

Inflation and Gravitational Wave Cosmology

Misao Sasaki

Kavli IPMU, University of Tokyo

Inflation

What is Inflation?

Brout, Englert & Gunzig '77, Starobinsky '79, Guth '81, Sato '81, Linde '81,...

- Inflation is a **quasi-exponential expansion** of the Universe at its very early stage; perhaps at $t \sim 10^{-34}$ sec.
- It was meant to solve **the initial condition (singularity, horizon & flatness, etc.) problems** in Big-Bang Cosmology:
if any of them can be said to be solved depends on precise definitions of the problems.

- **Quantum vacuum fluctuations** during inflation turn out to play the most important role. They give the initial condition for **all the structures in the Universe**.
- **Cosmic gravitational wave background** is also generated.

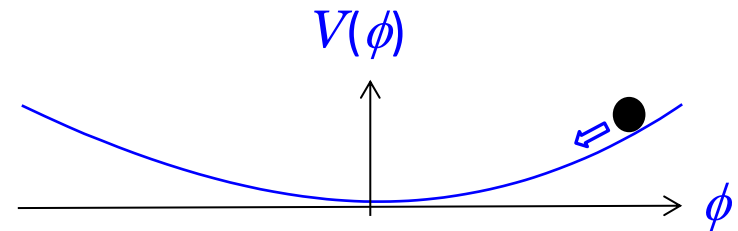
Inflation in a nutshell

- **Inflation** is driven by a **potential** energy $V(\phi)$ of a scalar field ϕ

- ϕ evolves very slowly: $V \approx \text{const.}$ $\Rightarrow \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \approx V(\phi)$

(expansion induces effective friction
 \Rightarrow friction-dominated, over-damped)

\Rightarrow energy density ρ remains almost constant in time



- Friedmann eq. (time-time component of Einstein eq.)

$$3H^2 = 8\pi G\rho \quad H = \frac{\dot{a}(t)}{a(t)} : \text{expansion rate of the Universe}$$

\swarrow cosmic scale factor $\text{Volume} \propto a^3$

- if $\rho = \text{const.}$, $H = \text{const.}$ $\Rightarrow a(t) \propto \exp[Ht]$

Volume expands exponentially = **Inflation!**

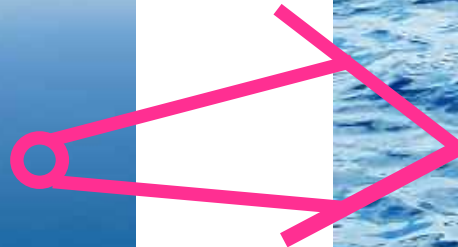
Quantum fluctuations in Inflationary Universe

vacuum fluctuations due to uncertainty principle $\Delta p \Delta x = \hbar$

$$\hbar = 7 \times 10^{-16} \text{ eV} \cdot \text{s}$$

they exist only on **micro** scales; **never** appear on **macro** scales

But,



exponential expansion stretches vacuum fluctuations to macroscopic scales and fluctuations become frozen

$$\Delta x \rightarrow \infty, \Delta p \rightarrow 0$$

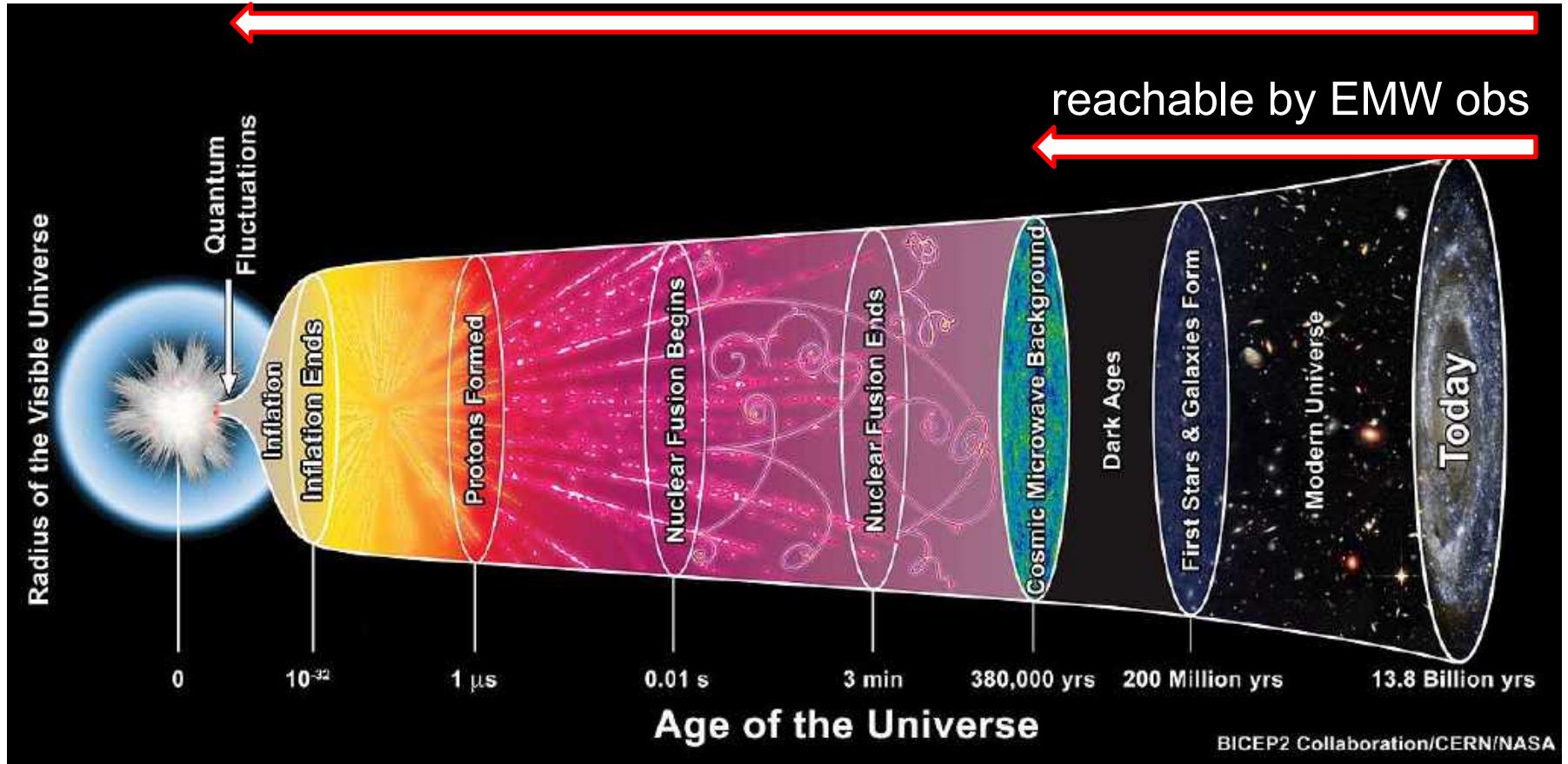


origin of all the structures
(stars, life, etc.)
in the Universe

Cosmological thermal history

reachable by GW obs

reachable by EMW obs



thermal equilibrium

end of inflation

end of hot universe

Planck Cosmic Microwave Background Map

Planck 2015/2018

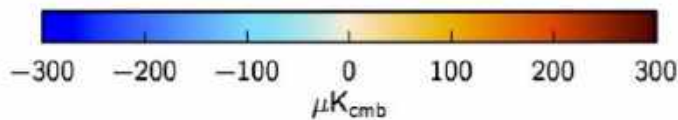
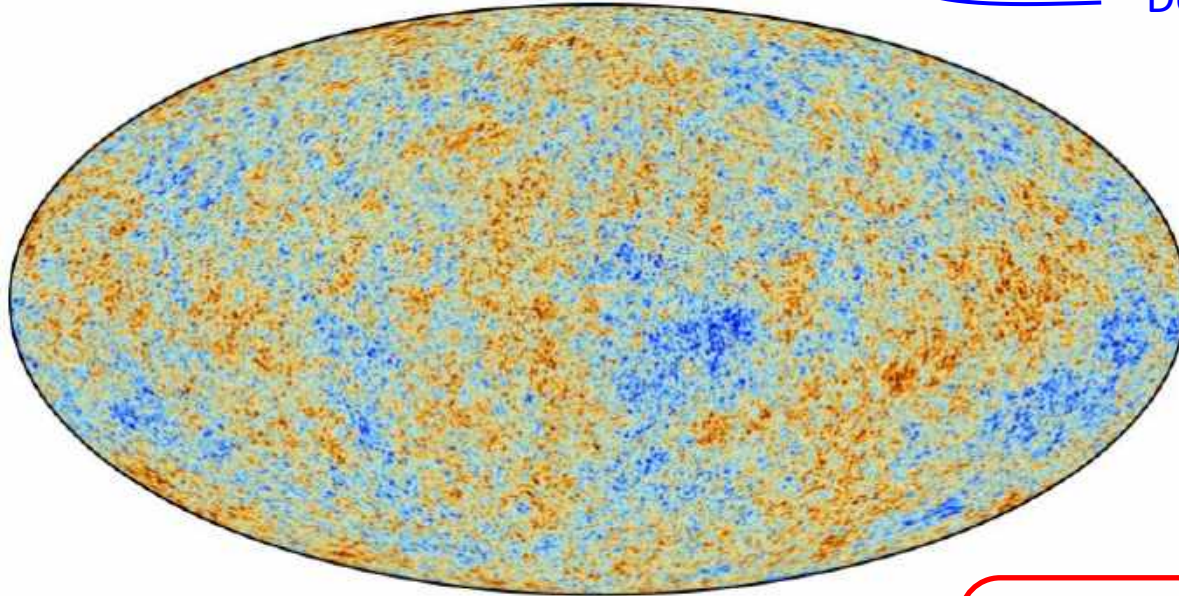
Sachs-Wolfe effect

(gravitational redshift)

$$\frac{\Delta T}{T_0} = \frac{1}{3} \Psi - \mathbf{n} \cdot \mathbf{v} + (\text{small corrections}) \sim 10^{-5}$$

but contains important info!

Doppler effect

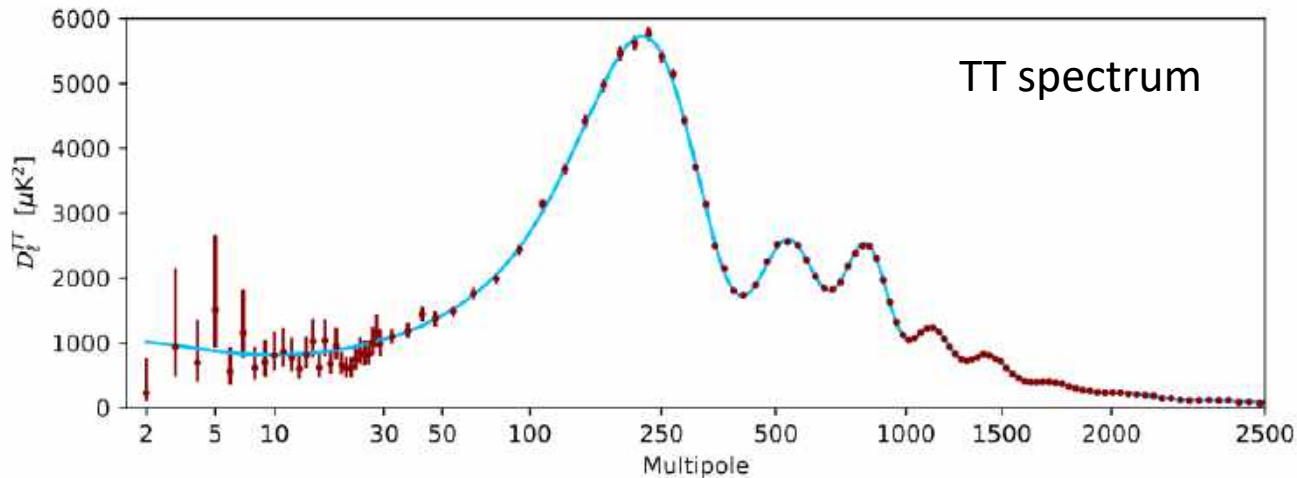


$$T_0 = 2.73 \text{ K}$$

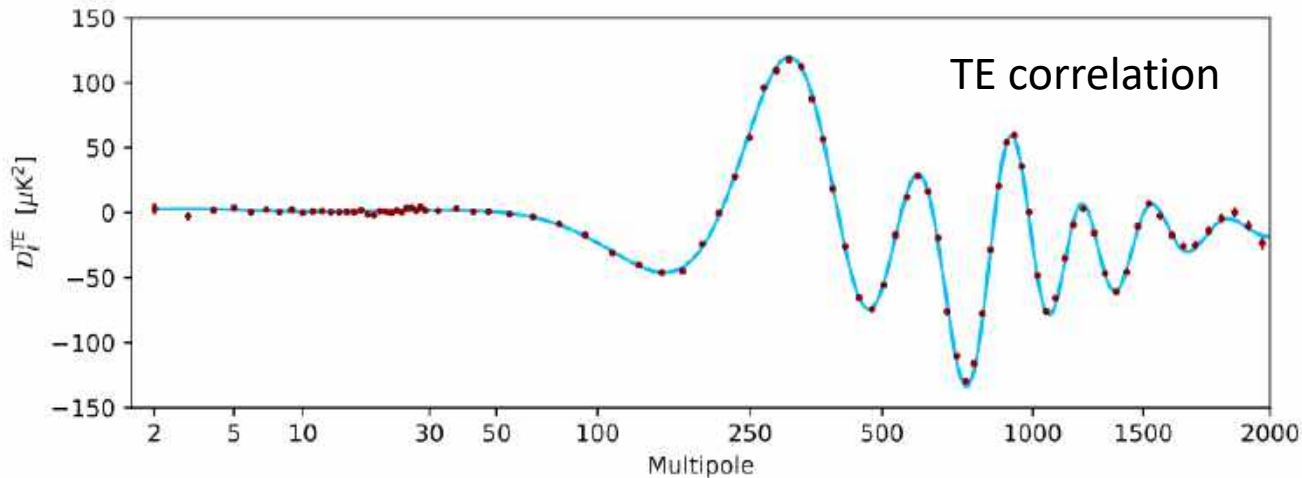
primordial vacuum fluctuations
produce Ψ and \mathbf{v}
(and induce fluctuations in matter)

Planck CMB anisotropy spectrum

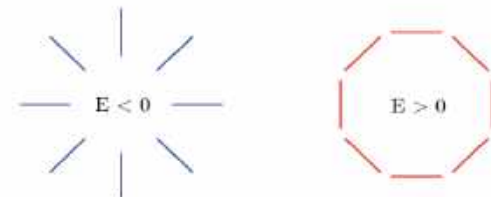
<https://www.cosmos.esa.int/web/planck/picture-gallery>



— theory
(best fit model)



E: E-mode polarization



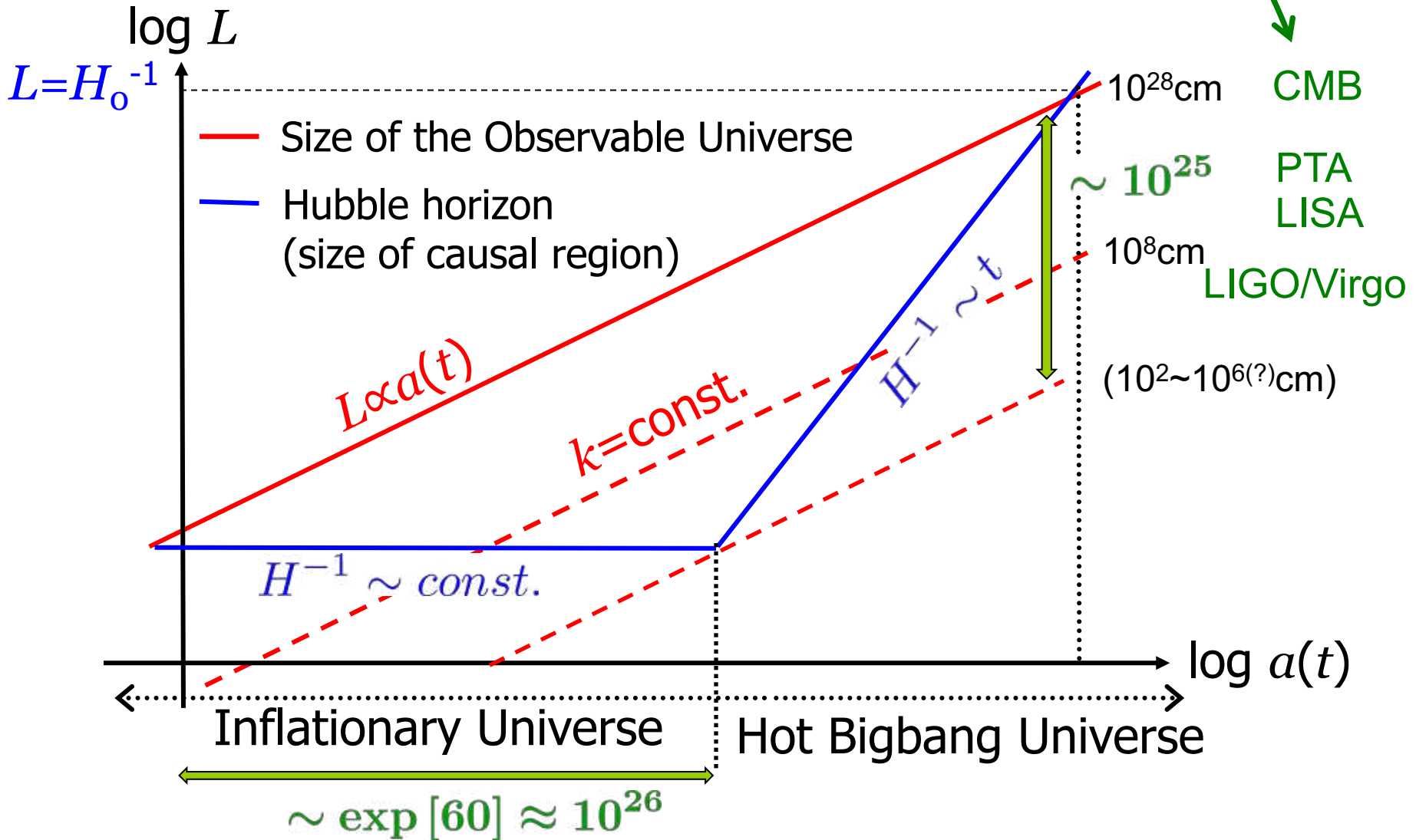
<https://arxiv.org/abs/0811.3919>

observational data **perfectly agree** with inflationary predictions

Inflation as GW source

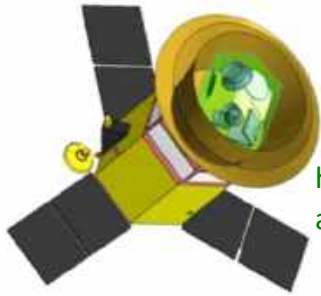
length scales of the inflationary universe

↔ targets for multi-frequency GW cosmology



Current and future GW detectors

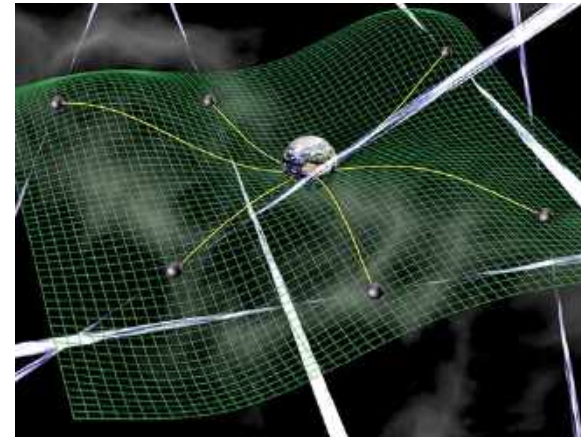
CMB B-mode: SP4, LiteBIRD...



$\sim 10^{-17}$ Hz

<https://www.ipmu.jp/en/research-activities/research-program/LiteBIRD>

Pulsar Timing Array (PTA)

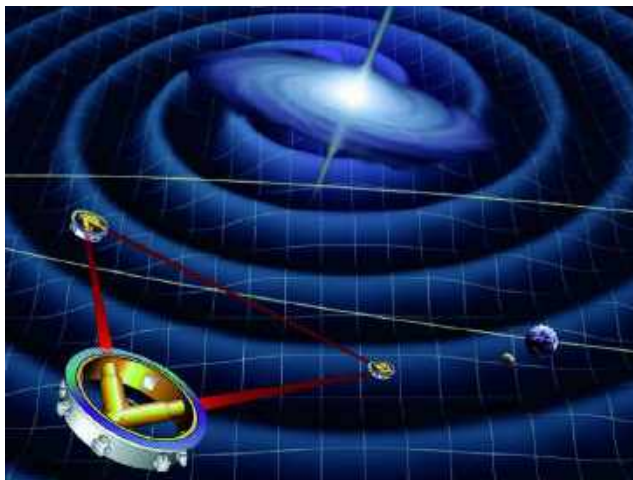


$\sim 10^{-8}$ Hz

credit: David Champion

Space-based IFO's: LISA, Taiji ...

$\sim 10^{-3}$ Hz



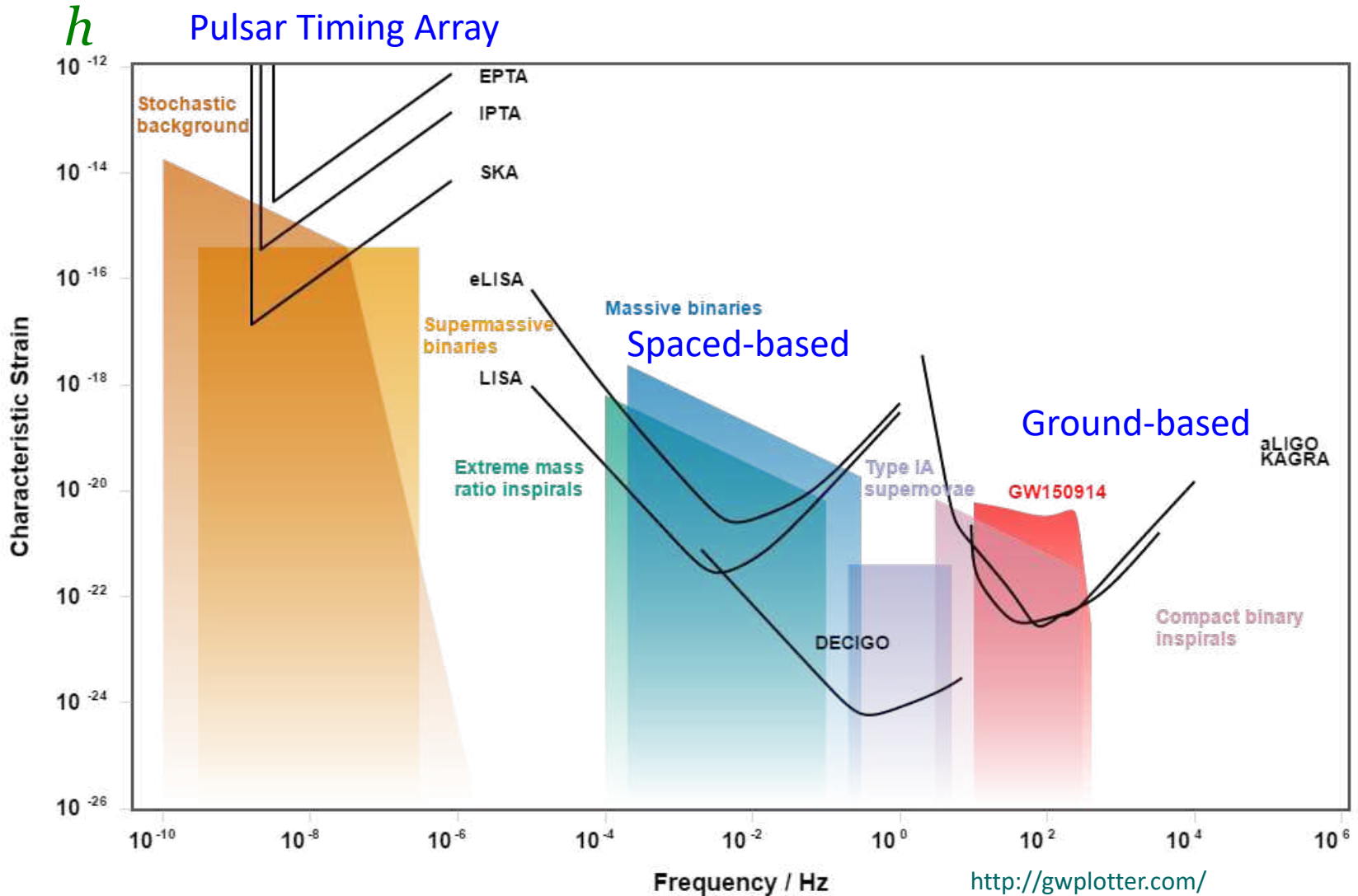
credit: ESA

Ground-based IFO's:
LIGO/Virgo/KAGRA ... $\sim 10^2$ Hz



<https://www.ligo.caltech.edu/>

GWs: New window to explore the Unknown Universe!

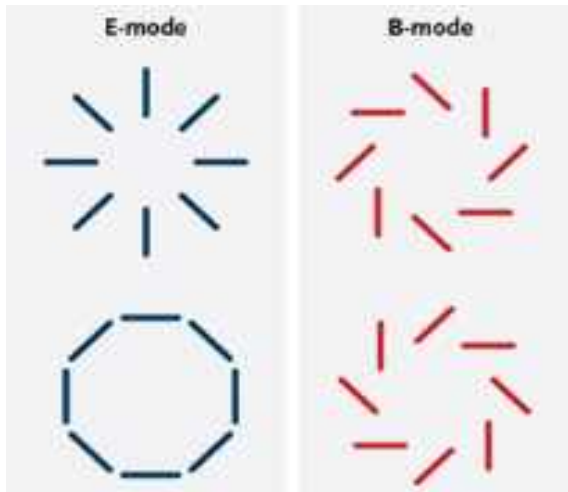
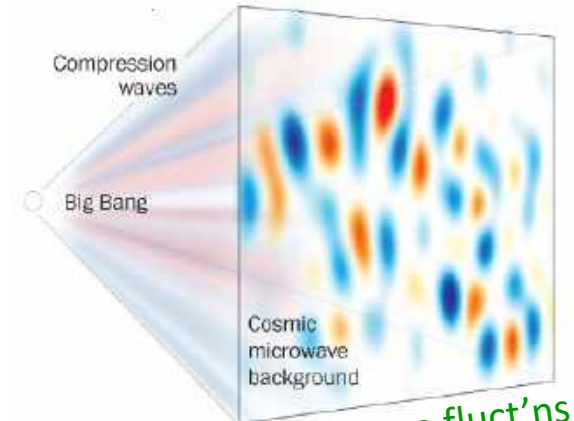


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GWs from Inflation

➤ scalar field(s) produce density fluctuations
 -> CMB temp + E-mode

➤ tensor (GW) fluctuations
 -> CMB temp + E-mode + B-mode

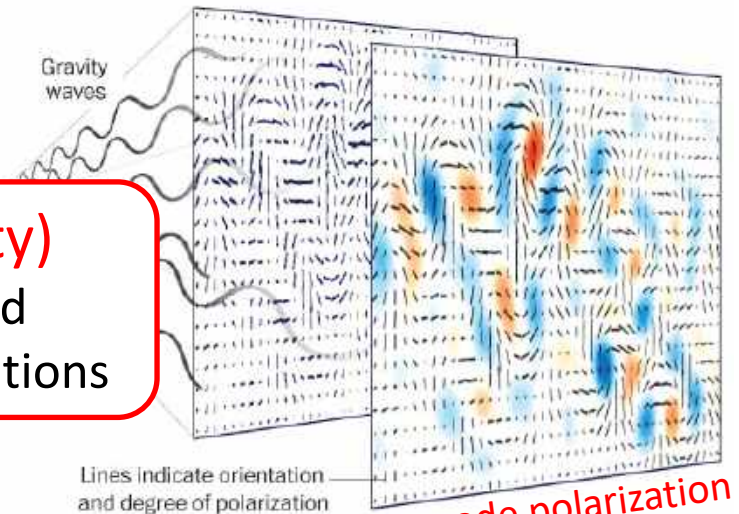


E-mode (even parity)



B-mode (odd parity)
 = cannot be produced
 from density fluctuations

<http://www.skyandtelescope.com/>

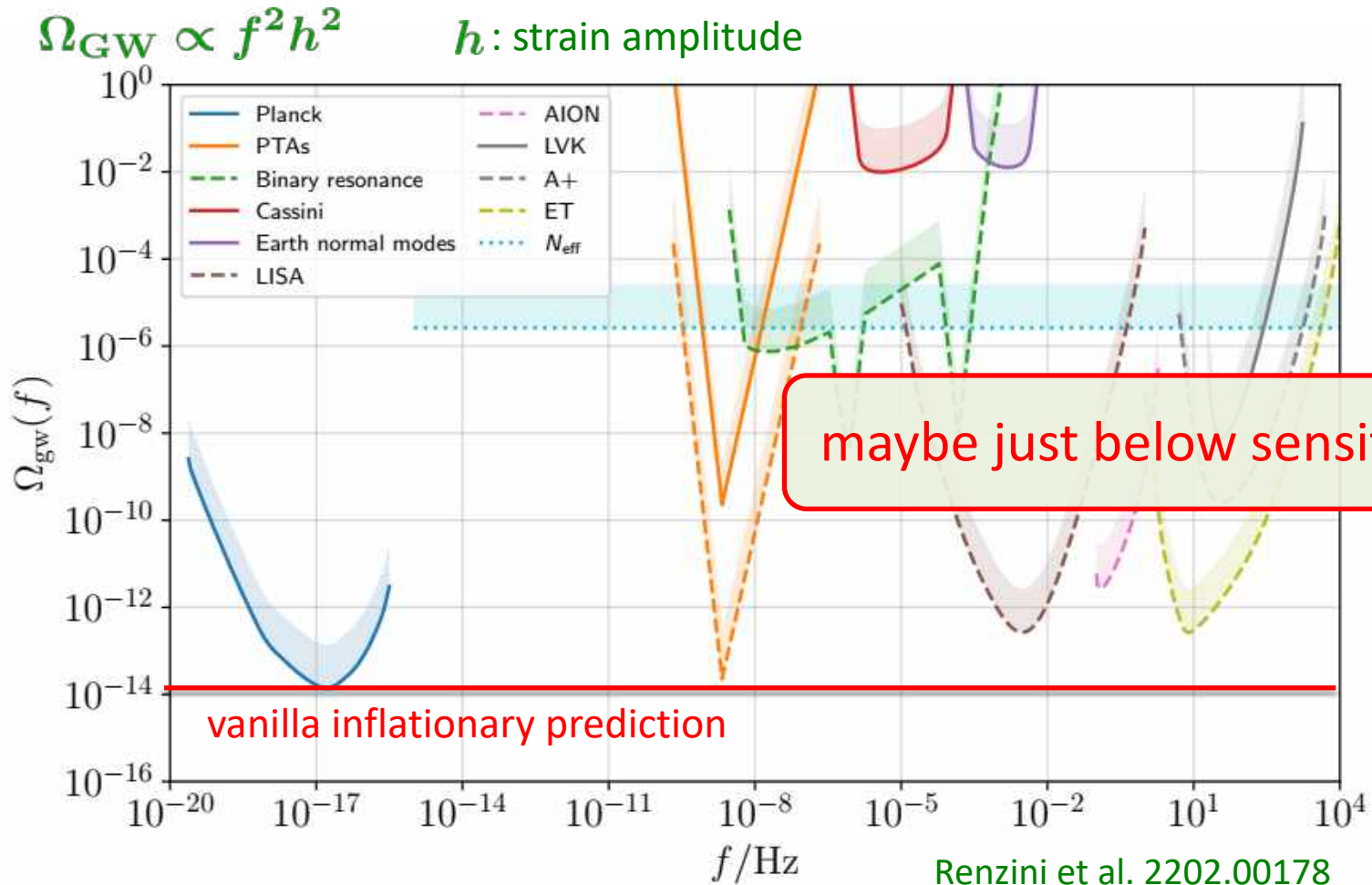


Harvard-Smithsonian Center for Astrophysics

CMB B-mode = cosmological GW detector!

GW spectrum from vanilla inflation

Can GWs from inflation be detected?



Blue-tilted GW spectrum?

possible e.g. in massive gravity inflation model [Lin & MS \(2015\)](#)

m_g^2 : tensor mass during inflation = fcn of inflaton ϕ

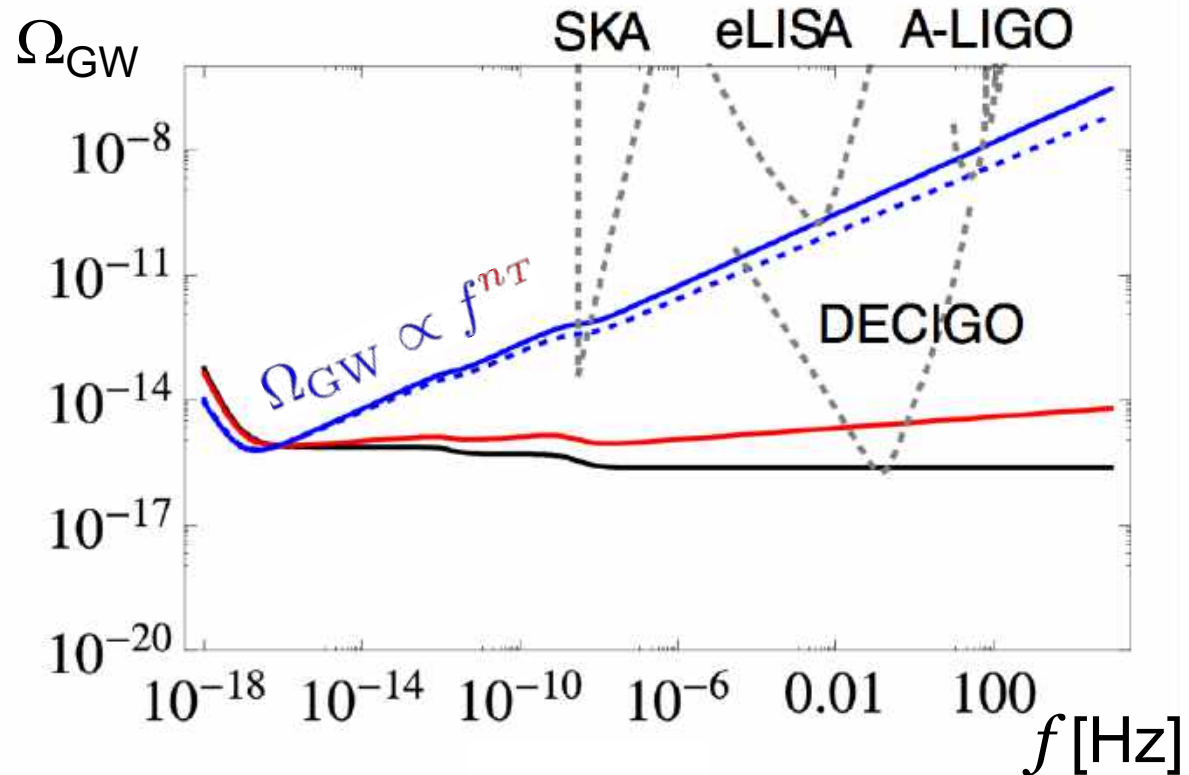
spectral index:

$$n_T = \frac{2m_g^2}{3H^2}$$

- : $m_g^2 / H^2 = 0.6$

- : $m_g^2 / H^2 = 0.1$

- : $n_T = 0$



➤ many other models that produce detectable GWs exist

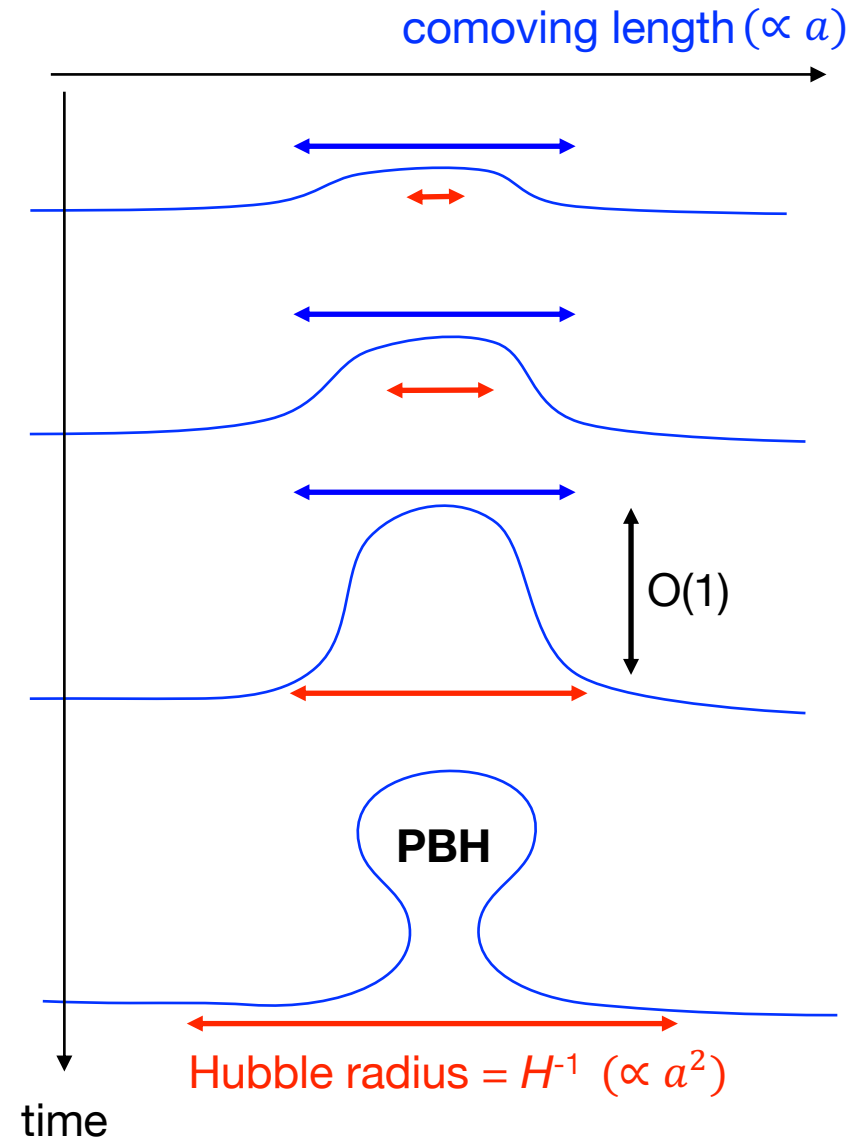
e.g., review by [Caprini & Figueroa 2018](#)

Primordial Black Holes and induced GWs

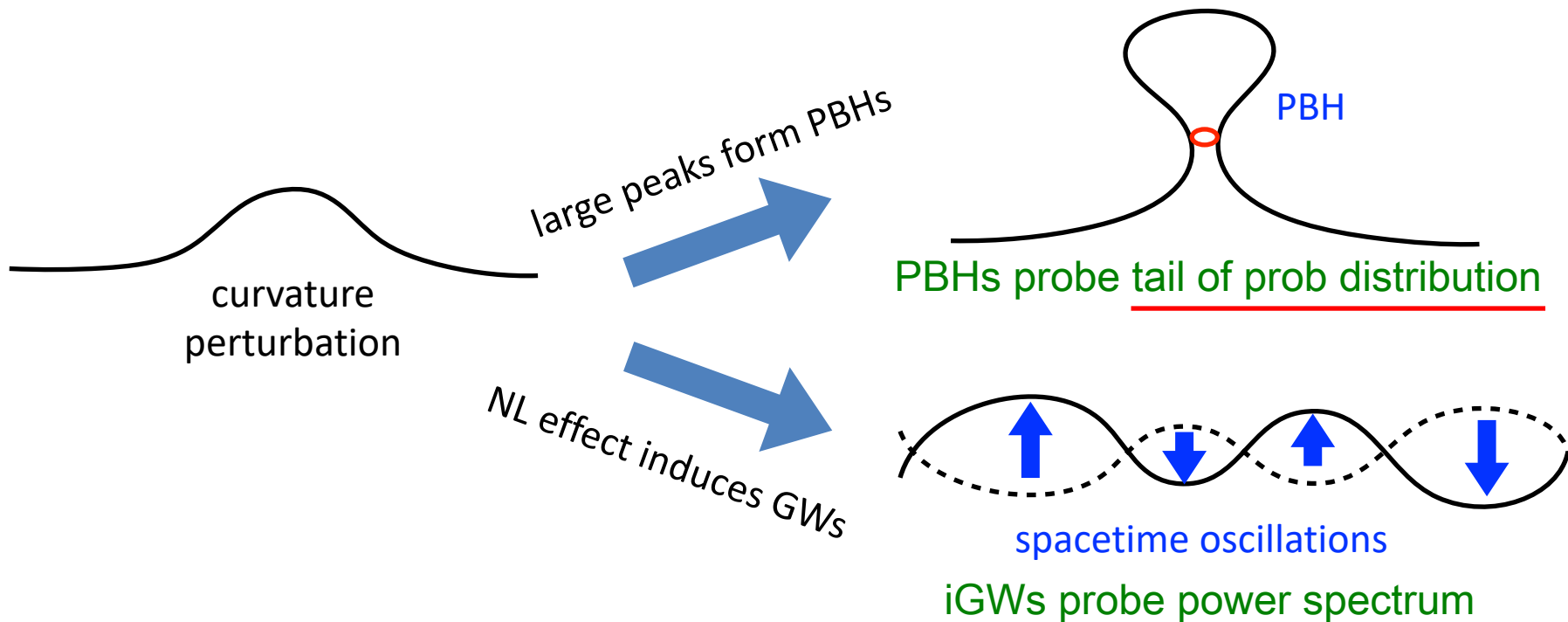
PBH formation in a nutshell

- **PBHs** are those formed in the very early universe, conventionally when the universe was **radiation-dominated**. Hawking '71, Carr & Hawking '74
- Presumably they originate from a large **positive curvature** perturbation **produced during inflation** (which hence should be a rare event).
- For a BH to form, the perturbation amplitude must be **$O(1)$ on the Hubble horizon scale**.

$$M_{\text{PBH}} \sim M_{\text{horizon}} \sim \left(\frac{100 \text{ MeV}}{T} \right)^2 M_{\odot} \sim \left(\frac{\ell}{1 \text{ pc}} \right)^2 M_{\odot}$$



iGWs can capture PBHs!



PBHs = CDM with $M_{\text{PBH}} \sim 10^{21} \text{g}$
generates GWs with $f \sim 10^{-3} \text{Hz}$

CDM=Cold Dark Matter

⇒ GWs in LISA band

PBHs = LV BHs with $M_{\text{PBH}} \sim 10 M_{\odot}$
generates GWs with $f \sim 10^{-8} \text{Hz}$

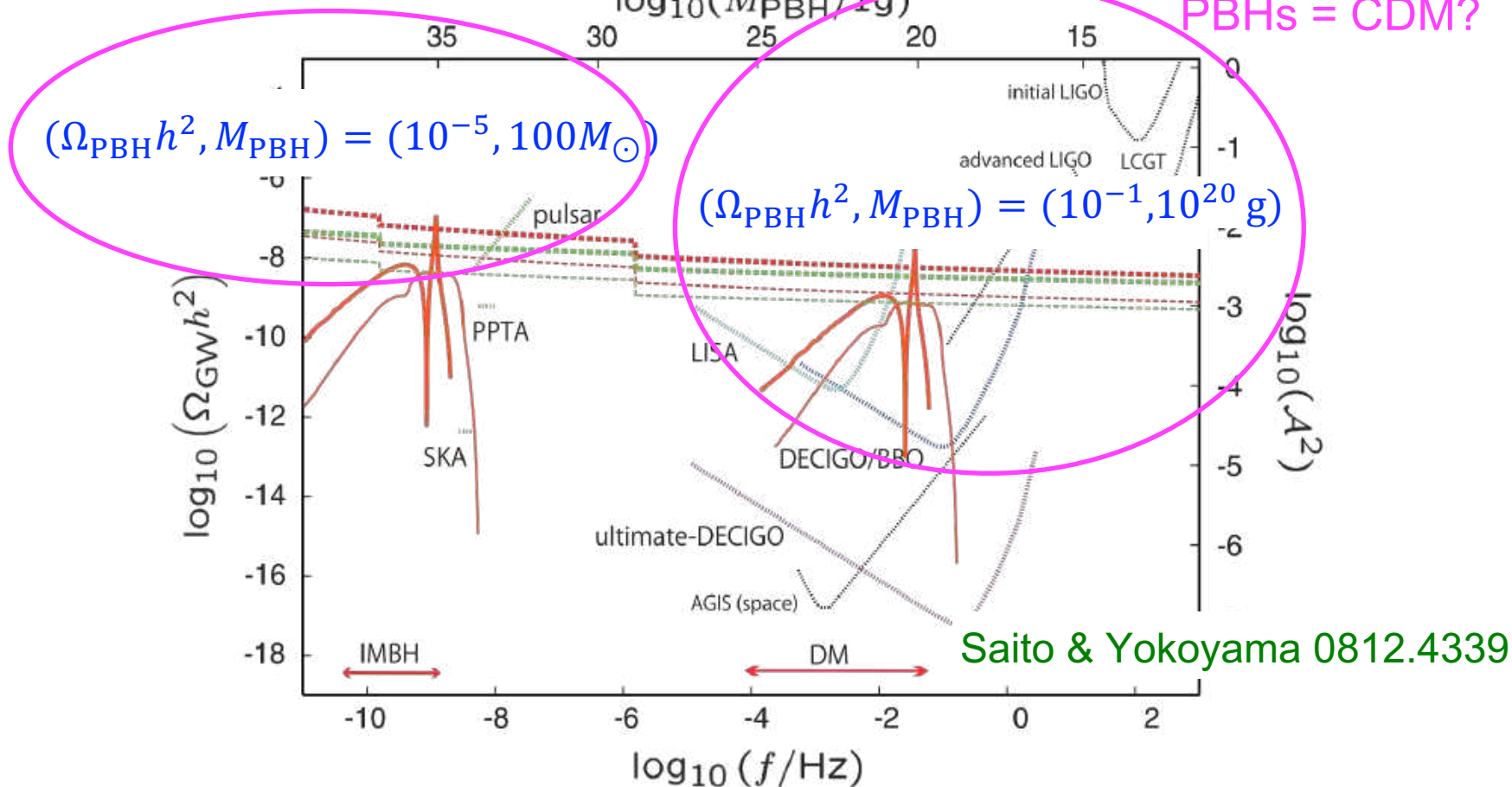
LV=LIGO-Virgo

⇒ GWs in PTA band

GWs can test PBH scenario!

PBHs = LV BHs?

PBHs = CDM?



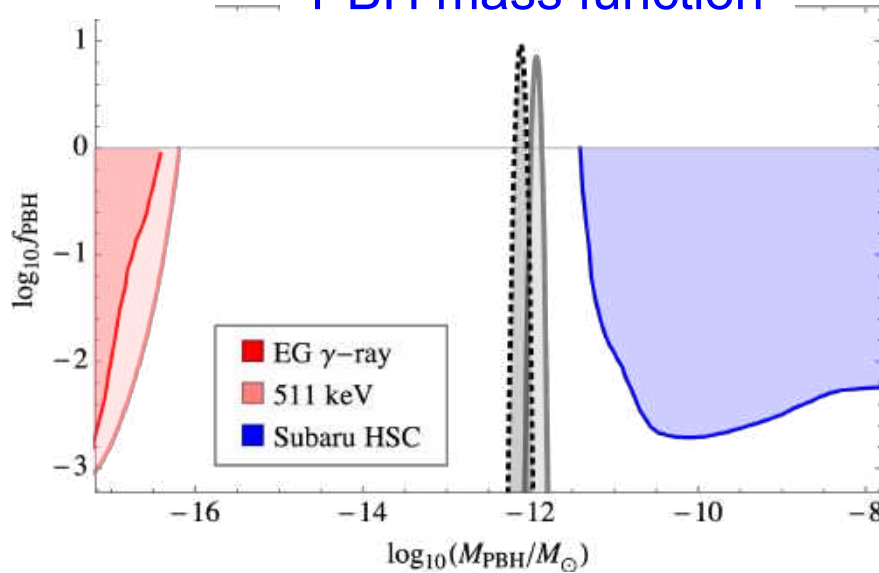
- PBHs = LV BHs scenario is already constrained by NANOGrav (PTA)
Cai, Pi, Wang & Yang 1907.06372

iGWs in non-minimal curvaton model

Pi & MS, 2112.12680

fraction
in CDM

PBH mass function

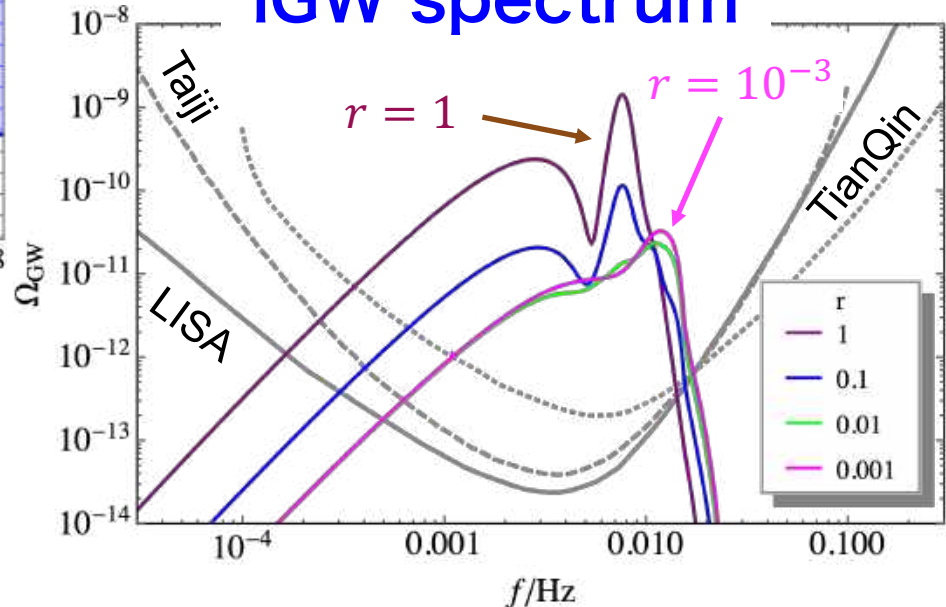


peak: $M_{\text{PBH}} \sim 10^{21} \text{ g}$

\updownarrow
 $R_{\text{BH}} \sim 10^{-7} \text{ cm}$

The model predicts a highly **non-Gaussian tail** in PDF for the curvature perturbation

iGW spectrum



$r \ll 1$ = stronger non-Gaussianity

PBH=CDM scenario can be clearly verified or falsified by LISA!

Summary

- * **Inflation** has become the **standard model** of the Universe
- * **Cosmological GWs** are the key to **confirmation of inflation**
- * Era of **multi-band GW cosmology** has begun!

CDM may be PBHs!

future GW detectors will prove/disprove the scenario.

- * **Entire stage** of inflation may be probed by **GWs**
- * **Astrophysical GWs** also play essential roles

GWs will be an essential tool to explore the Physics of the Unknown Universe!

Stay tuned!

Thank you!