Spinning charged operators in 3d CFTs

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based on 1711.02108 with A. De La Fuente, A. Monin, D. Pirtskhalava and R. Rattazzi
& on a work in progress with Z. Komargodski

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Stony Brook University

SIMONS CENTER FOR GEOMETRY AND PHYSICS

Overview

1. Introduction
2. The conformal superfluid EFT for large charge operators
3. Charged spinning operators and rotating superfluids in 3d CFT
4. Summary

Introduction

Universality of CFT data at large quantum numbers

- Large spin J

Alday Maldacena 2007, Komargodski Zhiboedov 2012, Fitzpatrick Kaplan Poland Simmons-Duffin 2012, Caron-Huot 2017,…

- Large internal charge Q

Hellerman Orlando Reffert Watanabe 2015, Monin Pirtskhalava Rattazzi Seibold 2016,…

- Large scaling dimension Δ

Lashkari Dymarsky Liu 2016, Cardy Maloney Maxfield 2017, Delacretaz 2020,…

Why universality at large quantum numbers?

$$\mathcal{O}_{\mathbb{R}^d}(x) \quad \xleftrightarrow{\textit{State-operator map}} \quad |\mathcal{O}\rangle_{\mathbb{R}\times S^{d-1}}$$

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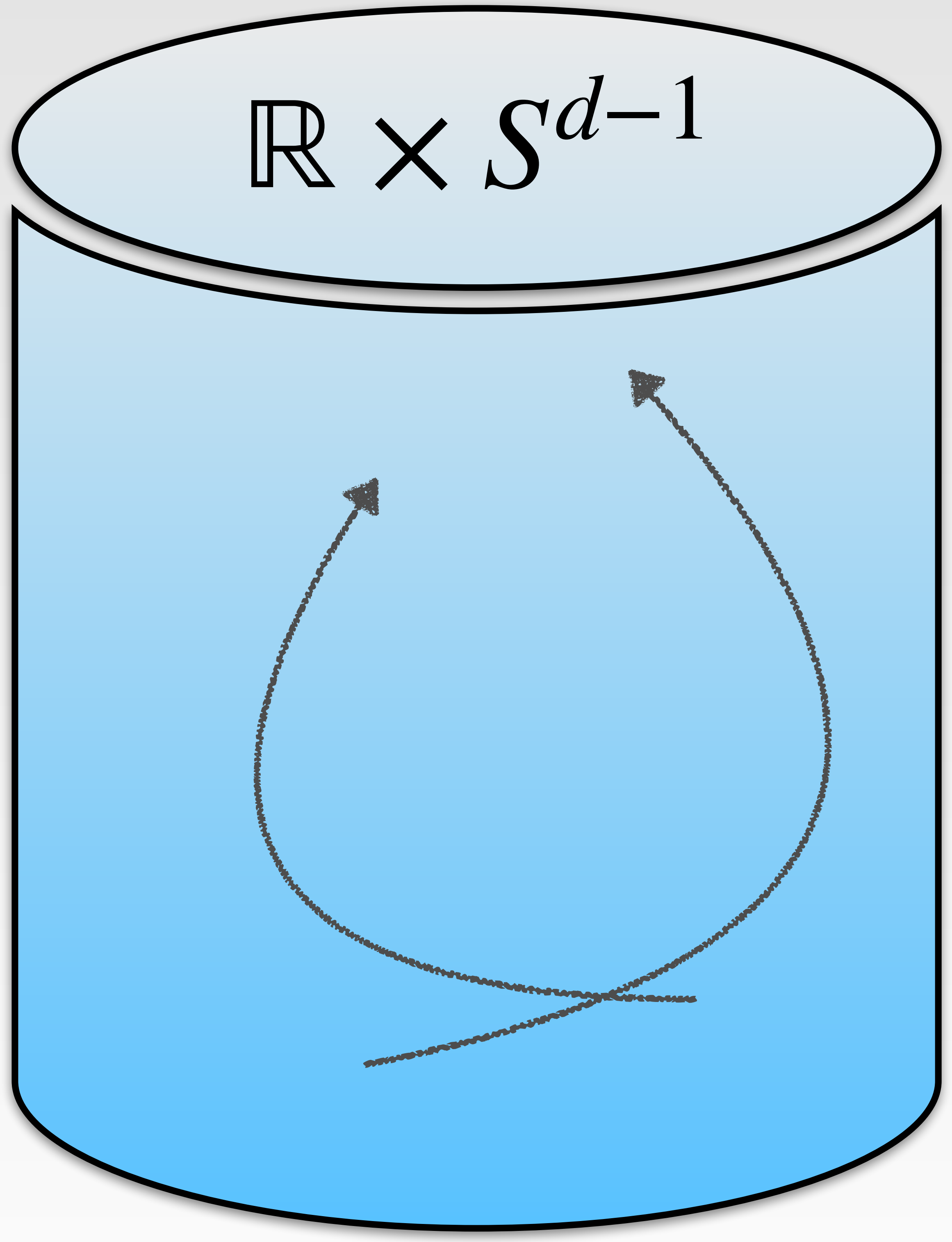
Ex.: Alday-Maldacena "EFT" for large spin double-trace operators

Alday Maldacena 2007

- To leading order, free moving particles
- Corrections due to "Yukawa" potential propagating at distance $\Delta\chi \sim \log J$ in $AdS_3 \times S^{d-3}$



$$\Delta_{\phi\partial^J\phi} = E_{free} + \# e^{-\tau_{\min}\Delta\chi}$$

$$= J + 2\Delta_\phi + \frac{\#}{J^{\tau_{min}}}$$

Large charge expansion in CFT

- d-dimensional CFT with internal symmetry G
- $\{Q_I\}$: global Cartan charges of G (+ spin)
- Lowest dimension operator at given charges Q : $\mathcal{O}_{\{Q_I\}}(x)$
- For $|Q_I| \gg 1$ compute *semiclassically*

$$\langle \mathcal{O}_{\{Q'_I\}}(x_{out}) \underbrace{\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\ldots\mathcal{O}_n(x_n)}_{\text{light operators}} \mathcal{O}_{\{Q_I\}}(x_{in})\rangle$$

Hellerman Orlando Reffert Watanabe 2015, Monin Pirtskhalava Rattazzi Seibold 2016

The conformal superfluid EFT for large charge operators

Large charge state

Consider a CFT w. $U(1)$ symmetry. Properties of $|Q\rangle$:

- charge density $j_0 \sim Q/R^{d-1} \propto \mu^{d-1}$
- scale separation $\mu \gg 1/R$ suggests *EFT* description
- condensed matter phase: *nonlinearly* realizes spacetime symmetries

→ low energy hydrodynamic modes are Goldstone bosons

Nicolis Penco Piazza Rattazzi 2015

The conformal superfluid

Summary: $SO(d+1,1) \times U(1) \longrightarrow SO(d) \times \bar{D}$ with $\bar{D} = D + \mu Q$

- 1 Goldstone mode $\chi(x) = \mu t + \pi(x)$

- "radial" modes: generically gapped at $\mu \sim \frac{Q^{\frac{1}{d-1}}}{R}$

(But moduli in SCFTs: Hellerman Maeda Orlando Reffert Watanabe 2017-2021)

Can write $\mathscr{L}\left(\chi\right)$ systematically in a derivative expansion $\partial/\mu \propto E/Q^{\frac{1}{d-1}}$

$$
\begin{aligned}
\mathcal{L} = \ & c(\partial\chi)^d && \Big\} = Q^{\frac{d}{d-1}} \\
& +c_1(\partial\chi)^{d-2}\left\{\mathcal{R} + (d-2)(d-1)\frac{[\partial_\mu(\partial\chi)]^2}{(\partial\chi)^2}\right\} && \multirow{2}{*}{$\Bigg\} = Q^{\frac{d-2}{d-1}}$} \\
& +c_2(\partial\chi)^{d-2}\mathcal{R}_{\mu\nu}\frac{\partial^\mu\chi\partial^\nu\chi}{(\partial\chi)^2} && \\
& + \dots && \Big\} = Q^{\frac{d-4}{d-1}}
\end{aligned}
$$

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Simple prediction for the ground state energy:

$$\Delta_0(Q) = Q^{\frac{d}{d-1}}\left[\alpha_1 + \alpha_2 Q^{-\frac{2}{d-1}} + \dots\right] + \text{1-loop}$$

$$\text{1-loop} = \begin{cases} -0.0937\dots & d=3 \\ -\frac{1}{48\sqrt{3}}\log Q & d=4 \end{cases}$$

Hellerman Orlando Reffert Watanabe 2015, Monin 2016, GC 2020

Sound speed of the phonon fluctuations fixed by conformal invariance:

$$S[\pi] = c\mu^{d-2}\frac{d(d-1)}{2}\int d^d x\sqrt{g}\left[\dot{\pi}^2 + \frac{1}{d-1}(\partial_i\pi)^2\right]$$

$$\omega_J^2 = \frac{1}{d-1}\frac{J(J+d-2)}{R^2}$$

Non-trivial informations about the spectrum

$$\Delta(Q,\{n_J\}) = \Delta_0(Q) + \sum n_J R\omega_J + \dots$$

$J = 1$ descendants $\begin{cases} R\omega_1 = 1, \\ a_{1,m} \propto K_m, \quad a_{1,m}^\dagger \propto P_m; \end{cases}$

$J > 1$ new primaries.

Hellerman Orlando Reffert Watanabe 2015

Charged spinning operators in 3d CFTs and rotating superfluids

Adding spin to the superfluid

A natural question is what happens to the large charge ground state energy as we increase the spin, e.g., in the 3d O(2) model. We may try to look at phonon states:

1 phonon with spin=J

$$\delta E_Q\, R = \frac{J}{\sqrt{2}}\sqrt{1+\frac{1}{J}}$$

$$J \ll \sqrt{Q}\,.$$

$$<$$

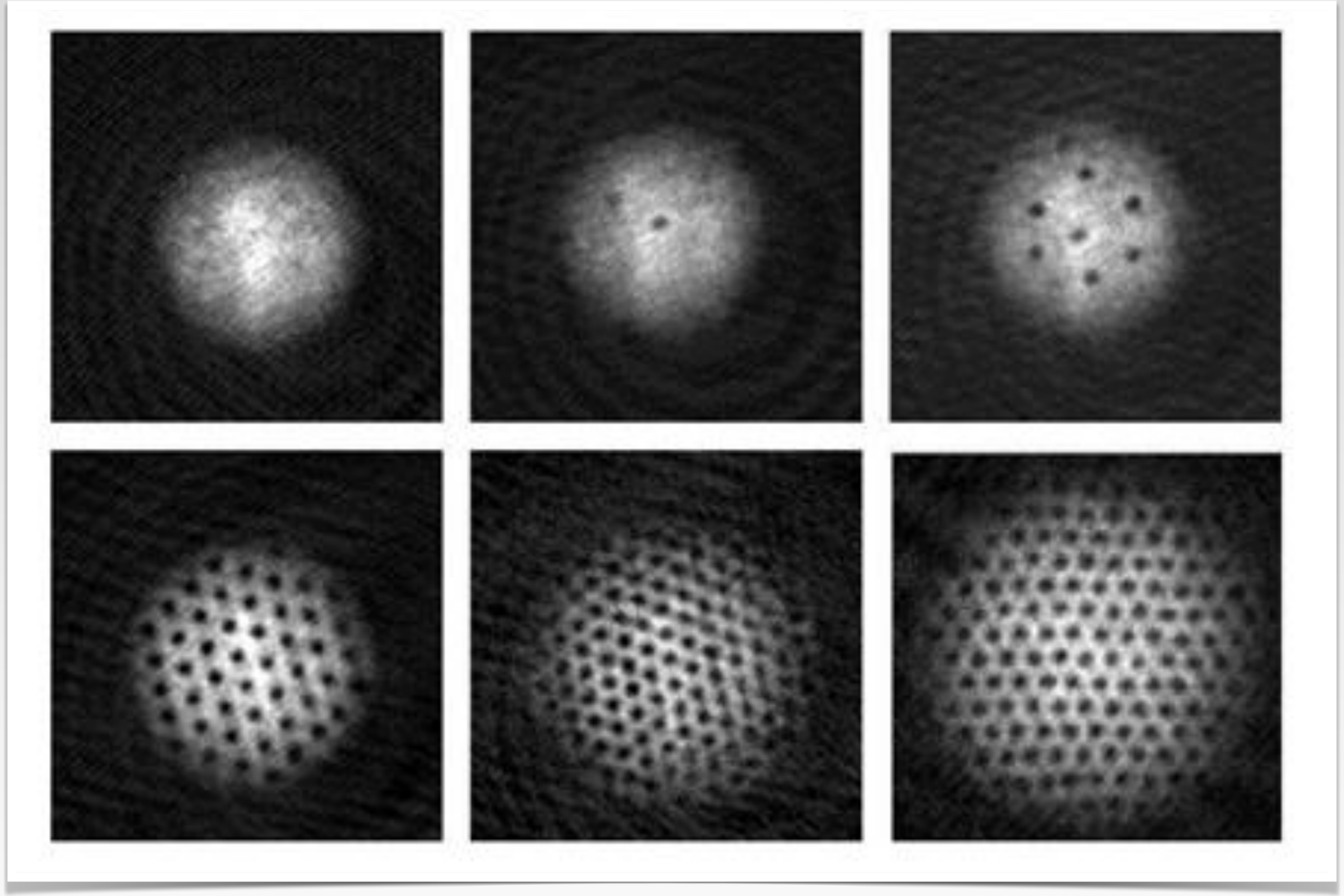
n phonons with spin=J/n

$$\delta E_Q\, R = \frac{J}{\sqrt{2}}\sqrt{1+\frac{n}{J}}$$

$$\begin{cases} J/n \ll \sqrt{Q}, \qquad n \ll Q, \\ \Longrightarrow \qquad J \ll Q^{3/2}. \end{cases}$$

$$\overset{?}{\Longrightarrow} \qquad \Delta(Q,J) = \alpha_1 Q^{3/2} + \frac{J}{\sqrt{2}}$$

… but experiments shows that spinning superfluids develop vortices!



Bose-Einstein condensates in a magnetic trap develop an increasing number of vortices as the angular velocity is increased

Vortex EFT and particle-vortex duality

It is convenient to introduce a dual gauge field:

$$\mathcal{L} = c(\partial\chi)^3 \quad \Longleftrightarrow \quad \mathcal{L} = -\kappa F^{3/2} \qquad \begin{cases} F = \sqrt{F_{\mu\nu}F^{\mu\nu}} \\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\ \kappa = \frac{1}{2^{5/4}(3\pi)^{3/2}\sqrt{c}} \end{cases}$$

The U(1) current provides the explicit relation:

$$j^\mu = 3c(\partial\chi)\partial_\mu\chi = \frac{1}{4\pi}\frac{\epsilon^{\mu\nu\lambda}}{\sqrt{g}}F_{\nu\lambda}$$

$$\langle j_0 \rangle = \frac{Q}{4\pi R^2} \quad \Longleftrightarrow \quad \langle F_{\theta\phi} \rangle = B\sin\theta = \frac{Q}{2R^2}\sin\theta\,.$$

Cutoff: $\Lambda \sim \sqrt{Q}/R \sim \sqrt{B}$. $B =$ monopole field.

Vortices=charged particles

$$S = -\kappa \int d^3x \sqrt{g} F^{3/2} - \sum_p q_p \int A_\mu dX_p^\mu - \sum_p \gamma_p \int d\tau \sqrt{F} \sqrt{\dot{X}_p^2} + \dots$$

Horn Nicolis Penco 2015

- Effective vortex mass: $m_p = \gamma_p \sqrt{B} \sim \sqrt{Q}$
- Physically: Landau levels (LLs) separated by $\omega_L = B/m_p \sim$ cutoff
- Integrate out all LLs but the first ⟶ EFT for the lowest LL
- In practice treat mass term as higher derivative term (leading single derivative kinetic term from the monopole connection)

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Classical analysis: electrostatics on the sphere

To leading order, a simple electrostatic problem:

$$E^i = (\dot{X}_p)_j F^{ji}, \qquad \frac{1}{e^2}\nabla_i E^i = \rho, \qquad (e^2 \sim \sqrt{Q})$$

$$\implies \qquad \vec{E} \sim \sqrt{Q}/d, \qquad \dot{\vec{X}} \sim \frac{1}{d\sqrt{Q}}.$$

Gauss law implies $\sum_p q_p = 0$.

GC de la Fuente Monin Pirtskhalava Rattazzi 2017

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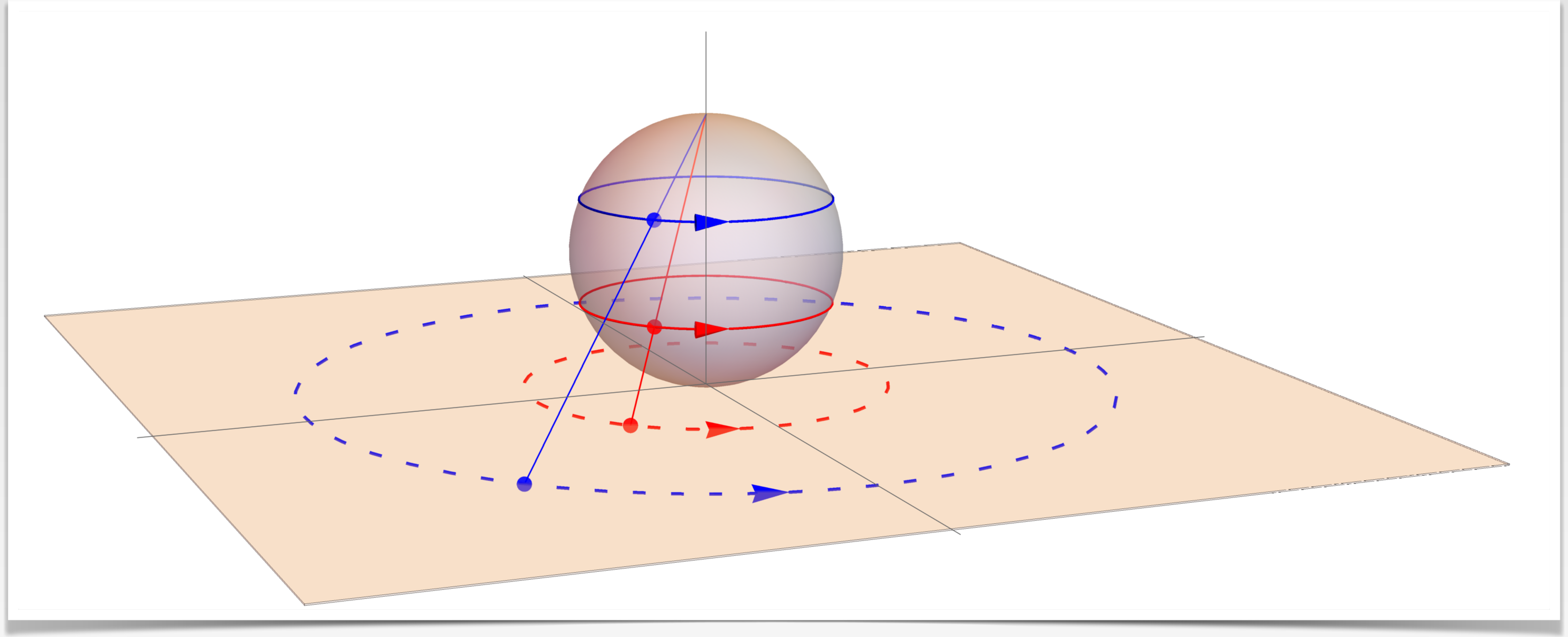
$$\Delta = \alpha Q^{3/2} - \frac{\sqrt{Q}}{12\alpha}\sum_{p\neq r} q_p q_r \log Q(\vec{R}_p - \vec{R}_q)^2 + \ldots$$

$$\vec{J} = -\frac{Q}{2}\sum_p q_p \vec{R}_p = \sum_p \vec{J}_p$$

GC de la Fuente Monin Pirtskhalava Rattazzi 2017

Results: vortex-antivortex pair

$$q = \pm 1 \qquad \Longrightarrow \qquad \vec{J} = \frac{Q}{2}\left(\vec{R}_- - \vec{R}_+\right)$$



$$\Delta = \alpha Q^{3/2} + \frac{\sqrt{Q}}{6\alpha}\log\frac{J^2}{Q}\,, \qquad \sqrt{Q} \ll J \leq Q.$$

Quantization: fuzzy sphere

Quantization from $SU(2)$ algebra:

$$\vec{J} = -\frac{Q}{2}\sum_p q_p \vec{R}_p = \sum_p \vec{J}_p \qquad \Longrightarrow \qquad [J_p^i, J_p^j] = i\epsilon_{ijk} J_p^k$$

- R_p^i are non-commuting coordinates
- Quantum corrected energy for a vortex-antivortex pair

$$\Delta = \alpha Q^{3/2} + \frac{\sqrt{Q}}{6\alpha} \log \frac{J(J+1)}{Q}$$

Results: vortex distribution

- $J \geq Q \quad \Longrightarrow \quad n_V > 1$
- $J \gg Q \quad \Longrightarrow \quad n_V \gg 1$: approximate by smooth distribution ρ

Minimize Δ at fixed J :

$$\rho = \frac{3}{2\pi R^2} \frac{J}{Q} \cos\theta$$

$$\Delta = \alpha Q^{3/2} + \frac{1}{2\alpha} \frac{J^2}{Q^{3/2}}, \qquad Q \ll J \ll Q^{3/2}.$$

Constant angular velocity (~ rigid body).

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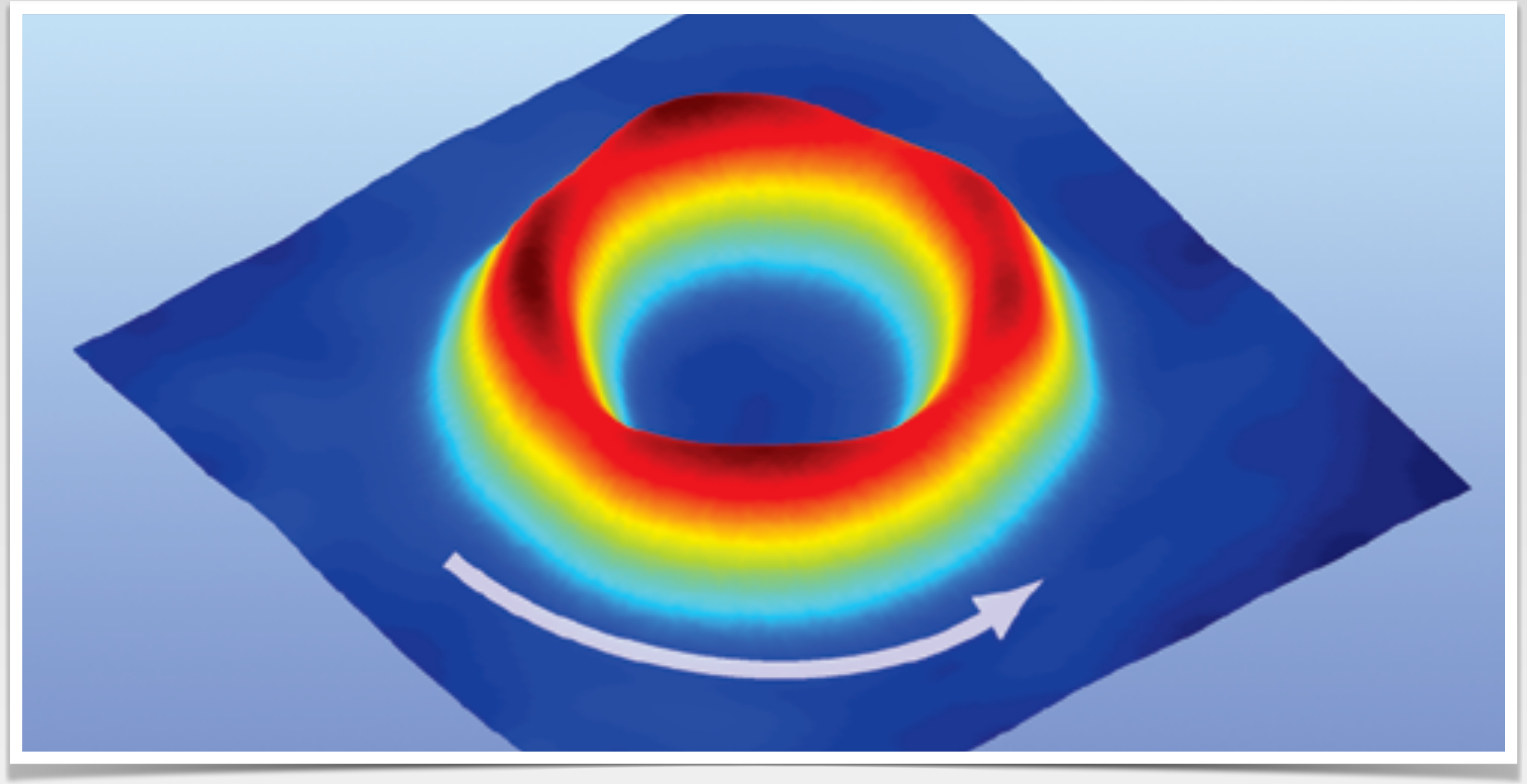
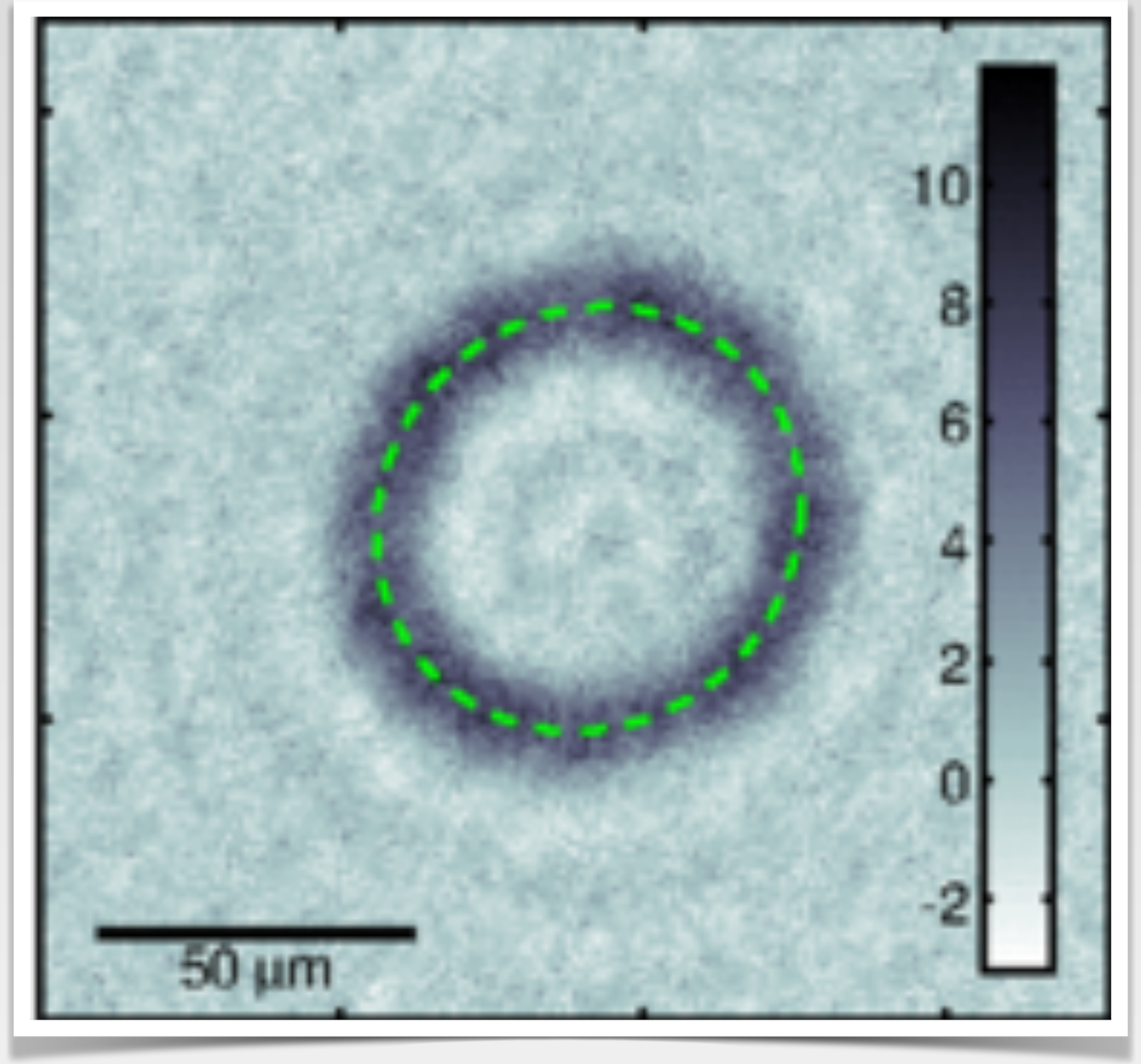
Constant angular velocity (~ rigid body). Can we go beyond this regime?

Let’s look again at experiments…





When the angular velocity exceeds the speed of sound in BECs the vortex lattice becomes unstable towards the formation of a coherent “giant” vortex annulus.

Theory: *Fischer Baym 2003, Fetter Jackson Stringari 2005*

Experiment: *Guo Dubessy de Herve Kumar Badr Perrin Longchambon Perrin 2019*

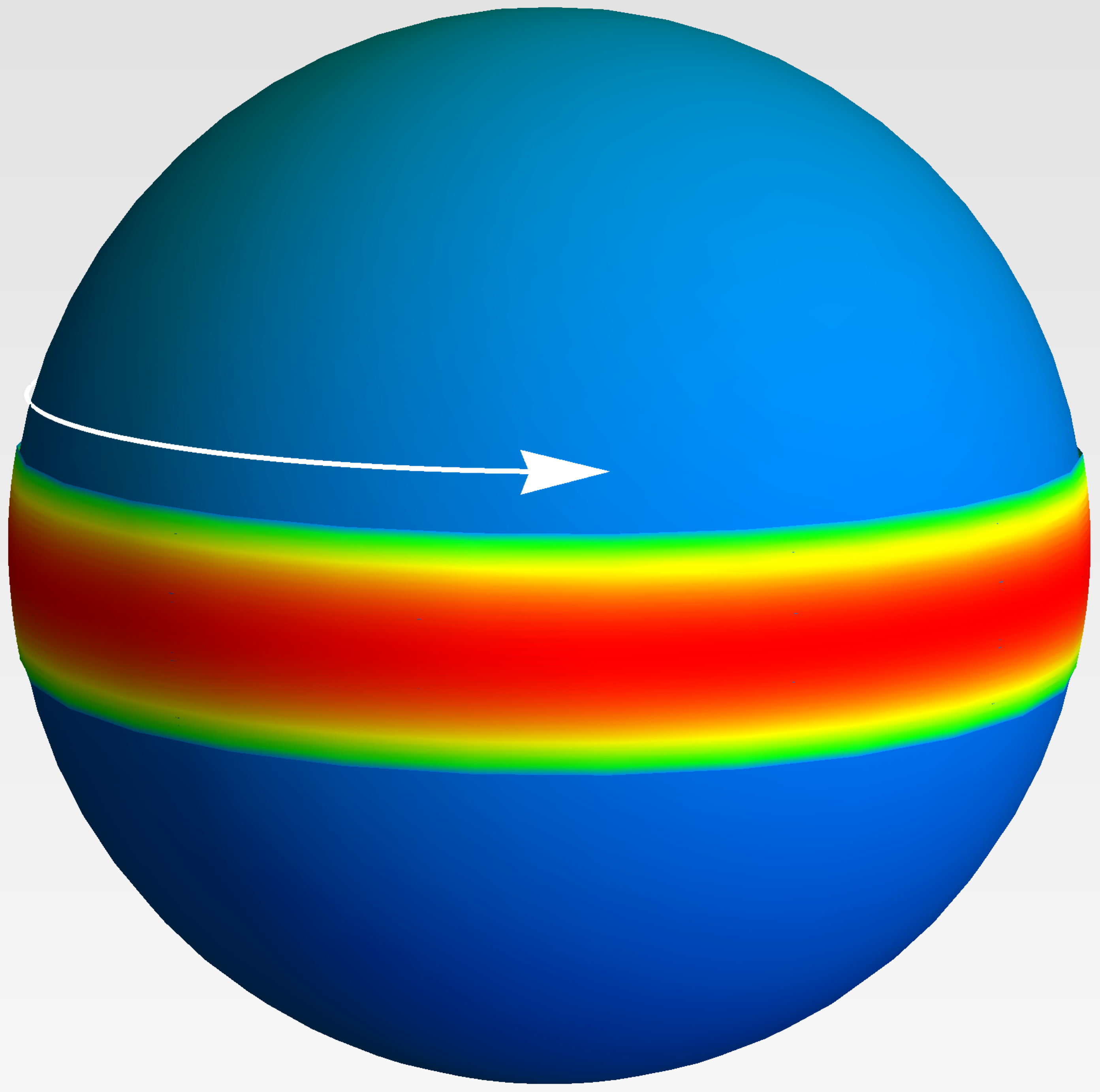
Non-technical review: *Sophia Chen - Physics 2020*

A giant vortex in the O(2) model

For $J/Q \in \mathbb{Z}$ a natural candidate for the "giant vortex" profile is

$$\chi = \mu t - \ell\phi \qquad \Longrightarrow \qquad J = \ell Q$$

$$\langle j_0 \rangle = \begin{cases} 3c\mu^2\sqrt{1 - \frac{\ell^2/\mu^2}{R^2\sin^2\theta}} & \sin^2\theta \geq \frac{\ell^2}{R^2\mu^2}\,, \\ 0 & \sin^2\theta < \frac{\ell^2}{R^2\mu^2}\,. \end{cases}$$



GC Komargodski in progress

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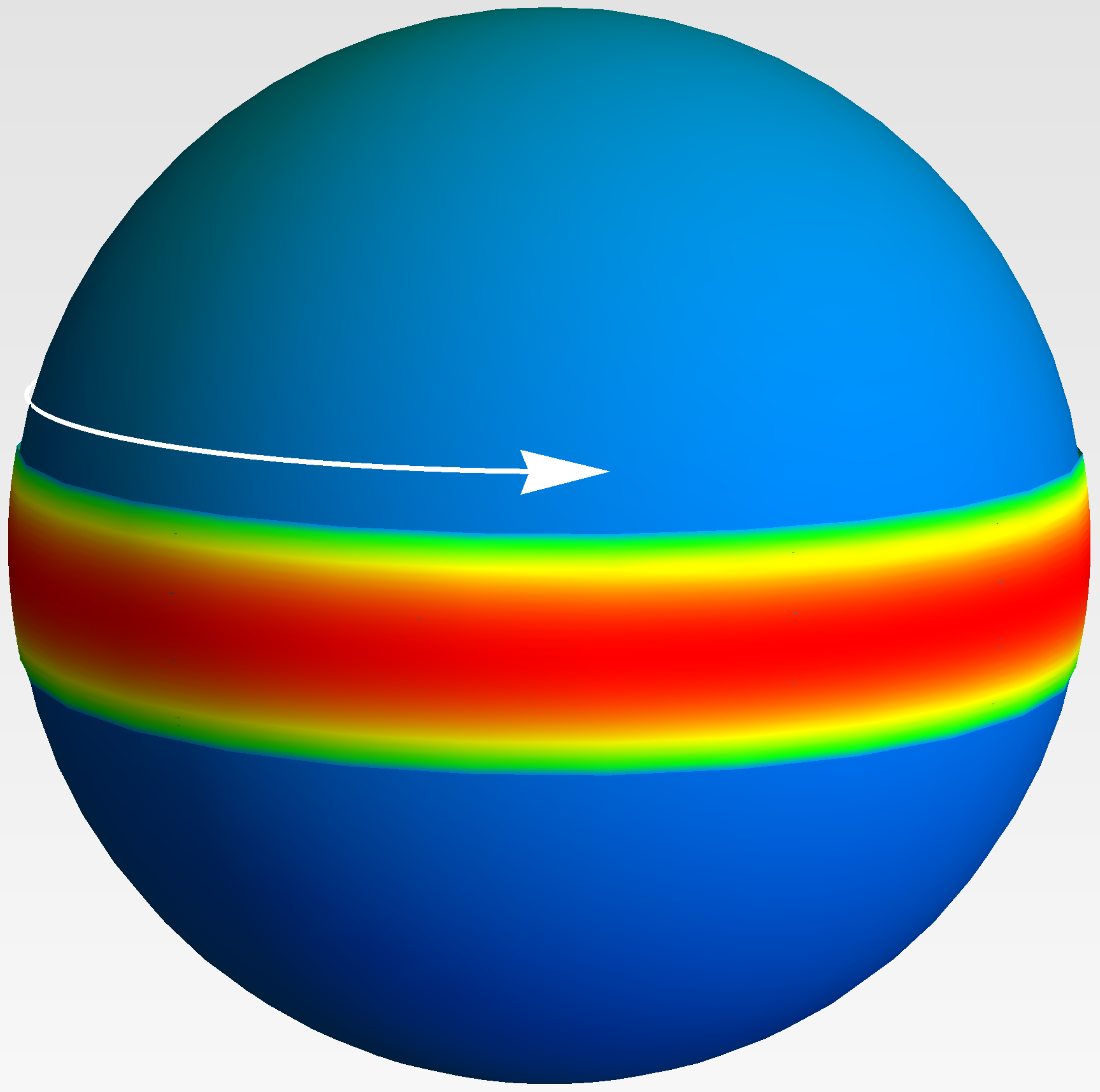


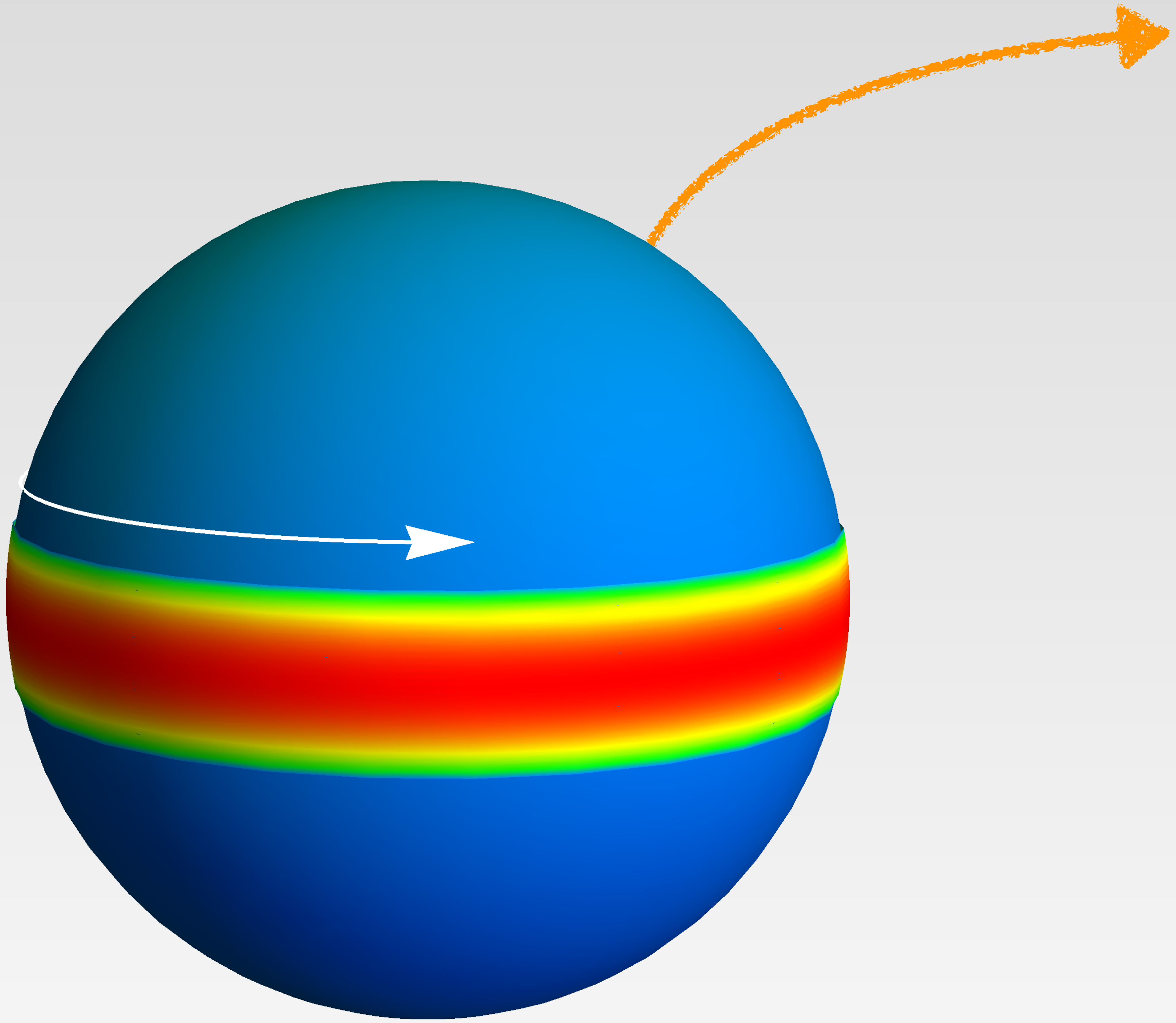
For $J \gg Q^{3/2}$:

$$R\mu = \frac{J}{Q}\left[1 + \frac{Q^3}{6\pi^2 c J^2} + \dots\right]\,,$$

$$\frac{\ell}{R\mu} = 1 - \frac{Q^3}{6\pi^2 c J^2} + \dots$$

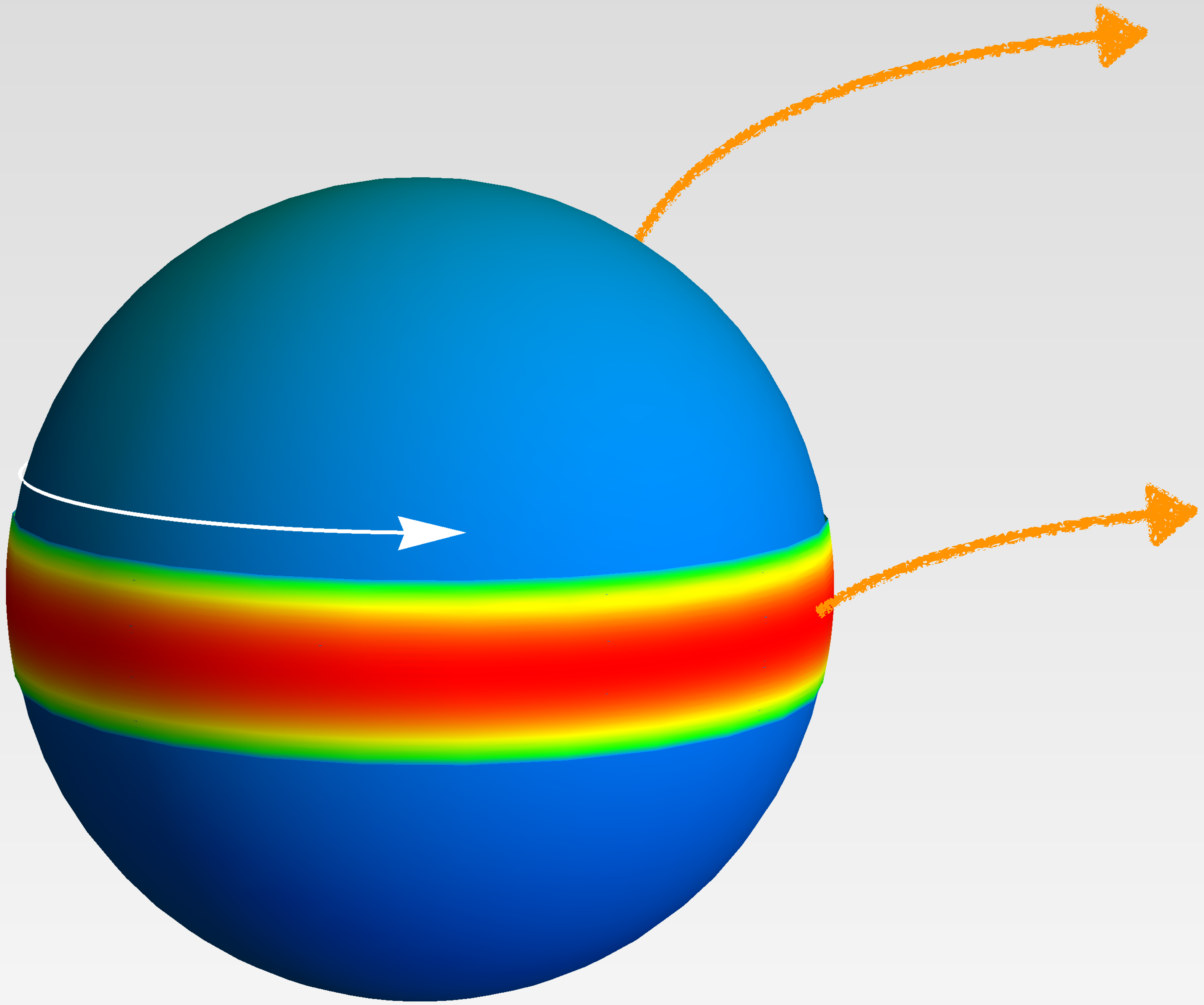
GC Komargodski in progress

Three physically distinct regions



- Away from the equator the centrifugal potential $V(\theta) \sim \dfrac{\ell^2}{\sin^2\theta}$ gaps all excitations .

Three physically distinct regions



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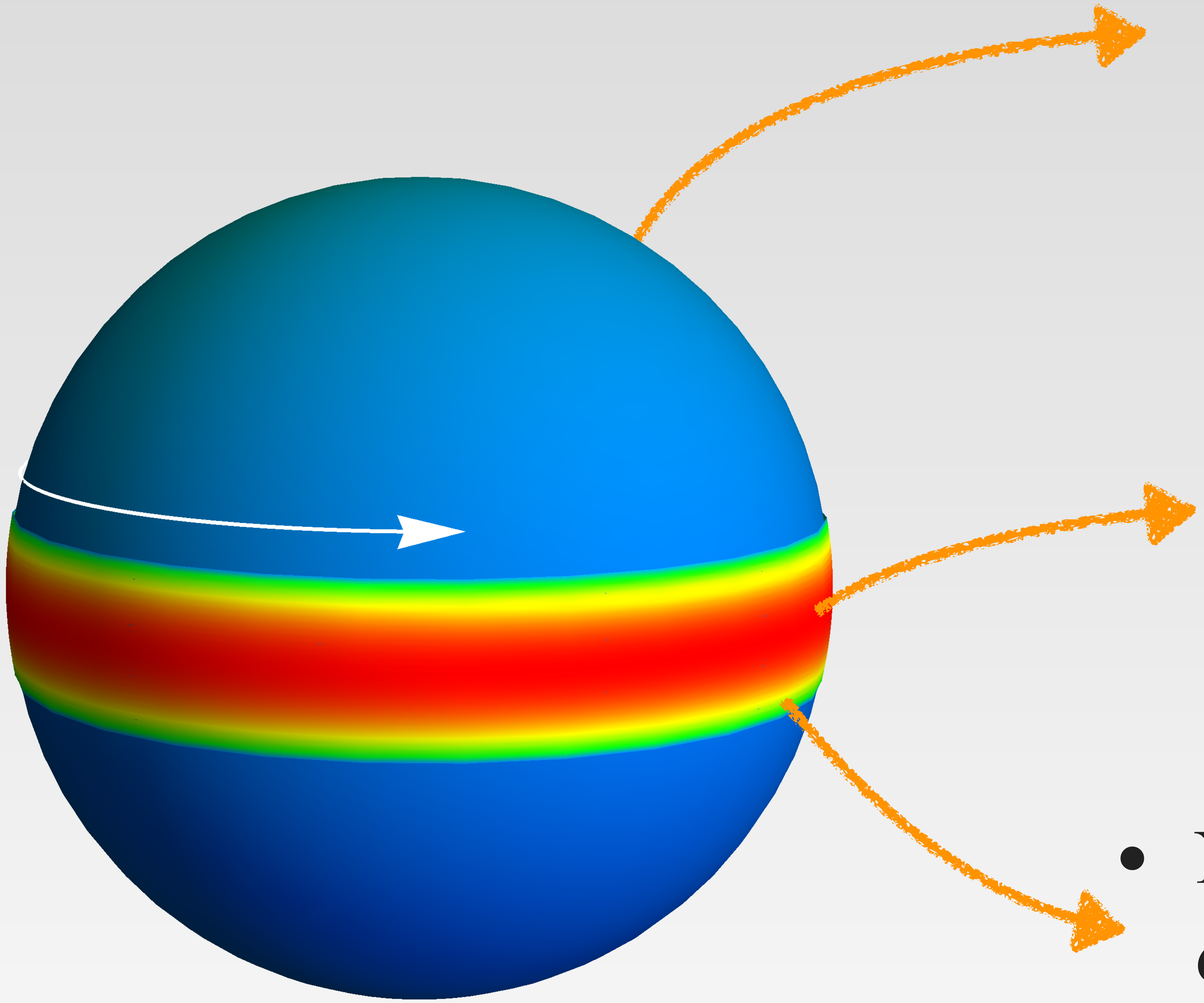
- The charge is localized around the equator, where the derivative expansion is controlled by the density: $(\partial\chi)^3 \gg \partial^2(\partial\chi) \quad \Longrightarrow \quad Q^2 \gg J$.

Three physically distinct regions



- Away from the equator the centrifugal potential $V(\theta) \sim \frac{\ell^2}{\sin^2\theta}$ gaps all excitations .
- The charge is localized around the equator, where the derivative expansion is controlled by the density: $(\partial\chi)^3 \gg \partial^2(\partial\chi) \implies Q^2 \gg J$.
- Near $\sin^2\theta \sim \ell^2/R^2\mu^2$ the charge density rapidly decreases and the gap of radial modes is controlled by the steepness of the profile $[\partial_\theta(\partial\chi)^2]^{1/3} \sim J^{1/3}/Q^{1/6}$.

Technically similar to rotating effective strings and NRCFTs in harmonic trap.

Hellerman Swanson 2013, Hellerman Swanson 2020

Results: the giant vortex energy

For $Q^{3/2} \ll J \ll Q^2$ we can compute the energy of the ground state:

$$\Delta = J\left[1 + \frac{9\alpha^2 Q^3}{4\pi J^2} + \frac{\beta_b}{(JQ)^{1/3}} + \mathcal{O}\left(\frac{Q^6}{J^4}, \frac{1}{Q}\right)\right]$$

- In the limit $J \gg Q^{3/2}$ the result approaches the expectation for $\phi\partial^{J/Q}\phi\ldots\partial^{J/Q}\phi$
- The $\left(Q^3/J^2\right)^n$ corrections arise from the LO action and depend on the same parameter α controlling the homogeneous superfluid energy $\Delta_{J=0} = \alpha Q^{3/2} + \ldots$
- The parameter β_b parametrizes energy corrections from the edge of the profile and it is reproduced in the EFT by a boundary operator $\mathcal{O}_b \sim \delta\left((\partial\chi)^2\right)\left[\partial_\theta(\partial\chi)\right]^{5/3}$

The spectrum of fluctuations

The quadratic action becomes tractable expanding in $\delta^2 = \dfrac{Q^3}{6\pi^2 c J^2}$:

$$S^{(2)} = \frac{3}{2} c\mu \int dt d\phi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dy \left[(\dot{\pi} + \partial_\phi \pi)^2 - (\partial_y \pi)^2 + \mathcal{O}\left(\delta^2\right) \right]$$

- The y coordinate is related to the original one as $\cos\theta = \delta\sqrt{2-\delta^2}\,\sin y$
- Regularity implies $\partial_y \pi = 0$ at $y = \pm \dfrac{\pi}{2}$
- To this order EOMs depend only on $(\partial_t + \partial_\phi)$ and ∂_y:

$$-(\partial_t + \partial_\phi)^2 \pi + \partial_y^2 \pi = 0$$

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In the limit $Q^3/J^2 \to 0$ the spectrum of fluctuations is then given by

$$\omega = m + n \,, \quad m \in \mathbb{Z} \,, \; n \in \mathbb{N} \,.$$

- Expected spectrum of a (free) multi-trace!
- Expansion holds for $m, n \ll \delta^{-2} \sim J^2/Q^3 \ll Q$
- Corrections in Q^3/J^2 lift the apparent degeneracy, e.g. the gap of the next-to-lowest dimensional state with the same spin J of the giant vortex is:

$$\delta\Delta = \frac{18\alpha^2}{\pi} \frac{Q^3}{J^2}$$

Summary of results

Summary

The lowest dimensional operator at fixed $Q \gg 1$ and J in the O(2) model corresponds to

- $0 \leq J \ll \sqrt{Q}$: homogeneous superfluid +1 phonon $\qquad \Delta = \alpha Q^{3/2} + \frac{\sqrt{J(J+1)}}{\sqrt{2}}$
- $\sqrt{Q} \ll J \leq Q$: vortex-antivortex pair $\qquad \Delta = \alpha Q^{3/2} + \frac{\sqrt{Q}}{6\alpha} \log \frac{J^2}{Q}$
- $Q \ll J \ll Q^{3/2}$: regular vortex distribution $\qquad \Delta = \alpha Q^{3/2} + \frac{1}{2\alpha} \frac{J^2}{Q^{3/2}}$
- $Q^{3/2} \ll J \ll Q^2$: giant vortex state $\qquad \Delta = J + \frac{9\alpha^2 Q^3}{4\pi J}$
- $Q^2 \ll J$: Alday-Maldacena multi-trace $\qquad \Delta = J + \# \frac{Q^2}{J^{\tau_{min}}}$

Some open questions

- Giant-vortex for $J/Q \notin \mathbb{N}$?
- Supperadiant transition for $J \sim Q^{3/2}$ at weak coupling
- Can we approach $J \to Q^2$ from the Alday-Maldacena description systematically?
- What about 4d?

THANK YOU!