Timely pandemic countermeasures reduce both health damage and economic loss: Generality of the exact solution

ISCO2023@OIST

J. Phys. Soc. Jpn., in press (2023)

Mathematics

Mathematical Models of Infectious Diseases

Physics

Thermodynamics?

Medicine: Public health

Transmission route

Optimal **pandemic** control

Airborne transmission

aerosol nature

Pandemics
by SARS-CoV-2

Economics

(Medical economics)

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Question mark over policy "Countermeasures at the last minute" (2020～)

***"To save economics, no serious measure until Stage 4** (explosively increasing)"*

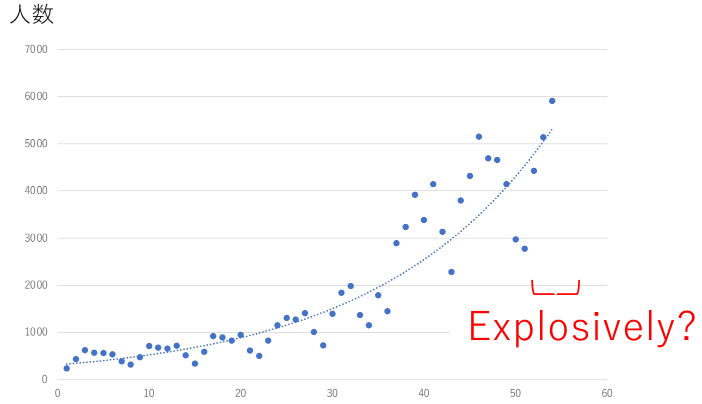
Is it economical?

Stage 1: infection is very rare
Stage 2: infection is increasing slowly
Stage 3: infection is increasing fast
} neglect

Stage 4: infection is explosively increasing

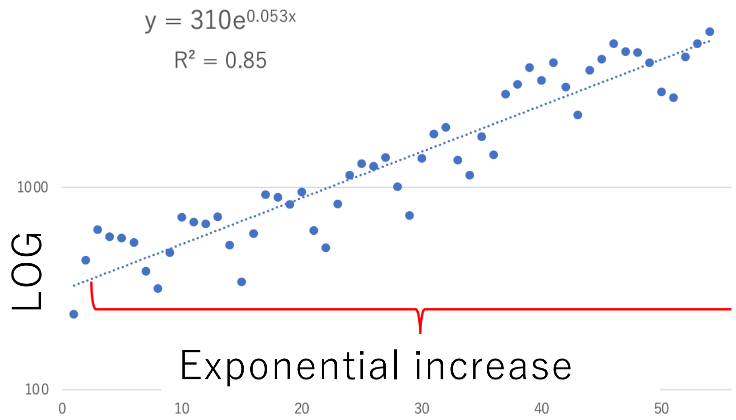
It's still Japan's policy today, while this question may be ***intuitively*** *obvious.*

Daily cases in Tokyo



Days after June 20, 2021

Daily cases in Tokyo



Days after June 20, 2021

Pioneering & suggestive

JOURNAL ARTICLE

A cost–benefit analysis of the COVID-19 disease

Robert Rowthorn ✉, Jan Maciejowski ← Control engineer

Originally **mathematician**!

Oxford Review of Economic Policy, Volume 36, Issue Supplement_1, 2020, Pages S38–S
https://doi.org/10.1093/oxrep/graa030
Published: 29 August 2020

Abstract

The British government has been debating how to escape from the lockdown without provoking a resurgence of the COVID-19 disease. There is a growing recognition of the damage the lockdown has caused to economic and social life. This paper presents a simple cost–benefit analysis inspired by optimal control theory and incorporating the SIR model of disease propagation. It also reports simulations informed by the theoretical discussion. The optimal path for government intervention is computed under a variety of conditions. These include a cap on the permitted level of infection to avoid overload of the health system, and the introduction of a test and trace system. We quantify the benefits of early intervention to control the disease. We also examine how the government's valuation of life influences the optimal path. A 10-week lockdown is only optimal if the value of life for COVID-19 victims exceeds £10m. The study is based on a standard but simple epidemiological model, and should therefore be regarded as presenting a methodological framework rather than giving policy prescriptions.

Optimal pandemic control

Timing and strength of countermeasure

Numerical approach

Economists

many papers

Lacks predictability:

Socio-economic parameters: divers

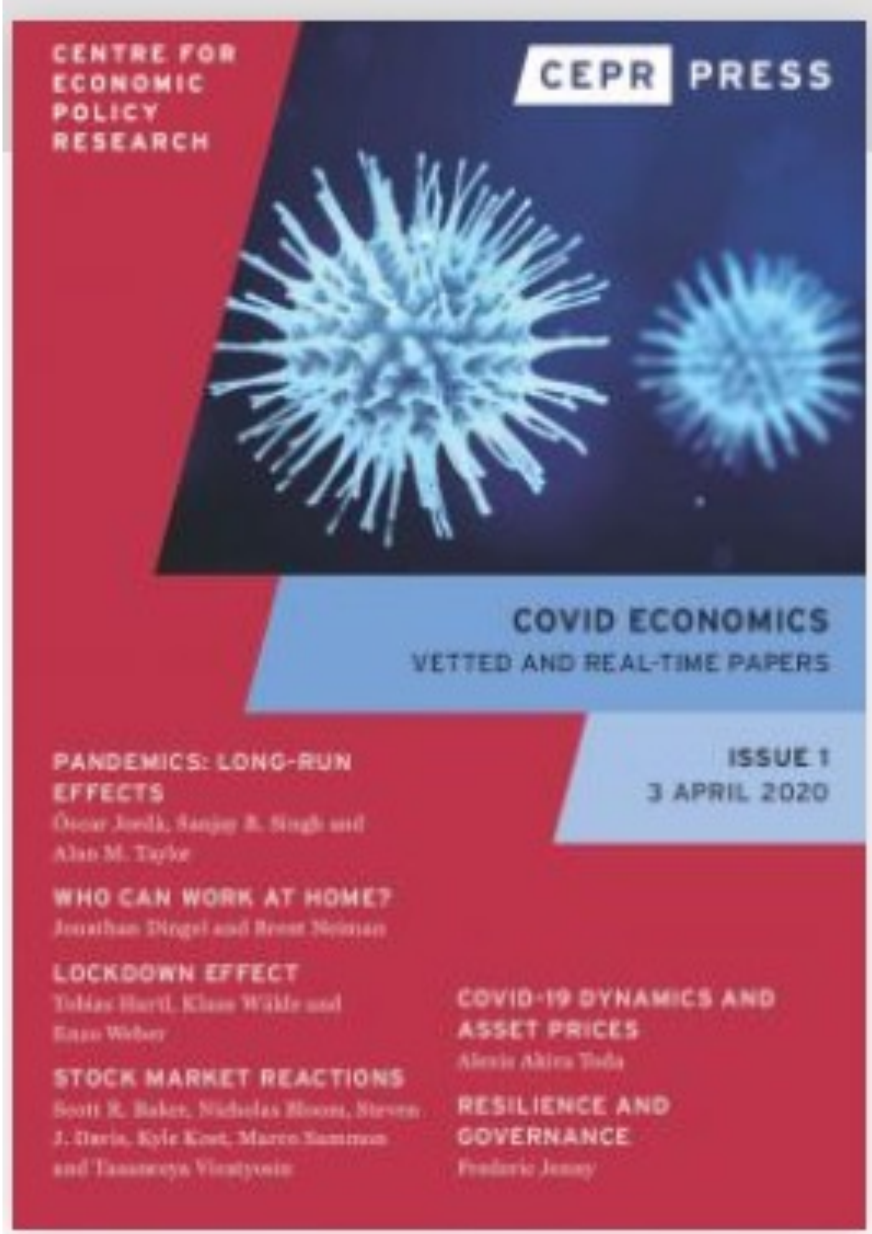
For **predictable** knowledge

Exact solution

Is indispensable

Not only for COVID

But also for future pandemics



Inspiration from Rowthorn & Maciejowski

- SIR models
- U.K. socio-economic condition
- How to regulate after lockdown
- Cost-benefit analysis

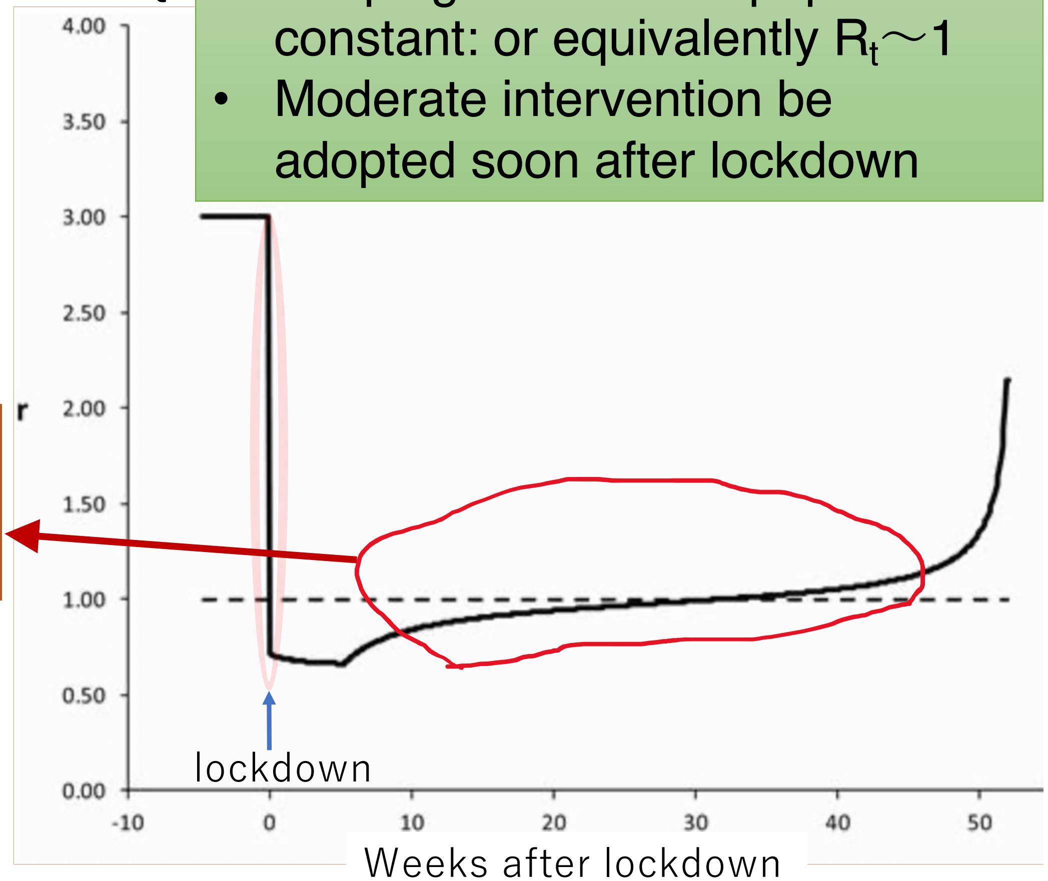
Analogy to "quasi-static process" in thermodynamics.

or

isentropic process

Effective reproduction number R_t

- Keeping the infected population constant: or equivalently $R_t \sim 1$
- Moderate intervention be adopted soon after lockdown



Typical isentropic process
Carnot's cycle

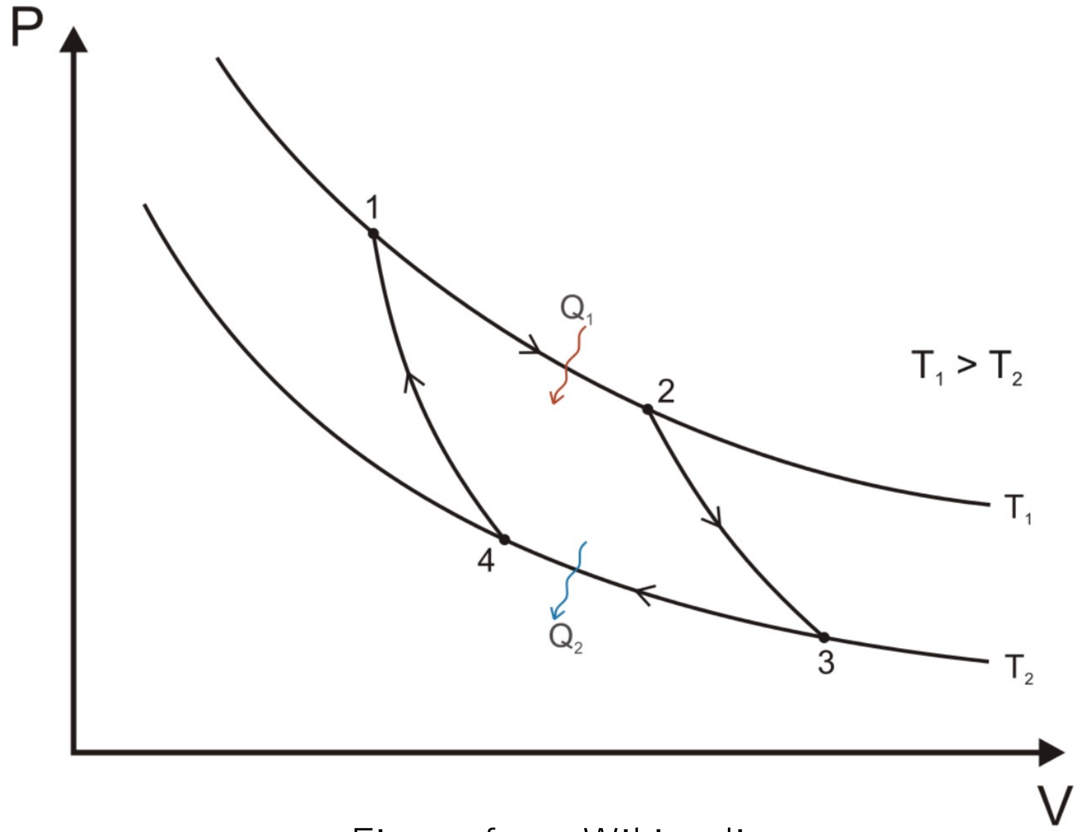


Figure from Wikipedia

Reversible process

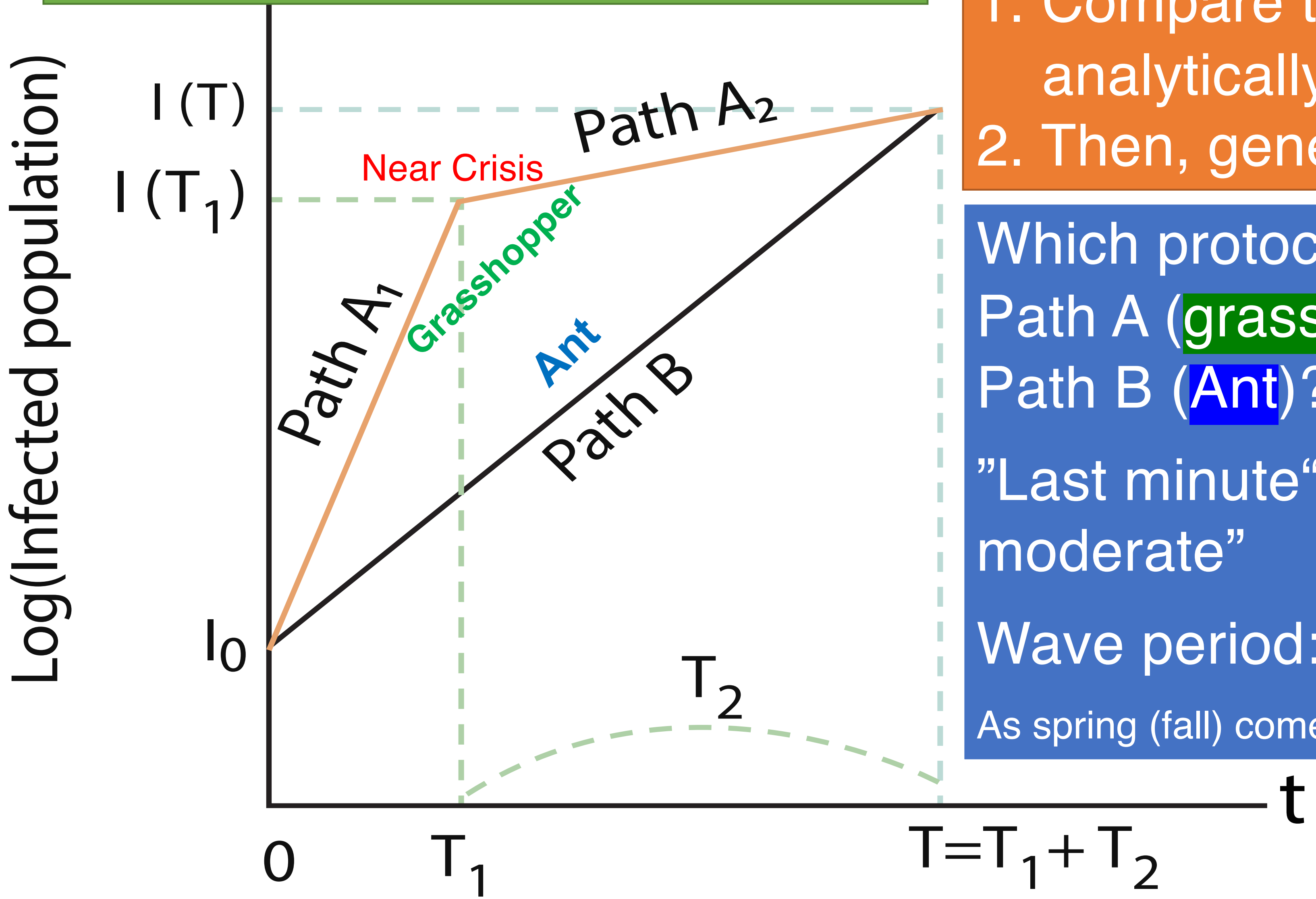
$\Delta S_{total}=0$

Irrevessible process

$\Delta S_{total}>0$

S: entropy

To answer the previous question



1. Compare two processes analytically
2. Then, generalize it

Which protocol is better, Path A (grasshopper) and Path B (Ant)?

"Last minute" or "early and moderate"

Wave period: assumed to be **Finite**

As spring (fall) comes, it naturally relaxes.

airborne nature

Simple assumptions for theory

- **Dynamics:**

  exponential expansion
  infected population I(t)

$$\frac{dI(t)}{dt} = \gamma \Delta(t)\, I(t)$$

$$\Delta(t) = R_t - 1$$

R_t: Effective reproduction number

Infected population
$\Delta(t)>0$: Increase
$\Delta(t)<0$: Decrease

Derived from mathematical models of infection diseases under small & moderate infection rate (except near herd immunity) see next slide

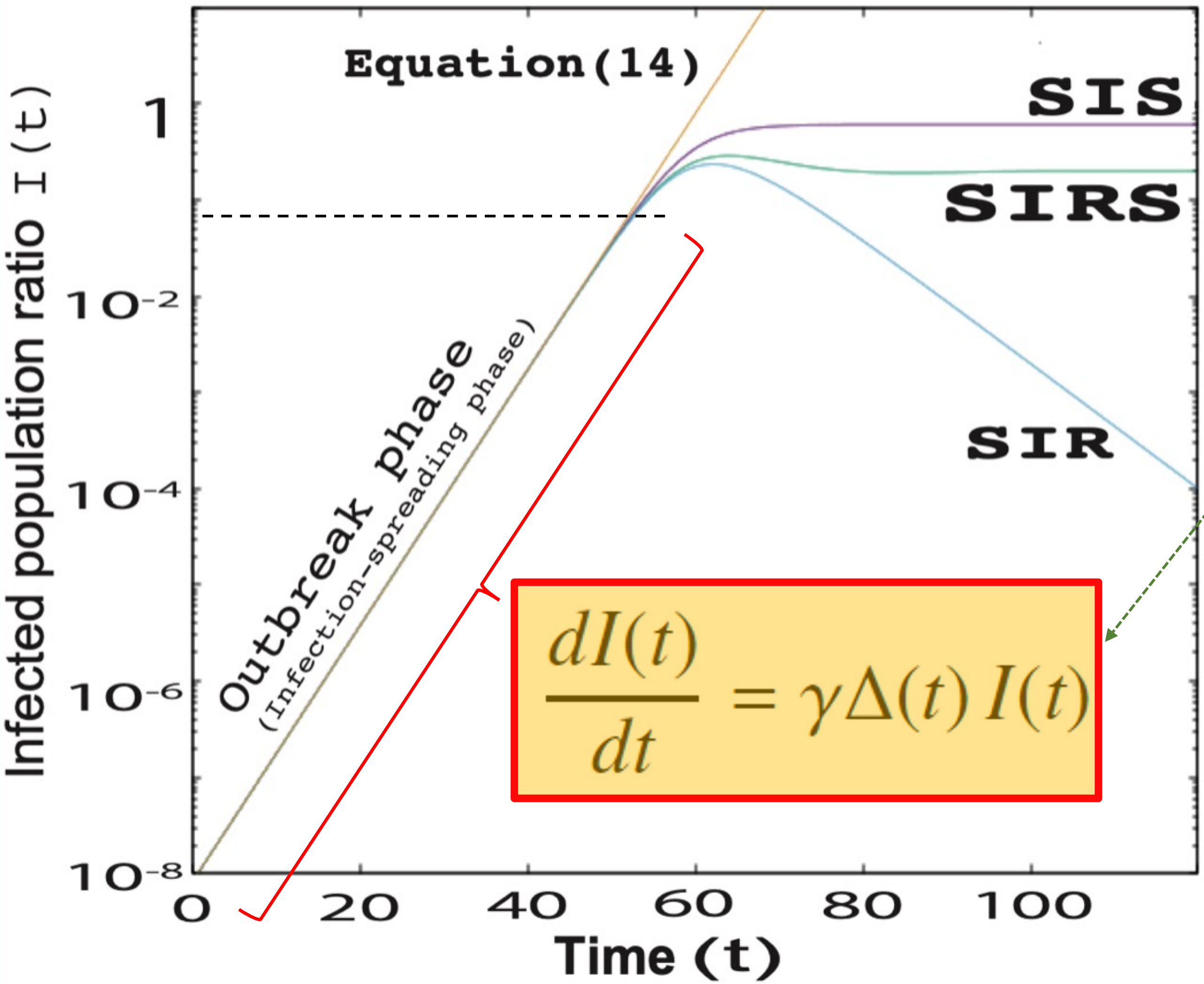
- **<u>Cost-benefit analysis</u>**: ***conventional*** in COVID economics

  Total Cost: Intervention cost *C*(*Δ*) + Medical cost *M*(*I*(*t*))

1. Intervention cost (C(Δ)) : economic damage by countermeasure later slide
2. Medical cost (M): increasing function of infected population(I)

$$\frac{dM}{dI} \geq 0$$

Validity of assumption 1: Exponential dynamics



$$\frac{dI(t)}{dt} = \gamma \Delta(t)\, I(t)$$

Except near (temporal) herd immunity, exponential dynamics is valid

Numerical simulation just for demonstration

$S(t) \fallingdotseq S(0)$ unless "I(t) is order of 1"

S(t): susceptible persons at time t

Ref.

- Vynnycky and White: An Introduction of Infectious Disease Modeling (Oxford Univ. Press, 2010)
- Hondou, J.Phys.Soc.Jpn 2021

Cost Benefit Analysis
Intervention cost $C(R_t)$: Convex downward

- Total cost ＝Intervention cost （C）＋Medical cost （M）
- Assume: Countermeasures: in order of cost-effectiveness

⇩

- Intervention cost of unit time

  function of the effective reproduction n

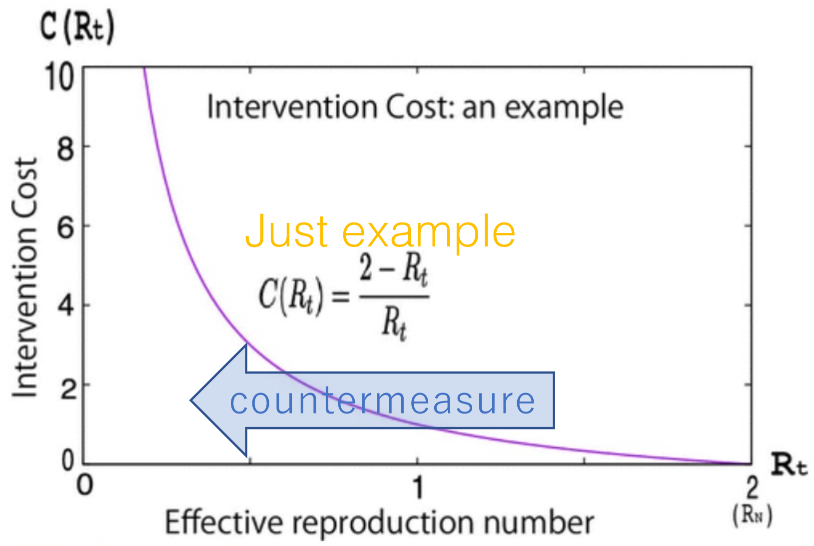
  $C(R_t)$

$C(R_t)$: convex downward

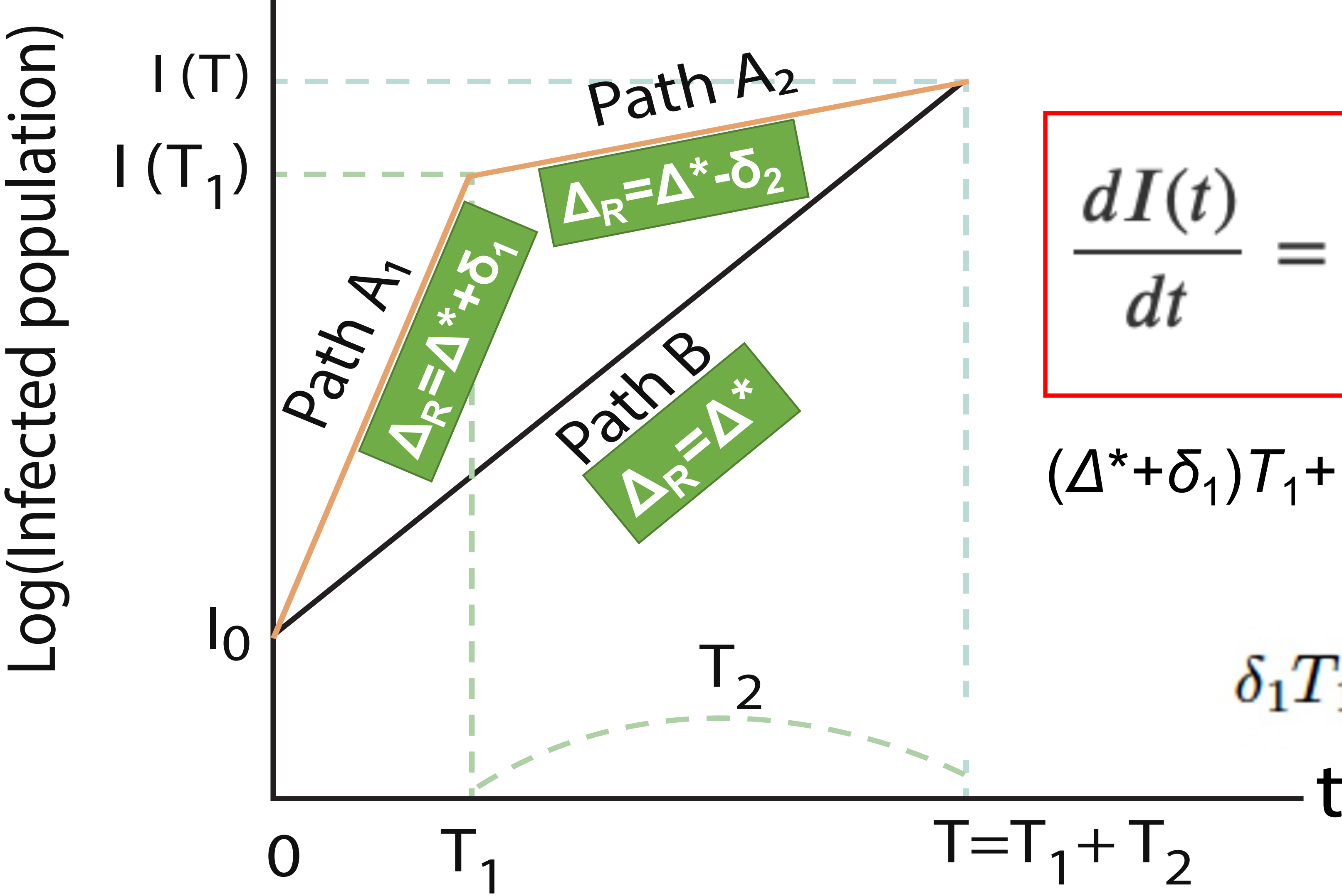
"diminishing utility" in economics

$C(R_N) = 0$ if no countermeasure

(R_N: natural effective reproduction number)



Now: compare 2 processes



$$\frac{dI(t)}{dt} = \gamma \Delta_R \, I(t).$$

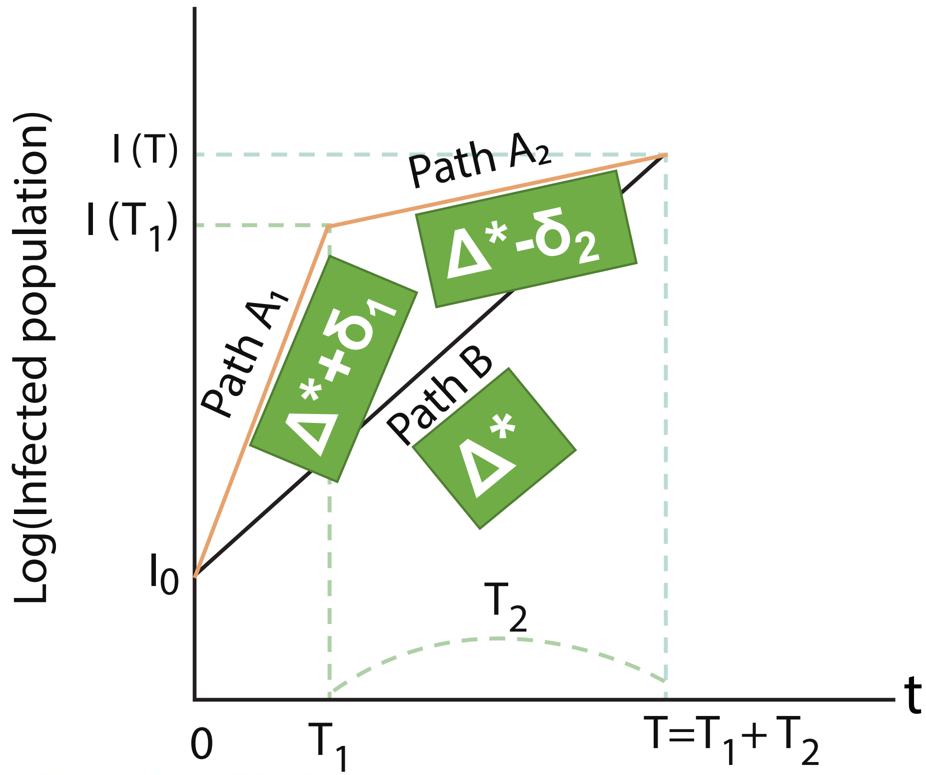
$$(\Delta^{*}+\delta_1)T_1+(\Delta^{*}-\delta_2)T_2=\Delta^{*}T$$

$$\delta_1 T_1 = \delta_2 T_2$$

$(\Delta_R = R_t - 1)$

Intervention cost, *C(Δ)* for paths A & path B

$(\Delta_R = R_t - 1)$



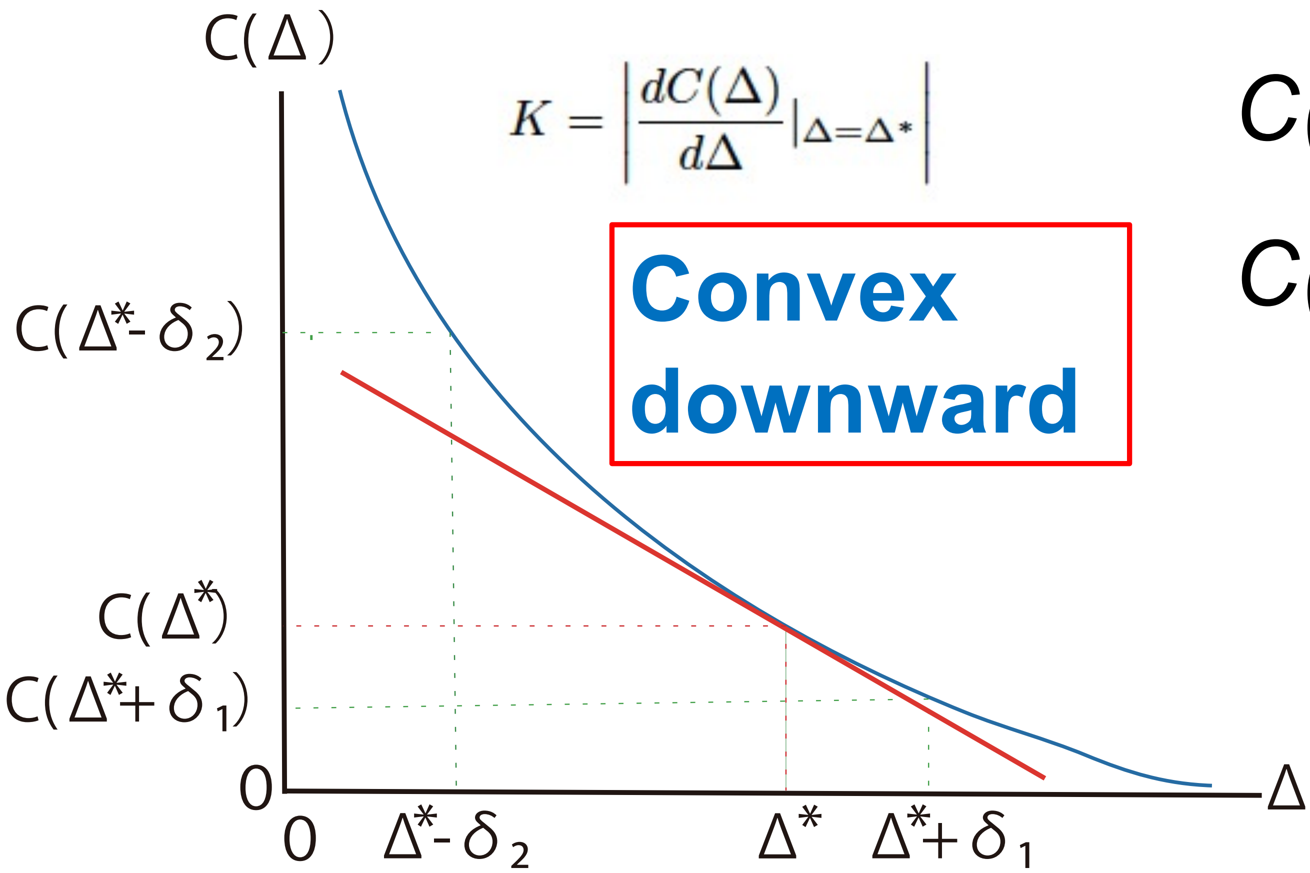
$$\int_{\text{path A}} C(\Delta(t))dt = C(\Delta^* + \delta_1)\,T_1 + C(\Delta^* - \delta_2)\,T_2$$

$$\int_{\text{path B}} C(\Delta(t))dt = C(\Delta^*)T = C(\Delta^*)T_1 + C(\Delta^*)T_2.$$

With the equality

$$\delta_1 T_1 = \delta_2 T_2$$

Since $C(\Delta)$ is convex downward



$$K = \left| \frac{dC(\Delta)}{d\Delta} |_{\Delta=\Delta^*} \right|$$

Convex downward

$$C(\Delta^* - \delta_2) > C(\Delta^*) + K\delta_2$$

$$C(\Delta^* + \delta_1) > C(\Delta^*) - K\delta_1$$

Note for mathematicians

Differentiability of C(Δ) is not necessary. Continuity is sufficient.

See https://arxiv.org/abs/2209.12805 for general proof.

Inequality holds for intervention costs

By using the convexity of the intervention cost, $C(\Delta)$ (Eq. 4) , one finds

$$\int_{\text{path A}} C(\Delta(t))dt - \int_{\text{path B}} C(\Delta(t))dt \geq [C(\Delta^*) - K\delta_1 - C(\Delta^*)]T_1 + [C(\Delta^*) + K\delta_2 - C(\Delta^*)]T_2$$
$$= K(-\delta_1 T_1 + \delta_2 T_2) = 0,$$

$$\int_{\text{path A}} C(\Delta(t))dt \geq \int_{\text{path B}} C(\Delta(t))dt,$$

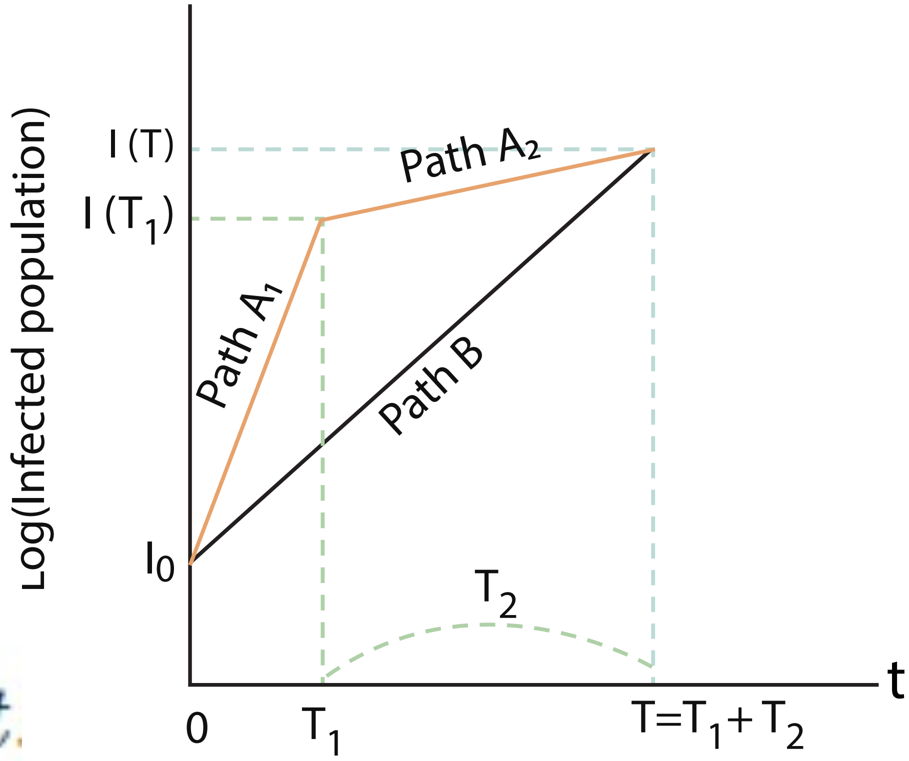
In "last minute" policy, intervention cost is larger than in moderate & early measure.

Medical & Total Cost

$I(t)$: path A ≧ path B

⬇

Medical cost: $M_{\text{path A}}(t) \geqq M_{\text{path B}}(t)$



Total Cost:

$$\int_{t'}^{t''} [C(R_t(t)) + M(I(t))]dt.$$

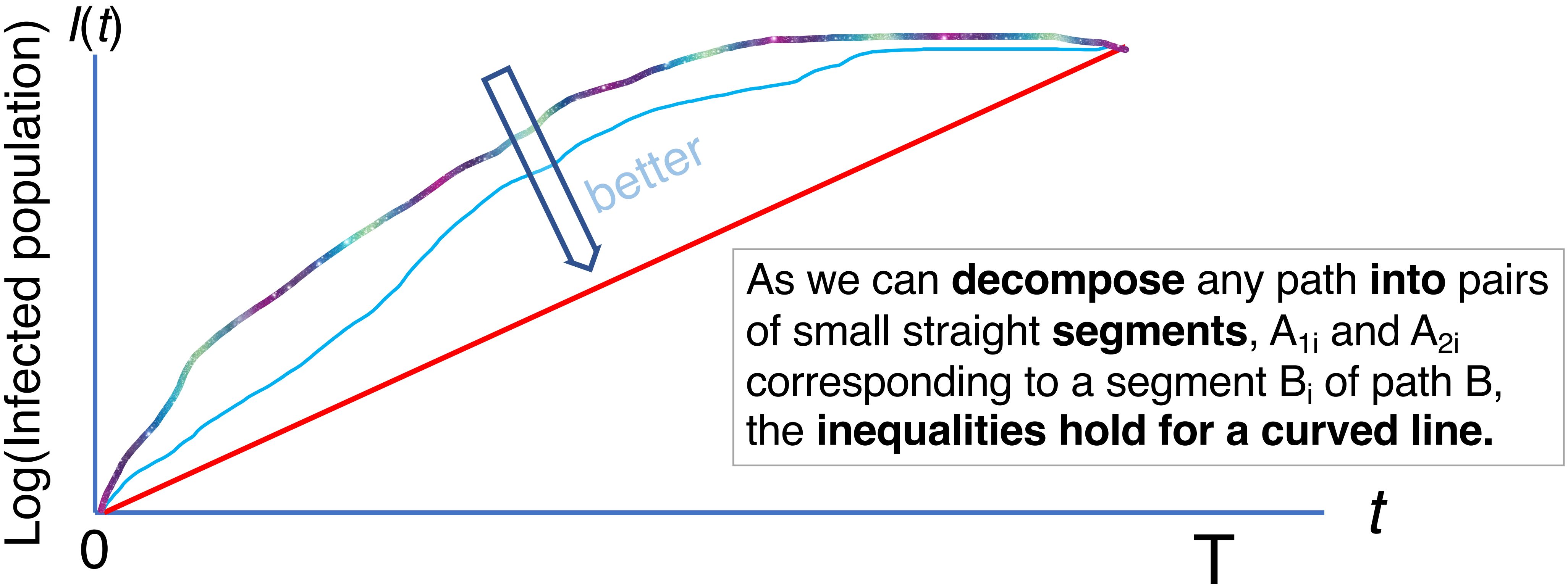
Total cost: Path A ≧ Path B

for any δ_1, δ_2

Inequalities are also valid for curves

Valid for any integrable $R_t(t)$ that

$I_{\text{path A}}(t) \geqq I_{\text{path B}}(t)$ and $I_{\text{path A}}(T) = I_{\text{path B}}(T)$



Conclusion & Discussion 1/2

Early and moderate intervention is better than last minute policy for either cost C, M and thus total cost.

<u>Theorem</u> (inequality for pandemic measure)

$$\int_{\text{path A}} [C(t) + M(t)] \geq \int_{\text{path B}} [C(t) + M(t)]dt$$

Even for curved line

The same inequalities hold for Intervention cost *C* or Medical cost *M* alone.

for $I_{\text{path A}}(t) \geqq I_{\text{path B}}(t)$

- This includes the previous result (J. Phys. Soc. Jpn. 2021) as a special solution to the cyclic condition: $I(0) = I(T)$
- <u>Excess infected population</u> in Path A compared to path B(t)
  ⇒ <u>irreversible costs</u> "economic irreversibility"

  Similar to thermodynamics (entropy production)

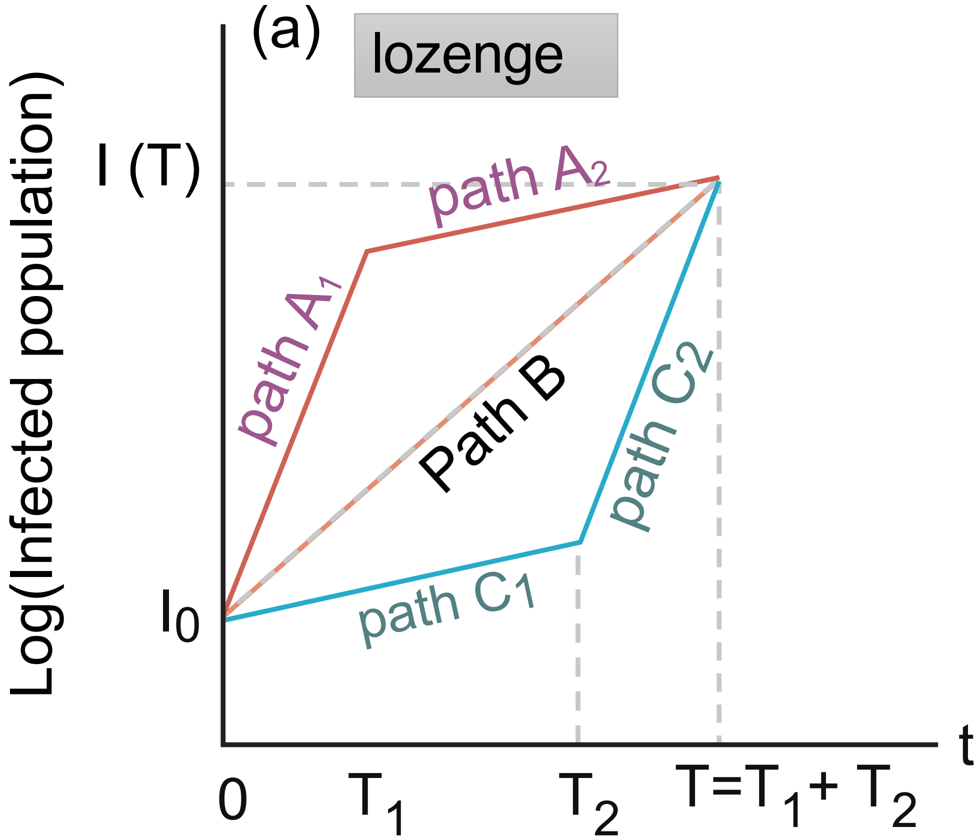
Conclusion & Discussion 2/2

- Robustness comes from simple & natural assumptions:
  - Medical cost : $\frac{dM}{dI} \geq 0$ (can be non-linear)
  - Exponential dynamics for Infected population as to Δ_t (=R_t-1)
  - Intervention cost C: convex downward as to Δ=R_t-1

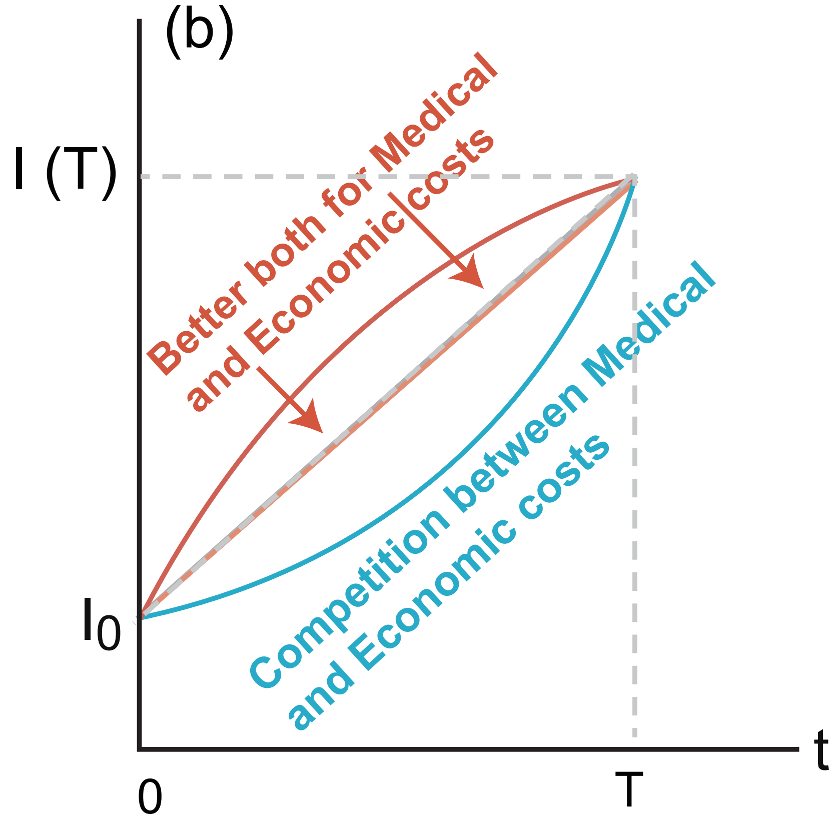
Differentiability: not necessary, continuity is sufficient

- Early and moderate measure: better for both Life and Economy
  - Dogma: ~~Life vs Economy~~ ⇐ *often unproductive*
- Applicable to future pandemic, irrespective of herd immunity
- Core knowledge for complex real-world systems: Role of exact solutions

More general perspective



Total cost: Path C < path A
(benefit for early measure)



Bottom right:

This theorem includes the previous result (2021) as a special solution to the cyclic condition: $I(0) = I(T)$

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Economic Irreversibility in Pandemic Control Processes: Rigorous Modeling of Delayed Countermeasures and Consequential Cost Increases

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