BCFT at Large Charge

[Cuomo, MM, Raviv-Moshe 2108.06579]

A puzzle

How to describe a Dirichlet BCFT in the large charge EFT?

Context:

- large charge EFT in the bulk [Hellerman et al.; ...]

$$\mathcal{L} = c_1 (\partial \chi)^d + \dots$$

$$j_\mu = \frac{\partial \mathcal{L}}{\partial(\partial^\mu \chi)} = c_1 d \, (\partial\chi)^{d-2} \partial_\mu \chi$$

- classical UV model (3d)

$$\mathcal{L} = |\partial\phi|^2 - \frac{g^2}{6}|\phi|^6 \xrightarrow{\text{turning on } \mu} \frac{1}{2}(\partial\rho)^2 + \frac{1}{2}\rho^2(\partial\chi)^2 - \frac{g^2}{48}\rho^6$$

$$\uparrow$$

$$\phi \equiv \frac{\rho}{\sqrt{2}} e^{i\chi} \qquad\qquad j_\mu = \rho^2 \partial_\mu \chi$$

$$\chi \equiv \mu t + \pi$$

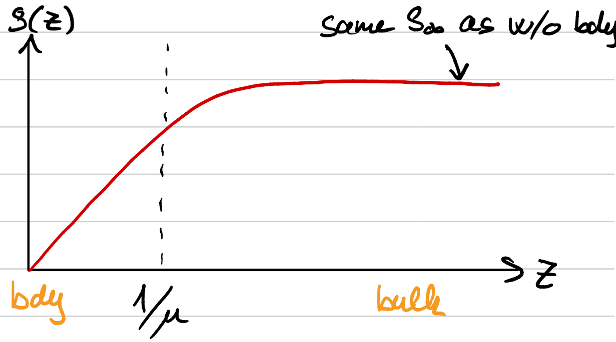
Neumann: $\partial_z \rho \big|_{z=0} = j_z \big|_{z=0} = 0$

Dirichlet: $\rho\big|_{z=0} = 0 \quad \Rightarrow \quad j_\mu \big|_{z=0} = 0$

- N: $\rho = \text{const} \approx \# (\partial\chi)^{1/2}$ indep. of z

EFT: $S_{EFT}^{(N)} = \frac{\#}{g} \int_{z\geq 0} d^3x\, (\partial\chi)^3 + \dots$

- D: ρ has profile



$\boxed{\text{What BC for } \chi\,?}$

- $j^\mu_{EFT}\big|_{z=0} = 0$? ↯

  doesn't allow for non-trivial χ profile (need $\chi_\infty = \mu t + \dots$)

- couple to new bdy dof ? ↯

- concentrate on capturing observables in EFT regime ✓

$$Q(L) = \int_0^L dz\, j^0_{UV} = \frac{\#}{g}\left[L\mu^2 + \frac{b}{\sqrt{8}}\mu + O(e^{-L\mu}) \right]$$

$$= \int_0^L dz\, j^0_{EFT}$$

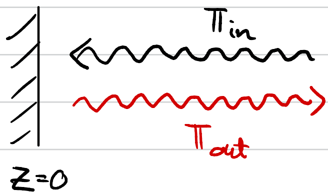
can be produced from bdy. cont. to j^0_{EFT}

$$S_{EFT}^{(D)} = \frac{\#}{g}\int_{z\geq 0} d^3x\, (\partial\chi)^3 + \frac{b}{2g}\int_{z=0} d^2x \left(\hat{\partial}\chi\right)^2 + \dots$$

$$j^{\mu}_{EFT} = \frac{\#}{g}(\partial\chi)\partial^{\mu}\chi + \boxed{\frac{b}{g}\,\delta(z)\,\hat{\partial}^{\mu}\chi} + \dots$$

The EFT w/ bdy term reproduces other observables:

Phase shift:



$$e^{i\delta(k)}$$

$$\delta(k) \underset{\substack{\uparrow \\ \text{from UV comp.}}}{=} \frac{b}{\sqrt{8}}\frac{k}{\mu} + O\left(\frac{k^2}{\mu^2}\right)$$

✓ reproduced by EFT

Note: BC for EFT is perturbed Neumann

$$\partial_z \pi\Big|_{z=0} = \frac{b}{\sqrt{8}}\underbrace{\frac{\hat{\partial}^2\pi}{\mu}}_{O\left(\frac{k^2}{\mu}\right)}\Bigg|_{z=0}$$

Outline

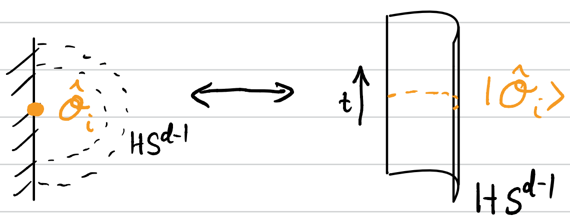
- A puzzle ✓
- Prediction for $\hat{\Delta}_Q$
- Comparison w/ $O(2)$ in $4-\varepsilon$ expansion
- Discussion on other bulk & bdy large charge phases

Prediction for $\hat{\Delta}_Q$

General facts about BCFT:

- local bdy op's $\hat{O}_i$, $\hat{\Delta}_i$
- state/operator correspondence:

$$\hat{O}_i \longleftrightarrow |\hat{O}_i\rangle_{HS^{d-1}\times R}$$



Large charge operator spectrum from EFT

Computation in bulk CFT:

$$R\mu = \# Q^{1/(d-1)} + O(Q^{-1/(d-1)})$$

$$\Delta_Q^{(\text{class})} = \# \mu^d + \# \mu^{d-2} + \dots \leftarrow \text{classical cont.}$$

$$= \alpha_1 Q^{d/(d-1)} + \alpha_2 Q^{(d-2)/(d-1)} + \dots$$

$$\Delta_Q^{(\text{1-loop})} = \frac{1}{2}\sum_\ell \underset{\substack{\uparrow \\ \text{degeneracy}}}{n_\ell} \omega_\ell \leftarrow \text{eigenfrequencies from } \ddot{\pi} - \frac{1}{d-1}\nabla^2_{S^{d-1}}\pi = 0$$

$$= \begin{cases} c_d + \dots & d \text{ odd [Hellerman et al.]} \\ \gamma_d \log Q + \dots & d \text{ even [Cuomo]} \end{cases}$$

$$\boxed{\Delta_Q = \Delta_Q^{(\text{class})} + \Delta_Q^{(\text{1-loop})} + \dots}$$

Computation in BCFT:

$$R\mu = \#(2Q)^{1/(d-1)} - \# b + O(Q^{-1/(d-1)})$$

$$\hat{\Delta}_Q^{(\text{class})} = \# \tfrac{1}{2}\mu^d - \# b\, \mu^{d-1} + \# \tfrac{1}{2}\mu^{d-2} + \dots$$

$$= \frac{1}{2}\alpha_1 (2Q)^{d/(d-1)} - \# b (2Q) + \left(\frac{1}{2}\alpha_2 + \# b^2\right)(2Q)^{(d-2)/(d-1)} + \dots$$

$$\hat{\Delta}_Q^{(\text{1-loop})} = \frac{1}{2}\sum_\ell \underset{\substack{\uparrow \\ \text{degeneracy}}}{n_\ell^+} \omega_\ell = \log Q \begin{cases} \# & d \text{ odd} \\ \frac{1}{2}\gamma_d & d \text{ even} \end{cases}$$

Reason for log Q term: $\exists$ operator that renormalizes the $O(Q^0)$ term in the action

- d even: bulk op.
  e.g. $d=4$ $\mathcal{O} = \sqrt{\tilde{g}}\, \hat{\tilde{R}}^2$ $\quad \tilde{g}_{\mu\nu} \equiv (\partial\chi)^2 g_{\mu\nu}$

- d odd: body op.
  e.g. $d=3$ $\hat{\mathcal{O}} = \sqrt{\hat{\tilde{g}}}\, \hat{\tilde{R}}$
  body quant. $\quad$ bulk quant evaluated at body

Main result: $\boxed{\hat{\Delta}_Q = \frac{1}{2}\Delta_{2Q} - \# b\, Q + \text{log term}}$

charge accumulation at body

D classically: $b<0$
Quantum mechanically b can take any sign

Can also use EFT to compute n-pt fn's:

- $\hat{\lambda}_{q,\delta} \equiv \langle Q+q \,|\, \hat{\mathcal{O}}_{q,\delta} \,|\, Q\rangle \propto Q^{\delta/(d-1)}$
  dim

- $\hat{\lambda}_{\hat{D}} \equiv \langle Q \,|\, \hat{D} \,|\, Q\rangle = \#\alpha_1 (2Q)^{d/(d-1)}$
  unique scalar in $T_{\mu\nu} \xrightarrow[z\to 0]{} \# \hat{D} + \ldots$

Comparison w/ $O(2)$ in $4-\varepsilon$ expansion

Double scaling limit $\varepsilon \to 0$, $Q \to \infty$, $\varepsilon Q =$ fixed
Can compute for all εQ, match to EFT for large εQ
[Badel et al.]

Comment: - similar situation in $O(N)$ model:
$N \to \infty$, $Q \to \infty$, $Q/N =$ fixed [Alvarez-Gaumé et al.]

- in $U(1) + N_f$ flavors instead
$N_f \to \infty$, $Q \ll N_f$ is already described by EFT
[Cuomo et al.; de la Fuente; Dupuis et al.]

$$S = \int d^d x \sqrt{g} \left(|\partial \phi|^2 + m_d^2 |\phi|^2 + \frac{\lambda_0}{4} |\phi|^4 \right) \qquad \lambda_* = O(\varepsilon)$$

$$\Delta_Q \underset{\substack{\uparrow \\ \text{double scaling}}}{=} \frac{1}{\lambda_*} f_{-1}(\lambda_* Q) + f_0(\lambda_* Q) + \dots \quad \text{same for } \hat{\Delta}_Q$$

Compute $\lim_{T \to \infty} \langle \varphi_Q | e^{-HT} | \varphi_Q \rangle = \# \, e^{-\Delta_Q T}$ using semiclassics

Look for time indep saddle of $S^{(UV)} + \int_{-T/2}^{T/2} d^d x \; i \# Q \dot{\chi}$:

$\chi = -i \mu \tau$

$g(\theta)$ solves a nonlinear ODE + $Q = \# \int_{S^{d-1}/HS^{d-1}} \mu g^2$ constraint

N: $g = \mathrm{const}$

$$\hat{f}_{-1}(\lambda_* Q) = \frac{1}{2} f_{-1}(2\lambda_* Q)$$

$$\hat{f}_0(\lambda_* Q) = \frac{R}{2} \sum_{\ell=0}^{\infty} n_\ell^+ \left[\omega_+(\ell) + \omega_-(\ell)\right] \leftarrow$$

divergent, renormalize in dim. reg. subtraction leaves finite sum

For $\lambda_* Q \ll 1$ reproduce diagrammatics:

$$\Delta_Q = Q \underset{\substack{\uparrow \\ (\frac{d}{2}-1)}}{\Delta_\phi} + \frac{\varepsilon}{10} (\underset{\substack{\uparrow \\ \text{classical}}}{Q^2} - \underset{\substack{\uparrow \\ \text{1-loop}}}{Q}) + \dots$$

$$\hat{\Delta}_Q = Q \Delta_\phi + \frac{\varepsilon}{5} (Q^2 - 2Q) + \dots$$

For $\lambda_* Q \gg 1$ reproduce EFT w/ Wilson coeffs determined:

$$c_i = \frac{\#}{\varepsilon} + \# + \dots \, , \qquad b = \cancel{\frac{\#}{\varepsilon}} + \underset{\substack{\uparrow \\ \text{charge accumulation near body}}}{0.0844982} + \dots$$

Note: not just fitting coeffs, many nontrivial relations obeyed

D: ρ has nontrivial profile

For $\lambda_* Q \ll 1$ find profile perturbatively

$$\frac{1}{\lambda_*}\hat{f}_{-1}(\lambda_* Q) = 2Q + \frac{\varepsilon}{5}Q^2 + \ldots \quad \leftarrow \text{semiclassics}$$

$$\hat{\Delta}_Q = 2Q + \frac{\varepsilon}{5}(Q^2 - 3Q) \quad \leftarrow \text{diagrams}$$

For $\lambda_* Q \gg 1$ find profile in matched asymptotic series expansion

$$b = -\frac{5}{6\sqrt{2}\pi^2\varepsilon} + O(\varepsilon^0)$$

Discussion on other bulk & bdy large charge phases

$\boxed{\text{How universal is the large charge EFT?}}$

In condensed matter phletora of other $T=0$, μ finite phases (besides superfluid / superconductor)

- BEC (realized in free scalar & SUSY examples)
  Becomes superfluid in the presence of interactions

- Fermi liquid (realized in free fermion & $\boxed{?}$)
  Very robust to interactions in NR case

What happens in the relativistic & conformal case?

- should be possible, since free fermion works
- should be impossible, since no Schrödinger Fermi liquid [Rothstein, Shrivastava]

– non-Fermi liquids

– extremal RN BHs

- wildly believed to be unstable, but IMO very weak arguments

- well-developed EFT for them: Schwarzian + $U(1)$ + $SO(3)$ analog of keeping just the zero mode of χ on $S^{d-1}\times\mathbb{R}$ [Almheiri, Polchinski; Maldacena et al; Mertens et al.; Yang; Iliesiu, Turiaci; ...]

Can compute:

$$\langle H_Q | \mathcal{O}_q(\tau,\theta)\, \mathcal{O}_{-q}(0) | H_Q \rangle$$

$$= \sum_J \langle \mathcal{O}_{qJ}(\tau)\, \mathcal{O}_{-qJ}(0) \rangle^{(EFT)}_{E=\Delta_Q}\, C_J(\theta)$$

Same correlator was analyzed in bootstrap [Jafferis et al.; Komargodski, MM, Pal, Raviv-Moshe]
Preliminary results suggest that crossing is trivially satisfied [Hare Krishna, Litvinov, MM wip]

- [Jafferis et al.] found many strongly coupled solutions to the bootstrap eqs

$\boxed{\text{Are they realized in any CFT?}}$

In BCFT we found many controlled examples that go beyond superfluid w/ perturbed N BC

- free bulk scalar w/ interacting bdy

$$S = \int d^dx \, |\partial\phi|^2 + \int_{z=0} d^{d-1}x \, \frac{\lambda}{4}|\phi|^4$$

$$\hat{\Delta}_Q^{(class)} = \begin{cases} Q\left(\frac{d}{2}-1\right) + \#\lambda Q^2 + \dots & \lambda Q \ll 1 \\ Q\frac{d}{2} + O\left(\frac{(\lambda Q)^{(d-1)/d}}{\lambda}\right) & \lambda Q \gg 1 \end{cases}$$

- free fermion has two conf. BCs

$$\hat{\Delta}_Q = \frac{1}{2}\Delta_{2Q}$$

- neutral bulk + charged bdy

4d example: $S_{EFT} = S_{\text{strongly coupled}} + \int_{z=0} d^3x \, c(\partial\chi)^3 + \dots$

$$+ \int_{z=0} d^3x \, c_{\hat{D}} \, \hat{D}/(\partial\chi) \leftarrow \text{irrelevant}$$