Universality of topological physics in quantum and classical systems

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Interdisciplinary Science Conference in Okinawa (ISCO 2023)
— Physics and Mathematics meet Medical Science —

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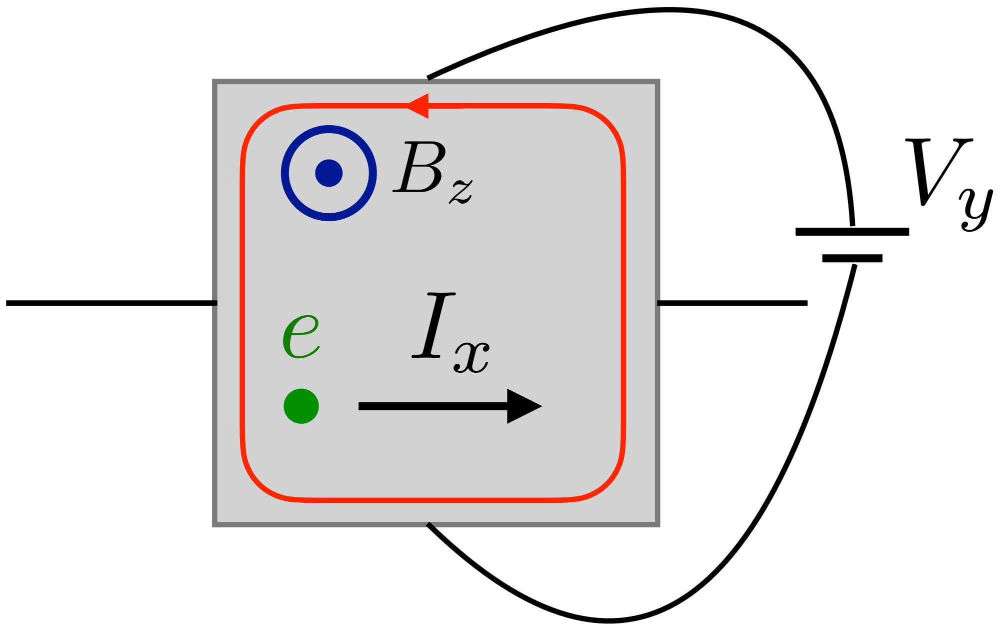
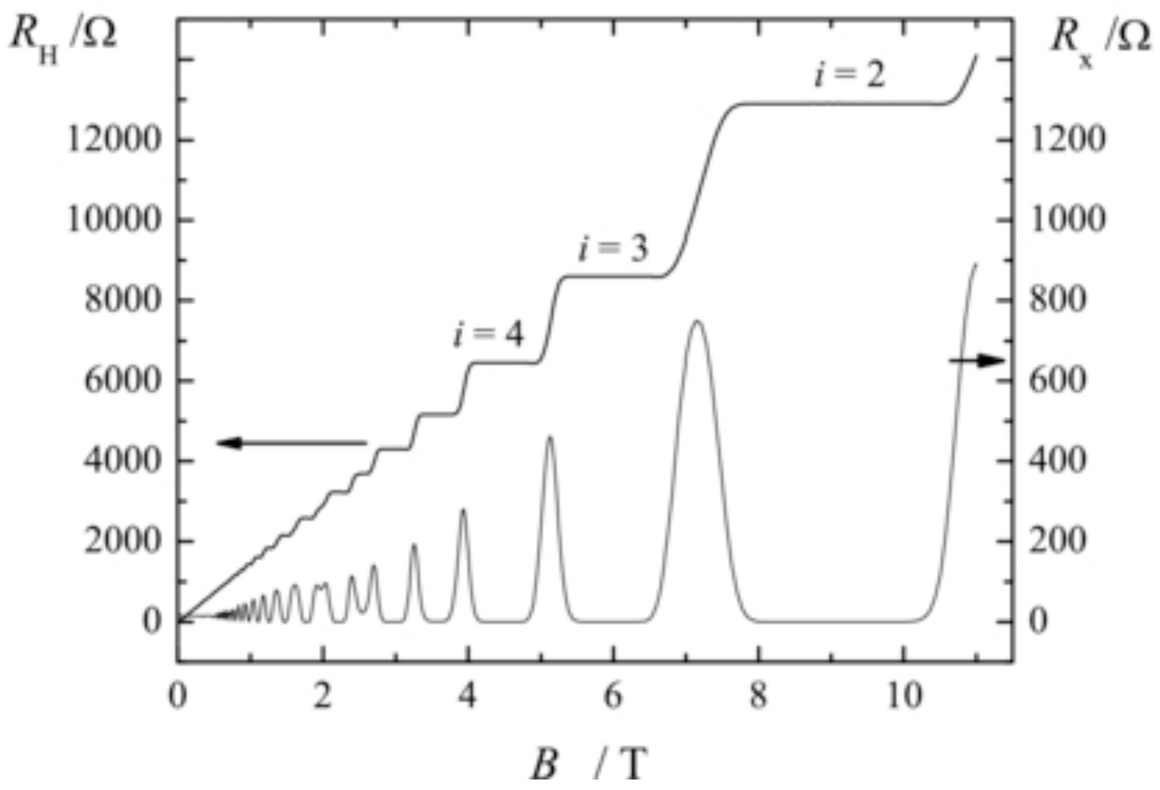
Quantum Hall effect



$$V_y = R_{xy} I_x$$

$$R_{xy} = \frac{h}{e^2} \cdot \frac{1}{\text{integer}}$$



In 1980, it was found that the Hall resistivity under a high magnetic field shows unexpected plateau

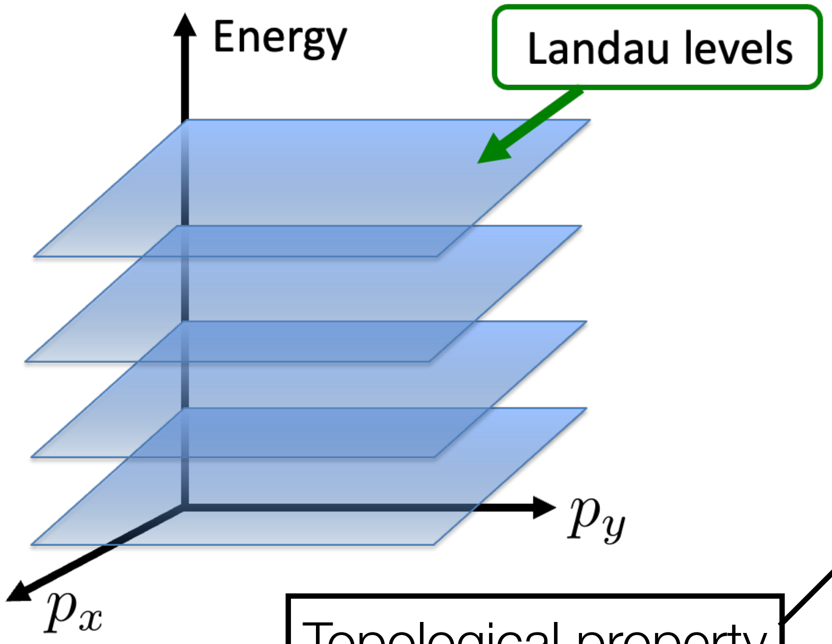
- Integer Quantum Hall Effect-

Main goal of my talk today are twofold:

- To discuss that this effect is related to “topology” of the “wave function”
- To show that this effect is more general than just in semiconductor electron physics

Landau level, topology, and bulk-edge correspondence

Possible energy of an electron in a magnetic field takes only discrete values called Landau levels

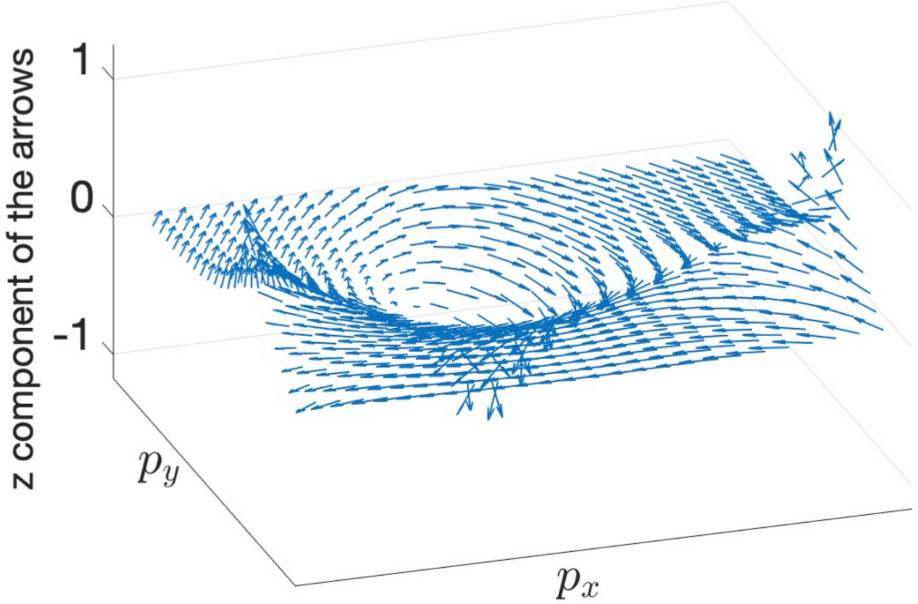


Energy is an **eigenvalue** of an operator called Hamiltonian
We can also consider its **eigenvectors**, which represent the corresponding **quantum states**

Eigenvectors can form a nontrivial shape in momentum space
How many times the eigenvectors **rotate** is an integer called the **Chern number**, and this is 1 for Landau levels

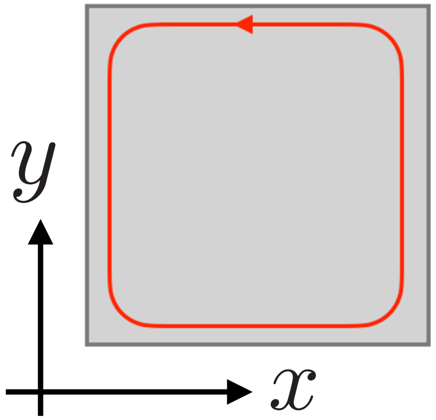
Topological property

Eigenvectors represented as arrows



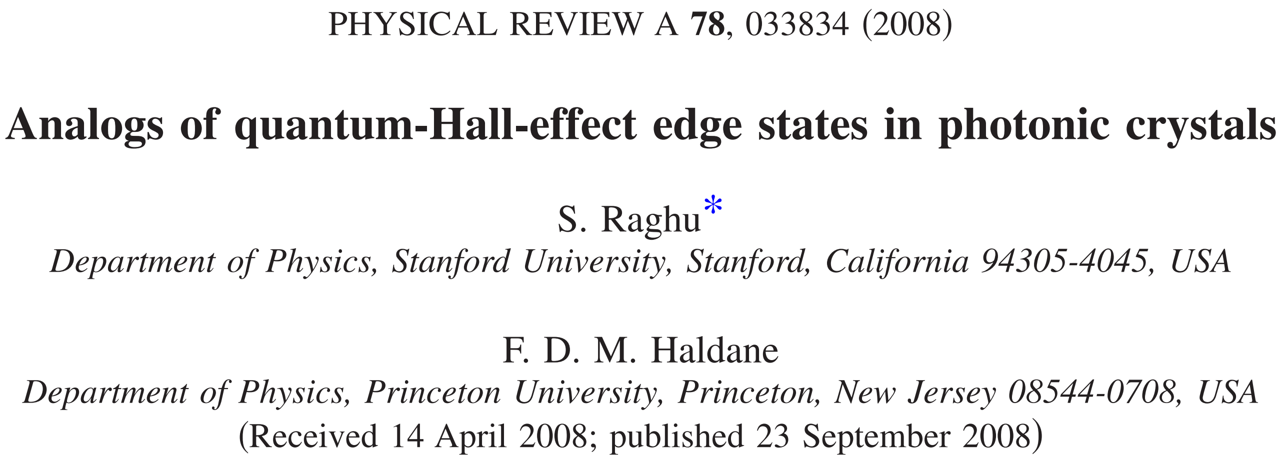
Chern number (calculated in momentum space) turns out to be equal to the number of edge-localized state in real space
- bulk-edge correspondence -

Hall resistivity is inversely proportional to the number of edge-localized modes



Topological photonics

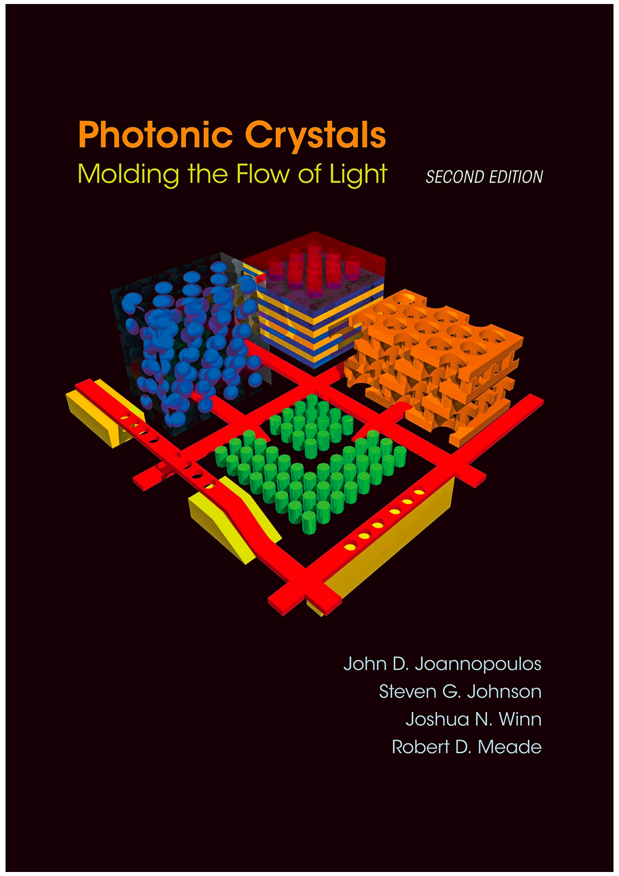
Eigenvectors in momentum space rotate also in other systems


PHYSICAL REVIEW A **78**, 033834 (2008)

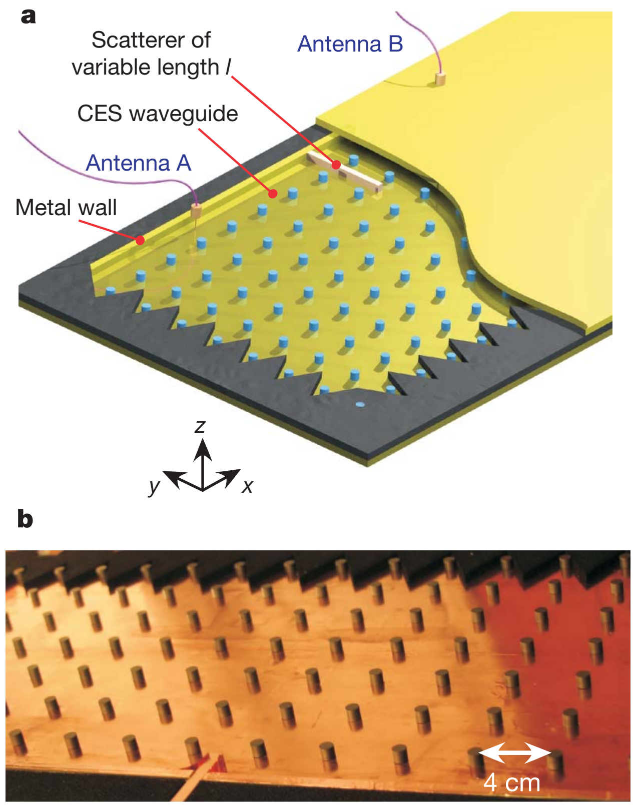
Analogs of quantum-Hall-effect edge states in photonic crystals

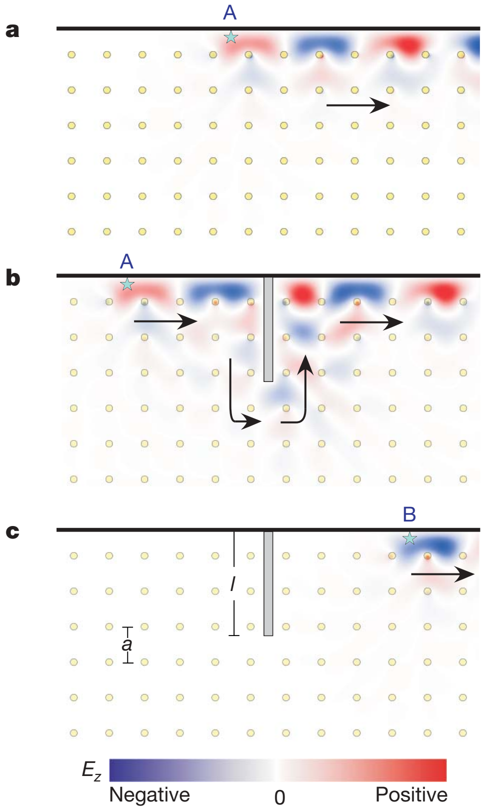
S. Raghu*
Department of Physics, Stanford University, Stanford, California 94305-4045, USA

F. D. M. Haldane
Department of Physics, Princeton University, Princeton, New Jersey 08544-0708, USA
(Received 14 April 2008; published 23 September 2008)


Photonic Crystals
Molding the Flow of Light
SECOND EDITION

John D. Joannopoulos
Steven G. Johnson
Joshua N. Winn
Robert D. Meade





Topological Photonics:
Topological energy eigenstates in photonics systems

Wang, Chong, Joannopoulos, & Soljačić (MIT), Nature **461**, 772 (2009)

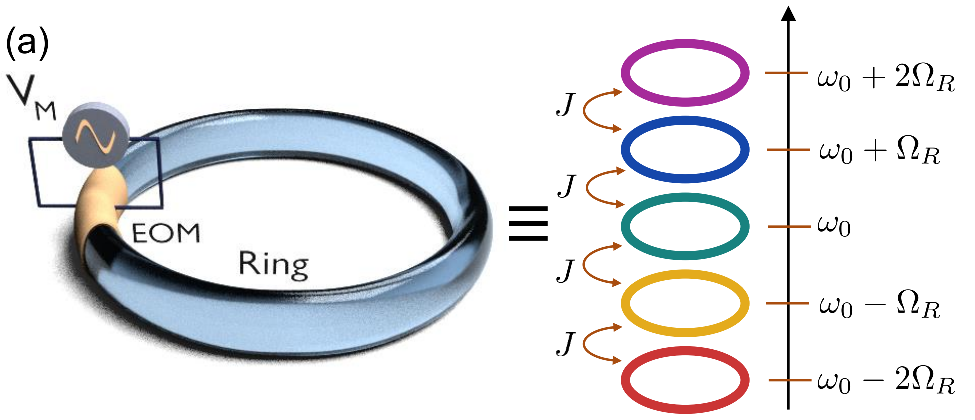
Topological photonics in synthetic dimensions

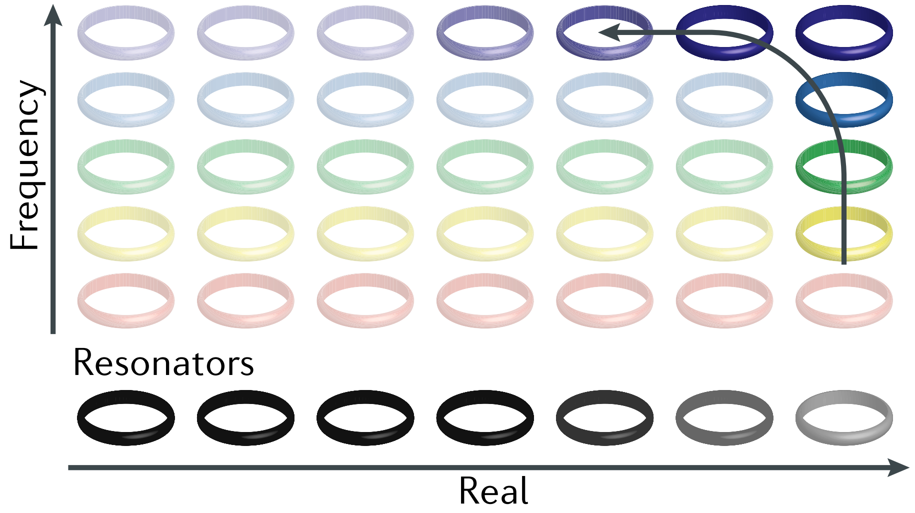
One can use non-spatial degrees of freedom as dimensions: **Synthetic dimensions**

We can use resonant modes of a resonator as a dimension and realize topological lattice models

Theory: TO, Price, Goldman, Zilberberg, & Carusotto, PRA **93**, 043827 (2016)
Yuan, Shi, & Fan, Opt. Lett. **41**, 741 (2016)

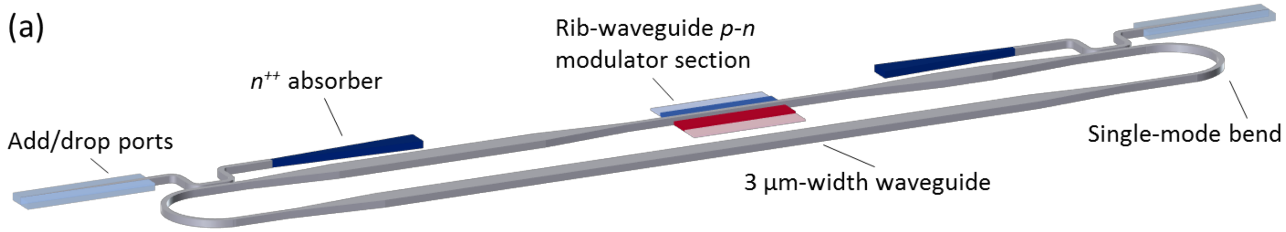
First experiment: Dutt, Minkov, Lin, Yuan, Miller, & Fan, Nature Comm. **10**, 3122 (2019)

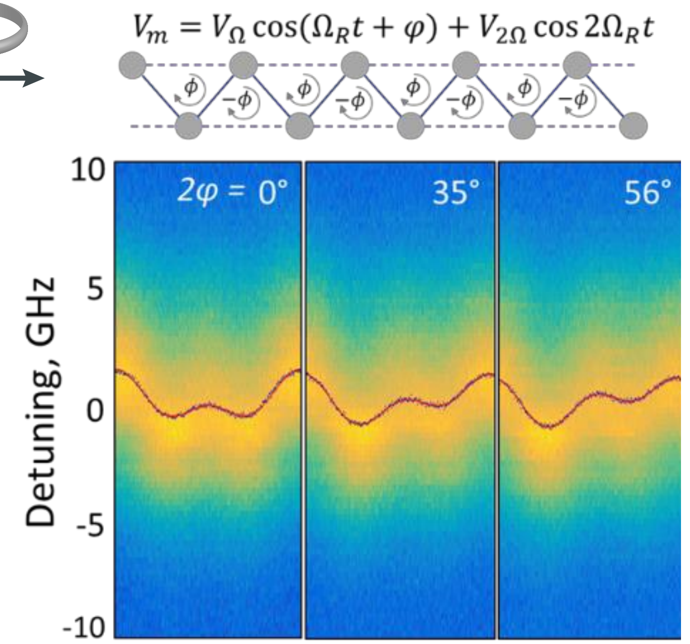




First experiment in silicon ring resonator:

Balčytis, TO, Ota, Iwamoto, Maeda, & Baba, Science Advances 8, eabk0468 (2022)

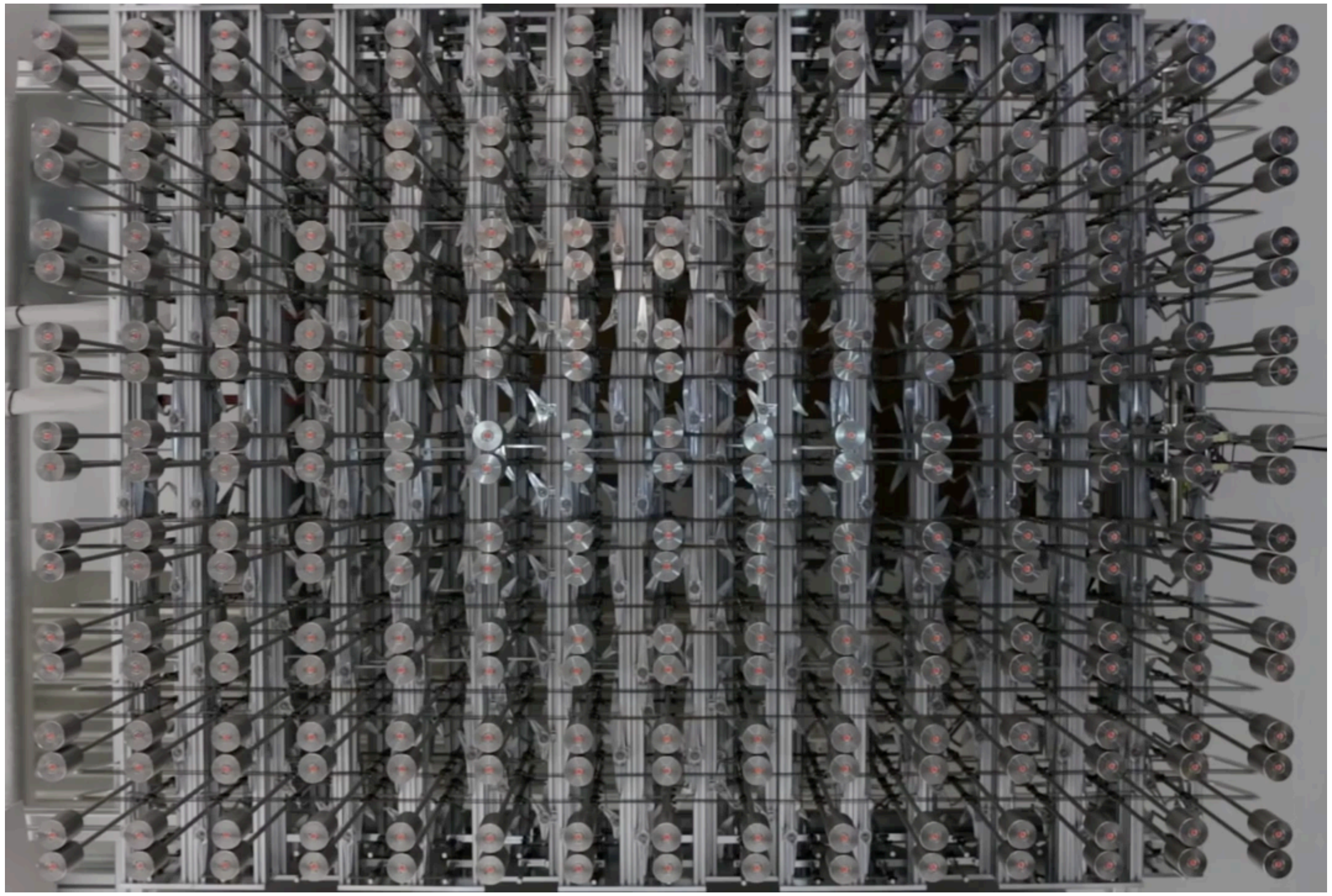




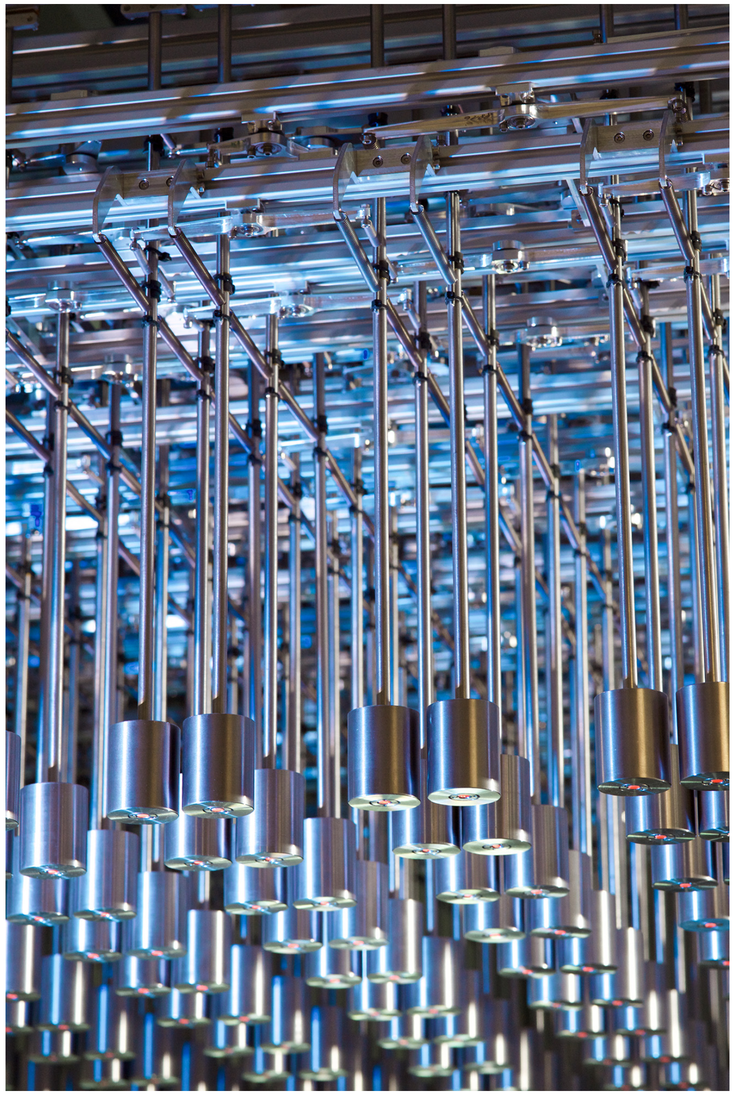
Topological mechanics

Situations similar to **quantum Hall effect** can also be realized with **Newtonian mechanics**

ETH Zurich: Süsstrunk & Huber, Science **349**, 47 (2015)



https://www.youtube.com/watch?v=__TGJEtFD-E



Two-pendulum experiment

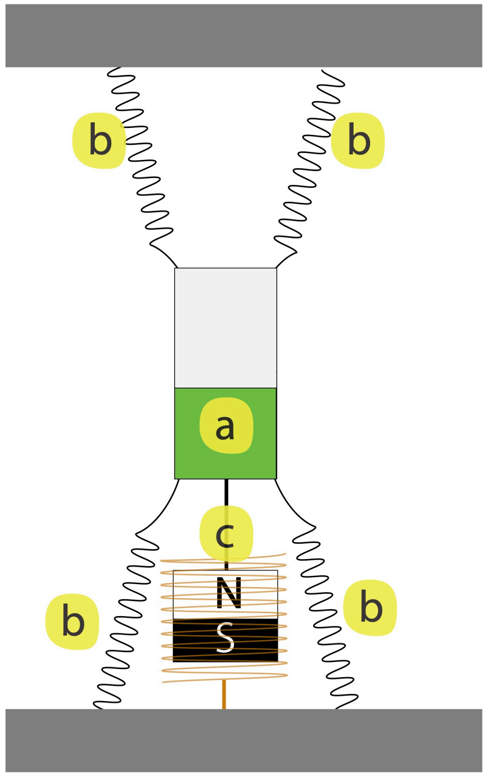
Two-pendulum experiment in collaboration with Gadway group @ University of Illinois

Natural frequency of a pendulum is 3Hz

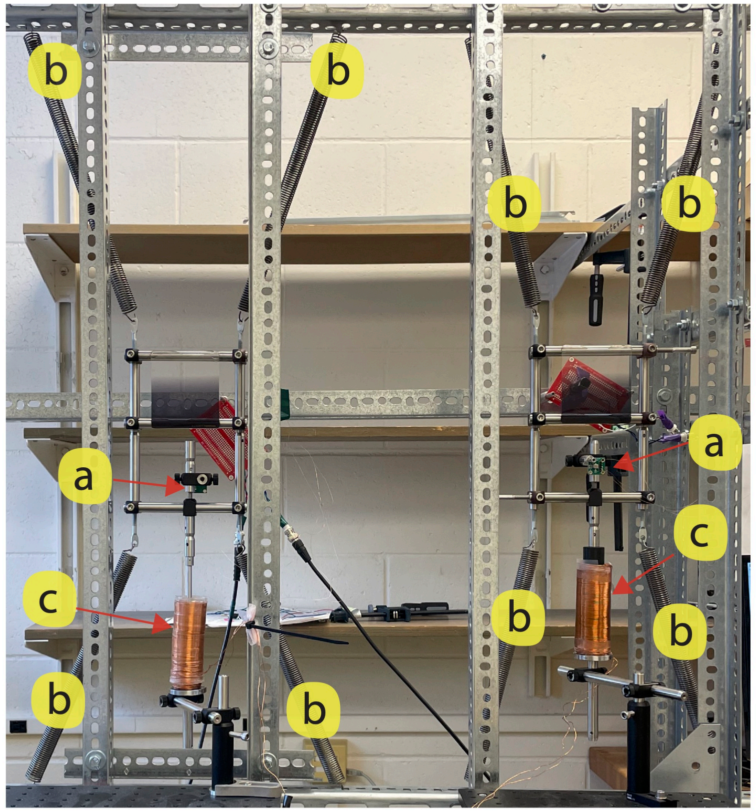
Pendulums are coupled to electromagnets, through which forces can be applied

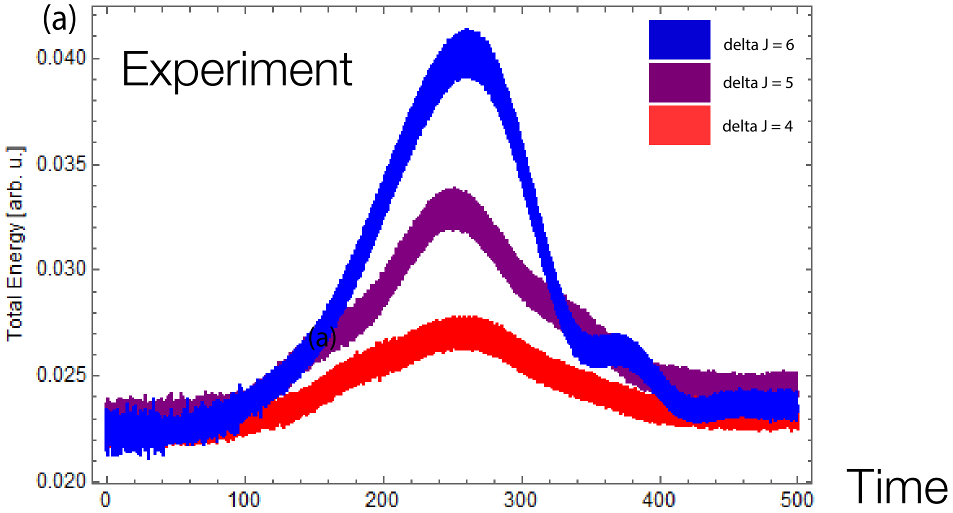
Measurement of non-Hermitian Berry phase

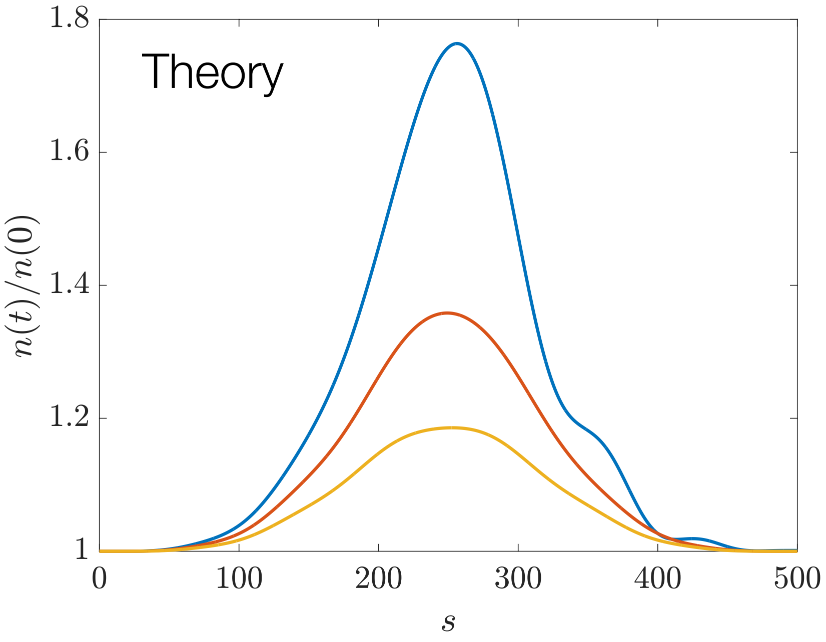
(a)



(b)



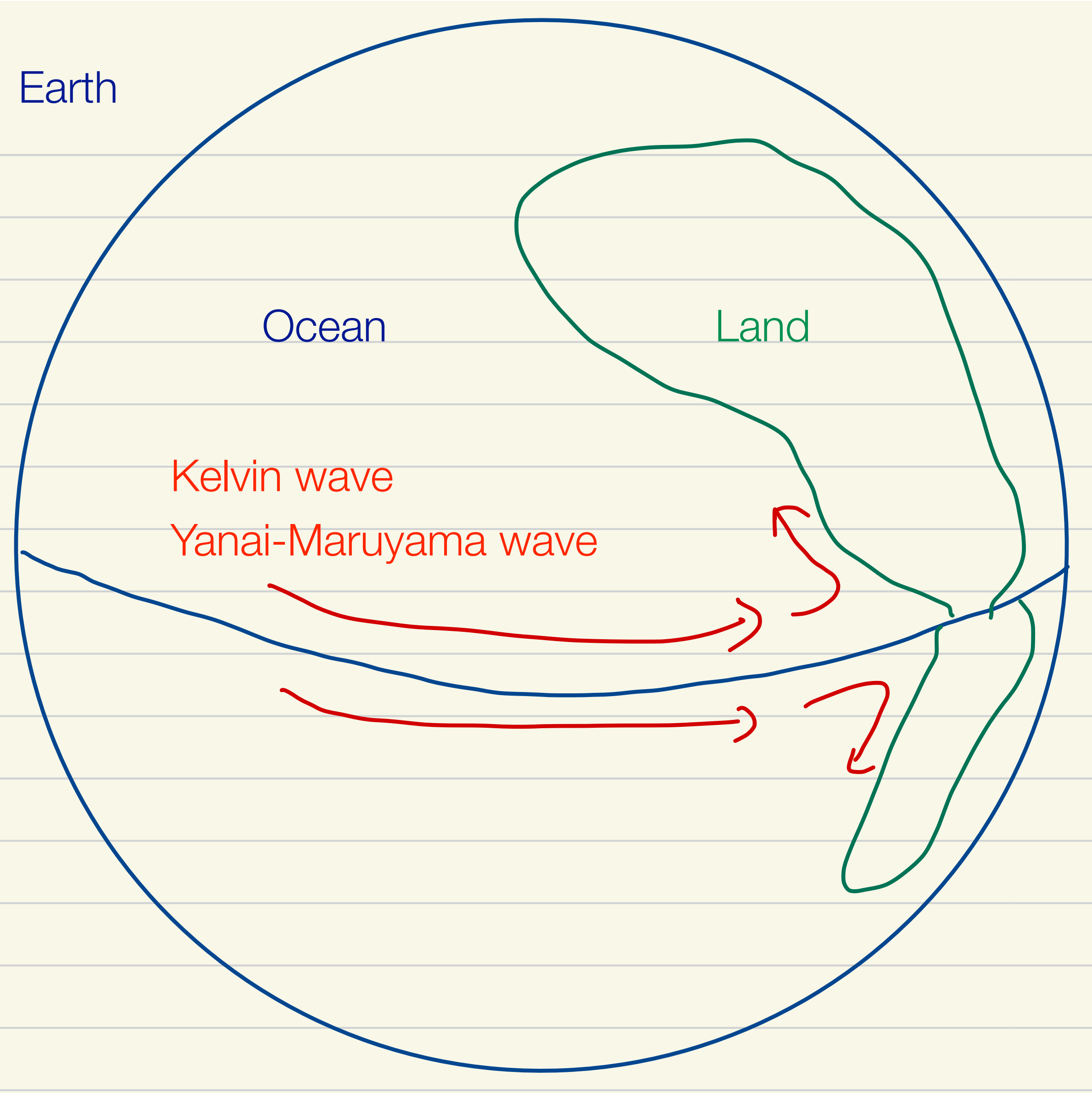




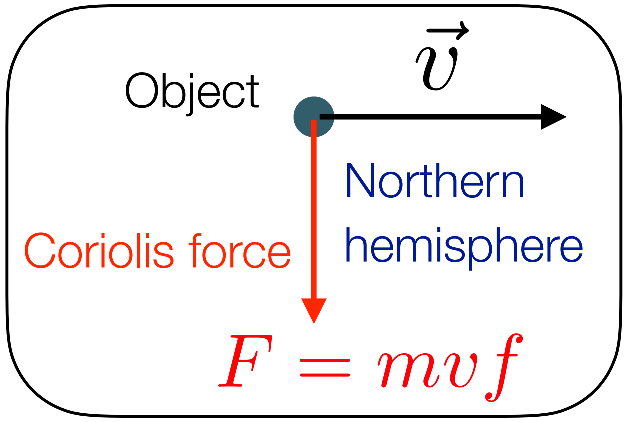
- Martello, Singhal, Gadway, <u>TO</u>, Price, arXiv:2302.03572
- Singhal, Martello, Agrawal, <u>TO</u>, Price, Gadway, arXiv:2205.02700
- Anandwade, Singhal, Paladugu, Martello, Castle, Agrawal, Carlson, Battle-McDonald, <u>TO</u>, Price, Gadway, arXiv:2107.09649

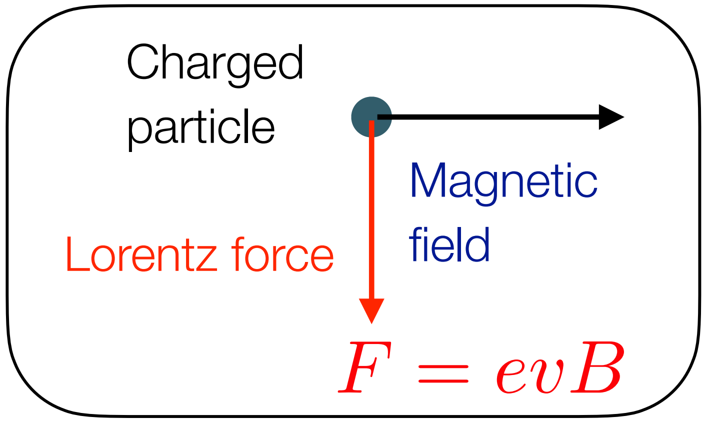
Equatorial wave

There are ocean and atmospheric waves called **Kelvin wave** and **Yanai-Maruyama wave**, which flow along the equator to the east



Moving objects on the earth feel the **Coriolis force** due to the rotation of the earth





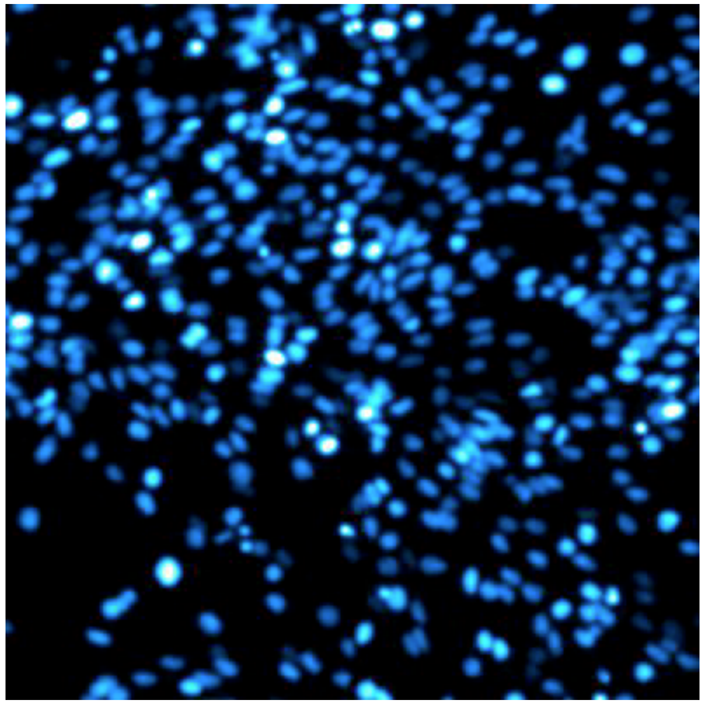
Coriolis force and Lorentz force are very similar

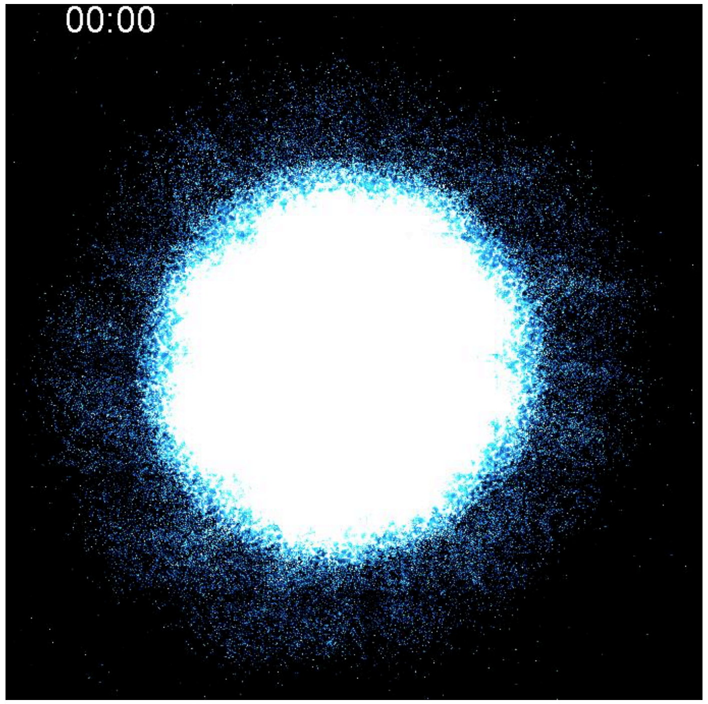
Kelvin wave and Yanai-Maruyama waves turned out to be the topological edge modes

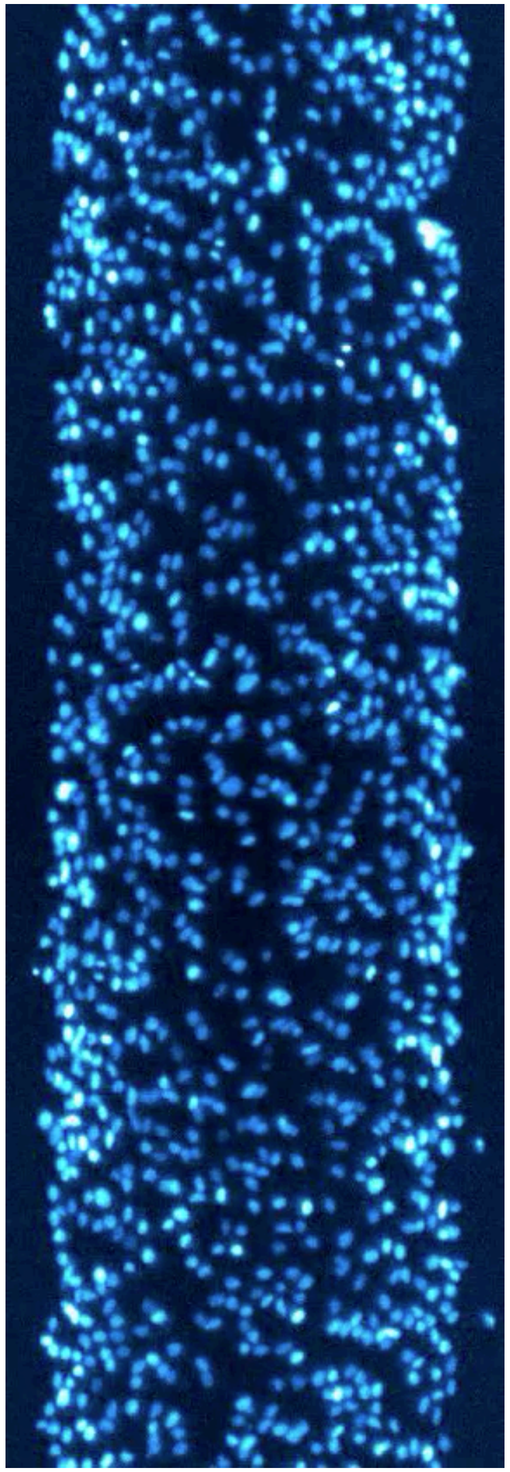
Delplace, Marston, & Venaille, Science **358**, 1075 (2017)

Neural stem cell

Stem cells of mice can move around







- They seem to bend in one direction (perhaps due to left-right asymmetry of cells)
- This motion looks similar to particles under Lorentz or Coriolis force
- When the cells are confined to move in a region with edges, they flow along the edges

Topological band structure of cell motion

One can model the system using the language of active matter

Linearizing the fluid equation, the resulting equation looks like the Schrödinger equation

$$i\partial_t \underbrace{|\Psi\rangle}_{\boxed{\text{Order parameter}}} = \underbrace{H}_{\boxed{\text{5-by-5 matrix}}}|\Psi\rangle + \underbrace{|s\rangle}_{\boxed{\text{Source term}}}$$

The “Hamiltonian” H resembles the one in the case of equatorial waves

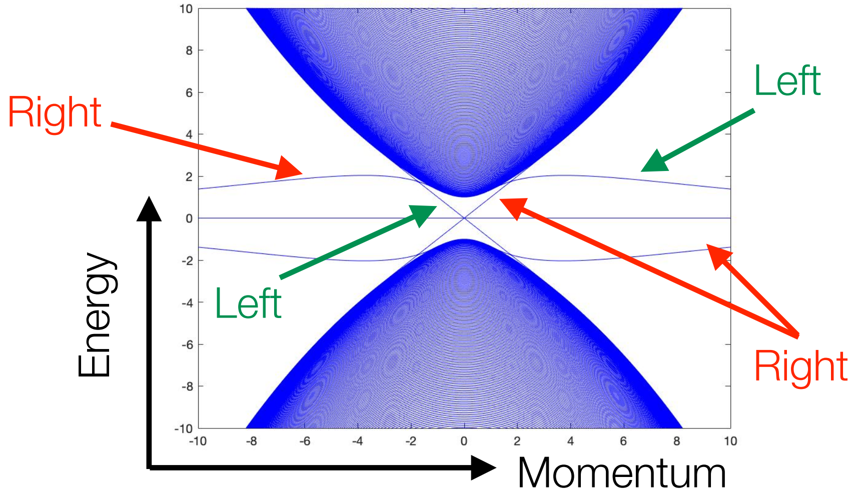
$$\mathcal{H}_{3\text{x}3} = \begin{pmatrix} 0 & v_0 p_x & v_0 p_y \\ v_0 p_x & 0 & i(f - \nu \mathbf{p}^2) \\ v_0 p_y & -i(f - \nu \mathbf{p}^2) & 0 \end{pmatrix}$$

v_0 : free velocity of cells

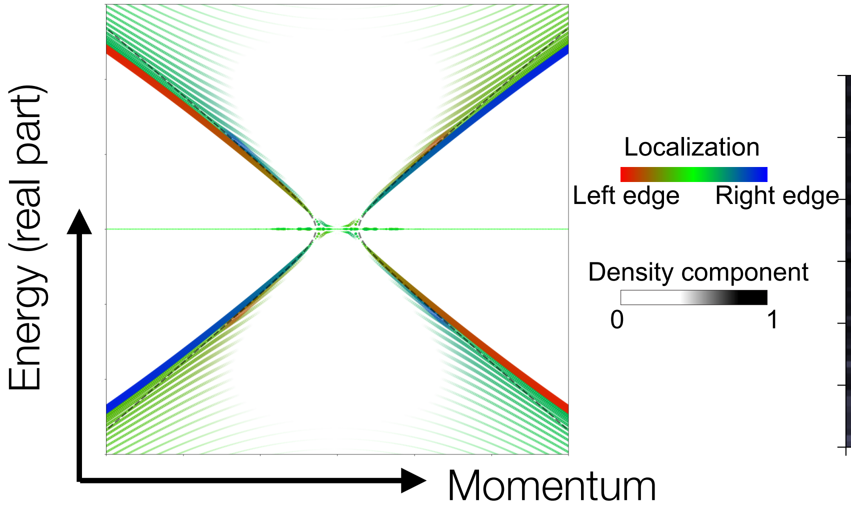
f : strength of chirality

ν : odd viscosity

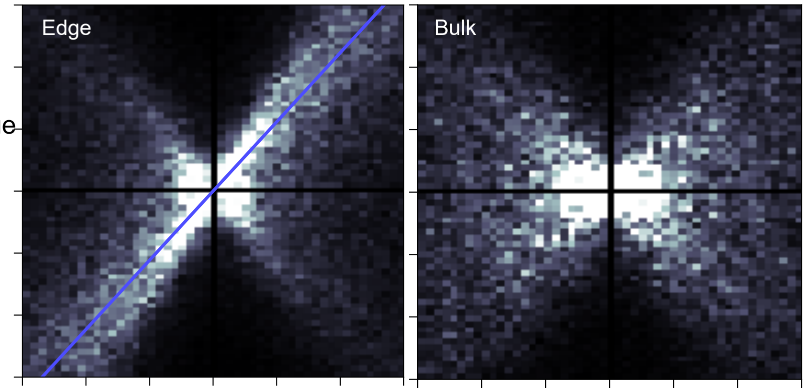
Band structure of the above Hamiltonian



More realistic situation

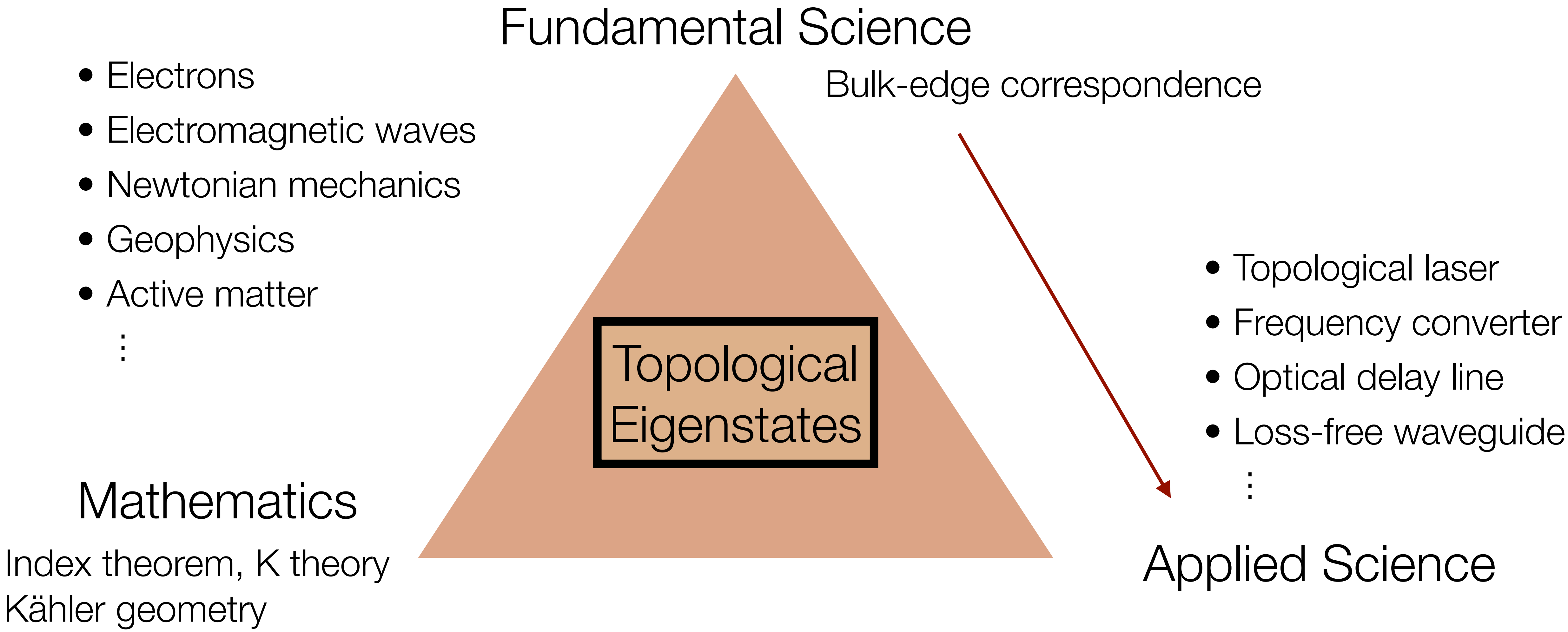


Experiment (edge & bulk)



Yamauchi, Hayata, Uwamichi, TO,& Kawaguchi, arXiv:2008.10852

Topological phases



Fundamental Science

- Electrons
- Electromagnetic waves
- Newtonian mechanics
- Geophysics
- Active matter

⋮

Bulk-edge correspondence

Topological Eigenstates

- Topological laser
- Frequency converter
- Optical delay line
- Loss-free waveguide

⋮

Mathematics

Index theorem, K theory
Kähler geometry

Applied Science

Some reviews I wrote:

- Topological photonics: TO et al., Rev. Mod. Phys. **91**, 015006 (2019).
- Synthetic dimensions: TO and Price, Nature Reviews Physics **1**, 349–357 (2019).
- Active topological photonics: Ota et al., Nanophotonics **9**, 547 (2020).
- Roadmap on topological photonics: Price et al., J. Phys. Photonics **4** 032501 (2022).