Regge trajectories for $\mathcal{N}=(2,0)$ Superconformal Field Theories

based on 2105.13361 w/ B. van Rees and X. Zhao

Outline :

1. Introduction & highligts
2. Analyticity in spin & inversion formula
3. 6d $\mathcal{N}=(2,0)$ SCFTs

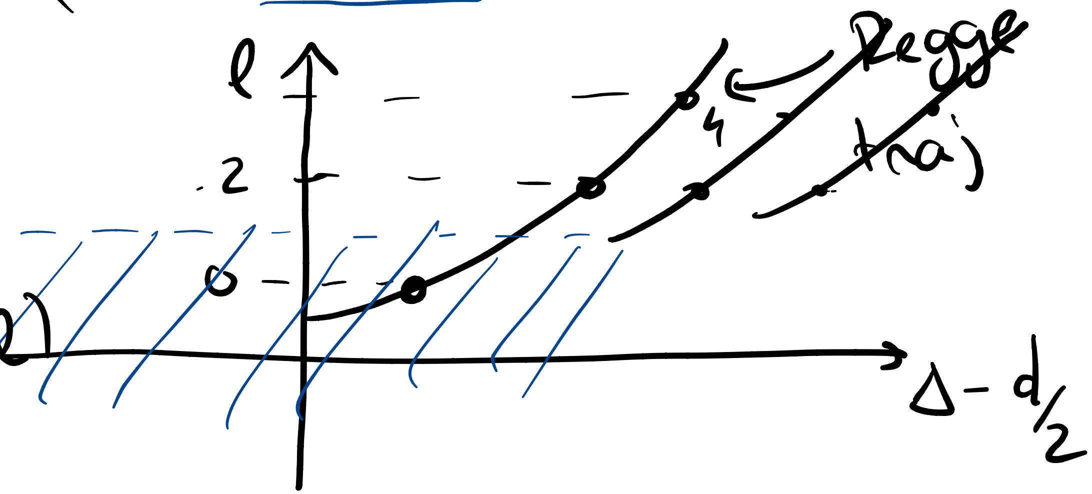
   ↳ Regge trajectories

   ↳ "Bootstrapping" the spectrum

1 Introduction & highlights.

- [Caron-Huot] CFT data organizes itself in Regge trajectories for $\boxed{\ell > 1}$

  $\Delta(\ell)$, $\lambda^2(\ell)$



$$\underline{\Phi \times \Phi} \sim \sum_{\ell, i} \lambda_i^2(\ell)\, \mathcal{O}_{\Delta_i(\ell)}$$

- Supersymmetry

  ↳ supercharges relate diff spins

6d $\mathcal{N} = (2,0)$ $\qquad Q: \ \ell \to \ell + 4$

naively

$\rightsquigarrow$ analyticity down to lower spins

$$\boxed{\ell > -3}$$

Depends on susy amount & on particular operators

↳ Exact results / protected operators

ops. w/ fixed Δ and sometimes λ^2

Will focus on 6d $\mathcal{N}=(2,0)$ SCFTs but a lot of what I'll say applies to other dims & susy

Focus on:

4-point function of stress-tensor multiplets

Analyticity in spin + SUSY !

- All ops. lie on Regge trajectories
  - analyticity for $\ell > -3$
- Intricate relations between trajectories of unprotected & protected ops

  Long $(\ell = \underline{-2}) \propto$ stress tensor multiplet
- Iterative procedure to "bootstrap" 4pt-functions through inversion formula

2. Analyticity in spin ⟿ Inversion formula [Caron-Huot]

$$x_{12}^{2\Delta} x_{34}^{2\Delta} \langle \underbrace{O(x_1) O(x_2)} O(x_3) O(x_4) \rangle \overset{(12)(34)\text{ OPE}}{=} g(z,\bar{z}) = \sum_{\ell=0}^{\infty} \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i} \, C(\Delta \ell) \, g_{\Delta \ell}(z,\bar{z})$$

$$+ (\text{non-norm})$$

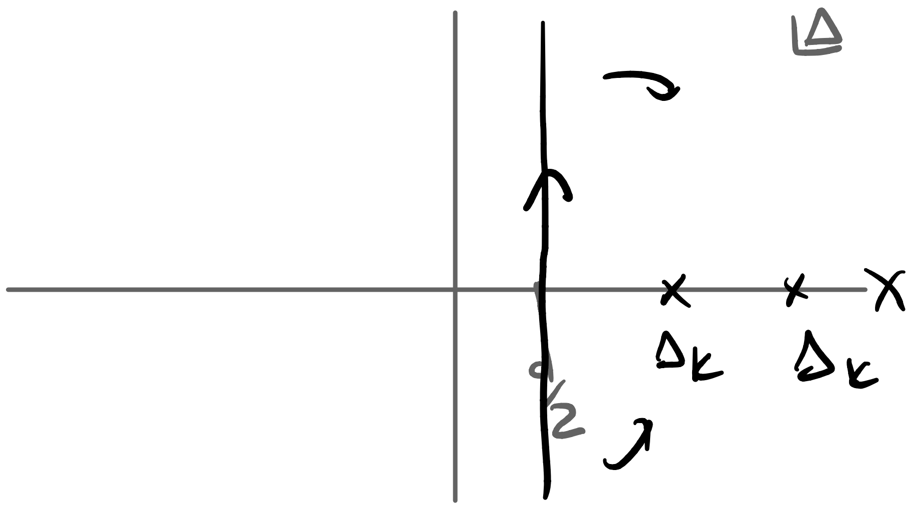
CFT data packaged in:

residues $\propto$ OPE coeff.

$$C(\Delta,\ell) \sim -\sum_k \frac{\lambda_{OOO_k}^2}{\Delta - \Delta_k}$$

poles where ops. are

Can be obtained through:



Euclidean inversion formula:

$$C(\Delta,\ell) = \int_{\text{Euclidean}} \text{Kernel} \; \underbrace{\langle O_1 O_2 O_3 O_4 \rangle x_{12}^{2\Delta} x_{34}^{2\Delta}}_{g(z,\bar{z})}$$

- valid for $\ell \in \mathbb{N}$
- Need to know full correlator

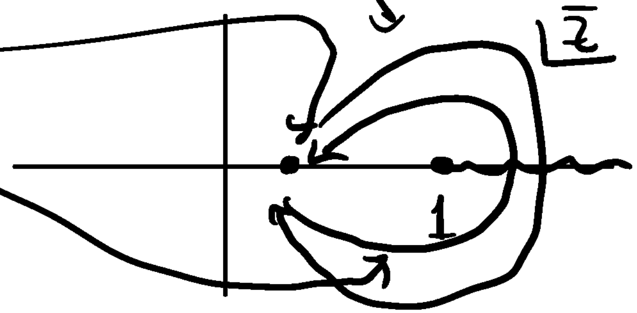
Lorentzian inversion formula [Caron-Huot]

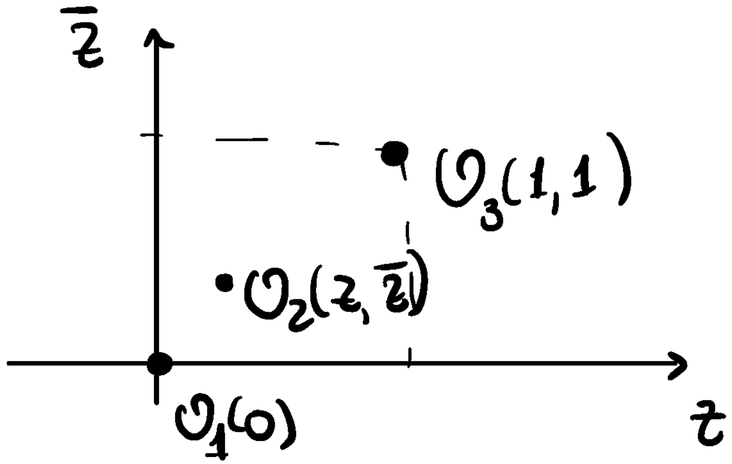
$$C(\Delta,\ell) = \int_{\text{Lorentzian}} \text{Kernel} \; \mathrm{dDisc}\left[g(z,\bar{z}) \right]$$

• No longer need $\ell \in \mathbb{N}$ — analytically con't CFT data in ℓ

Some arcs were dropped: $\boxed{\ell > 1}$ (w/o susy)

• dDisc: $g(z,\bar{z}) - \frac{1}{2}\left(g^{\circlearrowleft} + g^{\circlearrowright}\right)$





• $\mathcal{O}_4(\infty)$

Euclidean: $z = (\bar{z})^*$

Lorentzian, $z, \bar{z} \in \mathbb{R}$ & ind.

- represent $g(z,\bar{z})$ through t-channel OPE

$$\langle \mathcal{O}_1 (\mathcal{O}_2 \mathcal{O}_3) \mathcal{O}_4 \rangle$$

- inverting s-channel OPE:

$$\langle (\underline{\mathcal{O}_1 \mathcal{O}_2})(\mathcal{O}_3 \mathcal{O}_4) \rangle$$

- Nice props :

↳ $\ell \gg 1$ Kernel projects integral to

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle$$

approaching lightcone

$\mathcal{O}_2 \mathcal{O}_3$ OPE dominated by low twist $\Delta - \ell$

↳ Perturbation around generalized free theories: $(\mathcal{O}\,\Box^n \partial_{\mu_1} \ldots \partial_{\mu_\ell} \mathcal{O})$

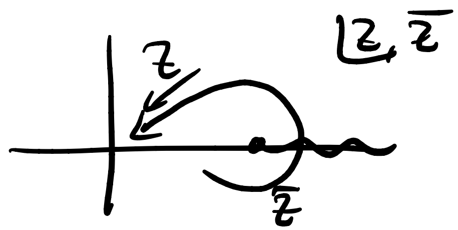
$$\mathrm{dDisc}\left[\begin{array}{c}\text{ops w/}\\ \Delta = 2\Delta_{\mathcal{O}} + 2n + \ell\end{array}\right] = 0$$

$$\mathrm{dDisc}\left[\Delta = 2\Delta_{\mathcal{O}} + 2n + \ell + \gamma\right] \propto \sin^2\left(\frac{\gamma\pi}{2}\right)$$

Finitely many ops have $\neq 0$ dDisc in pert theory (e.g. large N/c or ε-expansion [...])

- very nice results
- downside: for $\langle \ldots \rangle^{(n)}$ valid for $\ell > \#$
  ↳ low spin ambiguities

Rigidity in ℓ is related to Regge limit



[Caron-Huot]

$z, \bar z \to 0$ fixed $z/\bar z$

second sheet

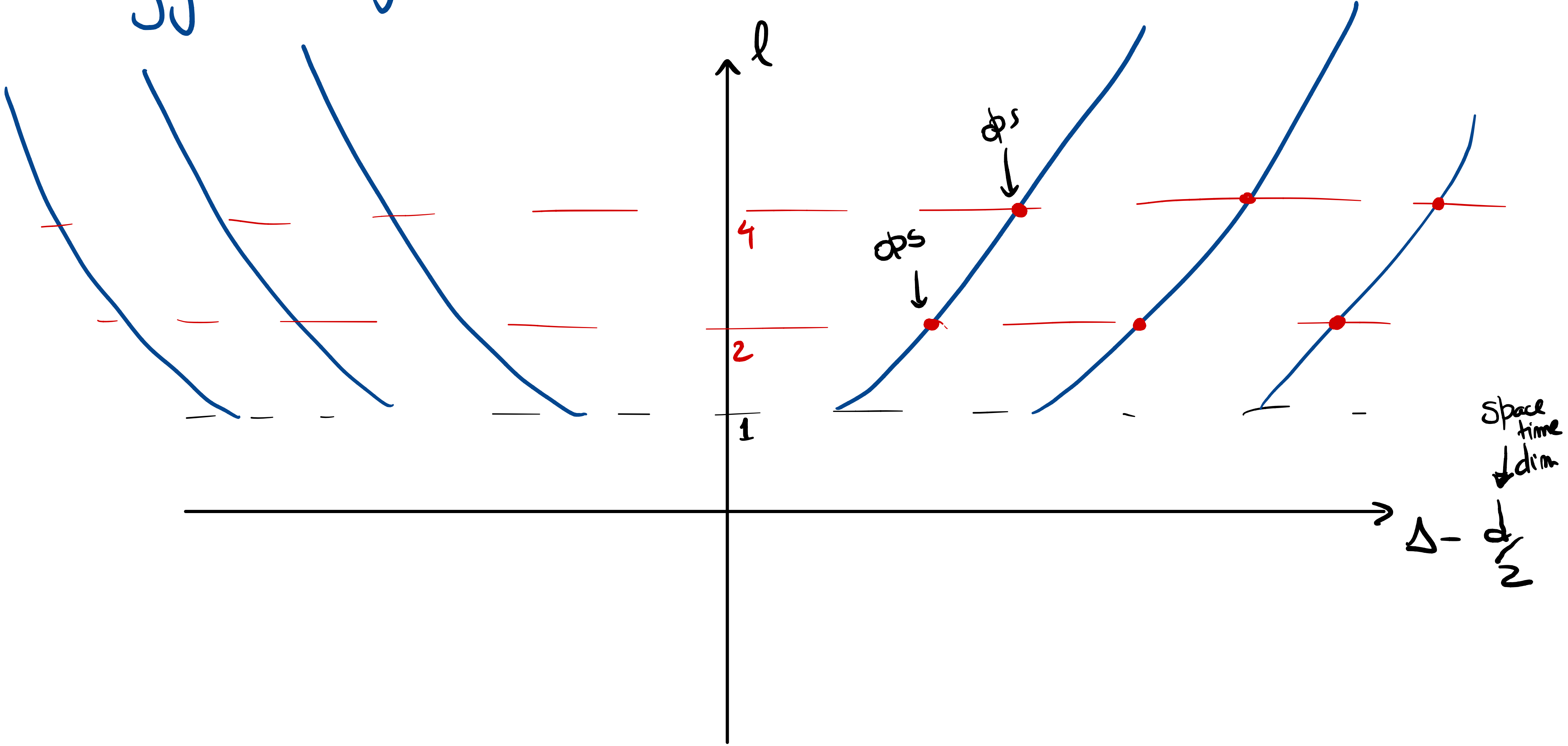
- 4-point-functions bounded by constant in this limit (use t-, u- channel)

- single s-channel block $\sim \frac{1}{z^{\ell-1}}$

- Regge growth of perturbative correlators behaves worse in Regge

Regge trajectories

$\mathcal{O}_1 \mathcal{O}_2$ OPE:



(Assuming identical ops $\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \to$ exchange even spin)

Add SUSY   Target: 6d $\mathcal{N}=(2,0)$ SCFTs

16 Q   16 S   - max susy in max dim.

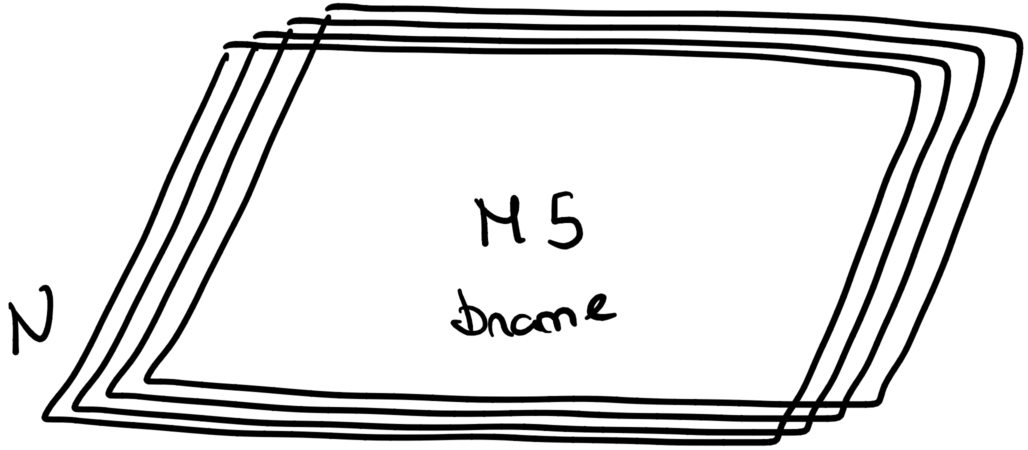
- Found in 90s as various decoupling limits of string & M-theory

- Central role: many lower dim theories obtained from its compactification

- No Lagrangian description available

- ADE classification: A_N, D_N, $E_{6,7,8}$

  w/ known $c \propto \langle T_{\mu\nu} T_{\rho\sigma} \rangle$



$\rightarrow$ low energy effective theory A_{N-1} + free tensor

- Isolated fixed point: no susy relevant or marginal defs. [Cordova Dumitrescu]

- Chiral Algebra of [Beem, Rastelli, van Rees] fixes ∞ many OPE coeffs. – chiral algebra short ops

- Numerical bootstrap [Beem, ML, Rastelli, van Rees]

- Studied in $1/c$ expansion [Arutyunov, Heslop, Sokatchev, Rastelli, Zhou, Alday, Chester, Raj, Lipstein, Abl, Perlmutter, ...]

The Spectrum of (2,0) Theories

At finite c

Study : Stress tensor supermultiplet

superprimary $\Phi_{\underline{14}} \xleftrightarrow{Q} T_{\mu\nu}$

$\Delta = 4$, scalar

$so(5)_R \sim$ R-symm. algebra

$\langle \Phi_{\underline{14}}(x_1)\, \Phi_{\underline{14}}(x_2)\, \Phi_{\underline{14}}(x_3)\, \Phi_{\underline{14}}(x_4) \rangle \Rightarrow$ Fixes 4point functions of all ops in supermultiplet
[Dolan, Gallot, Sokatchev]

$$\underline{14} \otimes \underline{14} = \underline{1} \oplus \underline{14} \oplus \underline{10} \oplus \underline{55} \oplus \underline{35}' \oplus \underline{81}$$

OPE of $\Phi_{\underline{14}} \Phi_{\underline{14}}$ has ops. transforming in these irreps

$\Rightarrow$ 6-channels:

conf. primaries

$$A_{\underset{R=1,\dots,6}{}}(z,\bar{z}) = \sum \underbrace{\lambda^2_{R,\Delta,\ell}}\; g_{\Delta,\ell}(z,\bar{z}) \to$$ 6d "normal" conformal blocks

$\rightsquigarrow$ Related by SUSY [Dolan Gallot Sokatchev]

A_R determined by: (through differential operators)

$$\begin{cases} a(z,\bar{z}) \\ h(z) \end{cases}$$

$a(z,\bar{z})$ — protected / unprotected

$h(z)$ – chiral algebra correlator

Can impose theory is interacting!

$h(z)$: chiral algebra correlator

$$h(z) = -\left(\frac{z^3}{3} - \frac{1}{z-1} - \frac{1}{(z-1)^2} - \frac{3}{3(z-1)^3} - \frac{1}{z}\right) - \frac{8}{c}\left(z - \frac{1}{z-1} + \log(1-z)\right) - \frac{1}{6} + \frac{8}{c}$$

$c \leftarrow \langle T_{\mu\nu} T_{\rho\sigma} \rangle$

$\Rightarrow$ OPE coeffs of short/protected operators $\langle O_1 O_2 O_3 O_4 \rangle$

↳ chiral algebra shorts

$$a(z,\bar{z}) = \sum \frac{1}{(z\bar{z})^6} \lambda^2_{\Delta,\ell}\, \boxed{g^{\Delta_{12}=0,\ \Delta_{34}=-2}_{\Delta+4,\ell}(z,\bar{z})}$$

$\Delta_1 - \Delta_2$, $\Delta_3 - \Delta_4$

superconf. blocks have shifts in dims!

$\lambda^2_{\Delta,\ell}$: absorbed some factors here

g: 6d conf blocks

- $\Delta = \ell+4,\ \ell$ — chiral algebra shorts ← know $\lambda^2_{\Delta,\ell}$
- $\Delta = \ell+6,\ \ell$ — non-chiral algebra shorts, unknown $\lambda^2_{\Delta,\ell}$
- $\Delta > \ell+6,\ \ell$ → long: Δ, λ^2 unknown.

Claim:

$\boxed{\text{CFT data contributing to } a(z,\bar{z}) \text{ is analytic in spin for } \ell > -3}$

→ adapted Caron-Huot's inversion formula for $a(z,\bar{z})$

dDisc contributions

chiral algebra shorts: 1, $T_{\mu\nu}$

non-chiral algebra shorts: —

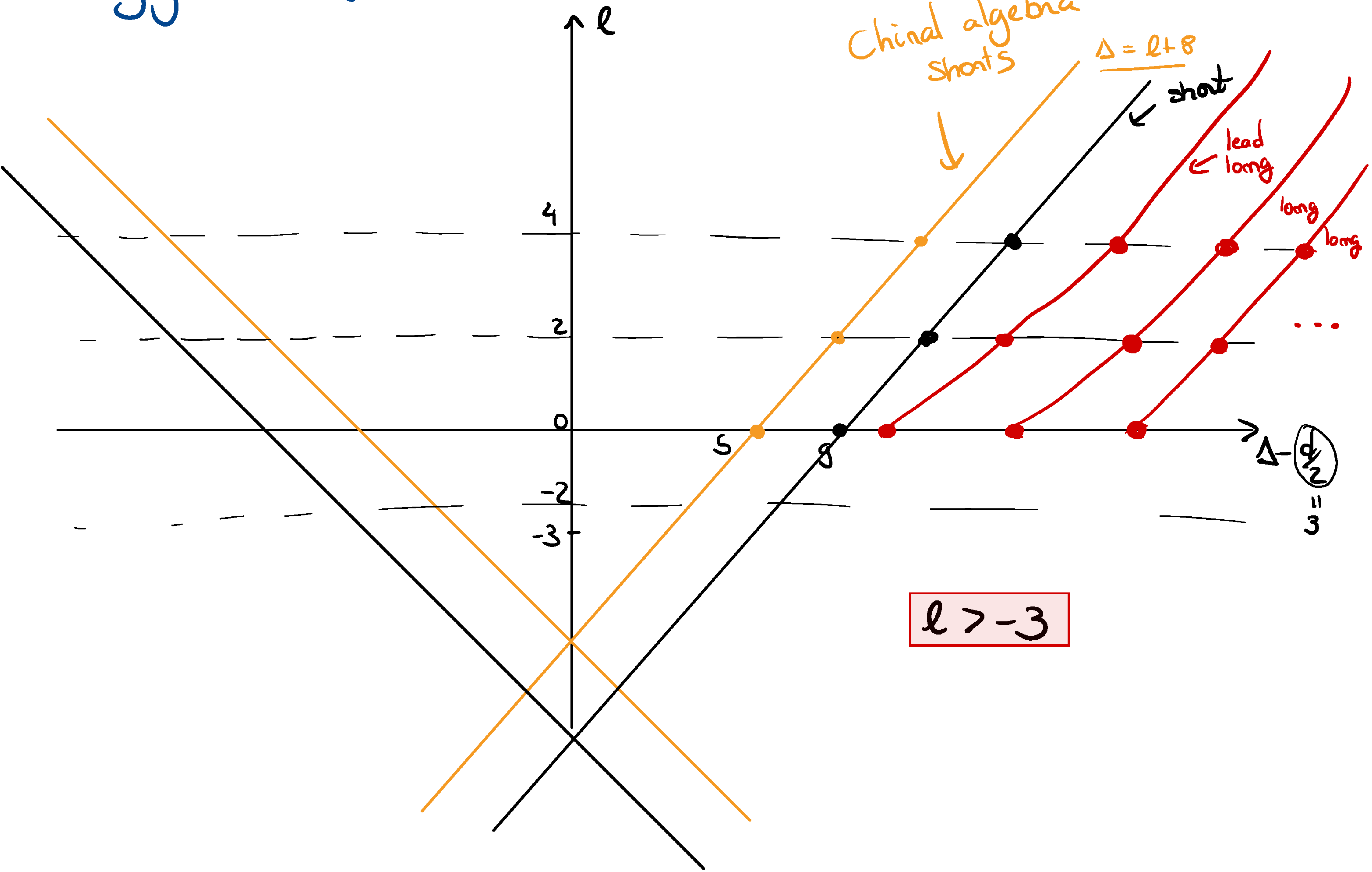
Longs: generic $\Delta \neq 8+2n+\ell$

Inversion output: chiral alg. shorts

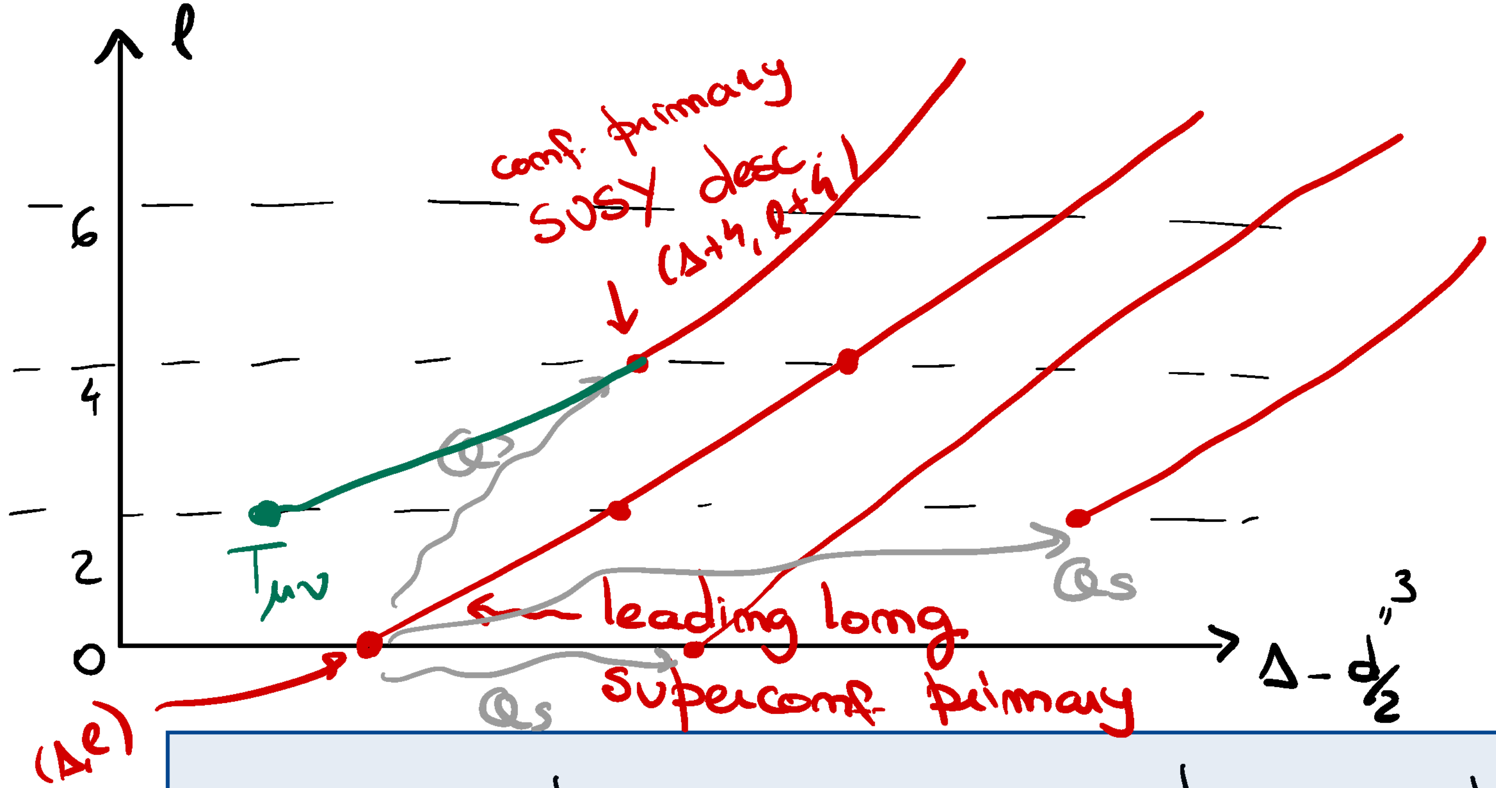
all non-chiral alg. shorts

all longs

Regge trajectories for $a(z,\bar{z})$



Regge trajectories for R = Singlet channel $A_R(z,\bar z)$

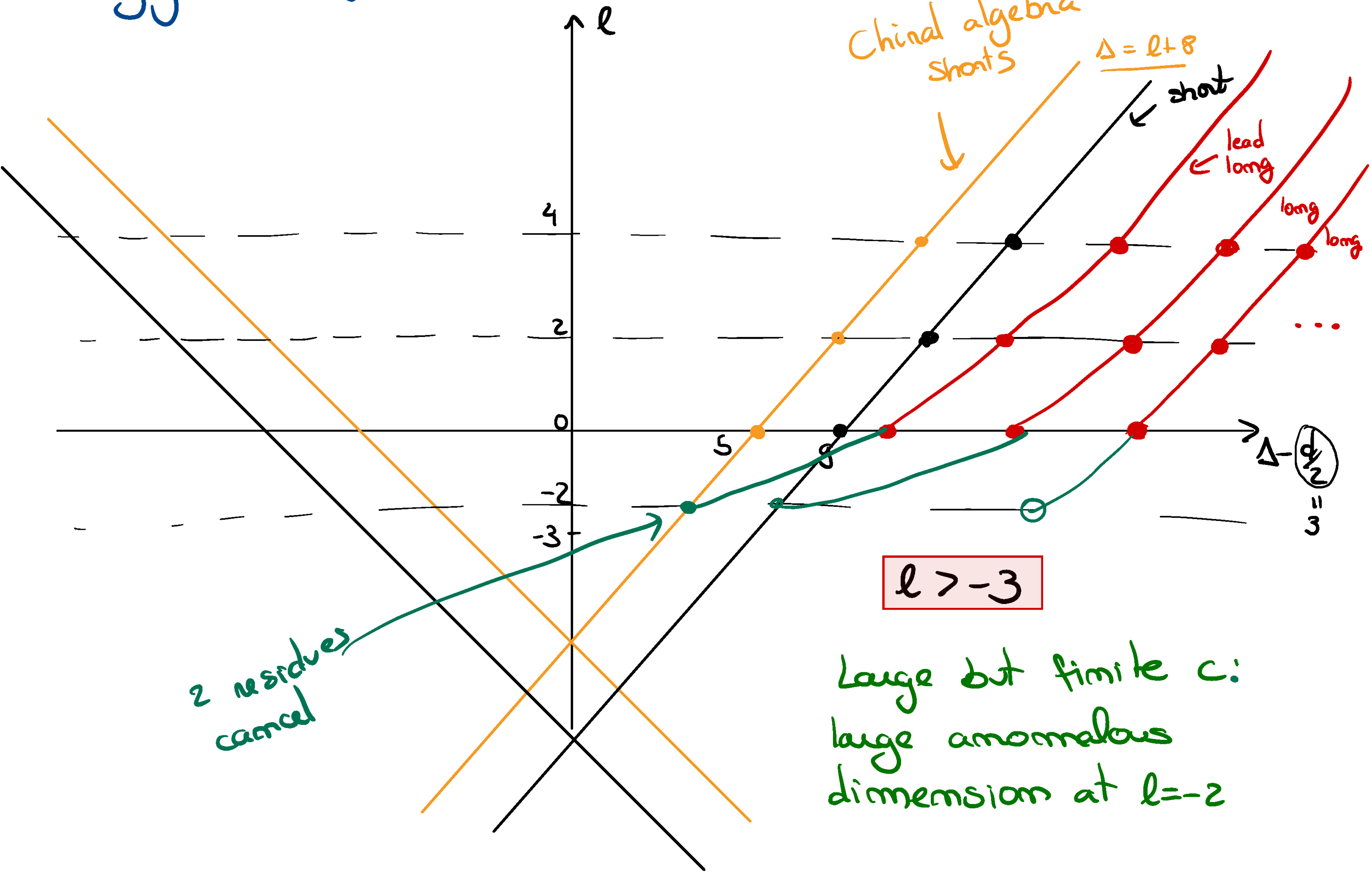


(just longs for simplicity)

$\boxed{\ell > 1}$

The leading long trajectory extends to $\ell = -2$ where it hits $\Delta = 2$ and has residue giving OPE coeff. $\propto -\frac{2}{c}$ $\Rightarrow$ gives stress tensor multiplet

Regge trajectories for $a(z,\bar{z})$



Large but finite c:
large anomalous dimension at $\ell = -2$

- Various "issues" in R-symm. channels lead to various other relations

$$\Phi_{\underline{14}} \times \Phi_{\underline{14}} \sim 1 + \overbrace{\mathscr{D}[2,0] + \mathscr{D}[4,0] + \mathscr{B}[2,0]_\ell}^{\text{chiral algebra shorts}} + \overbrace{\mathscr{D}[0,4] + \mathscr{B}[0,2]_\ell}^{\text{unfixed shorts}} + \text{Longs } (\Delta\,\ell)$$

$$\ell \to -4 \qquad \ell \to -2 \qquad \ell \to -1 \qquad (\Delta, \ell) \to (4, -2) \qquad (\Delta, \ell) \to (2, -2)$$

Bootstrapping the spectrum - results

Input: $1, \frac{1}{c}$ Stress tensor

Inversion

Output: shorts, longs

+ lead long

(non-susy supplement spin 0)

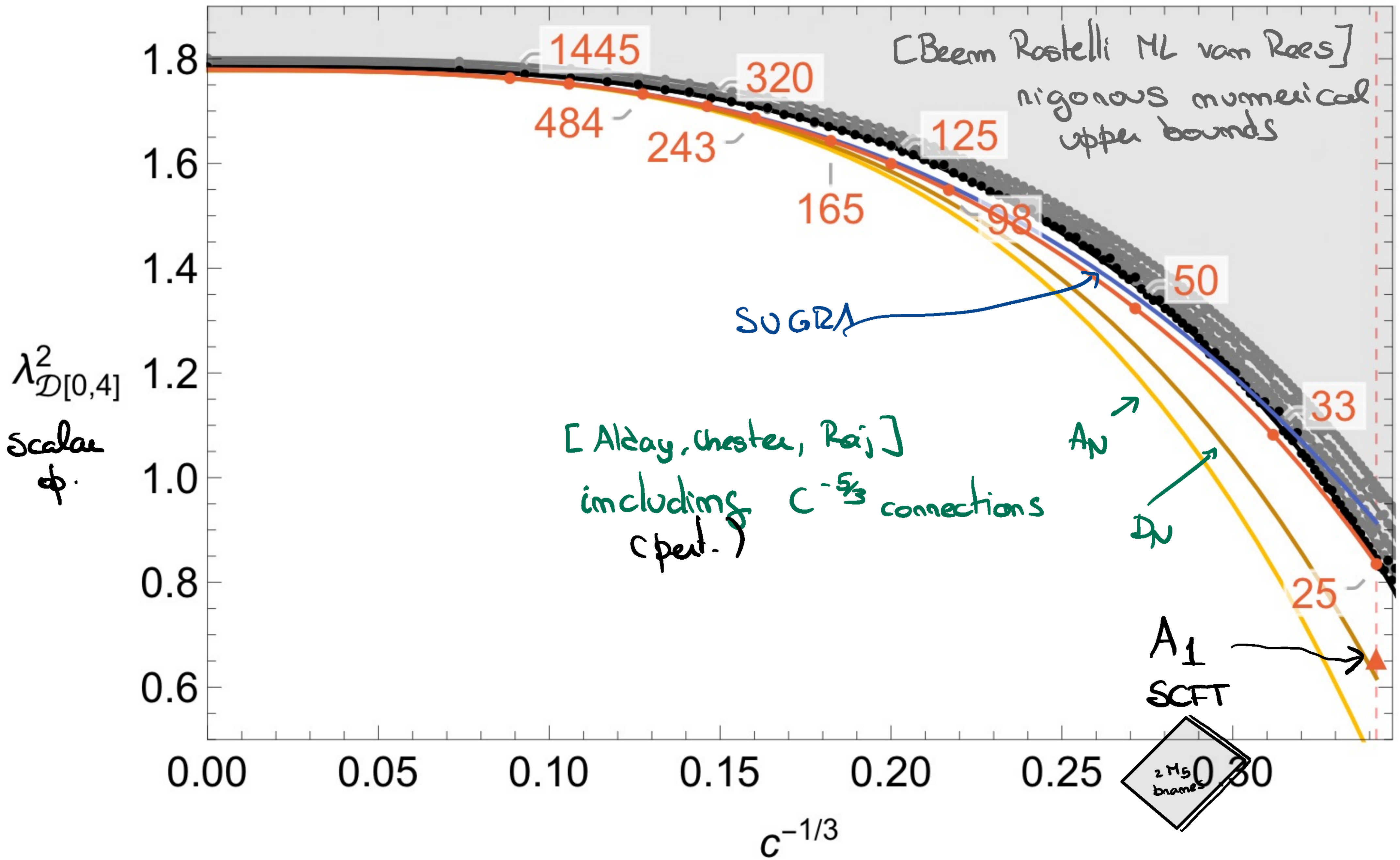
$$c(\Delta,\ell) \sim \int K \, \mathrm{dDisc}[\;\;]$$

Results for:

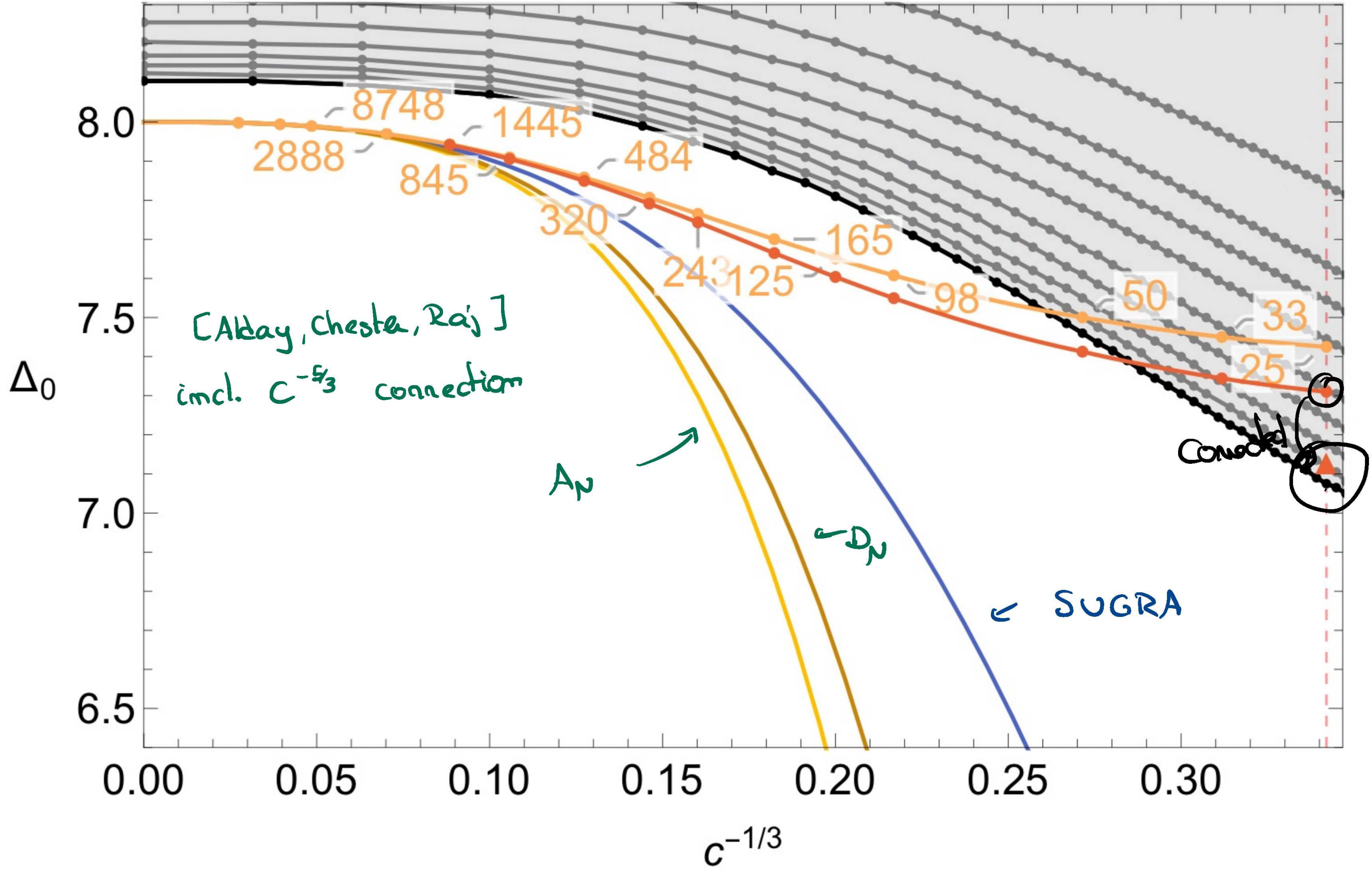
- leading longs of $\ell = 0, 2, 4, 6$ (Δ & λ^2)
- non-chiral algebra shorts OPE coeffs.

Note: inversion does not know crossing!

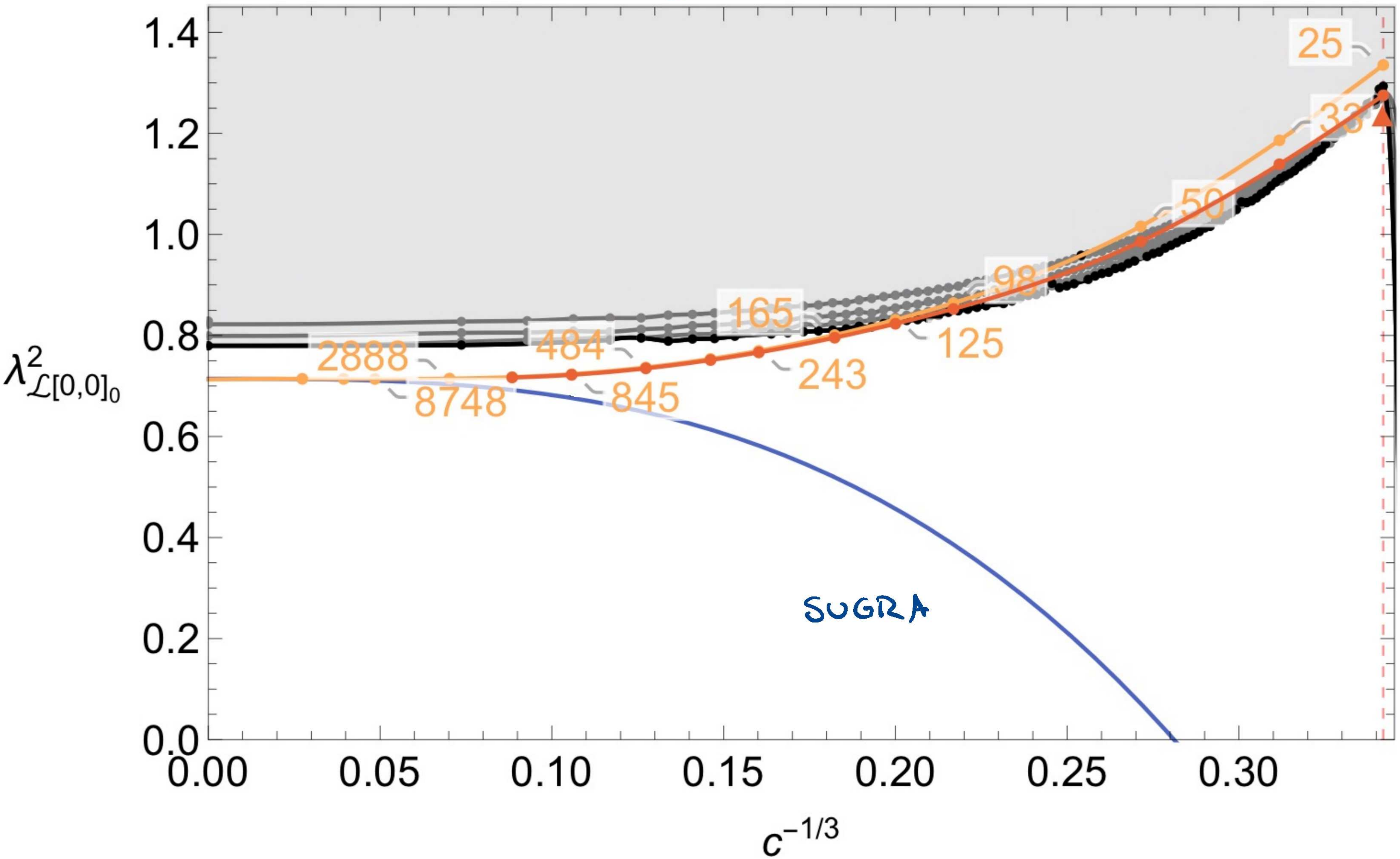
OPE coeff of non-chiral algebra short



Leading scalar long operator



OPE coeff. of leading scalar long



Outlook:

- First exploration of "bootstrapping" the (2,0) theories w/ inversion
  - incorporate better numerics / subleading traj.
  - incorporate $c^{-5/3}$ coeff. which distinguishes A_μ from D_μ
    2 fixed points of iterative procedure?
  - more correlators
- Would be nice to input structure of Regge trajectories (e.g. long at $\ell = -2$)
- Supersymmetric dispersion rels need no subtractions
  [Caron, Caron-Huot]

  Convergent sum rules of [Caron-Huot, Mazáč, Rastelli, Simmons-Duffin]?

- Short ops w/ unfixed coeffs ~ $\frac{1}{4}$-BPS ring relations for specific theories, e.g. A_1 - absent multiplet
  [Bhattacharyya Minwalla]

- Other SCFTs will have same interplay between Regge trajectories and SUSY

Thank You!