The Large Charge Sector of 3D parity violating CFTs

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arXiv: 2102.05046

Outline

- EFT without parity
  - Euler term at $\mathcal{O}(\partial^1)$
  - Vortices on the sphere
  - Spectrum and OPEs
- Anyon Superfluids
  - Derivation of the leading order EFT on a plane
  - Monopole operators
  - Vortices on the sphere as holes
  - Beyond EFT - Chern-Simons terms, double scaling, rotons

Large $U(1)$ charge EFT

$$S \sim c_1 \int d^3x \sqrt{g}\, |\partial\phi|^3 + \mathcal{O}(\partial^2) \quad \Leftrightarrow \quad S \sim \frac{1}{\sqrt{c_1}} \int d^3x \sqrt{g}\, |f|^{3/2} + \mathcal{O}(\partial^2)$$

Currents	ϕ	a_μ
$U(1)$	$\lvert\partial\phi\rvert^2 \partial_\mu \phi$	$\epsilon_{\mu\nu\rho} f^{\nu\rho}$
$U(1)^{(1)}$	$\epsilon_{\mu\nu\rho} \partial^\rho \phi$	$f_{\mu\nu}/\sqrt{\lvert f\rvert}$

Hellerman, Orlando, Reffert, Watanabe ’15
Monin, Pirtskhalava, Rattazzi, Seibold ’16

EFT without parity

- J^0_{Euler} integrated over space gives the Euler characteristic
- $\kappa \in \mathbb{Z}$ is quantized
- κ controls Hall viscosity
- Euler term breaks one-form symmetry
- Equation of motion: $\nabla_\mu J^{\mu\nu}_{1-form} = \kappa J^\nu_{Euler}$

$$S \sim \frac{1}{\sqrt{c_1}} \int d^3x \sqrt{g}\, |f|^{3/2} + \kappa \int d^3x \sqrt{g}\; a_\mu J^\mu_{Euler}$$

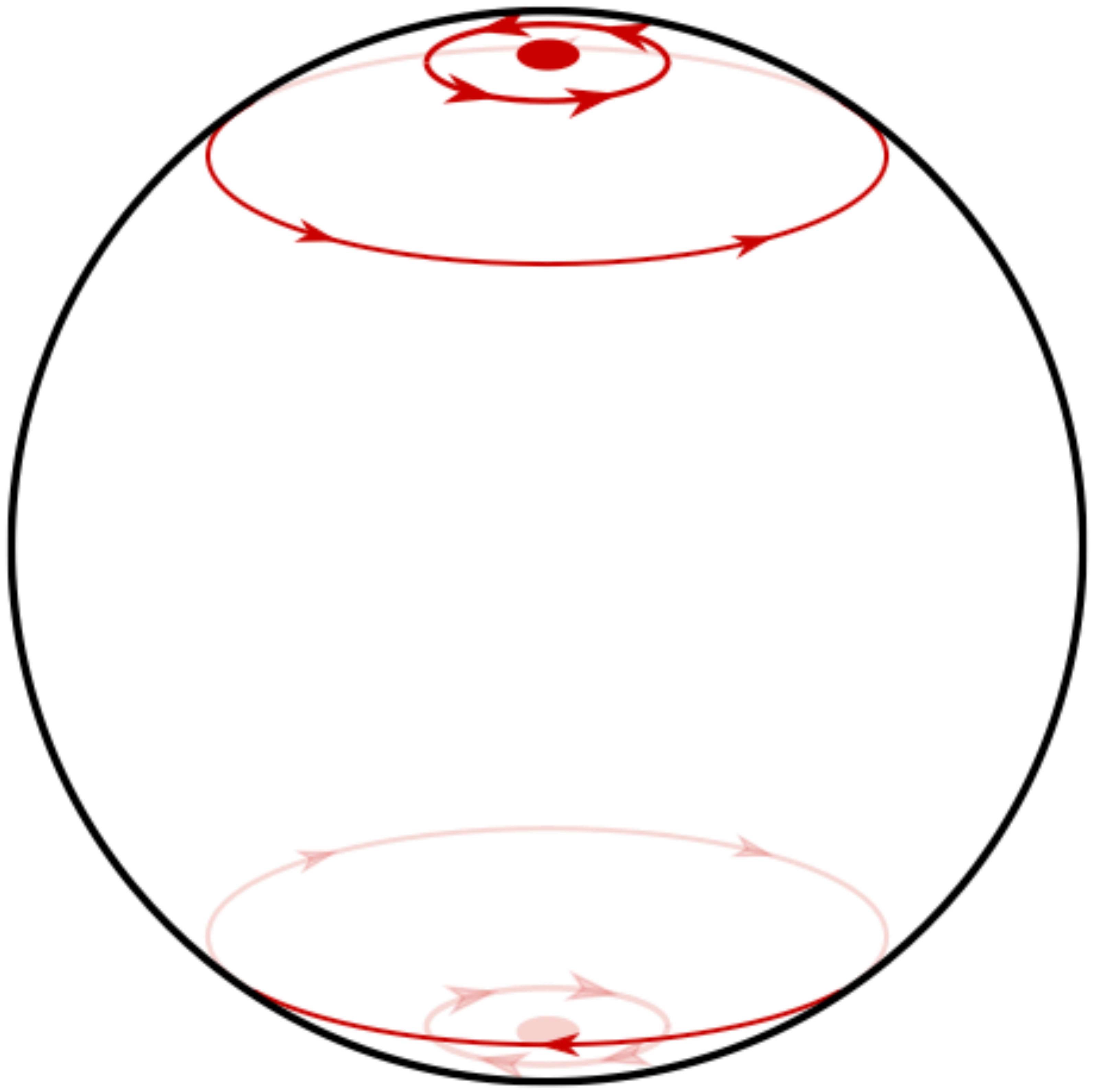
$$J^\mu_{Euler} = \frac{1}{8\pi} \epsilon^{\mu\nu\rho} \epsilon^{\alpha\beta\gamma} u^\alpha \left(\nabla_\nu u^\beta \nabla_\rho u^\gamma - \frac{1}{2} \mathscr{R}_{\nu\rho}{}^{\beta\gamma} \right)$$

$$u^\alpha = \frac{\epsilon^{\alpha\beta\gamma} f_{\beta\gamma}}{\sqrt{2}\,|f|}$$

Golkar, Roberts, Son '14

EFT on the sphere

- Integrated e.o.m. $\implies 0 = \kappa\chi_{S^2}/2 = \kappa$
  need κ vortices $\cong$ point charges
- $S_{vortices} = -\sum_{p=1}^{\kappa}\int_{X_p} a + \dots$
- Vortex angular momentum: $\vec{J} = \sum Q\vec{X}_p/2$
- Coulomb interaction: $H = -\sum \log|\vec{X}_p - \vec{X}_{p'}|^2$



Golkar, Roberts, Son '14

Spectrum: classical ground state

- Energy minimization = Whyte's problem - no general solution known except in special cases $\kappa = 1,2,3,4,5,6,12$

- Theorem: center of mass at sphere origin when $\kappa \neq 1$

- Main result: lightest operator has $J = 0$ for $\kappa \neq 1$ and $J = Q/2$ for $\kappa = 1$ with dimension

$$\Delta_{min}(Q) = \frac{1}{\sqrt{c_1}} Q^{3/2} + \kappa \sqrt{c_1} \sqrt{Q} \log Q + \dots$$

- Inhomogeneous ground state, large degeneracy

Dragnev, Legg, Townsend '02

see also Cuomo, de la Fuente, Monin, Pirtshkalava, Rattazzi, '17

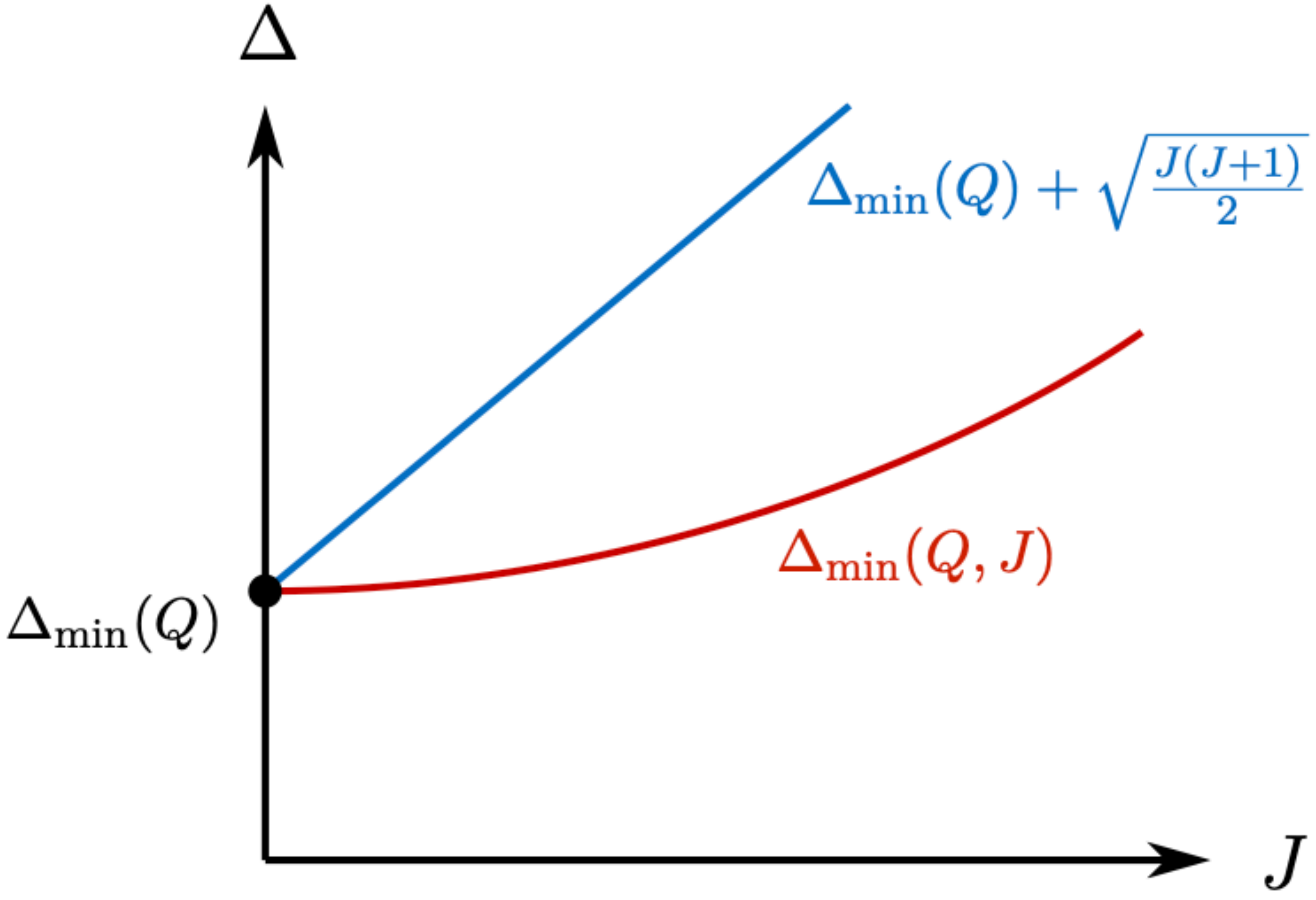
Spectrum: quantum ground state and vortex excitations

- Quantization $\Longrightarrow$ Landau levels on sphere (monopole harmonics)

- Quantization lifts degeneracy, eg $\kappa = 2$

$$\Delta_{min}(Q,J) = \Delta_{min}(Q) + \sqrt{c_1}\frac{J(J+1)}{Q^{3/2}}$$

- Much softer that phonon excitations with $\Delta_{Q,J} - \Delta_{min} = \sqrt{J(J+1)/2}$



Wu, Yang ’76

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Monin, Pirtskhalava, Rattazzi, Seibold ’16
Jafferis, Mukhametzhanov, Zhiboedov ’17

OPE coeffs and Hall viscosity

- Kubo formula: $\eta_H \sim \langle \bar{Q} T_{xy} T_{xx} Q \rangle$. The stress tensor is linear in Goldstone fluctuations

$$T \sim 1 + c_1 \partial\pi + \kappa \partial\partial\pi$$

- Therefore, we expect $\langle \bar{Q} T Q' \rangle_{odd} \sim \eta_H$ where $Q' = Q$ + single phonon

$$\langle \bar{Q} T Q_J \rangle_{even} \sim \frac{Q^{3/2}}{c_1^{1/4}} J^{1/2} \qquad \langle \bar{Q} T Q_J \rangle_{odd} \sim \kappa c_1^{1/4} Q^{3/2} J^{3/2}$$

Avron, Seiler, Zograf '95
Bradlyn, Goldstein, Read '12

Hellerman, Orlando, Reffert, Watanabe '15
Monin, Pirtskhalava, Rattazzi, Seibold '16
Jafferis, Mukhametzhanov, Zhiboedov '17

Candidate theories

- $U(N)_{k_1,k_2}$ or $SU(N)_k$ Chern-Simons matter theories?
- Global $U(1)$ - topological or fermion number respectively
- Seem to be a Fermi liquid at strict $N = \infty$, possibly superfluid for finite N
- Superfluid for $N = 1,\ k \to \infty$ - “anyon superfluids”

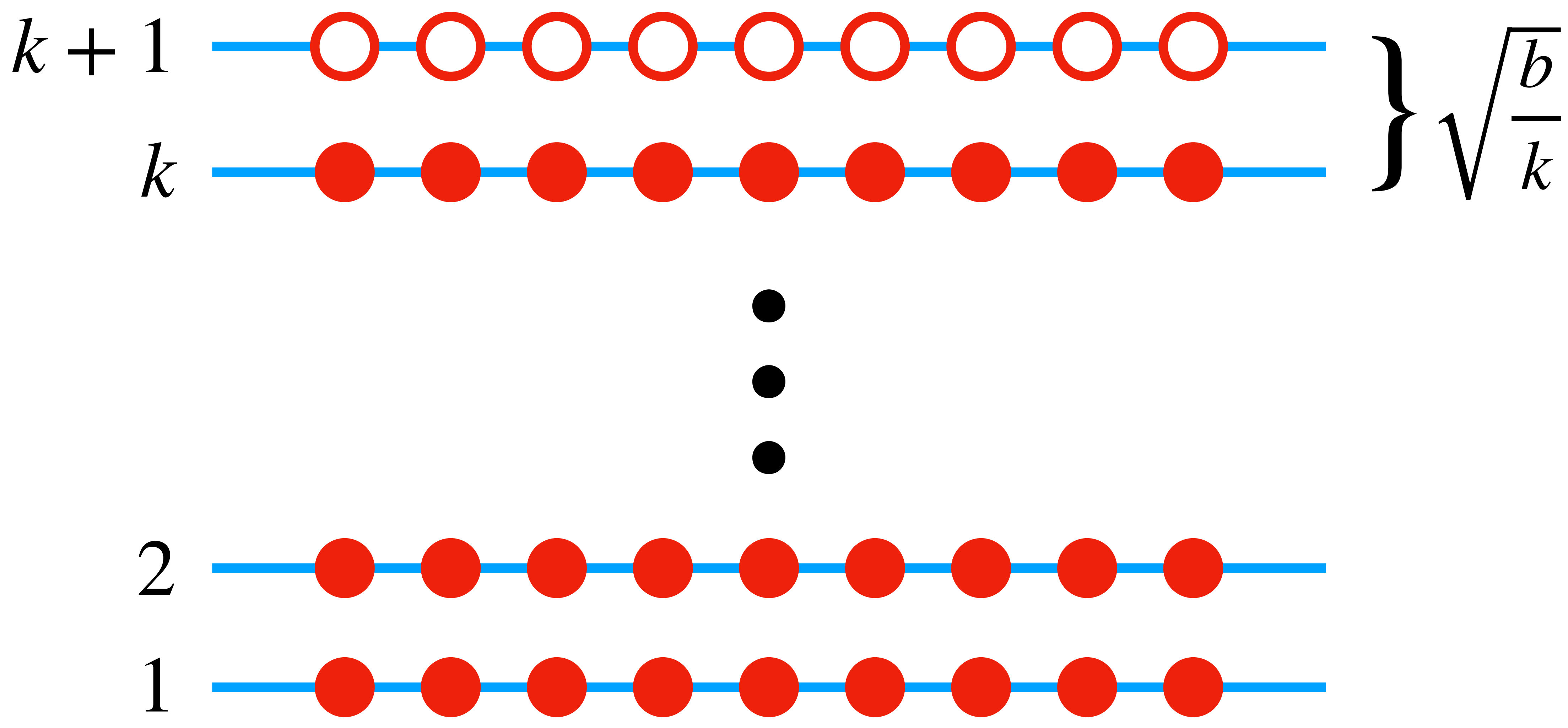
Geracie, Goykhman, Son ’15
Minwalla, Mishra, Prabhakar ’20

Chen, Wilczek, Witten, Halperin ’89
Wiegmann ‘90
Abanov, Wiegmann ’99, ’20

Anyon Superfluids - flat space

$$S = \int d^3x \left(\bar{\psi} i \gamma^\mu D_\mu \psi - \frac{k}{4\pi} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \right)$$

- Global $U(1)$ current $j^\mu = \frac{1}{4\pi} \epsilon^{\mu\nu\rho} f_{\nu\rho}$
- Gauss law: $n_\psi = \bar{\psi}\gamma^0\psi = kj^0 = \frac{kb}{2\pi}$
- k fully filled Landau levels
- Hall conductivity $\sigma_{xy} = \frac{k}{2\pi}$



Leading order EFT

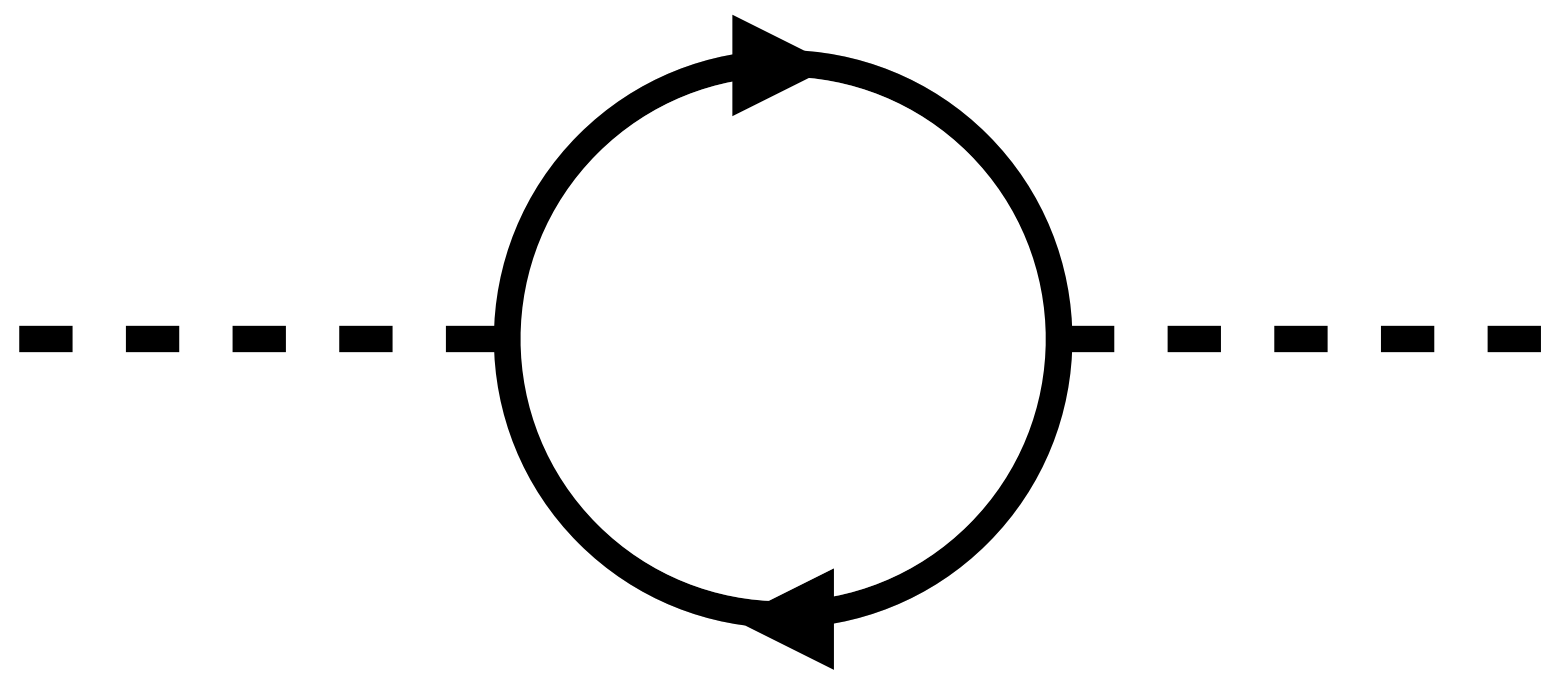
- Integrate out fermions to get

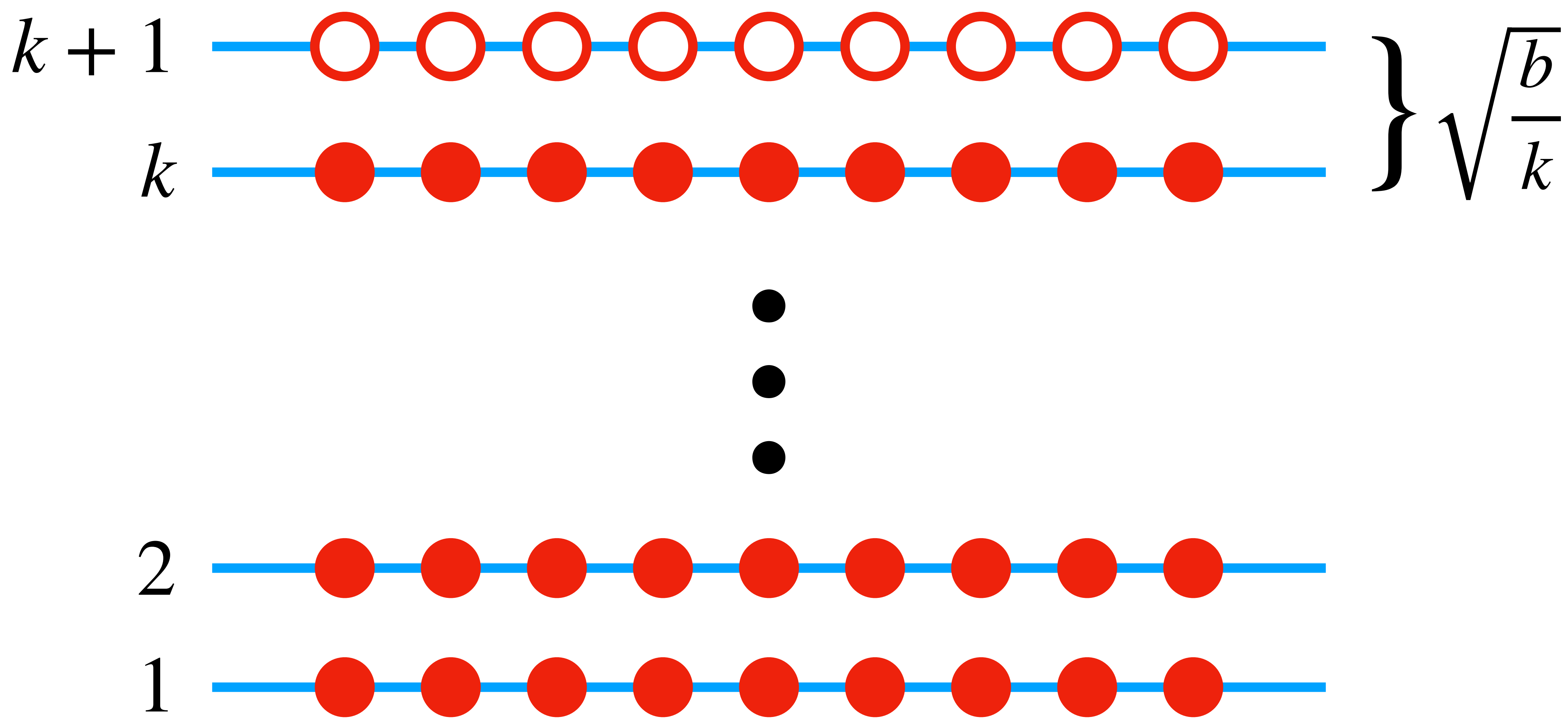
$$\delta S = \frac{k}{4\pi}\int ada$$

- Cancels CS term and generates higher order terms

$$S_{EFT} = \frac{k^{3/2}}{\sqrt{2\pi\rho}}\int\left(e^2 - \frac{1}{2}b^2\right) + \ldots \sim \int |f|^{3/2}$$

- Read off $c_1 = 1/2\pi k^3$





Nguyen, Gromov '16

Monopole operators and double scaling

- Charged operators are monopoles. Operator dimension from EFT ($1 \ll k \ll Q$) is given by

$$\Delta_{min}(Q) = \frac{2}{3}(kQ)^{3/2} + \mathcal{O}(1/k, k/Q)$$

from higher derivs

from higher loops

- Opposite limit $Q \ll k$

$$\Delta_{min}(Q) = \frac{2}{3}(kQ)^{3/2} + \ldots$$

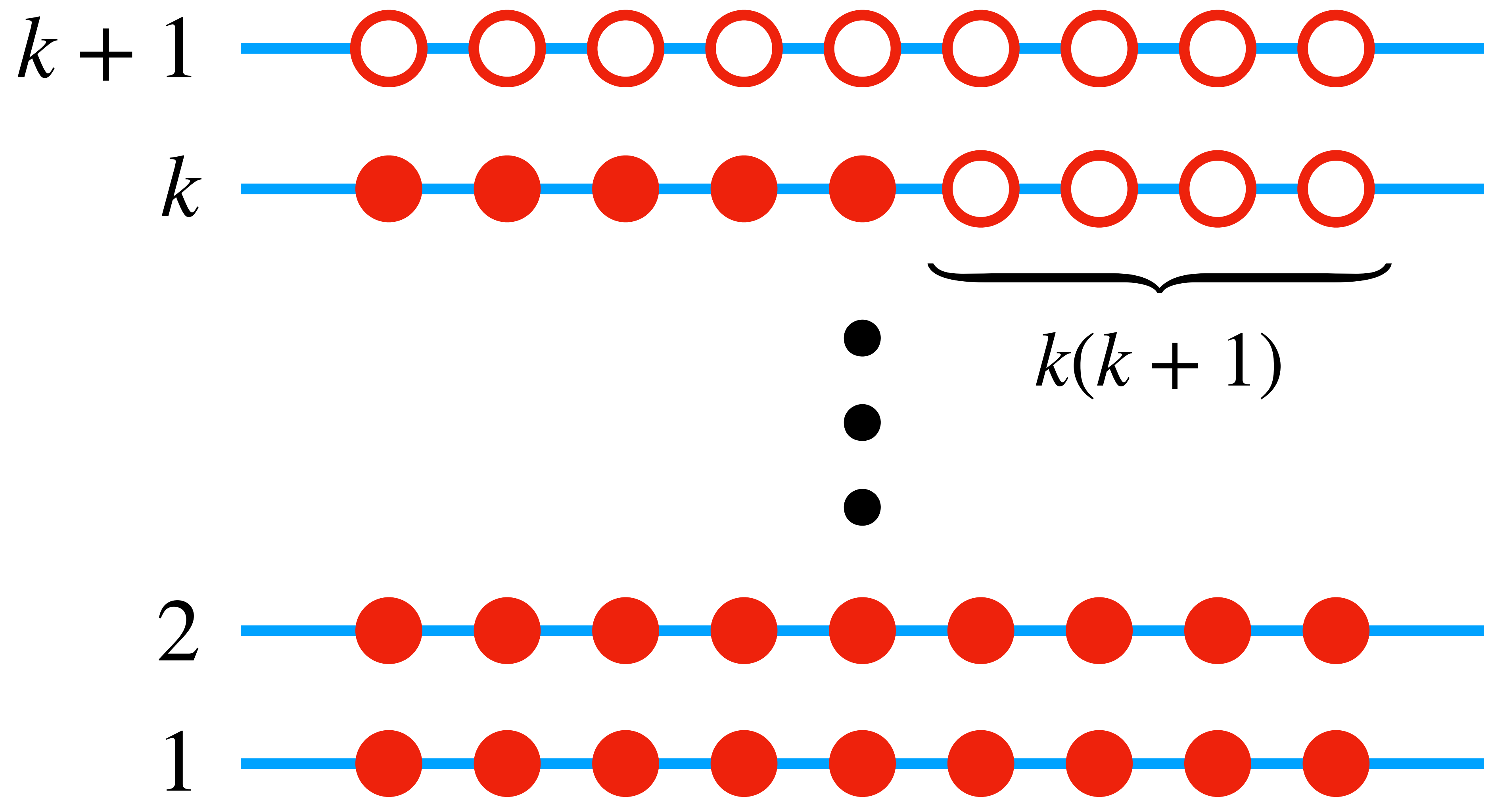
- Double scaling limit: $Q, k \to \infty$, Q/k fixed - organize Dirac particles in Landau levels obtained from the monopole background at leading order

$$\Delta_{min}(Q) = \frac{2}{3}k^3\left(\frac{Q}{k}\right)^{3/2}\left[1 + \mathcal{O}\left(\frac{1}{k}\right)\right]$$

Radicevic ’15
Chester, Iliesu, Mezei, Pufu ’17

Anyon superfluids on the sphere

- kQ Dirac fermions in sphere Landau levels
- Degeneracy $Q + 2|p|, p \in \mathbb{Z}$. k fully filled LLs requires $kQ + k(k+1)$ fermions
- $k(k+1)$ holes on the last one - superfluid vortices
- Identify $\kappa = k(k+1) \gg 1$



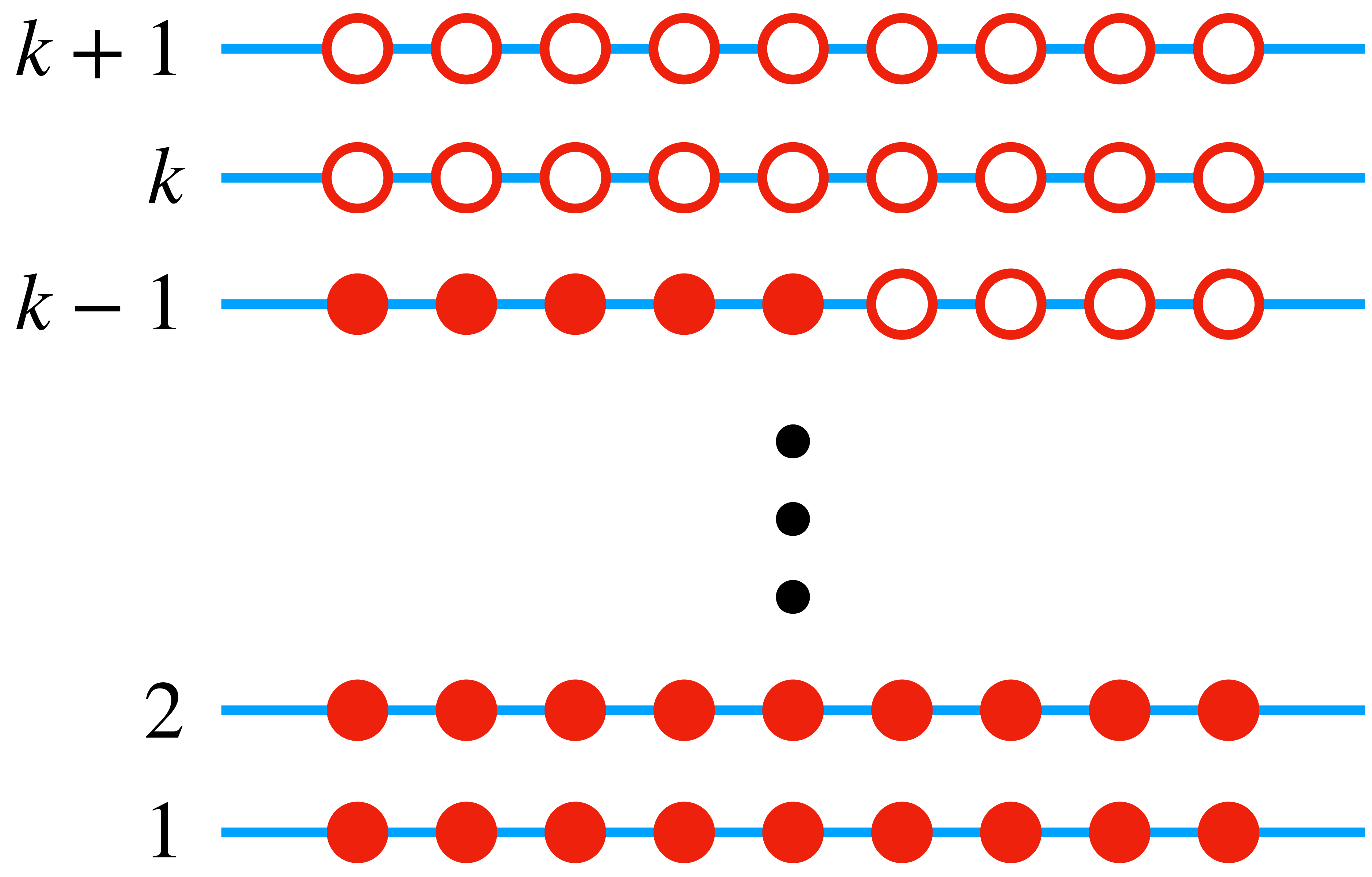
(assumed $k^2 \simeq \kappa \lesssim Q$)

Beyond EFT: Chern-Simons terms

- What happens when $k \ll Q \lesssim k^2$?
- Number of filled LLs $\sim k - \lfloor k^2/Q \rfloor$
- Integrating out ψ leaves residual Chern-Simons term

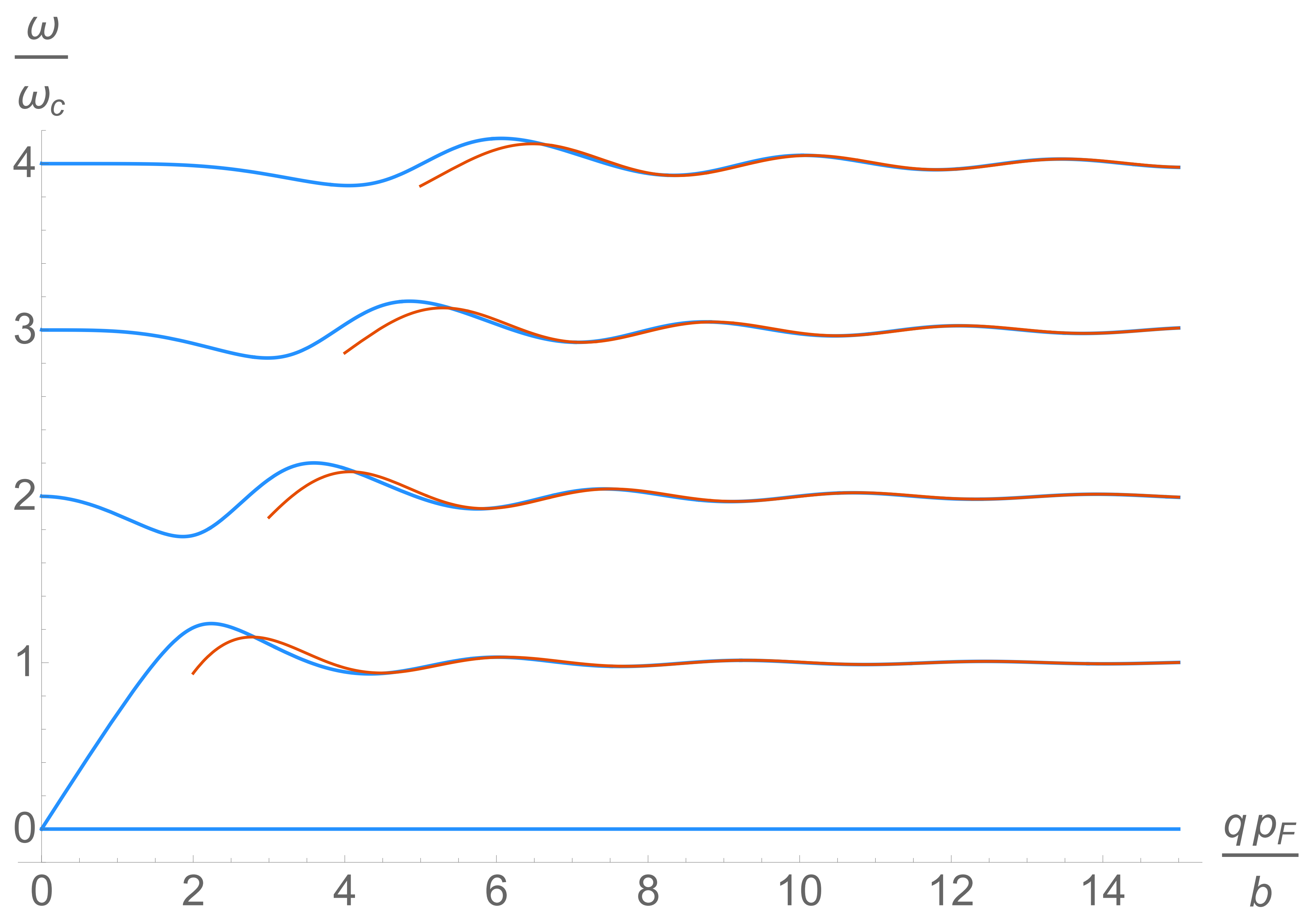
$$S_{EFT'} = S_{EFT} - \frac{\lfloor k^2/Q \rfloor}{4\pi} \int ada$$

- $\kappa \neq N_{vortices} \simeq k^2 - \lfloor k^2/Q \rfloor Q$; coefficient of subleading term in $\Delta_{min}(Q)$ changed from κ to $N_{vortices}$
- Superfluid phonons are gapped; $m_{phonon} \sim \sqrt{k/\rho} \ll 1/R_{S^2}$; gap invisible on sphere



Beyond EFT: rotons?

- Particle-hole excitations set the cutoff scale for EFT (and EFT') to $\sqrt{\rho/k}$
- Can compute flat space spectrum beyond EFT at weak coupling using a bosonized Fermi surface approach - "rotons"
- Rotons on the sphere? What CFT operators do they correspond to?



Du, **UM**, Son '20

Thank you!