CP Violation and Rare Decays in the Kaon System

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Based on 1911.06822 work with J. Brod, E. Stamou

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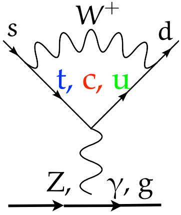
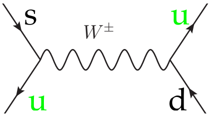
This talk

- CP violation and Flavour structure in the Kaon System
- ϵ_K
- ϵ'/ϵ
- $K \to \pi\bar{\nu}\nu$

~~CP~~ & Rare Kaon Decays: CKM Structure



Using the GIM mechanism, we can eliminate either $V_{cs}^* V_{cd}$ or $V_{us}^* V_{ud} \rightarrow - V_{cs}^* V_{cd} - V_{ts}^* V_{td}$



Z-Penguin and Boxes (high virtuality): power expansion in: $A_c - A_u \propto 0 + O(m_c^2/M_W^2)$

γ/g-Penguin (expand in mom.): $A_c - A_u \propto O(\mathrm{Log}(m_c^2/m_u^2))$

$$\mathrm{Im} V_{ts}^* V_{td} = -\mathrm{Im} V_{cs}^* V_{cd} = \mathcal{O}(\lambda^5) \qquad \mathrm{Im} V_{us}^* V_{ud} = 0$$

$$\mathrm{Re} V_{us}^* V_{ud} = -\mathrm{Re} V_{cs}^* V_{cd} = \mathcal{O}(\lambda^1) \qquad \mathrm{Re} V_{ts}^* V_{td} = \mathcal{O}(\lambda^5)$$

- CP Violation in Decay: $\mathrm{Im}A_K/\mathrm{Re}A_K = \lambda^4 \cdot \mathrm{loop}$
- $K \rightarrow \pi\bar{\nu}\nu$ (from Z & Boxes): Clean and suppressed
- Lattice input needed for all other decays

Meson-antimeson mixing

Restricting to $\{|K^0\rangle, |\overline{K}^0\rangle\}$, the time evolution is given by

$$\hat{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{11} \end{pmatrix} = \begin{pmatrix} \langle K^0|T|K^0\rangle & \langle K^0|T|\overline{K}^0\rangle \\ \langle \overline{K}^0|T|K^0\rangle & \langle \overline{K}^0|T|\overline{K}^0\rangle \end{pmatrix} = \hat{M} - \frac{i}{2}\hat{\Gamma}$$

- QCD $\rightarrow H_{11} = H_{22}$
- weak $\Delta F = 2$ interactions $\rightarrow H_{12}$ and H_{21}.

Eigenvectors $K_S = pK^0 + \bar{K}^0$ and $K_S = pK^0 - \bar{K}^0$.

Define: $\lambda_I = \dfrac{q}{p}\dfrac{\bar{A}_I}{A_I}$ for (isospin-)final state.

CP violation in $K \to \pi\pi$

- Experimental definition using $\eta_{ij} = \dfrac{\langle \pi^i \pi^j | K_L \rangle}{\langle \pi^i \pi^j | K_S \rangle}$

$$\epsilon_K = (2\eta_{+-} + \eta_{00})/3 \, , \qquad \epsilon' = (\eta_{+-} - \eta_{00})/3$$

- ϵ_K theory expression $\epsilon_K \simeq \dfrac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} =$

$$e^{i\phi_\epsilon} \sin\phi_\epsilon \frac{1}{2} \arg\left(\frac{-M_{12}}{\Gamma_{12}} \right) = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})^{Dis}}{\Delta M_K} + \xi \right)$$

$$\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle \to \mathrm{Im}(M_{12})^{Dis} \, , \; \frac{\mathrm{Im}\langle (\pi\pi)_{I=0} | K^0 \rangle}{\mathrm{Re}\langle (\pi\pi)_{I=0} | K^0 \rangle} \to \xi \;\; \phi_\epsilon \equiv \arctan \frac{\Delta M_K}{\Delta\Gamma_K / 2}$$

- ϵ' theory expression $\epsilon' \simeq \dfrac{1}{6} \left(\lambda_{00} - \lambda_{+-} \right)$

$$\epsilon_K$$

$\mathrm{Im}(M_{12})$

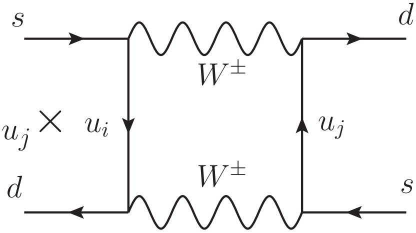
- We can factorise perturbatively calculated
  - short distance contributions at $\mu_t = m_t$,
  - from long distance effects calculated on Lattice

$$\langle H_{\text{eff}} \rangle = \langle Q^{|\Delta S=2|} \rangle(\mu_{\text{had}}) \quad U(\mu_{\text{had}}, \mu_c) \quad U(\mu_c, \mu_W) \quad C(\mu_W)$$

- factorising $U(\mu_{\text{had}}, \mu_c) = u^{-1}(\mu_{\text{had}}) u(\mu_c)$ we write:
- $\frac{2}{3} f_K^2 M_K^2 \hat{B}_K = \langle \bar{K}^0 | Q^{|\Delta S=2|} | K^0 \rangle u^{-1}(\mu_{\text{had}})$
- $\eta_{ij} S(x_i, x_j) = u(\mu_c) U(\mu_c, \mu_W) C(\mu_W)$
  is the short distance contribution
- $Q_{S2} = (\bar{s}_L \gamma_\mu d_L) \otimes (\bar{s}_L \gamma^\mu d_L)$

Kaon Mixing: CKM Structure

$$\sum_{u_i,u_j}^{u,c,t} \lambda_{u_i}\lambda_{u_j} \times$$



We define $\lambda_i = V_{id}V_{is}^*$

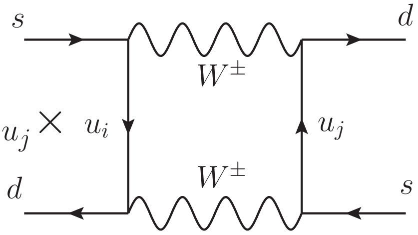
- Using $\lambda_u = -\lambda_c - \lambda_t$ we have

$$\begin{aligned} A &= \lambda_t^2(A_{tt} - 2A_{tu} + A_{uu}) + \\ &\quad 2\lambda_t\lambda_c(A_{tc} - A_{tu} + A_{uu} - A_{cu}) \\ &\quad \lambda_c^2(A_{uu} - 2A_{cu} + A_{cc}) \end{aligned}$$

- One could eliminate $\lambda_c = -\lambda_u - \lambda_t$.

Kaon Mixing: CKM Structure II

$$\sum_{u_i,u_j}^{u,c,t} \lambda_{u_i}\lambda_{u_j} \times$$



	Im	Re	$\mathcal{O}$
λ_t^2	$\sim \lambda^{10}$	$\sim \lambda^{10}$	m_t^2/M_W^2
$\lambda_c\ \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$	$m_c^2/M_W^2 \ln(m_t/m_c)$
λ_c^2	$\sim \lambda^6$	$\sim \lambda^2$	m_c^2/M_W^2
$\lambda_u\ \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$	$m_c^2/M_W^2 \ln(m_t/m_c)$
λ_u^2	0	$\sim \lambda^2$	m_c^2/M_W^2

Where $\lambda_i = V_{id}V_{is}^*$, $\lambda \equiv |V_{us}| \sim 0.2$ and we eliminated either: $\lambda_u = -\lambda_c - \lambda_t$ or $\lambda_c = -\lambda_u - \lambda_t$.

$\Delta S = 2$ Hamiltonian - Phase (In)Dependence

- Recall $\epsilon_K \propto \arg(-M_{12}/\Gamma_{12})$
- Trick: pull out λ_u^* and $(\lambda_u^*)^2$ from $H^{\Delta S=1}$ and $H^{\Delta S=2}$:
- Rephaseing invariant: $\lambda_i \lambda_j^* = V_{id} V_{is}^* V_{jd}^* V_{js}$
- $\Gamma_{12} \simeq A_0^* \bar{A}_0$ where $A_0 = \langle (\pi\pi)_{I=0} | K^0 \rangle$

$$\mathcal{H}_{f=3}^{\Delta S=2} = \frac{G_F^2 M_W^2}{4\pi^2 (\lambda_u^*)^2} Q_{S2} \Big\{ f_1 C_1(\mu) + iJ \left[f_2 C_2(\mu) + f_3 C_3(\mu) \right] \Big\} + \text{h.c.}$$

- $J = \mathrm{Im}(V_{us} V_{cb} V_{ub}^* V_{cs}^*)$, f_1, f_2 and f_3 are rephasing invariant
- Real part $f_1 = |\lambda_u|^4$ is unique
- Splitting of f_2 and f_3 not, but expect good convergence for C_2 and C_3.

Traditional Form

Traditionally the effective Hamiltonian is written as:

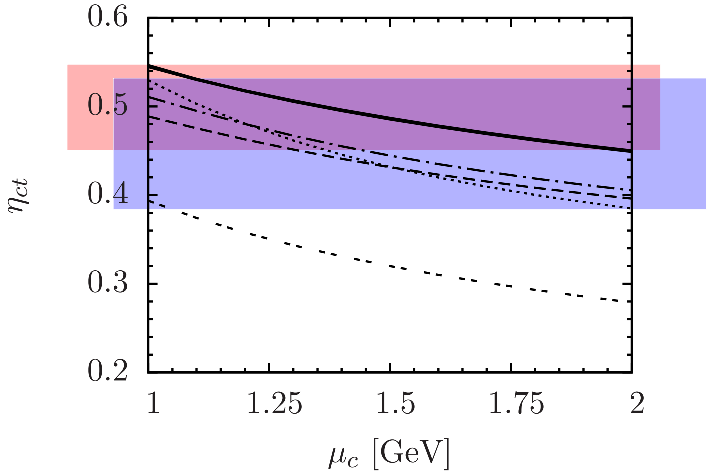
$$\mathcal{H}_{f=3}^{\Delta=2} = \frac{G_F^2 M_W^2}{4\pi^2}\Big[\lambda_c^2 C_{S2}^{cc}(\mu)+\lambda_t^2 C_{S2}^{tt}(\mu)+\lambda_c\lambda_t C_{S2}^{ct}(\mu)\Big]Q_{S2}+\text{h.c.}$$

where $f_2 = 2\mathrm{Re}(\lambda_t\lambda_u^*)$, $f_3 = |\lambda_u|^2$ and, using PDG convention and CKM unitarity,

$$C_{S2}^{cc} \equiv \mathcal{C}_1, \quad C_{S2}^{ct} \equiv 2\mathcal{C}_1 - \mathcal{C}_3, \quad C_{S2}^{tt} \equiv \mathcal{C}_1 + \mathcal{C}_2 - \mathcal{C}_3$$

- $\mathcal{C}_1 \leftarrow A_{uu} - 2A_{cu} + A_{cc}$ has bad short distance behaviour
- $\mathcal{C}_1$ determines ΔM_K via ReM_{12}
- But $\mathcal{C}_1$ contributes to ImM_{12} and hence ϵ_K

Residual scale dependence



- QCD corrections to $C_{S2}^{ct} \to \eta_{ct} = 0.497(47)$
- QCD corrections to $C_{S2}^{cc} \to \eta_{cc} = 1.87(76)$

$\mathrm{Im}M_{12}$ without ΔM_K pollution

- Using CKM unitarity and the PDG convention we can also write (as used in Lattice [Christ et.al.]):

$$\mathcal{H}_{f=3}^{\Delta=2} = \frac{G_F^2 M_W^2}{4\pi^2}\left[\lambda_u^2 C_{S2}^{uu}(\mu)+\lambda_t^2 C_{S2}^{tt}(\mu)+\lambda_u\lambda_t C_{S2}^{ut}(\mu)\right]Q_{S2}+\mathrm{h.c.}$$

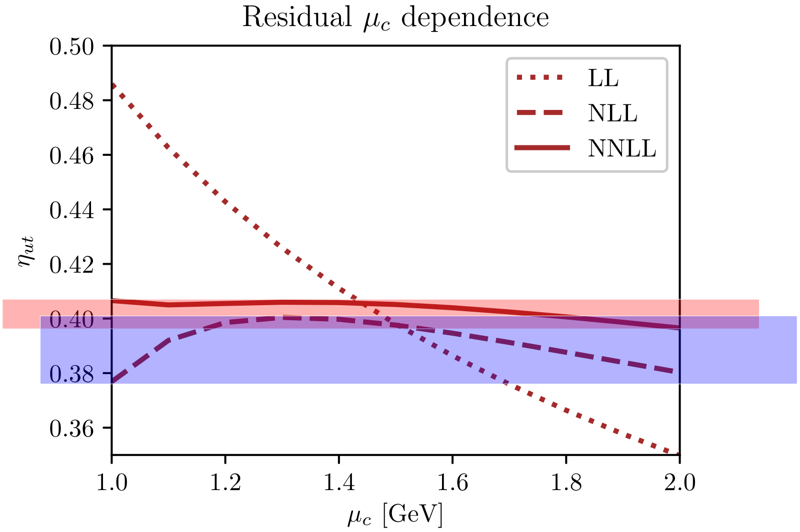
- Now real $\mathrm{Re}M_{12}$ and $\mathrm{Im}M_{12}$ are disentangled

$$C_{S2}^{uu} \equiv \mathcal{C}_1, \quad C_{S2}^{tt} \equiv \mathcal{C}_2, \quad C_{S2}^{ut} \equiv \mathcal{C}_3$$

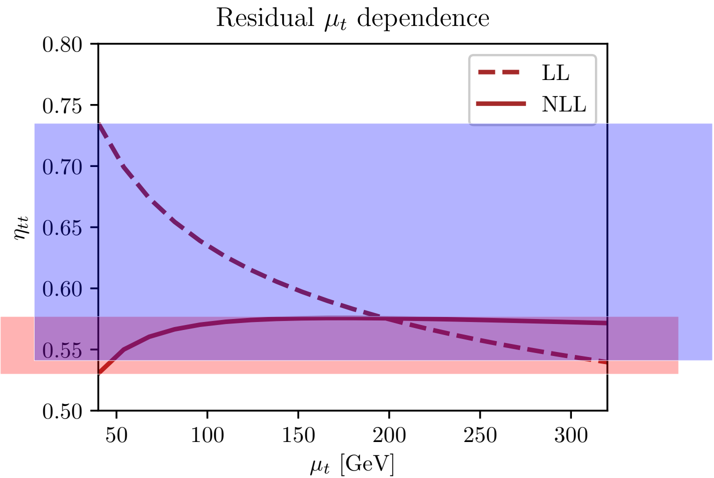
$$\begin{aligned} \mathcal{C}_3 &\leftarrow (A_{tu} - A_{tc} + A_{cc} - A_{cu}) \leftarrow \\ &\leftarrow (A_{uu} - 2A_{cu} + A_{cc}) - (A_{tc} - A_{tu} + A_{uu} - A_{cu}) \end{aligned}$$

- Extract anomalous dimensions and matching from old calculation and incorporate matching from η_{cc}

Residual scale dependence



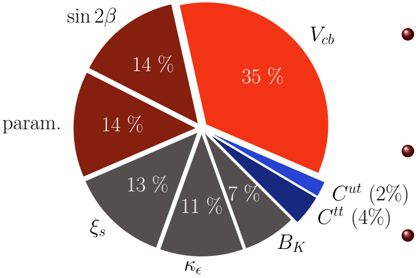
The top-quark: good convergence



Can be improved with NNLO calculation

SM prediction using PDG input

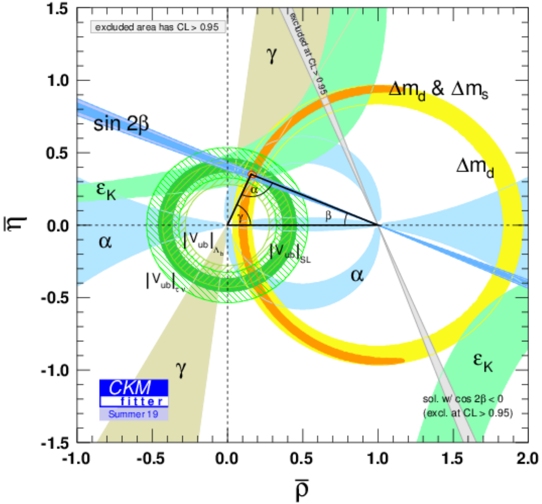
$$|\epsilon_K| = \kappa_\epsilon C_\epsilon \widehat{B}_K |V_{cb}|^2 \lambda^2 \bar{\eta} \times \left[|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt}(x_t) - \eta_{ut}(x_c, x_t) \right]$$



- $\hat{B}_K = 0.7625(97)$
  [FLAG 2019, 1902.08191]
- $|\epsilon_K^{\mathrm{SM}}| = 2.16(18) \times 10^{-3}$
- $|\epsilon_K^{\mathrm{exp}}| = 2.228(11) \times 10^{-3}$

CKMfitter 2019 update

Incorporating new formalism shows reduced uncertainty, but $\bar{\rho}$ and $\bar{\eta}$ not the (only) dominant CKM factors.



$$\epsilon'/\epsilon$$

ϵ'/ϵ: Isospin limit and breaking

$\epsilon' \simeq (\lambda_{00} - \lambda_{+-})/6$ in terms of charged pion final states.

a_0, a_2 & a_2^+ from experiment
[Cirigliano, et.al. `11]

a_0 & a_2: isospin amplitudes for isospin conservation

$$\langle \pi^0\pi^0|K^0\rangle = a_0 e^{i\chi_0} + a_2 e^{i\chi_2}/\sqrt{2}$$

$$\langle \pi^+\pi^-|K^0\rangle = a_0 e^{i\chi_0} - a_2 e^{i\chi_2}\sqrt{2}$$

$$\langle \pi^+\pi^0|K^+\rangle = 3a_2^+ e^{i\chi_2^+}/2$$

Current theory gives us only: $A_I = \langle(\pi\pi)_I|\mathcal{H}_{\rm eff}|K\rangle$

Normalise to K^+ decay (ω_+, a) and ε_K,
expand in A_2/A_0 and CP violation:

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) \simeq \frac{\epsilon'}{\epsilon} = -\frac{\omega_+}{\sqrt{2}\,|\epsilon_K|}\left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\left(1-\hat{\Omega}_{\rm eff}\right) - \frac{1}{a}\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right]$$

Adjusted to keep electroweak penguins in Im A_0 [Cirigliano, et.al. `11]

[Buras, Gerard 2005.08976]
Analysis of isospin breaking,
finds 40% reduction wrt RBC-QCD

Effective Hamiltonian for $N_f = 3$

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} \big(z_i(\mu) + \tau\, y_i(\mu) \big)\, Q_i(\mu)\,, \quad \tau \equiv -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$

current-current	$Q_{1,2/\pm} = (\bar{s}_i u_j)_{V-A}\, (\bar{u}_k d_l)_{V-A}$
QCD &	$Q_{3,\ldots,6} = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s} (\bar{q}_k q_l)_{V\pm A}$
electroweak penguins	$Q_{7,\ldots,10} = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_k q_l)_{V\pm A}$

C1-C6 @ NNLO [1611.08276] ; C7-C10 Partial NNLO [hep-ph/9911250]

- Fierz identities: $Q_4 = Q_3 + Q_2 - Q_1$, $Q_9 = 3/2 Q_1 - Q_3$ and $Q_{10} = Q_2 + Q_1 - Q_3$ plus Isospin: $\langle Q_{3/4} \rangle_2 = 0$

$\rightarrow$ Some matrix elements cancel in $\mathrm{Im}A_0/\mathrm{Re}A_0$ and $\mathrm{Im}A_2/\mathrm{Re}A_2$. [1507.06345]

Lattice calculations

- RBC UKQCD calculation [1502.00263] of A_2
- RBC UKQCD calculation [2004.09440] of A_0

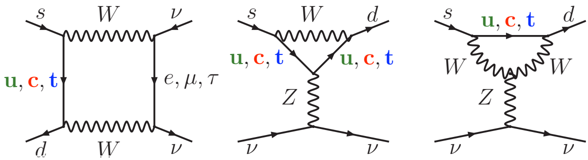
$$\mathrm{Re}(\epsilon'/\epsilon)_{RBCUKQCD} = 21.7(2.6)(6.2)(5.0) \cdot 10^{-4}$$

Uncertainties are statistical, systematic, and iso-spin breaking

$$\mathrm{Re}(\epsilon'/\epsilon)_{experiment} = 16.6(2.3) \cdot 10^{-4}$$

$$K \to \pi \bar{\nu} \nu$$

$K \to \pi\bar{\nu}\nu$ at M_W



$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td}(F(x_t) - F(x_u)) + V_{cs}^* V_{cd}(F(x_c) - F(x_u))$$

Quadratic GIM: $\boxed{\lambda^5 \frac{m_t^2}{M_W^2}}$ $\boxed{\lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}}$ $\boxed{\lambda \frac{\Lambda_{QCD}^2}{M_W^2}}$

Matching (NLO +EW):

Operator Mixing (RGE)

ChiPT & Lattice

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

- Below the charm: Only Q_ν, ME from K_{l3}
- semi-leptonic $(\bar{s}\gamma_\mu u_L)(\bar{\nu}\gamma^\mu \ell_L)$ operator: χ PT gives small contribution (10% of charm contribution)

Expressions for $K \to \pi\bar{\nu}\nu$

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+(1+\Delta_{\mathrm{EM}})\cdot\left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5}X(x_t)\right)^2 + \left(\frac{\mathrm{Re}\lambda_c}{\lambda}P_c(X) + \frac{\mathrm{Re}\lambda_t}{\lambda^5}X(x_t)\right)^2\right]$$

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) = \kappa_L \cdot \left(\frac{\mathrm{Im}\lambda_t}{\lambda^5}X(x_t)\right)^2$$

- $\mathrm{Im}\lambda_t = \eta A^2\lambda^5$, $\mathrm{Re}\lambda_t = \frac{\lambda^2-2}{2}A^2\lambda^2(1-\bar{\rho})$, $\mathrm{Re}\lambda_c = \lambda\frac{\lambda^2-2}{2}$
- $\kappa_+, \kappa_L, \Delta_{EM}$ strong and em iso-spin breaking [0705.2025]
- $\mathrm{P_c} = \mathrm{P_c^{pert.}} + \delta\,\mathrm{P_{c,u}}$ = 0.372(15) + 0.04(2) ← (NNLO + EW) [ph/0603079] [0805.4119] + χ PT & Lattice [ph/0503107] [1806.11520]

Uncertainty Analysis using UTfit values

$\mathcal{B}_+ \cdot 10^{11}$	Central:	8.510	$\mathcal{B}_L \cdot 10^{11}$	Central:	2.858
Error:	-0.543	0.555	Error:	-0.256	0.264
A	-0.34	0.352	A	-0.162	0.17
$\delta P_{c,u}$	-0.246	0.250	η	-0.162	0.167
X_t	-0.236	0.240	X_t	-0.113	0.115
ρ	-0.161	0.162	κ_l	-0.017	0.002
P_c	-0.185	0.187	λ	-0.001	0.00
κ_+	-0.041	0.041			
η	-0.037	0.039			
λ	-0.003	0.003			

- Precise theory prediction, suppression in standard model and current measurement at NA62 & KOTO $\rightarrow$ see talks in this session

CKM input: $A = 0.826(12), \bar{\rho} = 0.148(13), \bar{\eta} = 0.348(10)$

Conclusions

- Perturabtion theory allows for precise theory prediction of $K \to \pi \bar{\nu} \nu$ decay modes and, with remarkable progress from Lattice, in CP violting hadronic decays.
- Observables are highly suppressed in the standard model and their measurement constraints models of new physics.
- We can constrain high energy physics paramters $\to$ next talk
  - See also talk by Ulserik Moldanazarova in the DESY theory forum today on renormalised results for the $\Delta F = 1$ effective Hamiltonian in generic extensions of the Standard Model.