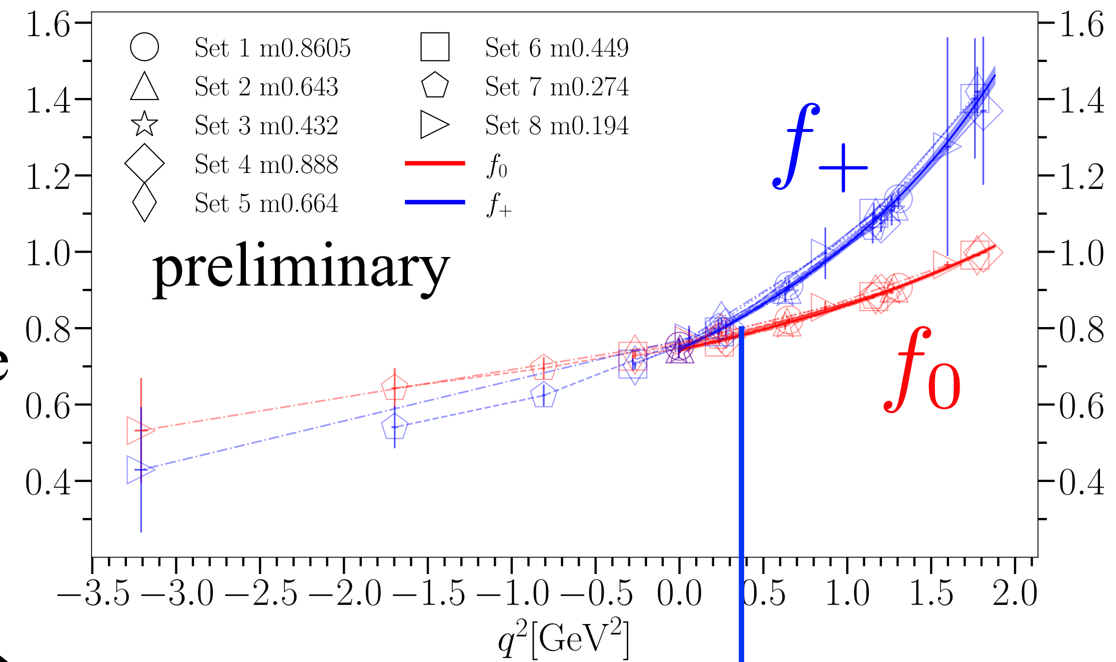
Charm form factors

UPDATE on Vcs from $D \to K \ell \overline{\nu}$

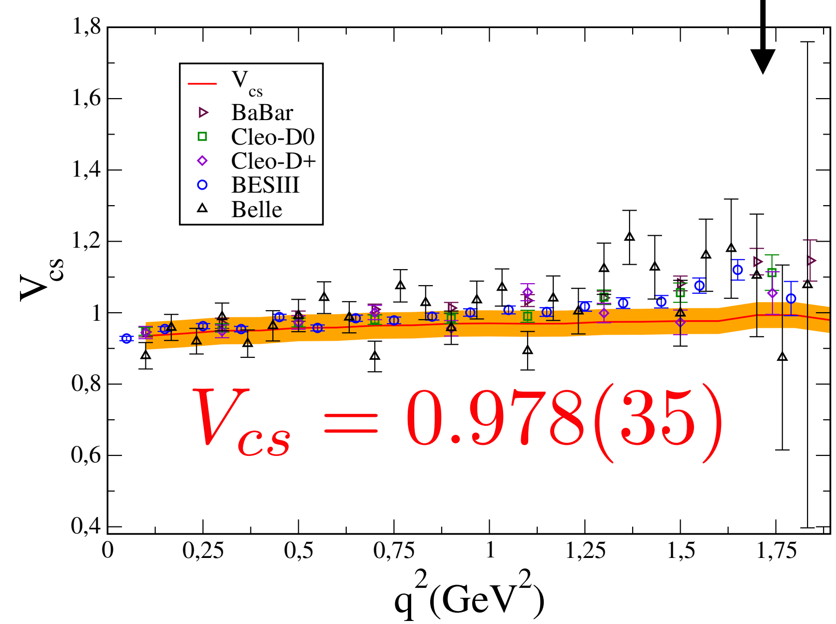
HPQCD: B. Chakraborty, W. Parrott et al, in prep.

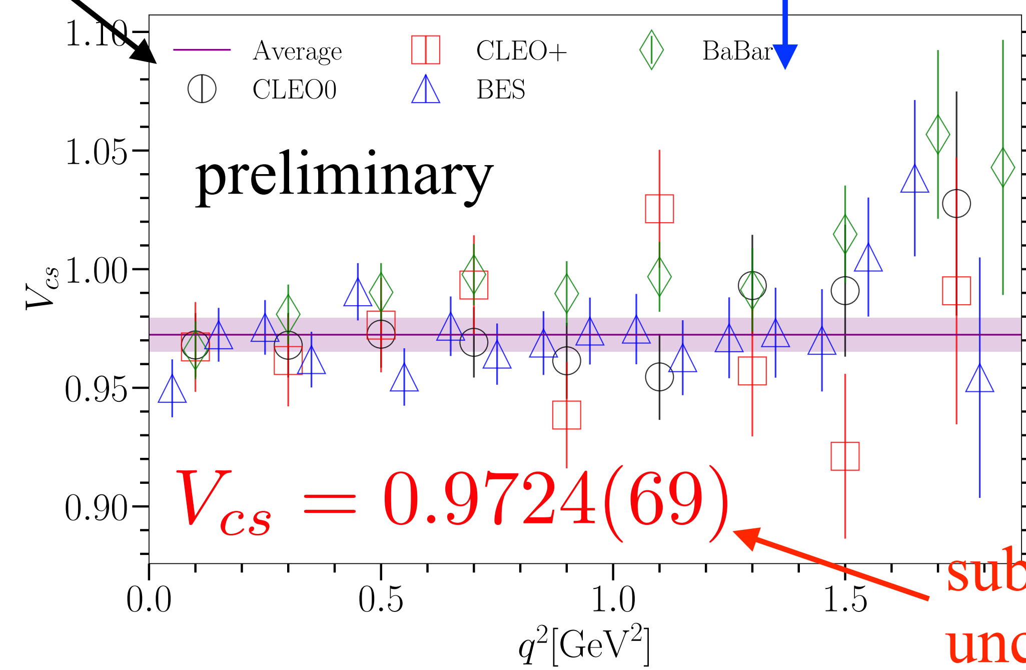
Small q² range - easily covered on lattice. Nonpert. normln. of current. Multiple lattice spacings, and sea quark masses, inc. physical u/d

Convert to z-space to fit, extrapolate to a=0. f_+ gives $d\Gamma/dq^2$ - bin-by-bin comparison w. expt. yields Vcs



ETMC:1706.03657
(also has D to π)



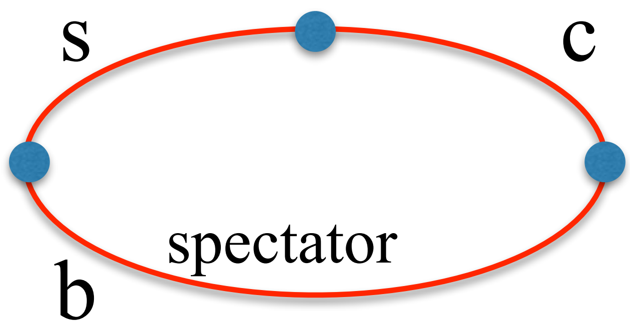


sub-1% uncty

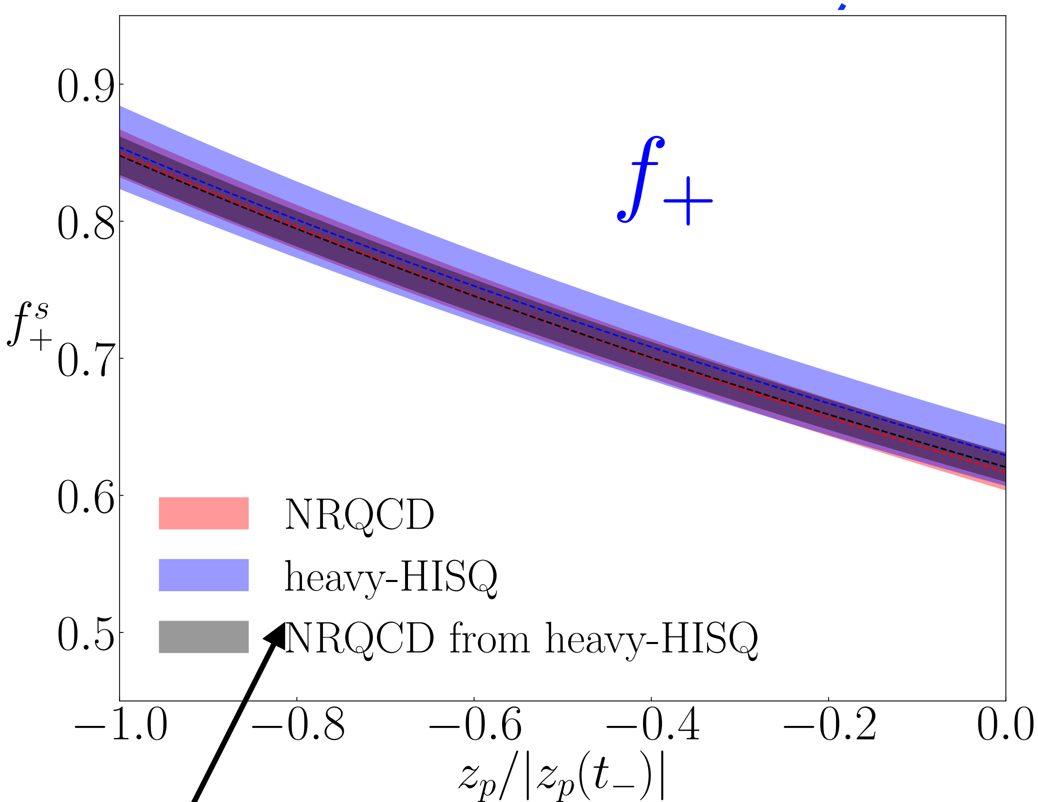
Charm form factors $B_c \to B_s \ell\nu$

on LHCb list for near future

FIRST lattice QCD calculation



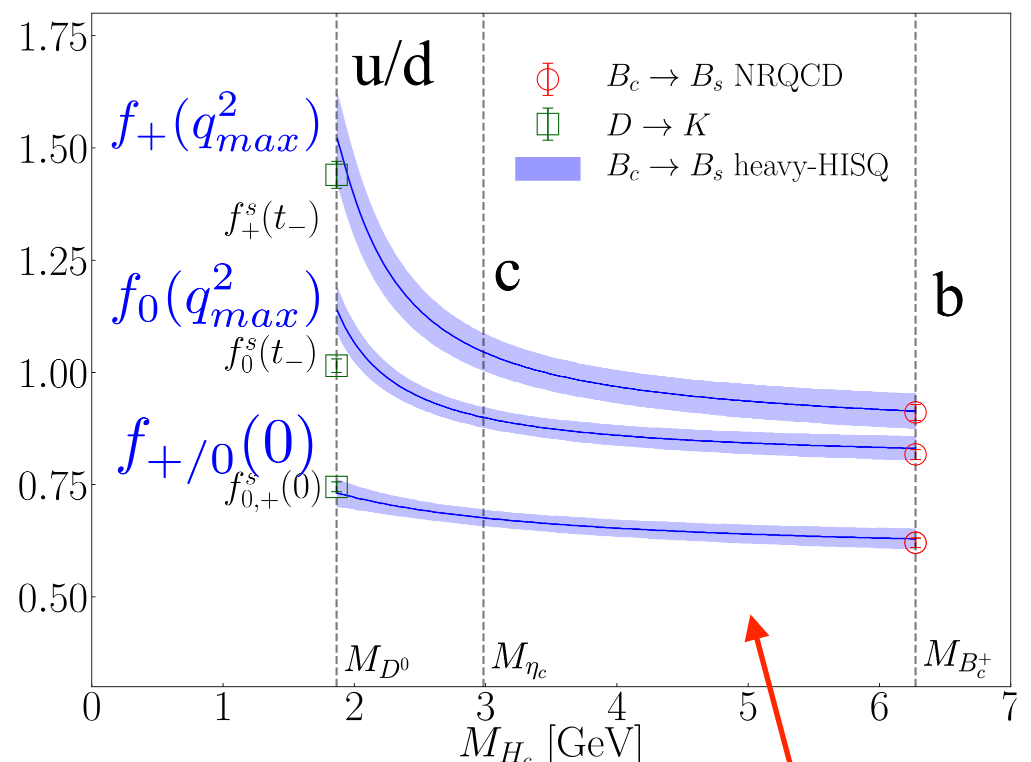
now spectator is b. Compare NRQCD b and HISQ 'b' (multiple m_h) - good agreement at b. Map out dependence on spectator mass



heavy HISQ and NRQCD b agree

~2% accuracy in f

L. Cooper et al, HPQCD, 2003.00914

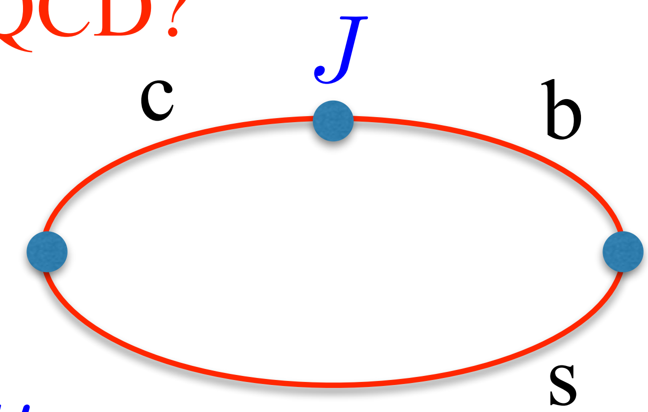


spectator mass dependence is mild

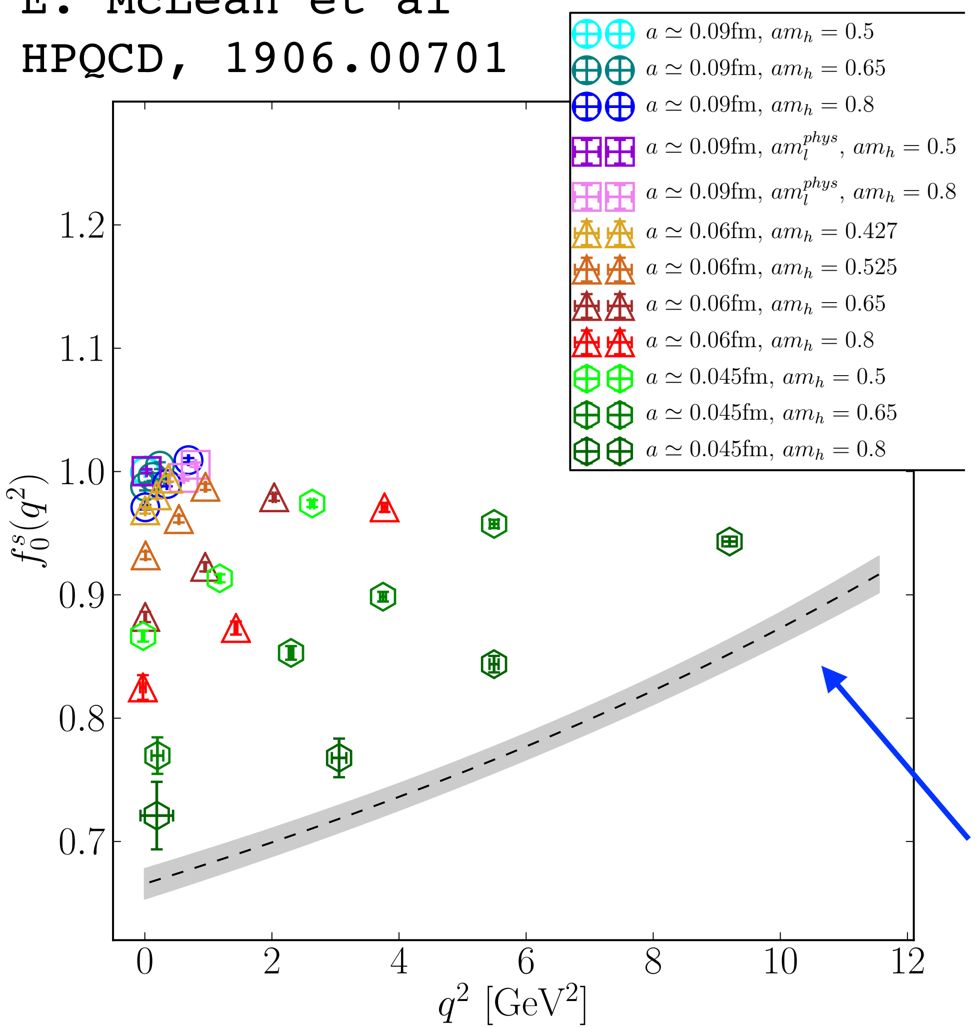
B meson form factors from b quark decay - much bigger q^2 range. How to cover this range in lattice QCD?

$$|\vec{p}_X|_{q^2=0} = \frac{M_B}{2} - \frac{M_X^2}{2M_B}$$

Large momenta need fine lattices



E. McLean et al
HPQCD, 1906.00701



$B_s \to D_s \ell \nu$ for Vcb

Use HISQ with multiple heavy quark masses ($am_h < 1$) for a range of fine lattice spacings.

full q^2 range is covered - range grows with m_h as a^{-1} grows

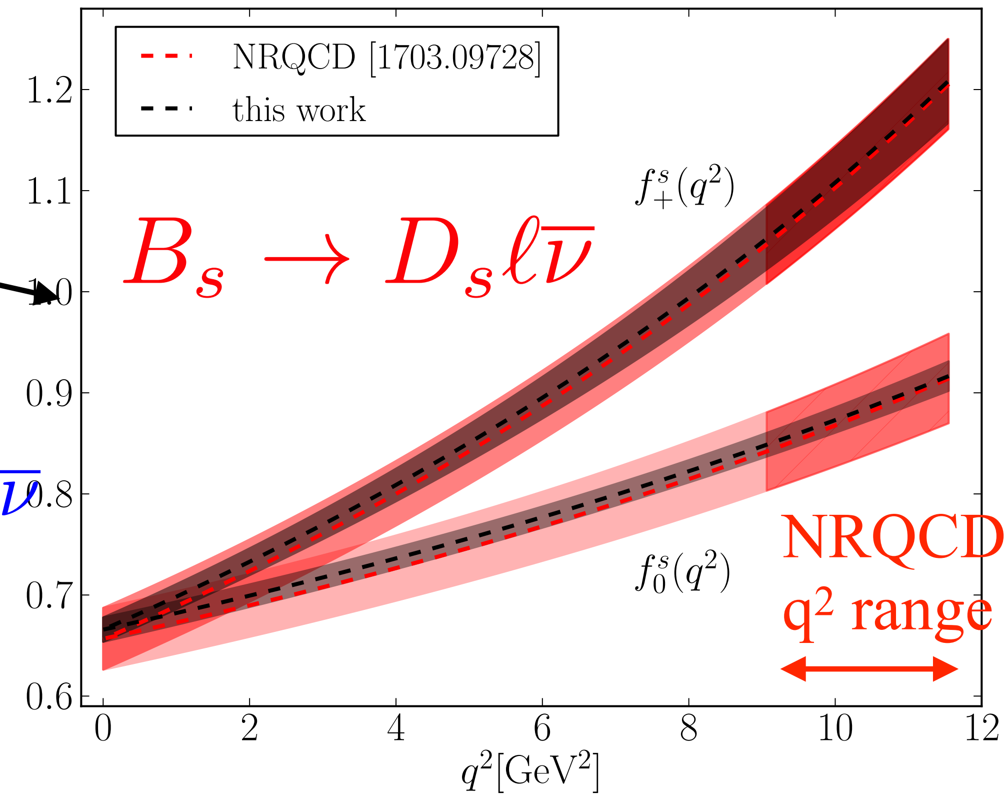
Advantage: accurate normln J

Fit $f(m_h, a, z)$ and extract physical curve at $m_h = m_b$.

E. McLean et al HPQCD, 1906.00701

Bottom form factors

'Heavy-HISQ' result more accurate than non-relativistic methods with J-normln uncty and q^2 close to zero-recoil.



LHCb 2001.03225 $B_s \to D_s^{(*)} \mu \overline{\nu}$

$|V_{cb}|_{\mathrm{CLN}} = 41.4(1.6) \times 10^{-3}$

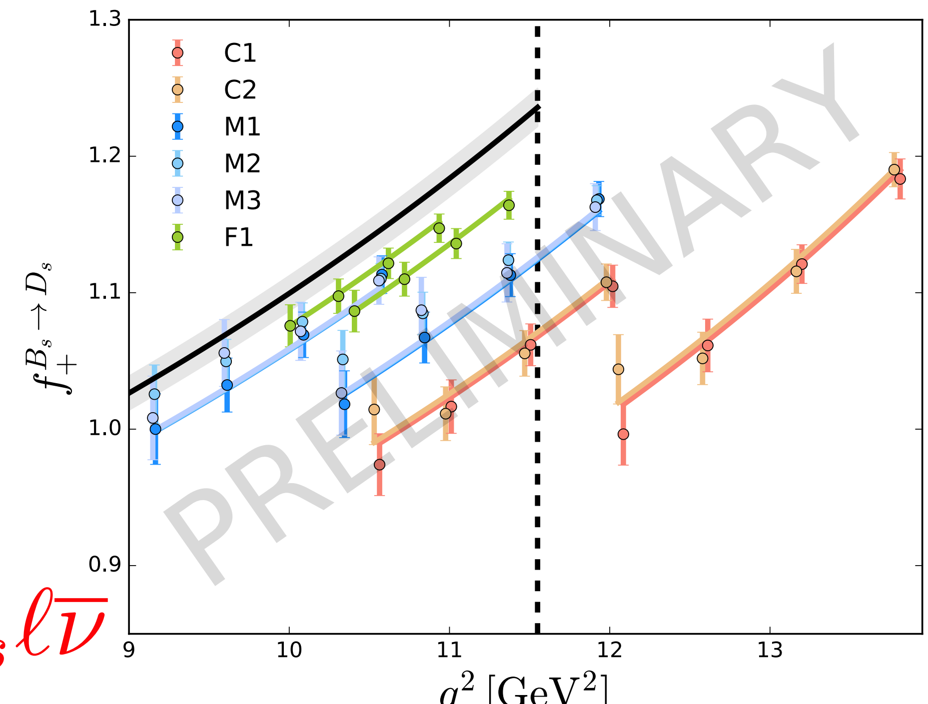
$|V_{cb}|_{\mathrm{BGL}} = 42.3(1.7) \times 10^{-3}$

agree with inclusive $b \to c$

Under way - RBC/UKQCD calculation with RHQ b and c quarks and domain-wall light quarks.

Tsang talk APLAT2020

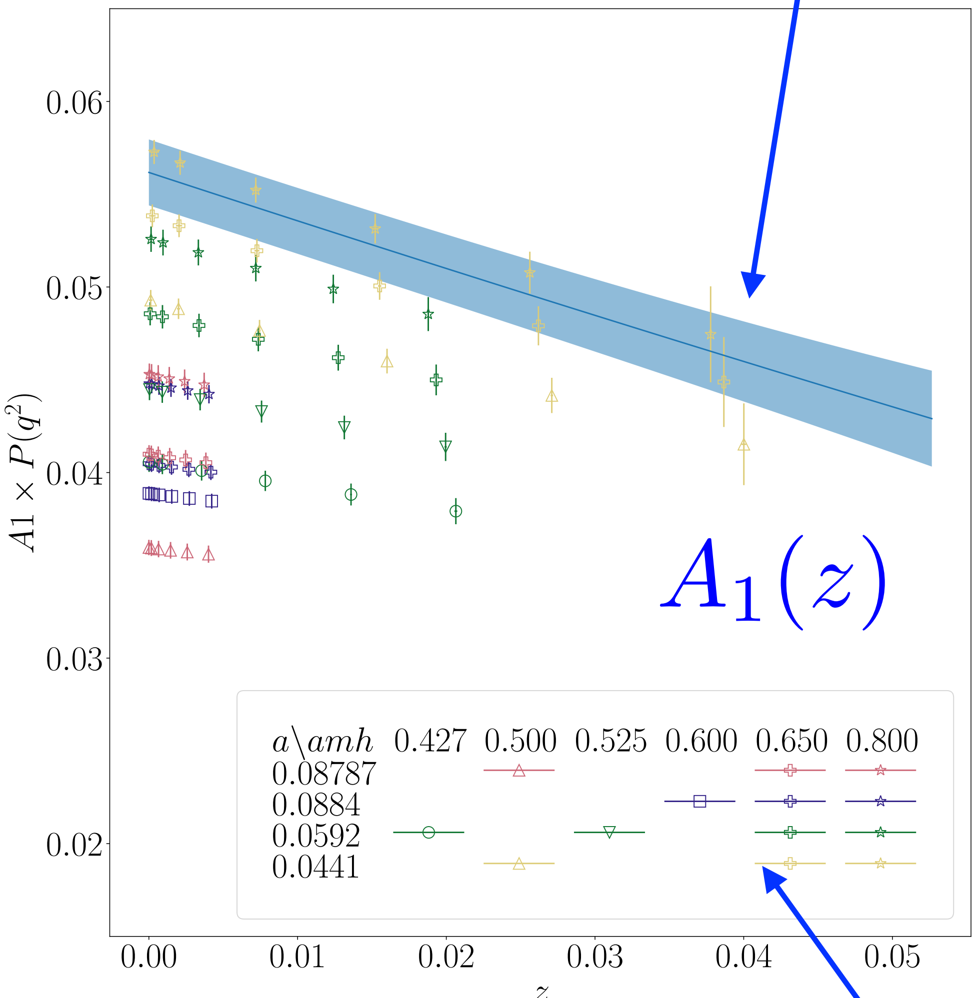
$B_s \to D_s \ell \overline{\nu}$

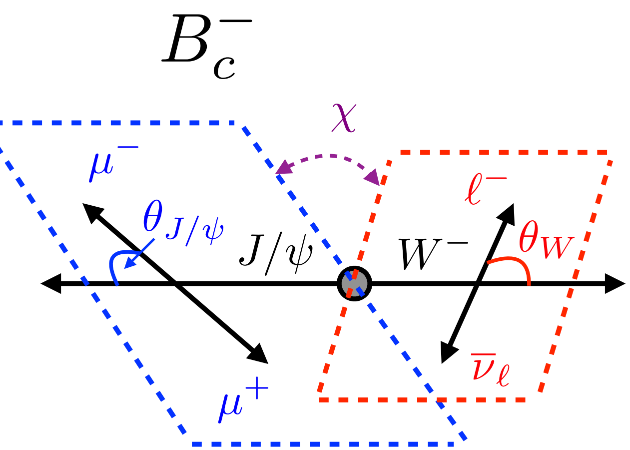


Bottom form factors $B_c \to J/\psi \ell \overline{\nu}$ *FIRST* lattice QCD calculation

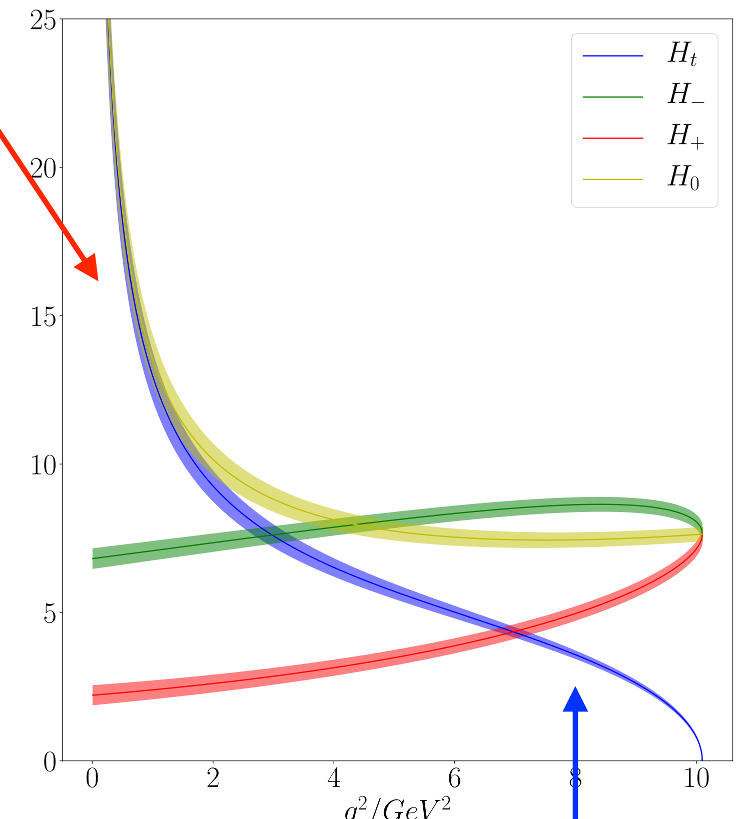
J. Harrison et al, HPQCD, 2007.06956, 2007.06957

Continuum result at b mass





Resulting helicity amplitudes combining all form factors



Contributes for τ

Heavy-HISQ approach covers full q^2 range - expands with mass range on finer lattices.
Fit in z-space - shows simple behaviour

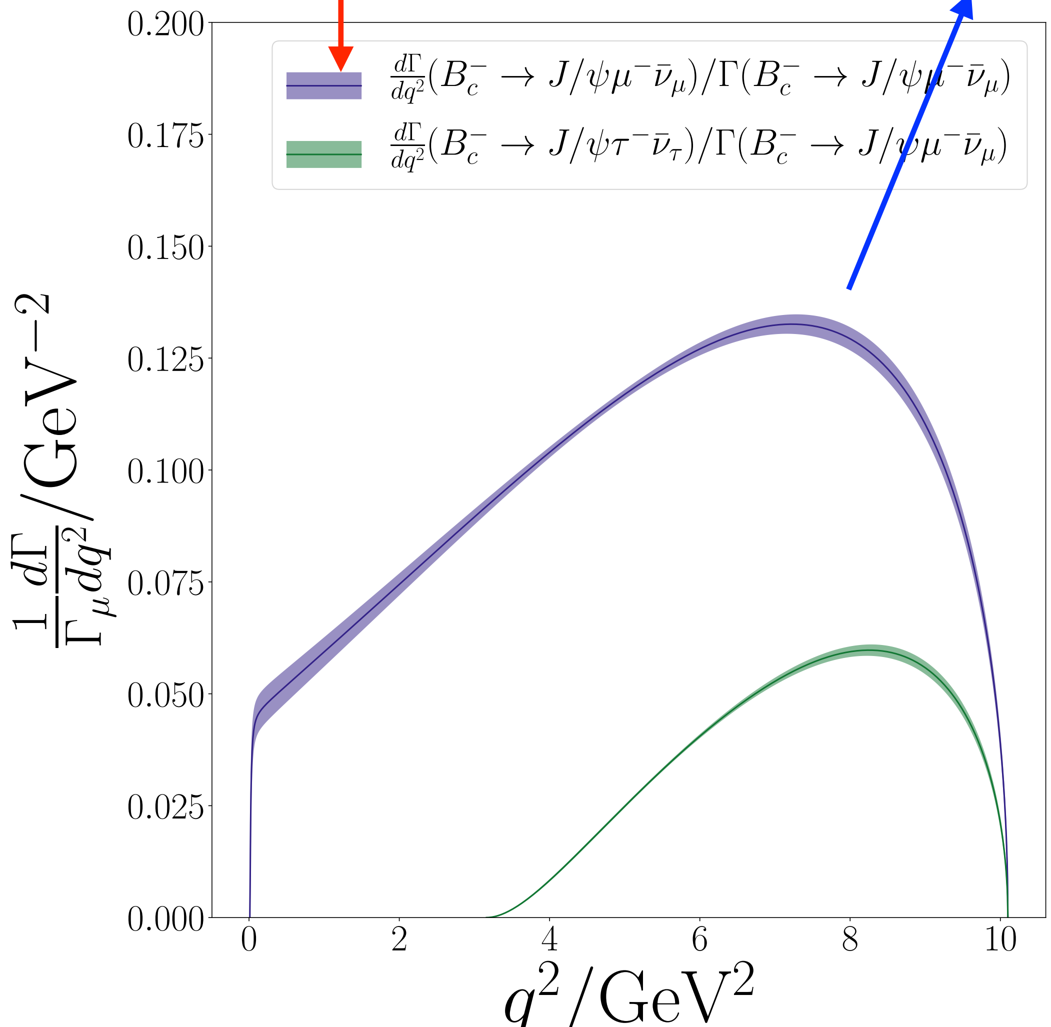
$B_c \to J/\psi \ell \overline{\nu}$

diffl. distns. for (massless) μ or (massive) τ

$$\frac{\Gamma(B_c \to J/\psi \mu \overline{\nu})}{|\eta_{EW} V_{cb}|^2} = 1.74(12) \times 10^{13} \mathrm{s}^{-1}$$

LQCD Vcb τ

$$\mathrm{Br}(B_c \to J/\psi \mu \overline{\nu}) = 0.0151(10)(10)(3)$$



J. Harrison et al, HPQCD, 2007.06956, 2007.06957

lepton universality test

$$R(J/\psi) = \frac{\mathrm{Br}(B_c \to J/\psi\, \tau \overline{\nu}_\tau)}{\mathrm{Br}(B_c \to J/\psi\, \mu \overline{\nu}_\mu)}$$

We find, in SM:

$$R(J/\psi) = 0.2601(36)$$

1.4%

close to SM value for R(D*)

$$R(D^*)|_{\mathrm{SM}} = 0.258(5)$$ HFLAV, 1909.12524

TENSION

$$R(D^*)|_{\mathrm{expt.av.}} = 0.295(14)$$

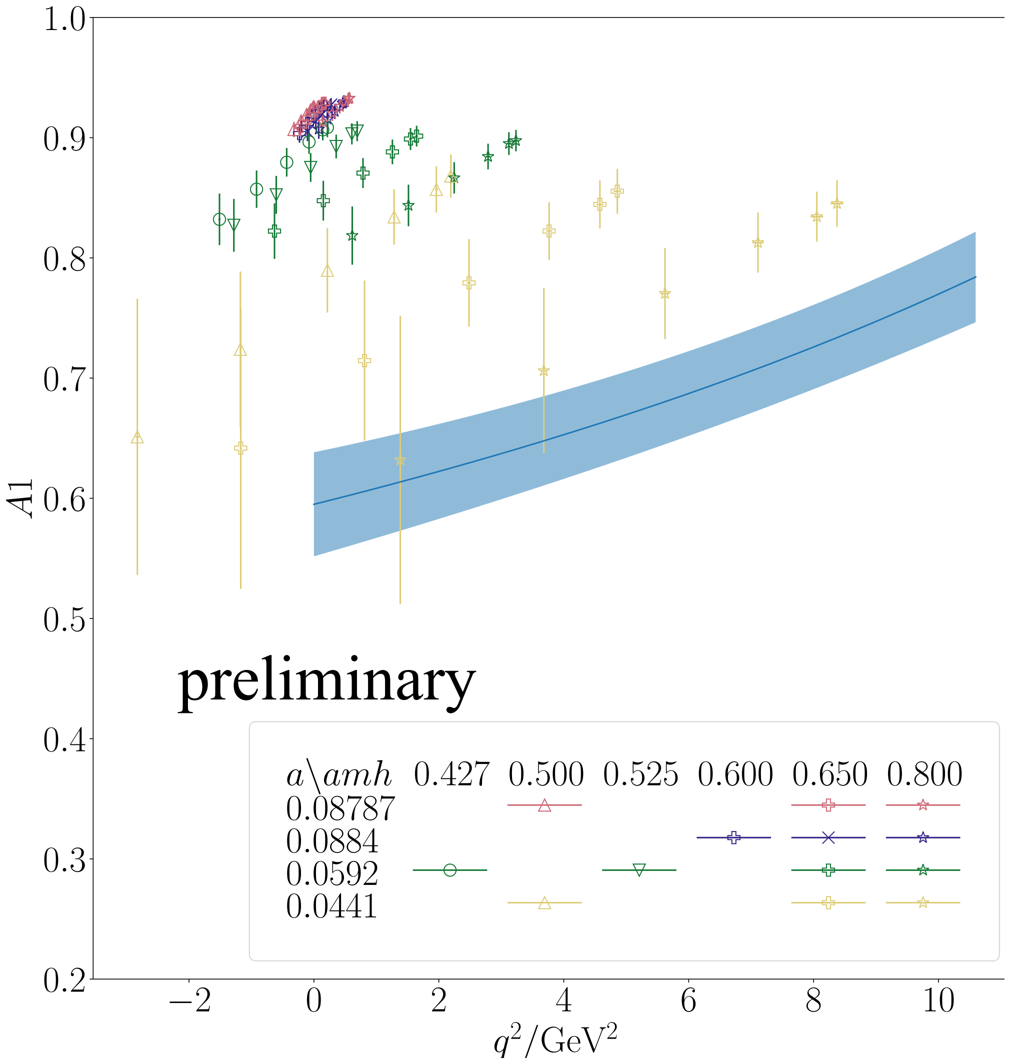
First LHCb result : Uncty from ff.

$$R(J/\psi) = 0.71(17)(18)$$

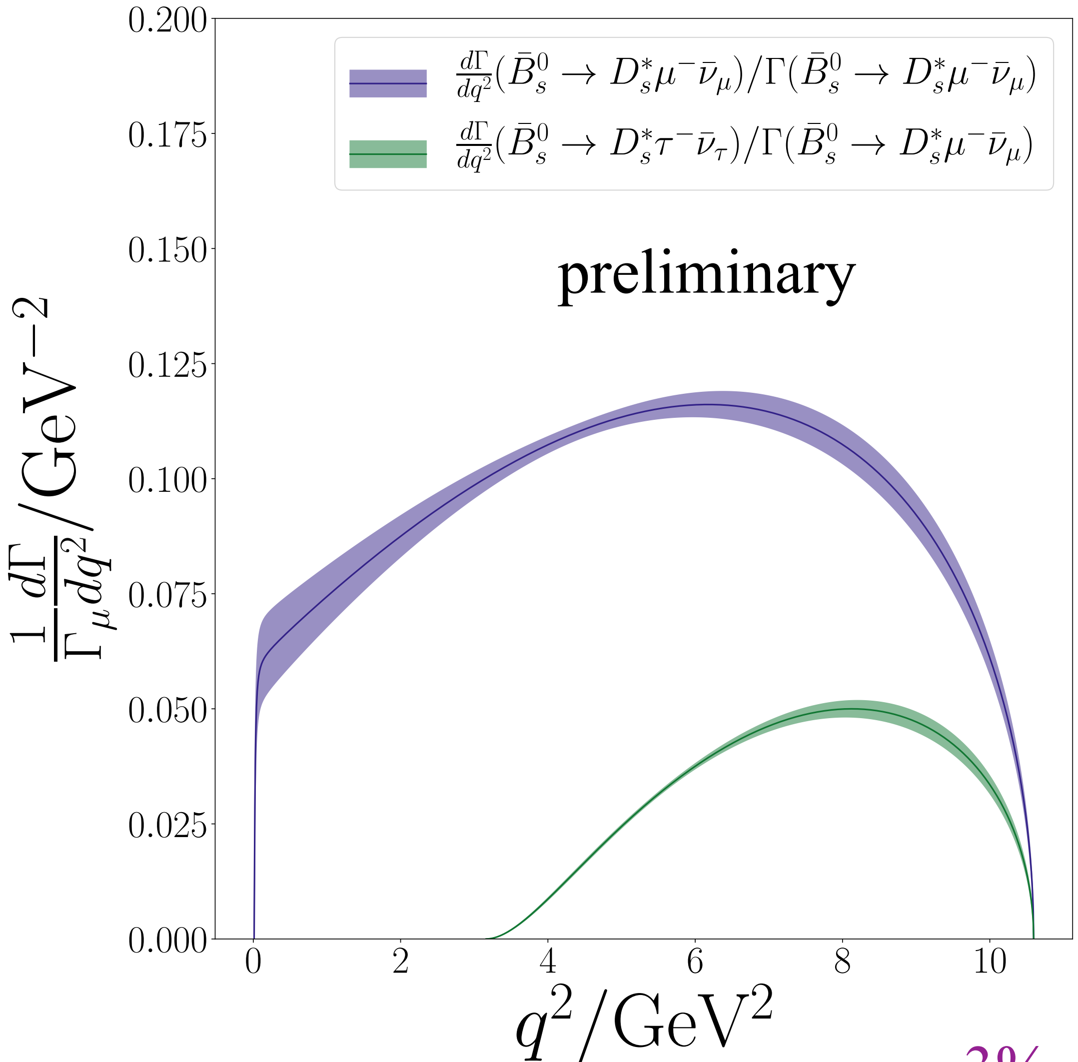
LHCb, 1711.05623

Bottom form factors $B_s \to D_s^* \ell \overline{\nu}$

J. Harrison et al,
HPQCD, in progress



Similar analysis to that for $B_c \to J/\psi \ell \overline{\nu}$



3%

$$R(D_s^*) = 0.246(7)$$

preliminary

Will also give total rate and branching fractions …..

Bottom form factors $B \to D^* \ell \overline{\nu}$ *away* from zero-recoil

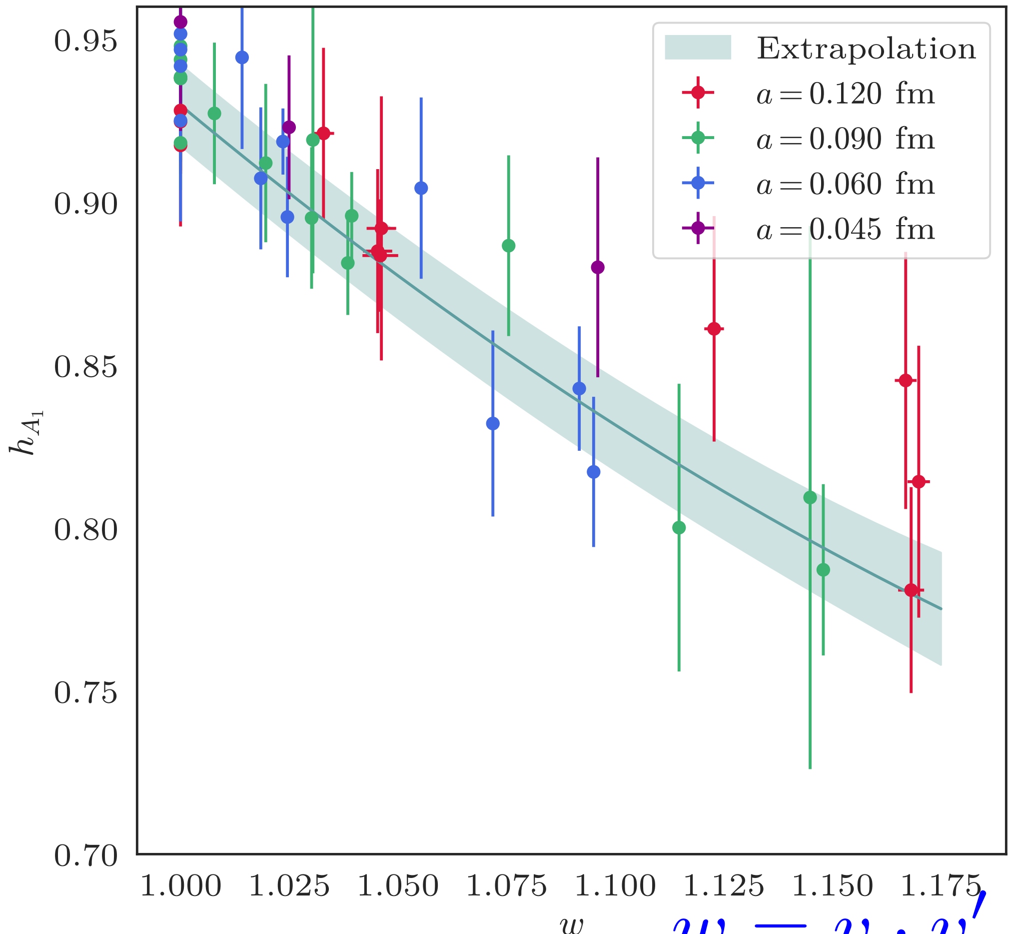
Work under way - FNAL/MILC using FNAL b and c quarks and asqtad light quarks, n_f=2+1. Analysis blinded

See FNAL/MILC 1912.05886

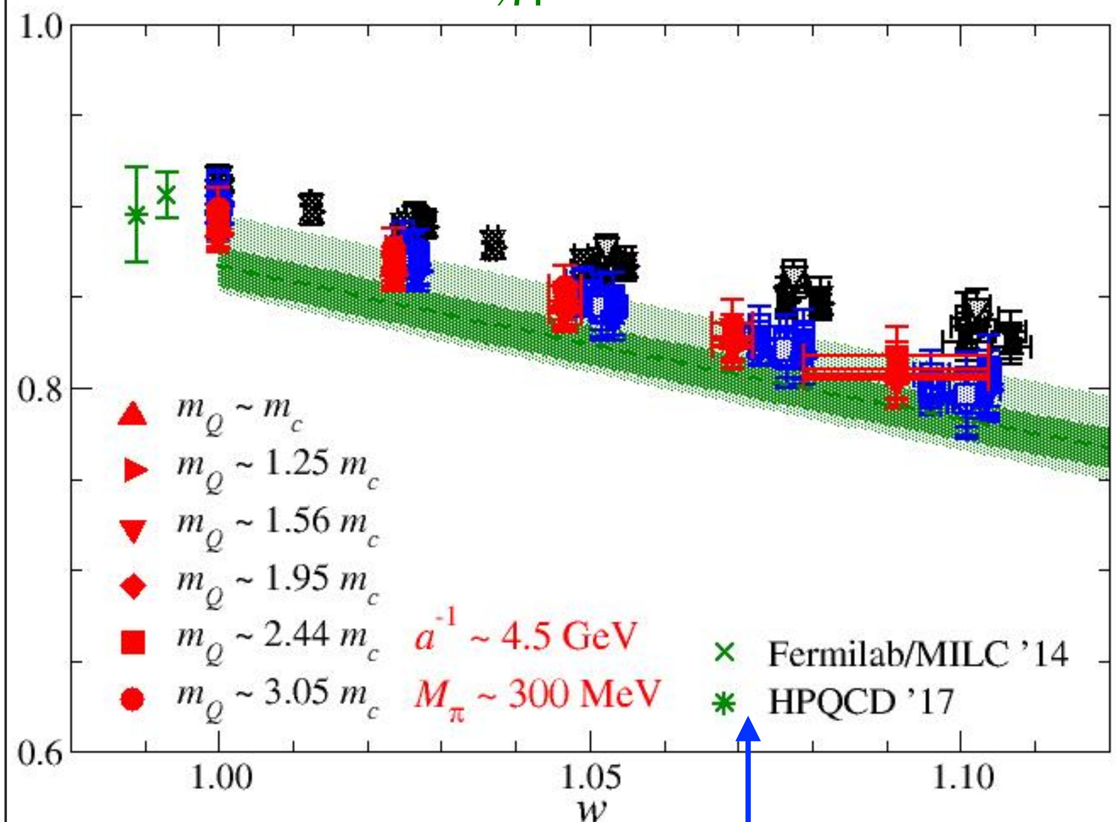
Work under way - JLQCD using Mobius domain-wall quarks, n_f=2+1.

See JLQCD 1912.11770 + Taneko (APLAT20)

h_{A1} vs w

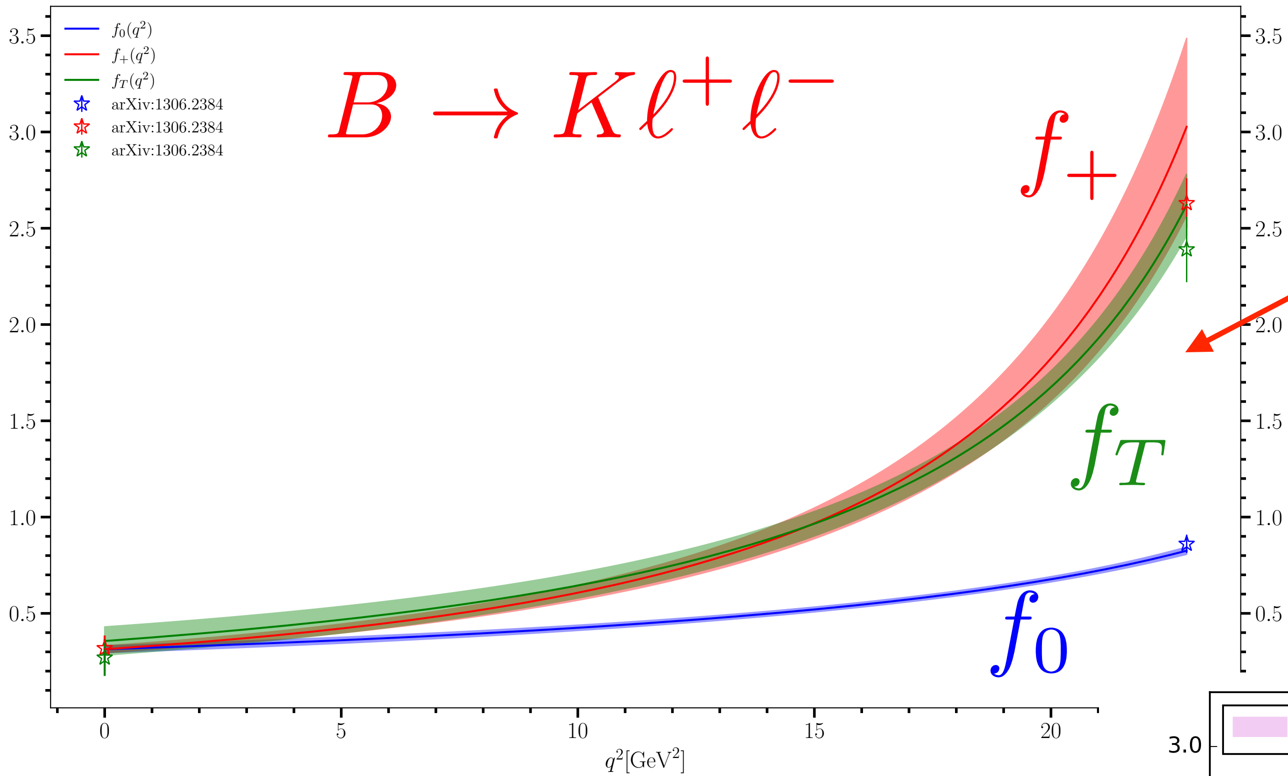


$w = v \cdot v'$



est. errors : 1-2% stat, 1-3% syst.

Bottom form factors b to light decay

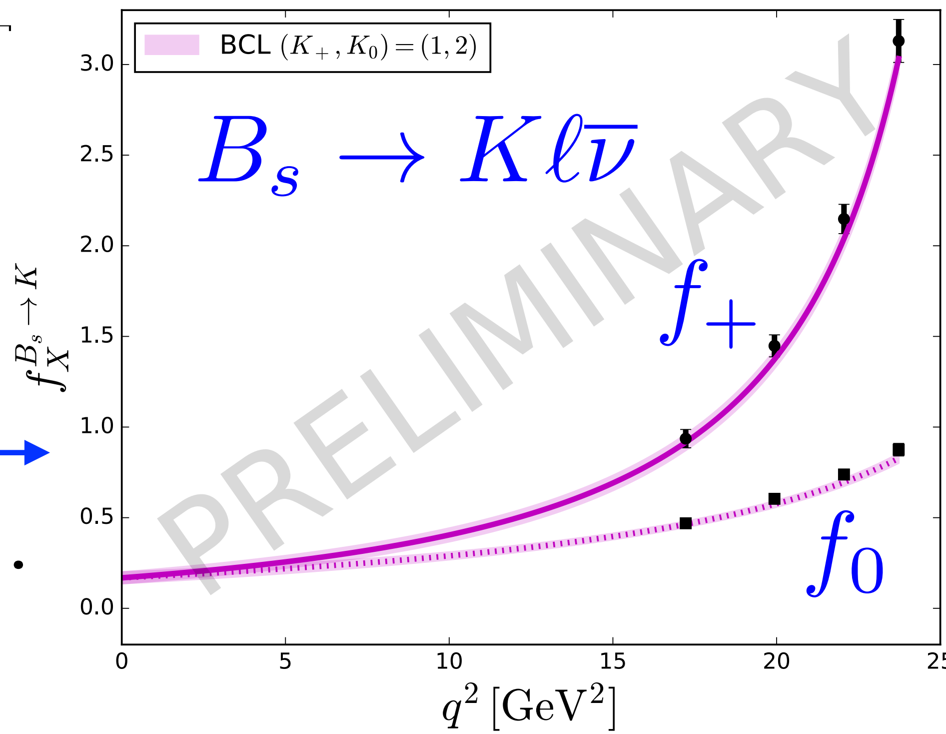


Work under way - HPQCD, heavy-HISQ. Tensor current normln. accurately using RI-SMOM + analysis of non-pert. effects HPQCD, 2008.02024

W. Parrott, APLAT20.

Work under way - RBC/ UKQCD calc. with RHQ b quark and domain-wall light quarks, n_f=2+1.

J. Tsang, APLAT20.



Also work on B to π ,

e.g. JLQCD 1912.02409

Conclusion

• 0.6% uncertainty on m_c, now includes effect of QED. <0.5% uncertainty on m_b - inclusion of QED underway.

• Lots of work on b and c form factors underway. Use of relativistic approach for b on fine lattices improves current normalisation and coverage of q^2 range.

• First $B_c \to J/\psi \ell \nu$ lattice calculation has 7% accuracy for total rate, 1.4% in R(J/ψ).

$B_s \to D_s^* \ell \nu$ under way.

• Results for $B \to D^* \ell \nu$ away from zero recoil available soon

• Improved calculations available soon for $D \to K \ell \nu$ $B \to K \ell^+ \ell^-$