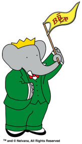
4-dimensional angular analysis of $\overline{B} \to D^{*}\ell^{-}\overline{\nu}_{\ell}$ *with hadronic tagging at* BABAR

[PRL 123, 091801 (2019)]

Biplab Dey

(on behalf of the $BaBar$ Collaboration)

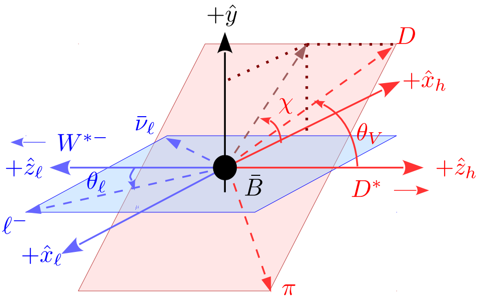


BEAUTY 2020, Japan (virtual)
September 23^{rd}, 2020

Motivation: $\overline{B} \to D^{*}\ell^{-}\overline{\nu}_{\ell}$ angular analysis

- Along with semi-tauonic case, $\ell \in \{e, \mu\}$ hold puzzles as well.
- $|V_{cb}|$ tension, higher order Heavy Quark Effective Theory (HQET) corrections for the form-factors, ...
- Experimentally very clean, and fully exclusive final-state with hadronic tagging.
- With the narrow D^{*} vector meson, excellent system to probe New Physics (NP) on the leptonic side.

The 4-d kinematic variables



- 4-body decay topology
- $\sqrt{q^2}$: di-lepton mass.
- 3 angles: $\Omega \in \{\theta_l, \theta_V, \chi\}$
- Spin-1 D^* retains full spin info of the recoiling W^* in $b \to cW^{*-}$, unlike spin-0 D, where this info is reduced $\Rightarrow$ richer pheno!

- Two complementary approaches employed in the analysis:
  - Angular observables extracted using moments method in 10 q^2 bins.
  - Unbinned z-expansion FF fit to the form-factors (today).
- In either case, we perform a full 4-d analysis.

The generic 4-d pdf [arXiv:1505.02873]

- Differential rate (4-d fit pdf):

$$\frac{d\Gamma}{dq^2 d\Omega} \propto \sum_{i=1}^{14} f_i(\Omega)\Gamma_i(q^2)$$

- Transversity q^2 amplitudes:

$$H_0(q^2) \equiv h_0$$
$$H_{\{\parallel,\perp\}}(q^2) \equiv h_{\{\parallel,\perp\}} \underbrace{e^{i\delta_{\{\parallel,\perp\}}}}_{\text{NP phase}}$$

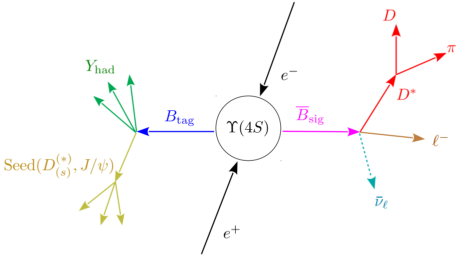
- Orthonormal angular basis:
  - $Y_l^m \equiv Y_l^m(\theta_l, \chi)$
  - $P_l^m \equiv \sqrt{2\pi}Y_l^m(\theta_V, 0)$

i	$f_i(\Omega)$	$\Gamma_i^{\rm tr}(q^2)/(\mathbf{k}q^2)$
1	$P_0^0 Y_0^0$	$h_0^2 + h_\parallel^2 + h_\perp^2$
2	$P_2^0 Y_0^0$	$-\frac{1}{\sqrt{5}}(h_\parallel^2 + h_\perp^2) + \frac{2}{\sqrt{5}}h_0^2$
3	$P_0^0 Y_2^0$	$\frac{1}{2\sqrt{5}}\left[(h_\parallel^2 + h_\perp^2) - 2h_0^2\right]$
4	$P_2^0 Y_2^0$	$-\frac{1}{10}(h_\parallel^2 + h_\perp^2) - \frac{2}{5}h_0^2$
5	$P_2^1\sqrt{2}\,Re(Y_2^1)$	$-\frac{3}{5}h_\parallel h_0 \cos\delta_\parallel$
6	$P_2^1\sqrt{2}\,Im(Y_2^1)$	$\frac{3}{5}h_\perp h_0 \sin\delta_\perp$
7	$P_0^0\sqrt{2}\,Re(Y_2^2)$	$-\frac{3}{2\sqrt{15}}(h_\parallel^2 - h_\perp^2)$
8	$P_2^0\sqrt{2}\,Re(Y_2^2)$	$\frac{\sqrt{3}}{10}(h_\parallel^2 - h_\perp^2)$
9	$P_0^0\sqrt{2}\,Im(Y_2^2)$	$\sqrt{\frac{3}{5}}h_\perp h_\parallel \sin(\delta_\perp - \delta_\parallel)$
10	$P_2^0\sqrt{2}\,Im(Y_2^2)$	$-\frac{\sqrt{3}}{5}h_\perp h_\parallel \sin(\delta_\perp - \delta_\parallel)$
11	$P_0^0 Y_1^0$	$-\sqrt{3}h_\perp h_\parallel \cos(\delta_\perp - \delta_\parallel)$
12	$P_2^0 Y_1^0$	$\frac{3}{\sqrt{15}}h_\perp h_\parallel \cos(\delta_\perp - \delta_\parallel)$
13	$P_2^1\sqrt{2}\,Re(Y_1^1)$	$\frac{3}{\sqrt{5}}h_\perp h_0 \cos\delta_\perp$
14	$P_2^1\sqrt{2}\,Im(Y_1^1)$	$-\frac{3}{\sqrt{5}}h_\parallel h_0 \sin\delta_\parallel$

The data sample: hadronic tagging

- We use $426\,\text{fb}^{-1}$ full *BABAR* data: $\Upsilon(4S) \to B_{\text{tag}}\overline{B}_{\text{sig}}(\to D^*\ell^-\overline{\nu}_\ell)$ corresponding to $471\text{M}\ B\overline{B}$ pairs.

- Hadronic B_{tag} reconstruction: 2968 modes compared to 651 in BABAR-09 $D\ell\overline{\nu}_\ell$ tagged analysis.
- Lower purity but higher efficiency.
- Skim-level tag-side requirement: $m_{\text{ES}} > 5.27$ GeV, $|\Delta E| < 72$ MeV.



- Similar to $\overline{B} \to D^*\tau^-\bar{\nu}_\tau$ *BABAR* paper, except no requirement on purity of tag-side modes, since our signal-side is very clean.

- Still cut-based ☹, unlike Belle II

The TreeFit to full topology

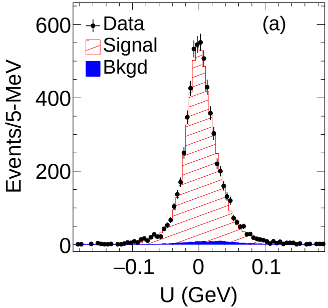
- Only $D^* \to D\pi_{\rm slow}$ considered. $|\vec{p}_{\pi_{\rm slow}}| < 400$ MeV in lab frame.
- $\Delta m \equiv (m_{D^*} - m_D)$ within $4\ \sigma$ of PDG expectation.
- Almost no uds continuum background for the charm system.
- `TreeFit` is critical to this analysis:
  - Mass-constrain the following: $\{B_{\textsf{tag}}, B_{\textsf{sig}}, D, D^*, \nu_{\rm miss}\}$
  - $\Upsilon(4S)$ is beam-spot constrained. D^* vertex-constrained after B flight.
- Final background $(\sim 3\%)$ very small. Estimated from generic $B\overline{B}$ MC and assigned as systematic.

The discriminating variable U

- Signal variable in B_{sig} rest frame for missing neutrino (from `TreeFit` w/o ν mass-constraint):

$$U = E_{\text{miss}} - |\vec{p}_{\text{miss}}| = E_\nu - |\vec{p}_\nu|$$

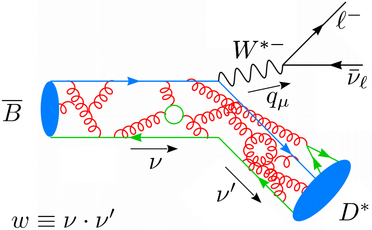
- Global comparison:



- Good comparisons found in other variables.
- $N_{\text{tot}} = 6112$, $N_{\text{bkgd}} \sim 180$

Heavy Quark Effective Theory for $\overline{B} \to D^*$

- w: relativistic γ factor of D^* in B RF
- At $\frac{\Lambda_{\rm QCD}}{m_{b,c}} \to 0$ limit, only w matters.
- Heavy Quark Symm (HQS): $\{b^\uparrow, b^\downarrow, c^\uparrow, c^\downarrow\}$ (spin-flavor symm.)
- Non-pert. QCD effects pushed into a single universal FF, $\zeta(w)$.



- In time scales $\ll \Lambda_{\rm QCD}^{-1}$ Dirac structure in the weak current irrelevant

$$\frac{\langle D^*(v',\varepsilon)|V^\mu|\overline{B}(v)\rangle}{\sqrt{m_B m_{D^*}}} = i h_V(w)\epsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu v'_\alpha v_\beta$$

$$\frac{\langle D^*(v',\varepsilon)|A^\mu|\overline{B}(v)\rangle}{\sqrt{m_B m_{D^*}}} = h_{A_1}(w)(w+1)\varepsilon^{*\mu} - h_{A_2}(w)(\varepsilon^*\cdot v)v^\mu - h_{A_3}(w)(\varepsilon^*\cdot v)v'^\mu$$

$$A_1 = \frac{w+1}{2} r' h_{A_1}$$

$$A_2 = \frac{r h_{A_2} + h_{A_3}}{r'} \equiv \frac{R_2 h_{A_1}}{r'}$$

$$V = \frac{h_V}{r'} \equiv \frac{R_1 h_{A_1}}{r'}$$

- HQS limit: $\{h_V, h_{A_1}, h_{A_3}\} \to \zeta(w)$ and $h_{A_2} \to 0$. $r' \equiv \frac{2\sqrt{m_{D^*} m_B}}{(m_{D^*}+m_B)}$

FF parameterization: BGL basis (Boyd *et al.*)

- Three versions: BGL95, BGL97 and BGL17.
- BGL FF's: $\{f_0, F_1, g\}$ related to conventional $\{A_1, A_2, V\}$ FF's and the helicity amplitudes $\{H_\pm, H_0\}$.

$$
\begin{aligned}
f_0 &= (m_B + m_{D^*})A_1 \\
F_1 &= \frac{1}{2m_{D^*}}\left[(m_B^2 - q^2 - m_{D^*}^2)(m_B + m_{D^*})A_1 - \frac{4m_B^2|\mathbf{k}|^2}{(m_{D^*} + m_B)}A_2\right] \\
&= \sqrt{q^2}H_0 \\
g &= \frac{2V}{m_B + m_{D^*}} \\
H_\pm &= (f_0 \mp m_B\mathbf{k}g)
\end{aligned}
$$

The BGL z-expansion

- z is a conformal map from $t \equiv q^2$ to unit circle.

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$
$$t_\pm = (m_B \pm m_{D^*})^2$$

- Choose t_0 so that z is small in the physical region $t \in [0, t_-]$

- Expansion of each FF from dispersion relations + analyticity

$$F_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^{N} a_n^i z^n$$

- Blasche-factor $P(z)$ from $B_c^{(*)}$ poles in non-physical $\sqrt{q^2} > m(BD^*)$ region.
- QCD ϕ_i outer functions.

- NB: $P_i(z)$ and $\phi_i(z)$ are non-trivial and carry significant part of the heavy-lifting in the q^2 shape. We follow Gambino'17.

FF parameterization: CLN [Caprini'97]

- CLN: similar expansion as BGL, but model-dependent extrapolation to $w \to 1$. Compact practical form with only 3 fit parameters:

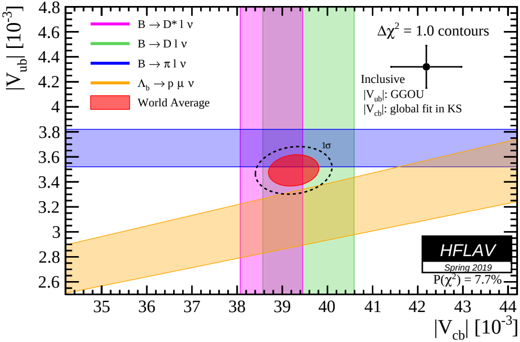
$$
\begin{aligned}
h_{A_1}(w) &= h_{A_1}(1)[1 - 8\rho^2_{D^*} z(w) + (53\rho^2_{D^*} - 15)z(w)^2 - (231\rho^2_{D^*} - 91)z(w)^3] \\
R_1(w) &= R_1(1) - 0.12(w-1) + 0.05(w-1)^2 \\
R_2(w) &= R_2(1) + 0.11(w-1) - 0.06(w-1)^2
\end{aligned}
$$

- Curvature and slope related for h_{A_1}. Too constrained?

- The expansions are derived from HQET+QCDSR, but experimentalists ignore the uncertainties.

- Calculations/uncertainties being carefully revisited by several groups now. $\mathcal{O}(\lambda_{\rm QCD}/m_c) \sim 20\%$ can't be ruled out.

The $|V_{ub}|$-$|V_{cb}|$ inclusive-exclusive tension saga



- $|V_{ub}|, |V_{cb}|$ tension: inclusive *persistently* $>$ exclusive at $> 3\sigma$.
- 2017 Grinstein/Gambino: fits to four 1-d projections (prelim. Belle)
- Claim: BGL form-factor parameterization resolves the $|V_{cb}|$ tension.
- Generated a lot of excitement...

BABAR BGL fits setup

- Linear BGL expansion till $N = 1$. ($N = 2$ terms not statistically significant but creates problems w/ unitarity)
- Two relations used to connect coefficients:
  - Lattice (MILC) constrains h_{A1} at zero-recoil and $f(q^2_{\mathrm{max}}) = 2\sqrt{m_B m^*_D} h_{A_1}$
  - Also at zero-recoil, $F_1(q^2_{\mathrm{max}}) = (m_B - m^*_D) f(q^2_{\mathrm{max}})$, so that $a_0^{F_1}$ is not an independent parameter.
- Slight isospin-dependence in above relations (negligible impact w/ current statistics)
- Unbinned non-extended ML fit using full 4-d rate and complete *BABAR* data in $q^2 \in [0.2, 10.2]$ GeV^2. No normalization.

BGL fits setup – incorporating $|V_{cb}|$

- Normalization needed to get $|V_{cb}|$ obtained from HFLAV:

Mode	$\mathcal{B}(\overline{B} \to D^* \ell^- \overline{\nu}_\ell)(\%)$	τ_B (ps)	$\Gamma_{\rm tot}$ $\times 10^{15}$ (GeV)
B^0	4.88 ± 0.10	1.518 ± 0.004	21.16 ± 0.38
B^+	5.59 ± 0.21	1.638 ± 0.004	22.46 ± 0.79

- Add the $\Gamma_{\rm tot} \equiv \int (d\Gamma/dw) dw = \mathcal{B}/\tau$ as Gaussian constraints.
- Nominal results quoted using the MILC result as a Gaussian constraint:

$$h_{A_1}(1)|_{\rm MILC} = 0.906 \pm 0.013$$

Unitarity, stability checks and global minima

- Six fit variables: $\{a_0^f, a_1^f, a_1^{F_1}, a_0^g, a_1^g, |V_{cb}|\}$. *BABAR*-only version (without $|V_{cb}|$) also provided.
- BGL setup is even more complicated than CLN. Significant effort to ensure stability and robustness of the global minimum.
- 1000 fit iterations with randomized start values within $[-1, 1]$ for the BGL coefficients.
- Fit parameters scaled appropriately such that `MINOS` cov. matrix diag. elements are all ~ 1. `HESSE` and `MINOS` cov. matrices matches.

Systematics and final results

- Most systematics cancel since *BABAR* part uses no normalization.
- Many cross-checks performed with isospin-separated (different slow pions), e/μ separated fits. Primary remnant source is the background.
- Final $N = 1$ BGL expansion results, including systematic uncertainties:

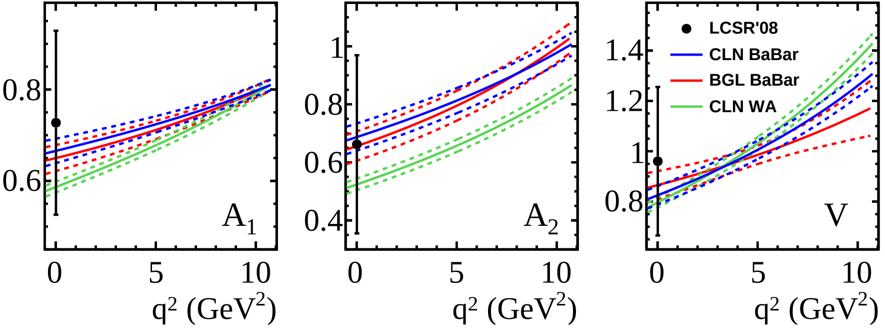
$a_0^f \times 10^2$	$a_1^f \times 10^2$	$a_1^{F_1} \times 10^2$	$a_0^g \times 10^2$	$a_1^g \times 10^2$	$\lvert V_{cb}\rvert \times 10^3$
1.29 ± 0.03	1.63 ± 1.00	0.03 ± 0.11	2.74 ± 0.11	8.33 ± 6.67	38.36 ± 0.90

- Final CLN results:

$\rho^2_{D^*}$	$R_1(1)$	$R_2(1)$	$\lvert V_{cb}\rvert \times 10^3$
0.96 ± 0.08	1.29 ± 0.04	0.99 ± 0.04	38.40 ± 0.84

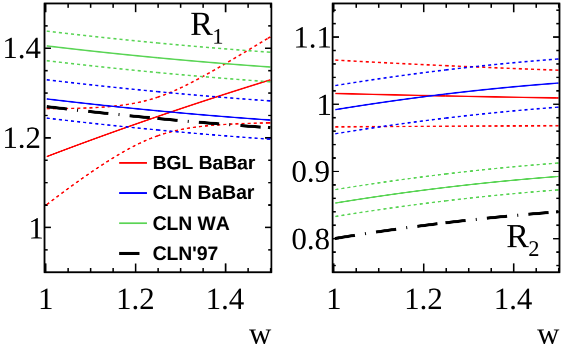
- Our BGL $|V_{cb}|$ is consistent with WA $|V_{cb}|_{\rm excl.}$ and remains in tension with WA $|V_{cb}|_{\rm incl.}$

BABAR results – FF shapes



- Dashed curves denote 1σ error bands
- *BABAR* FF's significantly different from CLN-WA. CLN p-value: 0.0017
- Within *BABAR*, CLN/BGL seem consistent.
- LCSR'08 at $q^2 = 0$ (large recoil) has huge errors, but central value more consistent with *BABAR*.

BABAR results – $R_{1,2}$ ratios



- Included the $R_{1,2}$ curves (w/o errors) from the original Caprini paper.

- *BABAR* BGL: $R_1(1) \approx 1.2$ and the slope is positive; $R_2 \approx 1$ and flat.
- Consistent with HQET, although no HQET constraints used in the fit. It's what the data prefers.
- Note: CLN $R_{1,2}$ *shape* is fixed. Too constrained?

Summary

- First tagged 4d $\overline{B} \to D^{*}\ell\overline{\nu}_{\ell}$ angular analysis using *BABAR* data
- *BABAR* FF's w/ BGL/CLN consistent and deviate with CLN-WA.
- *BABAR* + HFLAV-BF: $|V_{cb}|$ consistent between BGL and CLN-WA. $\sim 3\sigma$ exclusive-inclusive tension persists (in the $B \to D^{*}$ sector).
- Ongoing effort: *BABAR* BGL fit for $B \to D$ and joint $B \to D^{(*)}$ HQET fits. LQCD $B \to D^{*}$ $w > 1$ FFs urgently needed!

Summary

- First tagged 4d $\overline{B} \to D^* \ell \overline{\nu}_\ell$ angular analysis using *BABAR* data
- *BABAR* FF's w/ BGL/CLN consistent and deviate with CLN-WA.
- *BABAR* + HFLAV-BF: $|V_{cb}|$ consistent between BGL and CLN-WA. $\sim 3\sigma$ exclusive-inclusive tension persists (in the $B \to D^*$ sector).
- Ongoing effort: *BABAR* BGL fit for $B \to D$ and joint $B \to D^{(*)}$ HQET fits. LQCD $B \to D^*$ $w > 1$ FFs urgently needed!

Thank you!

Backup

References

- Gambino'17: arXiv:1703.06124, arXiv:1707.09509
- BGL: hep-ph/9508211, hep-ph/9705252, arXiv:1703.08170
- CLN: arXiv:9712417
- Belle τ/D^* pol.: arXiv:1709.00129, arXiv:1901.06380

The $b \to cW_L^{*-}(\to \ell^- \overline{\nu}_\ell)$ effective Hamiltonian

- In the SM, W_L^{*-} acts a left-handed off-shell spin-1 vector boson.
- The minimal effective Hamiltonian for light leptons is:

$$\mathcal{H}_{\text{eff}} = \frac{2G_F V_{cb}^L}{\sqrt{2}} \left[\left(g^V \bar{c}\gamma_\mu b - g^A \bar{c}\gamma_\mu\gamma_5 b \right) \bar{\ell}\gamma^\mu \nu_L + (...) \right]$$

- Additional scalar (charged $H^\pm$) and tensor (Leptoquarks) operators can occur.
- Heavy W_R^- can induce $g_{V,A} \equiv (1 \pm \epsilon_R)$ even for light leptons.
- The helicity amplitudes $\{H_\pm, H_0\}$ can acquire phases from complex ϵ_R.

NP searches: the $\sin\chi$ terms [arXiv:1505.02873]

- In SM, the amplitudes are relatively real. $\sin\chi$ terms are zero.
- Non-zero $\Gamma_{\{6,9,10,14\}}$ would be clear sign of NP!
- Formalism developed at *BABAR* for both the ρ and D^* vector mesons.
- $\pi\pi$ S-wave under the ρ also very interesting for RH current searches in $b \to u$.

i	$f_i(\Omega)$	$\Gamma_i^{\mathrm{tr}}(q^2)/(\mathbf{k}q^2)$
1	$P_0^0 Y_0^0$	$h_0^2 + h_\parallel^2 + h_\perp^2$
2	$P_2^0 Y_0^0$	$-\frac{1}{\sqrt{5}}(h_\parallel^2 + h_\perp^2) + \frac{2}{\sqrt{5}} h_0^2$
3	$P_0^0 Y_2^0$	$\frac{1}{2\sqrt{5}}\left[(h_\parallel^2 + h_\perp^2) - 2h_0^2\right]$
4	$P_2^0 Y_2^0$	$-\frac{1}{10}(h_\parallel^2 + h_\perp^2) - \frac{2}{5} h_0^2$
5	$P_2^1 \sqrt{2}\, Re(Y_2^1)$	$-\frac{3}{5} h_\parallel h_0 \cos\delta_\parallel$
6	$P_2^1 \sqrt{2}\, Im(Y_2^1)$	$\frac{3}{5} h_\perp h_0 \sin\delta_\perp$
7	$P_0^0 \sqrt{2}\, Re(Y_2^2)$	$-\frac{3}{2\sqrt{15}}(h_\parallel^2 - h_\perp^2)$
8	$P_2^0 \sqrt{2}\, Re(Y_2^2)$	$\frac{\sqrt{3}}{10}(h_\parallel^2 - h_\perp^2)$
9	$P_0^0 \sqrt{2}\, Im(Y_2^2)$	$\sqrt{\frac{3}{5}} h_\perp h_\parallel \sin(\delta_\perp - \delta_\parallel)$
10	$P_2^0 \sqrt{2}\, Im(Y_2^2)$	$-\frac{\sqrt{3}}{5} h_\perp h_\parallel \sin(\delta_\perp - \delta_\parallel)$
11	$P_0^0 Y_1^0$	$-\sqrt{3} h_\perp h_\parallel \cos(\delta_\perp - \delta_\parallel)$
12	$P_2^0 Y_1^0$	$\frac{3}{\sqrt{15}} h_\perp h_\parallel \cos(\delta_\perp - \delta_\parallel)$
13	$P_2^1 \sqrt{2}\, Re(Y_1^1)$	$\frac{3}{\sqrt{5}} h_\perp h_0 \cos\delta_\perp$
14	$P_2^1 \sqrt{2}\, Im(Y_1^1)$	$-\frac{3}{\sqrt{5}} h_\parallel h_0 \sin\delta_\parallel$

4-d rate expression

- Generic amplitude with complex $\mathcal{H}_i$'s for $\overline{B} \to D^*(\to D\pi)\ell^-\overline{\nu}_\ell$:

$$|\overline{\mathcal{M}}|^2 = \left| \sum_{\lambda\in\{0,\pm1\}} \sqrt{3}\mathcal{H}_\lambda d_{\lambda,0}(\theta_V) d^1_{\lambda,-1}(\theta_l) e^{i\lambda\chi} \right|^2$$

- Expanding this out, leads to:

$$\frac{d\Gamma}{dq^2 d\Omega} = \frac{\sqrt{8\pi}|V_{cb}|^2 \eta^2_{\mathrm{EW}} G_F^2 \mathcal{B}^{D^*\to D\pi}}{3m_B^2(4\pi)^4} \left\{ \sum_{i=1}^{14} f_i(\Omega)\Gamma_i(q^2) \right\}$$

- If the $\mathcal{H}_i$'s are real, it boils down to the usual expression:

$$\frac{d\Gamma}{dq^2 d\Omega} = \Big[\left(H_+^2 (1-\cos\theta_l)^2 + H_-^2 (1+\cos\theta_l)^2 \right) \sin^2\theta_V + 2H_0 \sin\theta_l \sin 2\theta_V \cos\chi \left(H_+(1-\cos\theta_l) - H_-(1+\cos\theta_l) \right) + 4H_0^2 \sin^2\theta_l \cos^2\theta_V + 2H_+H_- \sin^2\theta_l \sin^2\theta_V \cos 2\chi \Big] \times \frac{3}{8(4\pi)^4} G_F^2 \eta^2_{\mathrm{EW}} |V_{cb}|^2 \frac{kq^2}{m_B^2} \mathcal{B}(D^* \to D\pi)$$

BSemiExclAdd skim

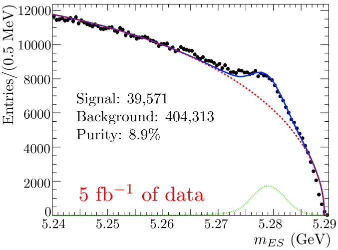
- Charmful seed $S \in \{D^{(*)0}, D^{(*)+}, D_s^{(*)+}, J/\psi\}$ and charmless $Y = n_1\pi^{\pm} + n_2 K^{\pm} + n_3 K_S^0 + n_4 \pi^0$,
- Hadronic B_{tag} reconstruction: 2968 modes compared to 651 in BABAR-09 $D\ell\overline{\nu}_\ell$ tagged analysis.
- Charmful seed $S \in \{D^{(*)0}, D^{(*)+}, D_s^{(*)+}, J/\psi\}$ and charmless $Y = n_1\pi^{\pm} + n_2 K^{\pm} + n_3 K_S^0 + n_4 \pi^0$,

- Purity cut removed.
- In the e^+e^- rest frame:

$$\Delta E = E_{\text{tag}} - \sqrt{s}/2$$

$$m_{\text{ES}} = \sqrt{s/4 - |\vec{p}_{\text{tag}}|^2}$$

- Skim-level tag-side requirement: $m_{\text{ES}} > 5.27$ GeV, $|\Delta E| < 72$ MeV.



Signal side reconstruction and MC samples

- Full exclusive event topology reconstructed. No additional charged tracks. Final *BABAR* ultimate PID selectors for $\{e, \mu, \pi, K, \pi^0\}$
- Leptons: $|\vec{p}_{\rm lab}| > 200$ and 300 MeV for e and μ. $\theta^{\ell}_{\rm lab} \in (0.4, 2.6)$ fiducial cut. Brem-recovery for e.
- For a given event all $\{D^0, D^{\pm}, D^{*0}, D^{*\pm}, +\text{charmless}\}\ell$ combinations sought in disjoint channels. Helps cleaning up the "specific" SL backgrounds.
- MC samples:
  - Generic $B\overline{B}$ MC: FF's and BF's reweighted to PDG. Used for background studies/estimation.
  - `FLATQ2` MC: tag-side same as in generic MC. Signal-side generated flat in $dq^2 d\Omega$ for acceptance calculation in the angular analysis.

The 12 individual $D^*(\to D\pi)\ell$ modes

- E_{extra}: additional good photons not used in the reconstruction.
- Only 3 cleanest D^0 modes used. Cuts on E_{extra} and CL from `TreeFit`.

Particle	D^0 Decay mode	Mode #	Cuts
$D^{*0} \to D^0\pi^0$	$K^-\pi^+$	1	$E_{\text{extra}} < 0.5$ GeV, CL > 0.001
	$K^-\pi^+\pi^0$	2	$E_{\text{extra}} < 0.4$ GeV, CL > 0.01
	$K^-\pi^+\pi^-\pi^+$	3	$E_{\text{extra}} < 0.4$ GeV, CL > 0.01
$D^{*+} \to D^0\pi^+$	$K^-\pi^+$	4	$E_{\text{extra}} < 0.6$ GeV, CL $> 10^{-6}$
	$K^-\pi^+\pi^0$	5	$E_{\text{extra}} < 0.6$ GeV, CL $> 10^{-6}$
	$K^-\pi^+\pi^-\pi^+$	6	$E_{\text{extra}} < 0.6$ GeV, CL $> 10^{-6}$

- 6 D^* modes $\Rightarrow$ 12 $D^*\ell$ modes.
- Truth-matching criterion: reconstructed and generated mode must match.
- Each mode has different acceptance/backgrounds. Analysed independently and combined at the end.
- Quite different from *BABAR* $R(D^{(*)})$ analysis (*many* modes lumped together)

Final mode-wise statistics

- Due to low tagging efficiency, limited MC statistics is critical.

$[D]\pi e^{\pm}$ mode	$N_{\rm tot}$	$N_{\rm bkgd}$	$N_{\rm sim}$
$[K^-\pi^+]\pi^0 e^{\pm}$	486	15	14396
$[K^-\pi^+\pi^0]\pi^0 e^{\pm}$	547	19	17806
$[K^-\pi^+\pi^-\pi^+]\pi^0 e^{\pm}$	350	13	3410
$[K^-\pi^+]\pi^+ e^{\pm}$	418	5	3990
$[K^-\pi^+\pi^0]\pi^+ e^{\pm}$	801	14	7795
$[K^-\pi^+\pi^-\pi^+]\pi^+ e^{\pm}$	453	8	4472

$[D]\pi\mu^{\pm}$ mode	$N_{\rm tot}$	$N_{\rm bkgd}$	$N_{\rm acc}$
$[K^-\pi^+]\pi^0 \mu^{\pm}$	442	18	14893
$[K^-\pi^+\pi^0]\pi^0 \mu^{\pm}$	574	29	18441
$[K^-\pi^+\pi^-\pi^+]\pi^0 \mu^{\pm}$	395	16	3577
$[K^-\pi^+]\pi^+ \mu^{\pm}$	417	10	4171
$[K^-\pi^+\pi^0]\pi^+ \mu^{\pm}$	768	22	7838
$[K^-\pi^+\pi^-\pi^+]\pi^+ \mu^{\pm}$	461	11	4446

- Final cut: $|U| < 90$ MeV.
- $N_{\rm tot} = 6112$, $N_{\rm bkgd} \sim 180$ over all 12 modes.
- Clean enough that background is a *systematic* from generic $B\bar{B}$ MC.
- No additional signal-background separation required.

"Model-independent" FF fits: pseudoscalars

- Pseudoscalar cases: z-expansion fits for the single $f_+(q^2)$ FF from both w/ and w/o hadronic tagging at B-factories.

- Rate $\propto \sin^2\theta_\ell |f_+^2(q^2)|$ factorizes $\Rightarrow$ effectively 1d analysis in q^2.

- Reliable theory inputs also exist:
  - $\overline{B} \to \pi\ell^-\overline{\nu}_\ell$: both LCSR (large-recoil) and Lattice (low-recoil)
  - $\overline{B} \to D\ell^-\overline{\nu}_\ell$: non-zero recoil Lattice data from both MILC (1503.07237) and HPQCD (1505.03925)

- Latest Belle hadronic tagged results: $\overline{B} \to \pi\ell^-\overline{\nu}_\ell$ (1306.2781) and $\overline{B} \to D\ell^-\overline{\nu}_\ell$ (1510.03657).

"Model-independent" FF fits: vector mesons

- Vector meson cases much more difficult than pseudoscalars:
  - Three FF's that require a full 4-d amplitude analysis to disentangle.
  - Charmless $\{\rho, \omega\}$: broad resonances and difficult for both LCSR and Lattice. ρ can have significant $\pi\pi$ S-wave. Plus ρ-ω mixing.
  - Charmful D^*: published Lattice data only at $w = 1$ yet. LCSR at large recoil unreliable for heavy D^*. HQET framework helps, though.

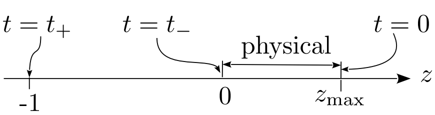
- Long time goal at *BABAR*. But hard, analysis-wise as well as little theory guidance on the QCD input in the z-expansion (till pre-2017). BGL papers for $B \to D^*$ were pre-2000.

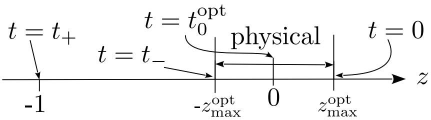
- Grinstein $B \to D^*$ paper in 2017 reinvigorated the discussion. Much follow-up theory activity by Gambino/Ligeti/Lattice $\Rightarrow$ very helpful!

BGL: optimal t_0 and unitarity conditions

- Nominal choice (CLN):
  $t_0 = t_-$

- Optimal choice:
  $t_0^{\rm opt} = t_+ - \sqrt{t_+(t_+ - t_-)}$





- $t_0^{\rm opt}$ approximately halves the $|z|$ range, for better convergence.

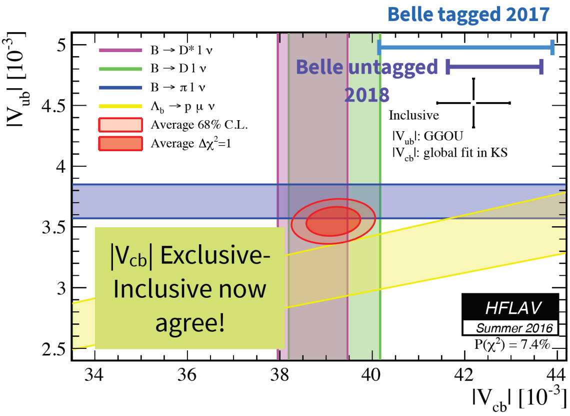
- The BGL coefficients must satisfy the unitarity constraints:

$$\sum_{n=0}^{N} |a_n^f|^2 + |a_n^{F_1}|^2 \equiv S_{1+} \leq 1, \quad \sum_{n=0}^{N} |a_n^g|^2 \equiv S_{1-} \leq 1$$

- In reality, far from saturation: $S \sim \mathcal{O}(10^{-2})$.

$|V_{cb}|$ tension and BGL fits: ICHEP'18

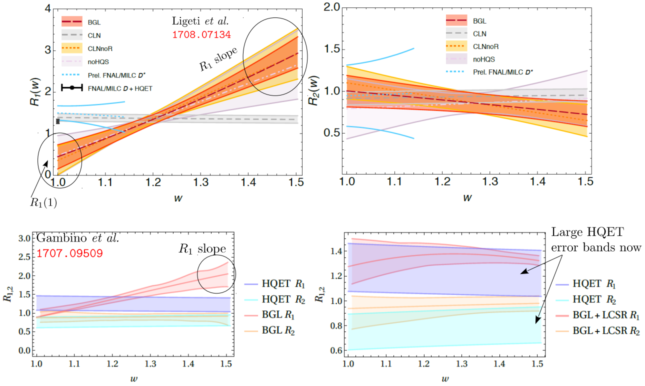
- ICHEP'18 plenary claim:



- But life is not so simple...

HQET implications from these BGL fits

- Ab initio, *huge* HQET breaking from BGL fits:

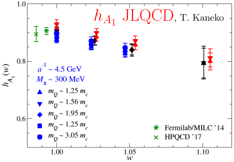


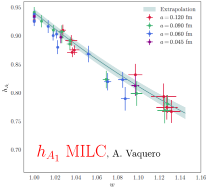
- HQS: FF ratios $R_{1,2} \to 1$.
- R_1 from Gambino looks quite weird
- LCSR+unitarity usage!

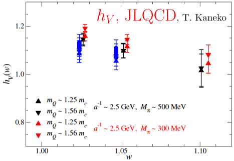
- HQET works pretty well in heavy quark spectroscopy, lifetimes etc. So, quite surprising, if true...

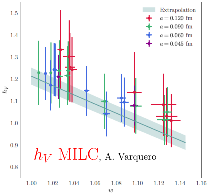
Preliminary non-zero recoil Lattice data

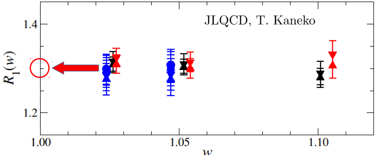
- Preliminary lattice data seems consistent with $h_{A_1}, h_V, R_1 \sim \mathcal{O}(1)$ expectation from HQET. Lattice uncertainties on these $\sim 3\%$.







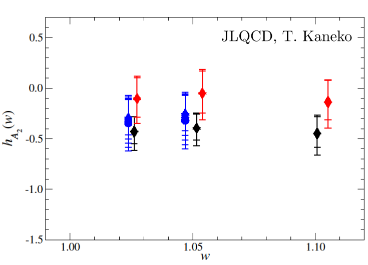


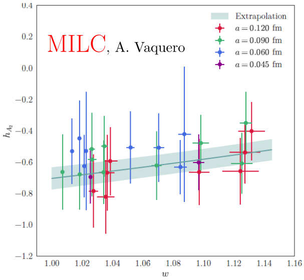


- $R_1(w)$ flat-ish and $R_1(1) \sim 1.3$
- MILC/JLQCD have different systematics and ensembles. Need both for confirmation.

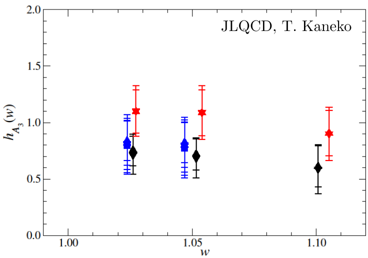
- R_2 uncertainties appear to be large in Lattice.

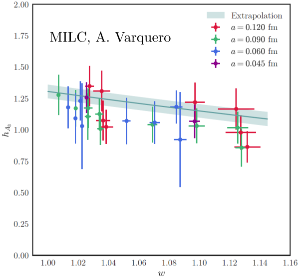
Preliminary non-zero recoil Lattice data (cntd.)





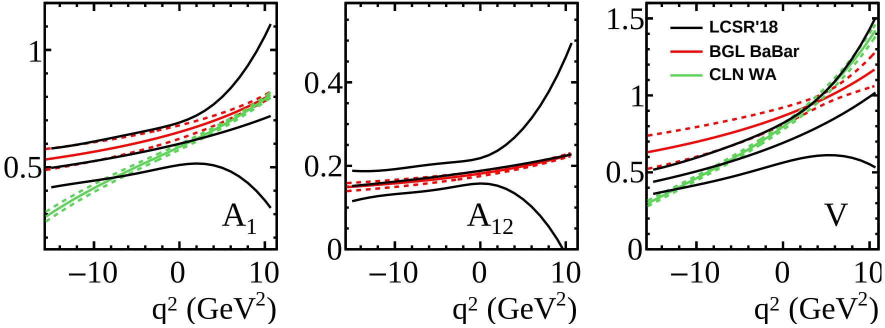
- HQET: $h_{A_2} \to 0$
- Error: $\sim 40\%$





- HQET: $h_{A_3} \to 1$
- Error: $\sim 30\%$

BABAR BGL results – FF shapes (cntd.)



- New LCSR data at large negative q^2 values from 1811.00983
- BABAR BGL "not inconsistent" with these LCSR data. NB: BABAR fits did not include any LCSR/HQET inputs.
- Curvature of A_1 from CLN-WA seems to change at $q^2 \to -15$ GeV^2.

Ambiguities in 'product of four 1-d' fits?

- Angle distributions: barred terms, q^2-info integrated away. Very little sensitivity to q^2-dependence.

$$\frac{d\Gamma}{d\chi} \sim \left[\left(\overline{|H_+|^2} + \overline{|H_-|^2} + \overline{|H_0|^2}\right) - \overline{|H_+H_-|}\cos 2\chi\right]$$

$$\frac{d\Gamma}{d\cos\theta_\ell} \sim \left[(1-\cos\theta_\ell)^2\overline{|H_+|^2} + (1-\cos\theta_\ell)^2\overline{|H_-|^2} + 2\sin^2\theta_l\overline{|H_0|^2}\right]$$

$$\frac{d\Gamma}{d\cos\theta_V} \sim \left[\sin^2\theta_V\left(\overline{|H_+|^2} + \overline{|H_-|^2}\right) + 2\cos^2\theta_V\overline{|H_0|^2}\right]$$

- $d\Gamma/dw$ has info only on sum of FF squares:

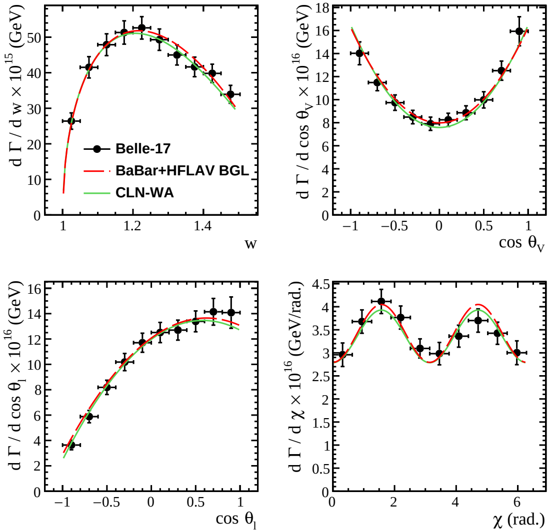
$$\frac{d\Gamma}{dw} \sim \sqrt{w^2-1}(1-2wr+r^2)\left[|H_+|^2 + |H_-|^2 + |H_0|^2\right]$$

- Can the individual FF's be extracted unambiguously in a fit to product of four 1-d projections?

- *BABAR*-BGL'19 instead employs a full 4-d fit.

Comparisons w/ Belle-17 data

Overlay, *not* a fit:

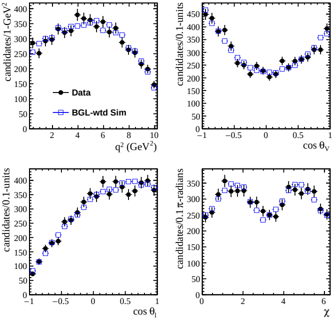


- Attempted to add unpublished Belle'17 data as a Gaussian constraint to the overall *BABAR* NLL.
- Fit significantly deviating from *BABAR*-only if entire 40×40 cov. matrix used. Single set of 10×10 cov. matrix ok $\Rightarrow$ not used in the end.
- Note: only mild diff. in 1-d projections between CLN-WA and *BABAR*-BGL.

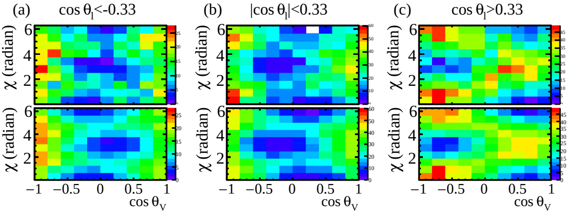
Data/MC comparisons: *BABAR*

- `FLATQ2` accepted MC (incl. eff. effects) weighted by the BGL results should match the data in all multi-dim. distributions.

1-d distributions:



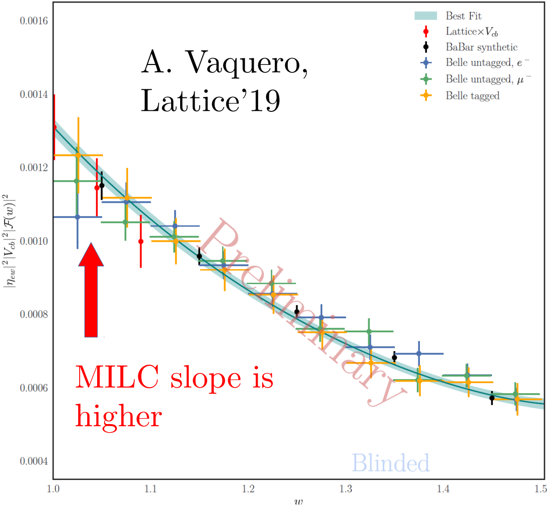
3-d in angles, q^2-integrated:



Top row: Data

Bottom row: Acc. MC wtd by BGL

Lattice'19 MILC slide



BABAR-BGL semi-tauonic predictions

- We mostly follow the Gambino'17 prescriptions with $\sim 15\%$ uncertainty on the HQET estimation of $P_1(1)$ (scalar FF).
- At maximum recoil, instead of LCSR, we use the BABAR-BGL prediction from $F_1(w_{\max})$.

$R(D^*)$ Description	Value
HFLAV'19 WA exp.	0.295 ± 0.014
Fajfer'12 CLN	0.252 ± 0.003
Ligeti'17 BGL	0.257 ± 0.003
Gambino'17 BGL	0.260 ± 0.008
HFLAV'19 SM avg.	0.258 ± 0.005
BABAR BGL	0.253 ± 0.005

- $\sim 2.8\sigma$ dev. in $R(D^*)$ w/ BABAR-BGL.
- Predictions stable, but disagreement on the errors.
- Effect of the FF's on exp. efficiencies?

- P_τ: -0.483 ± 0.027 (BABAR-BGL), $-0.38 \pm 0.51^{+0.21}_{-0.16}$ (`Belle'17`)
- $F_L^{D^*}$: $\underbrace{+0.454 \pm 0.011}_{\text{predictions}}$ (BABAR-BGL), $\underbrace{+0.60 \pm 0.08 \pm 0.035}_{\text{measurements}}$ (`Belle'19`)