Cancer Detection via Electrical Impedance Tomography and PDE Constrained Optimal Control in Sobolev Spaces

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$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq \mu \sum_{i=1}^{n} \xi_i^2, \; \forall \xi \in \mathbb{R}^n; \quad \mu > 0.$$

Complete Electrode Model

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Optimal Control Problem

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$$\mathcal{J}(v) = \sum_{l=1}^{m} \Big| \int_{E_l} \frac{U_l - u(x)}{Z_l} ds - I_l \Big|^2 + \beta |U - U^*|^2 \to min$$

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Theorem on Fréchet Differentiability

The functional $\mathcal{K}(v)$ is differentiable on V_R in the sense of Fréchet and the Fréchet gradient $\mathcal{K}'(\sigma, U) = \Big(\mathcal{K}'_A(A,U), \mathcal{K}'_U(A,U)\Big) \in \mathcal{L}' \times \mathbb{R}^m$ is

$$\mathcal{K}'_A(A,U) = -\Big(\sum_{j=1}^{m} \psi^j_{x_p} u^j_{x_q}\Big)_{p,q=1}^{n},$$

$$\mathcal{K}'_U(A,U) = \Big(\sum_{j=1}^{m}\sum_{l=1}^{m} 2\Big[\int_{E_l} \frac{U_l^j - u_j}{Z_l} ds - I_l^j\Big] \int_{E_l} \frac{\delta_{l,\theta_{kj}} - w^{\theta_{kj}}(s)}{Z_l} ds\Big)_{k=1}^{m}$$

where $\psi^j(\cdot), j = 1, ..., m$, be a solution of the adjoined PDE problem with $u(\cdot), U$ and I replaced with $w^j(\cdot), U^j$, I^j respectively, and

$$\theta_{kj} = \begin{cases} k - j + 1, & \text{if } j \le k, \\ m + k - j + 1, & \text{if } j > k. \end{cases}$$

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$$a_{ij}^{N+1}(x) = \begin{cases} \mu, & if \; \tilde{a}_{ij}^{N+1}(x) \le \mu, \\ \tilde{a}_{ij}^{N+1}(x), & if \; \mu \le \tilde{a}_{ij}^{N+1}(x) \le R, \\ R, & if \; \tilde{a}_{ij}^{N+1}(x) > R. \end{cases}$$

$$U_k^{N+1} = \tilde{U}_k^{N+1} - \frac{1}{m} \sum_{k=1}^{m} \tilde{U}_k^{N+1} \; k = 1, ..., m$$

Discretization via Finite Differences

$$\mathcal{J}_h([v]_h) = \sum_{l=1}^{m} \Big(\sum_{A(\hat{E}_{lh})} \Gamma_{l\alpha} \frac{U_l - u_\alpha}{Z_l} - I_l \Big)^2 + \beta |U - U^*|^2 \ \rightarrow \ \text{min}$$

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$$F_h^R := \Big\{ [v]_h = ([\sigma]_h, U) \Big| \sum_{l=1}^{m} U_l = 0, \ \|[\sigma]_h\|^2_{\tilde{H}^1(Q_h)} + |U|^2_{\mathbb{R}^m} \leq R^2,$$

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$$h^n \sum_{A(Q_h^+)} \sigma_\alpha \sum_{i=1}^{n} u_{\alpha x_i} \eta_{\alpha x_i} + \sum_{l=1}^{m} \frac{1}{Z_l} \sum_{A(\hat{E}_{lh})} \Gamma_{l\alpha} u_\alpha \eta_\alpha + J_h([u]_h, [\eta]_h) \\ = \sum_{l=1}^{m} \frac{U_l}{Z_l} \sum_{A(\hat{E}_{lh})} \Gamma_{l\alpha} \eta_\alpha$$

Convergence of the Method of Finite Differences

The sequence of discrete optimal control problems $\mathcal{E}_h$ approximates the optimal control problem $\mathcal{E}$ with respect to functional, i.e.

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Interpolation of the discrete control minimizers are

- is precompact in Tikhonov topology of $\tilde{H}^1(Q)\times\mathbb{R}^m$ formed with the product of the weak topology of $\tilde{H}^1(Q)$ and Euclidean topology of $\mathbb{R}^m$;
- is precompact in Tikhonov topology of $C^{0,\mu}(\overline{Q})\times\mathbb{R}^m$, $0<\mu<\frac{1}{2}$ formed with the product of the strong topology of Hölder space $C^{0,\mu}(\overline{Q})$ and Euclidean topology of $\mathbb{R}^m$;

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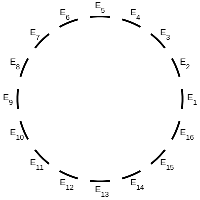
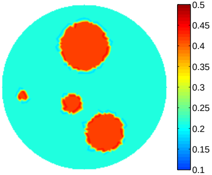
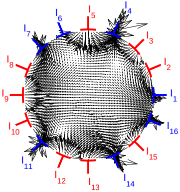
- Multilinear interpolations of the discrete state vectors $[u([v]_{h',\epsilon})]_{h'}$ converge to the optimal state vector $u=u(x;v_*)$ weakly in $H^1(Q)$, strongly in $L_2(Q)$, and almost everywhere on Q.

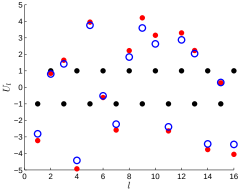
Numerical Results

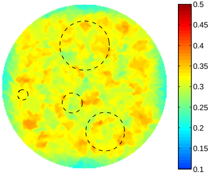






Cancer Detection though Electrical Impedance Tomography (EIT) and Optimal Control Theory





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