Adaptive Semi-linear Inversion: Sophisticated Lens Modeling of Large Strong-Lensing Data Sets

James Nightingale

Simon Dye

Introduction

- **Scientific Motivations**
- **Square and Adaptive Semi-linear Inversion**

  - Overview of methods.

  - Comparison of methods.

  - Data discretization – Unique and Unrelated Data Discretization.
- **Strong Lens Analysis Pipeline**

  - Multi-phase analysis – Example.

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Adaptive SLI - Overview

- **Adaptive SLI has two primary goals:**

1) Accurately perform modeling of degenerate lens models with just strong long data.

2) Do this in a fast, efficient and streamlined manner allowing analysis to be performed on the largest data sets possible.

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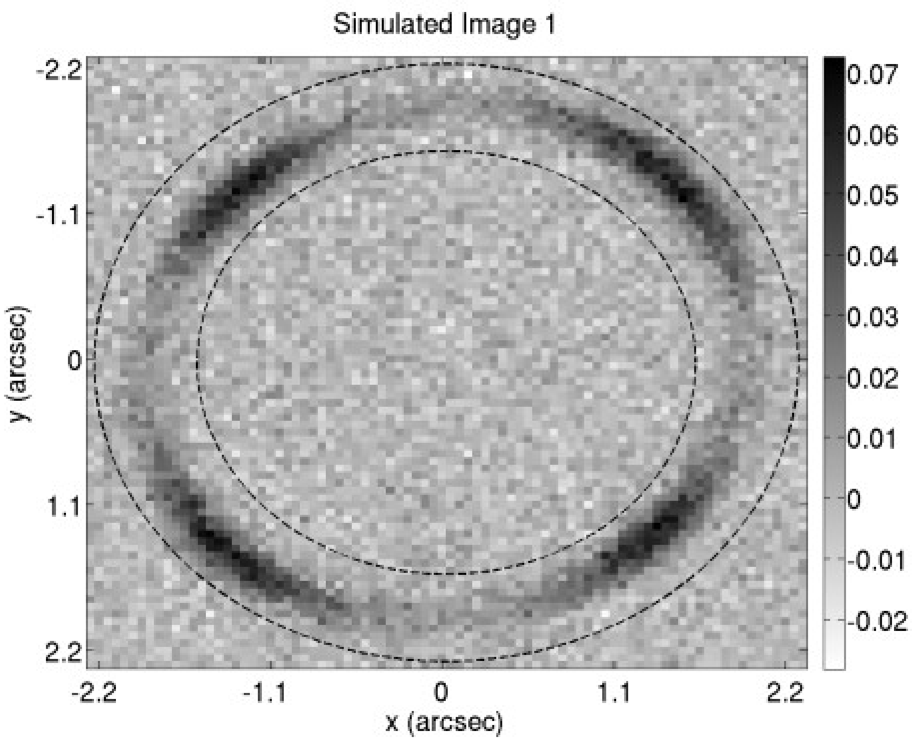
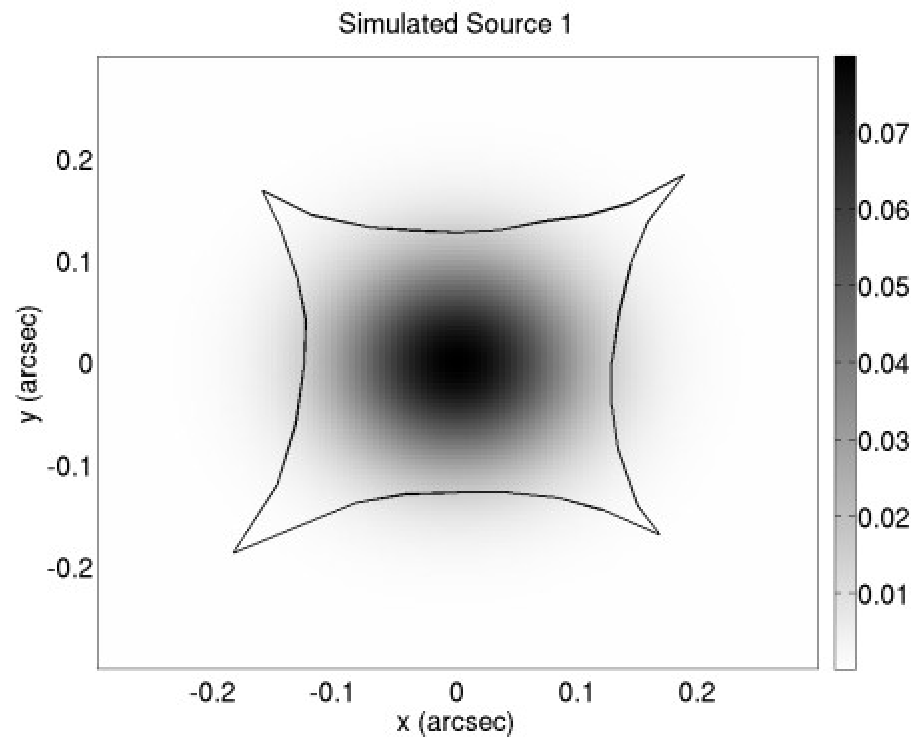
Square Semi-linear Inversion (SLI)

Simulated Image 1





Singular Power-Law Ellipsoid (SPLE) Lens model: $\rho(r) \propto r^{-\alpha}$

- (x,y) offsets = 0
- q = b/a = 0.8
- σ = 285 km/s
- Φ = 45°
- α = 2.0 (SIE)

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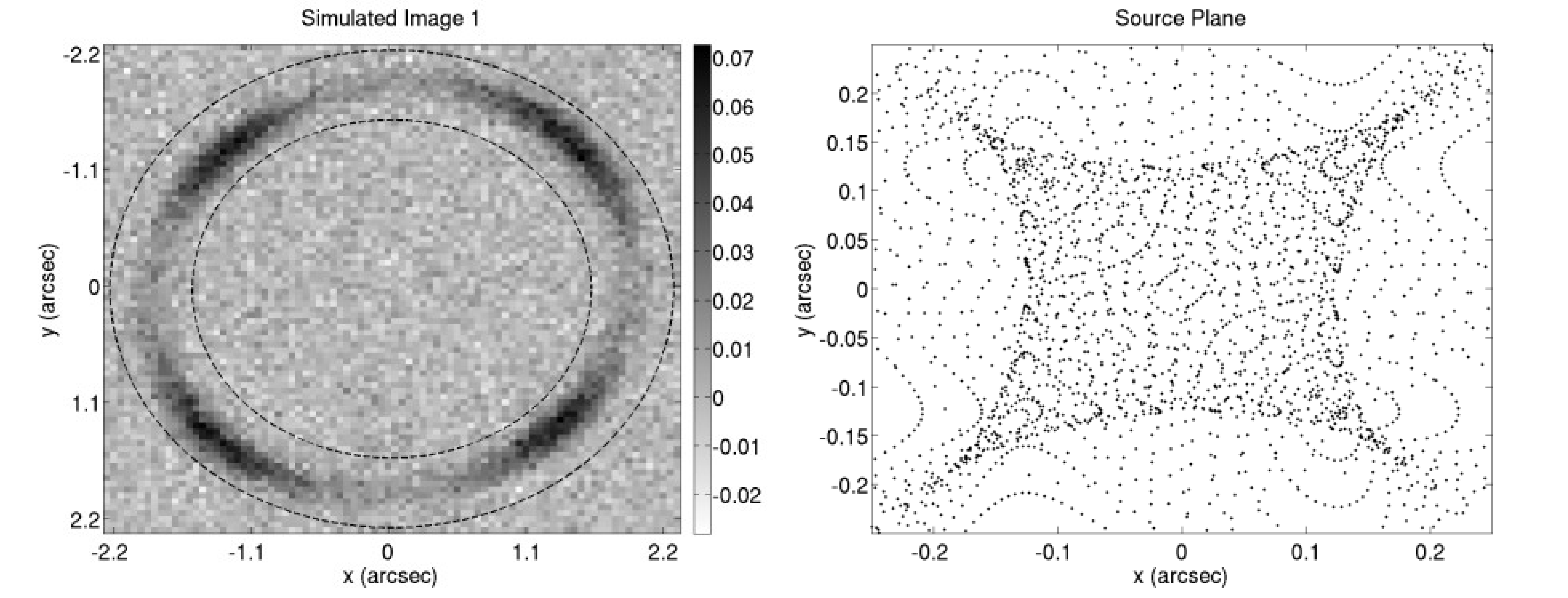
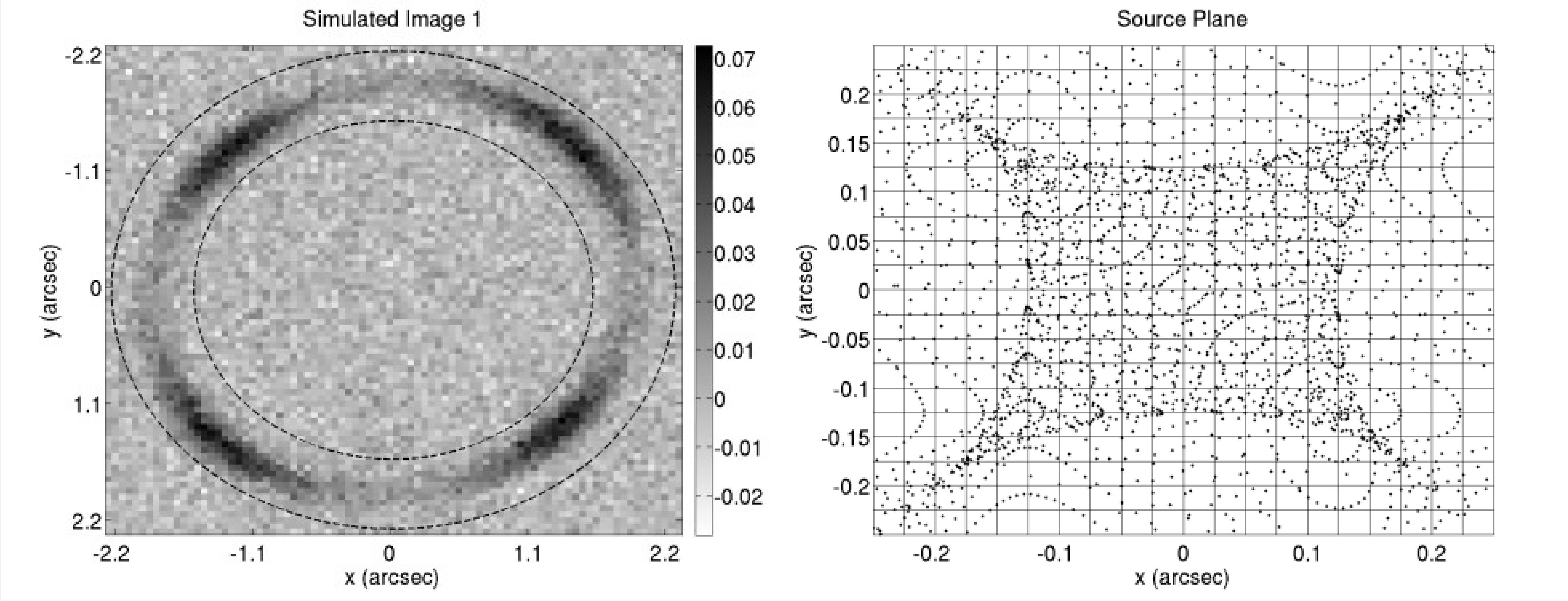
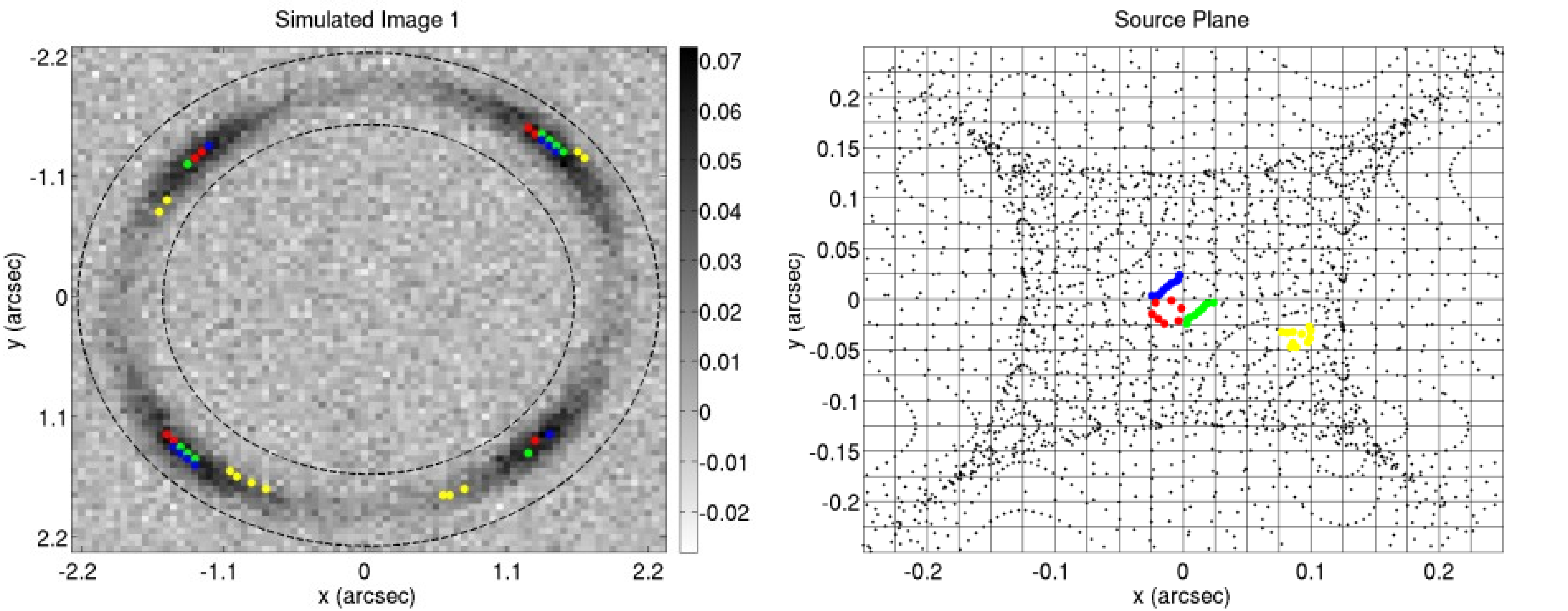
Step 1 – Trace Image Pixels to Source Plane



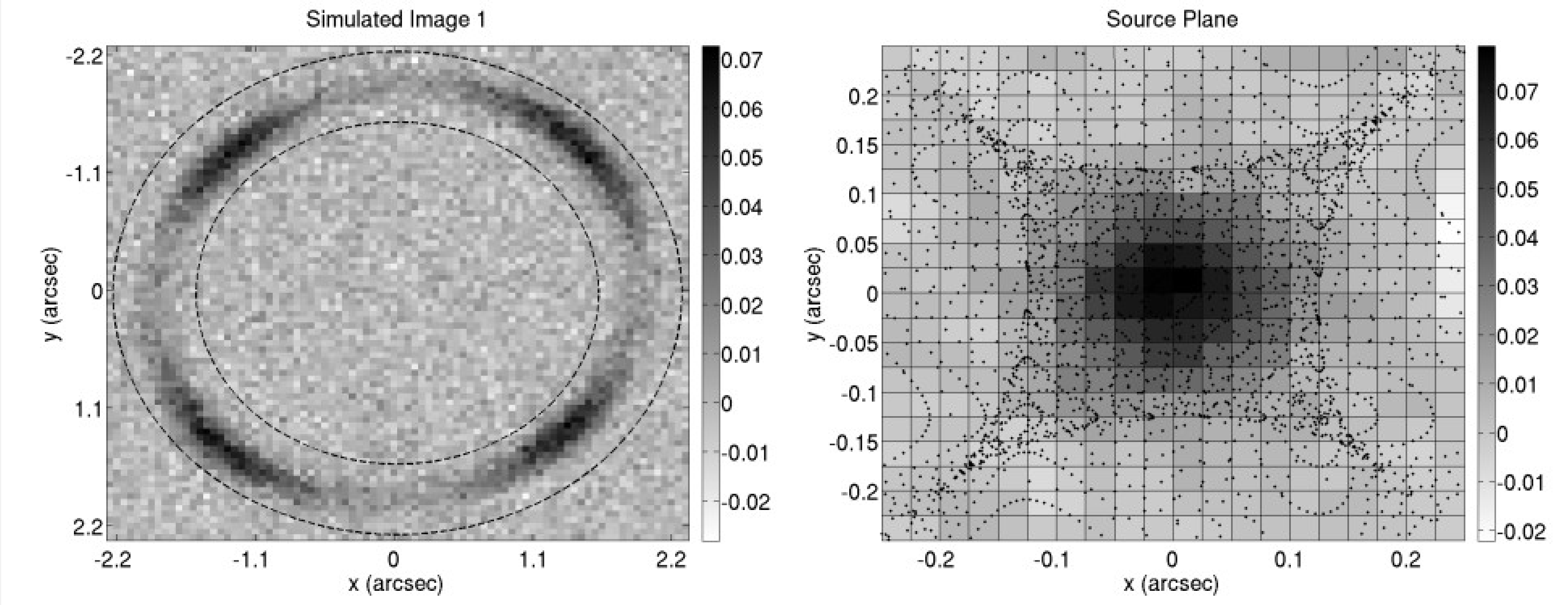
Step 2 – Overlay Square Grid



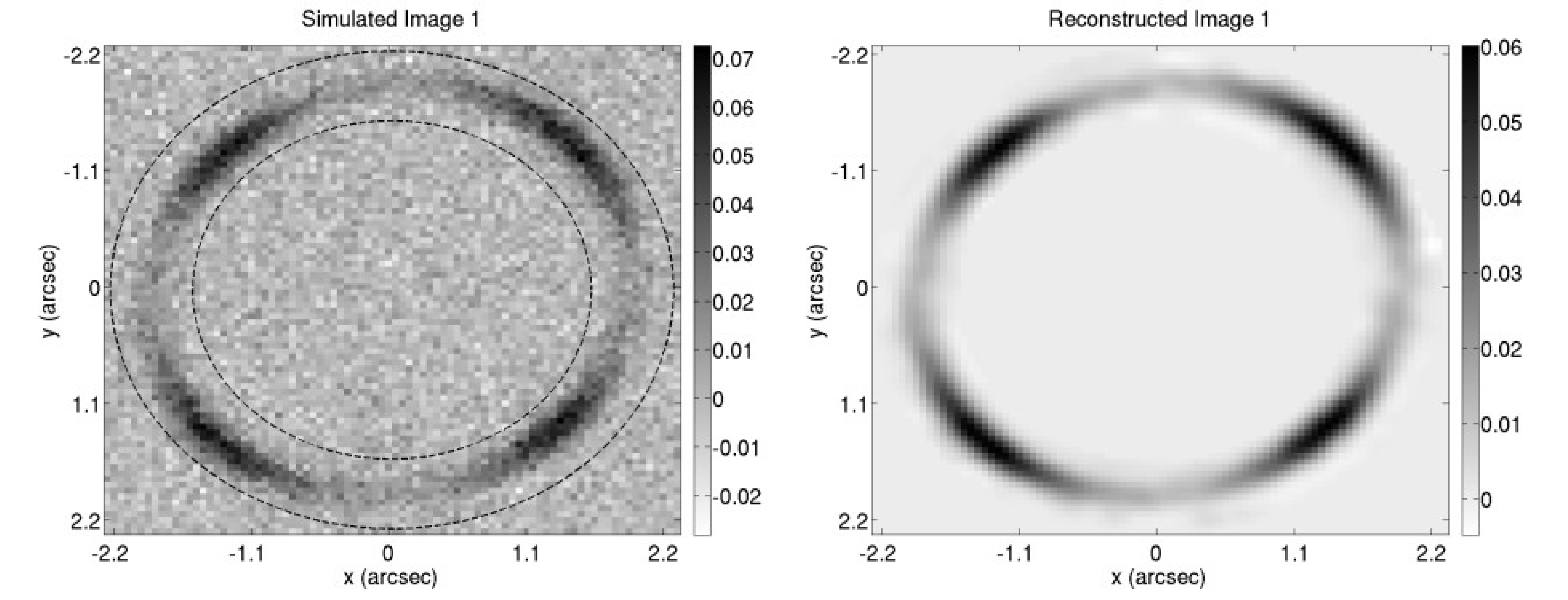
Step 3 – Image / Source Pixel Mappings



Step 3 – Reconstruct Source via Matrix Inversion



Step 4 – Compare Observed and Reconstructed Images



Adaptive Semi-linear Inversion (SLI)

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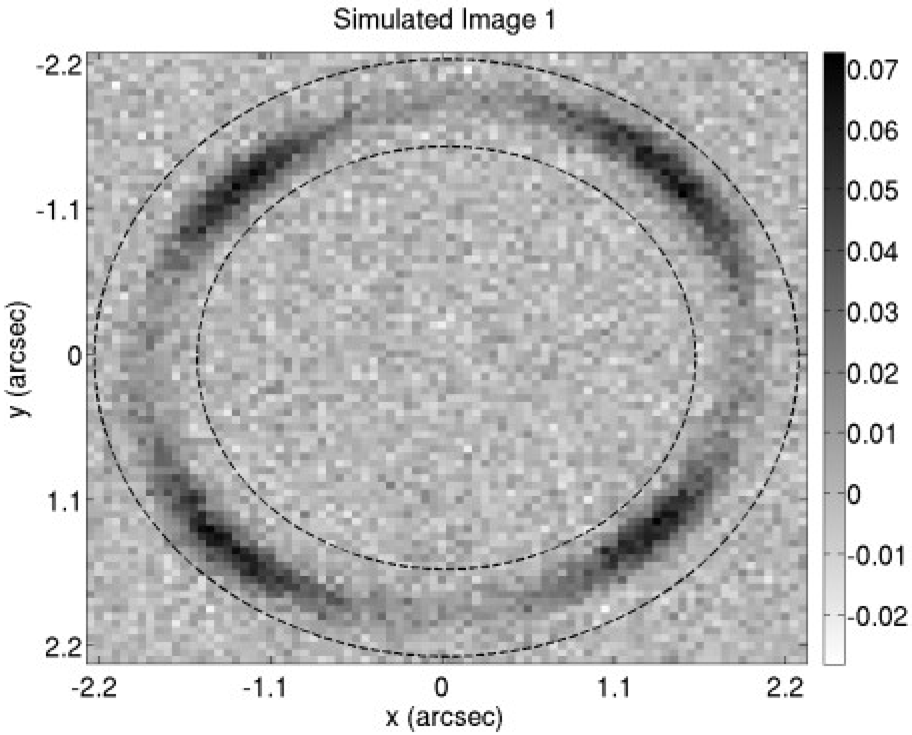
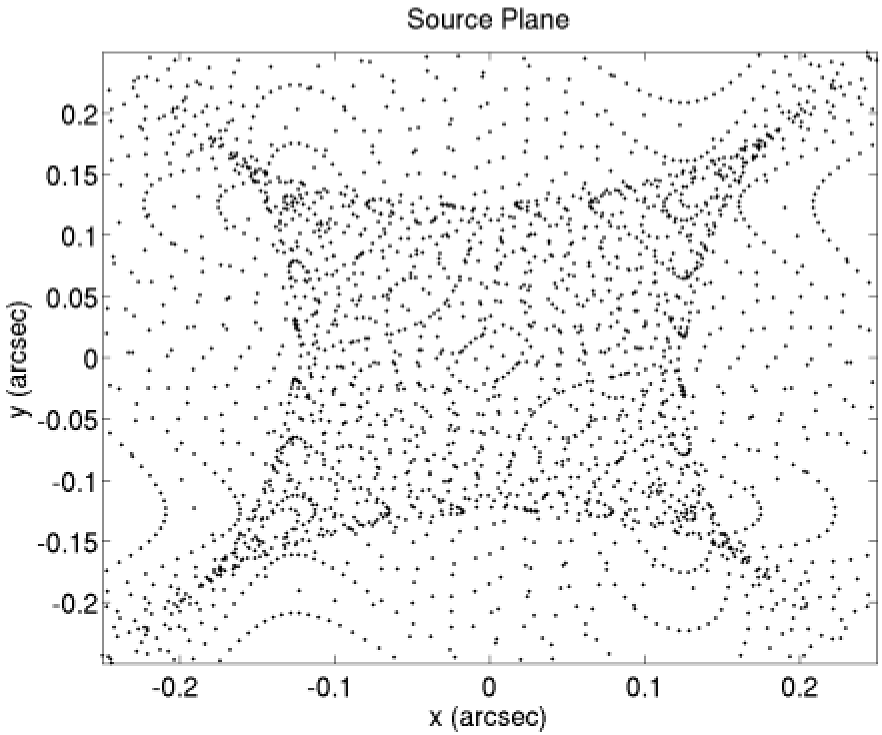
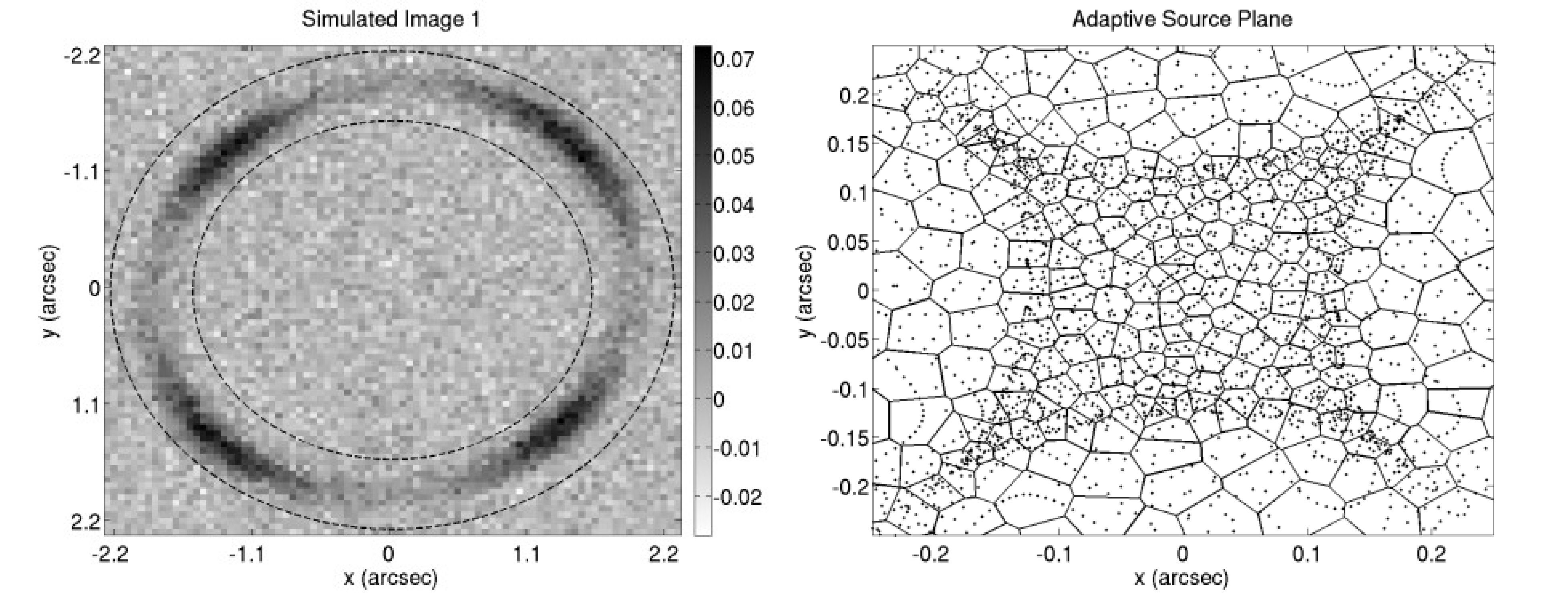




Step 2 – Input Into h-means Clustering Algorithm



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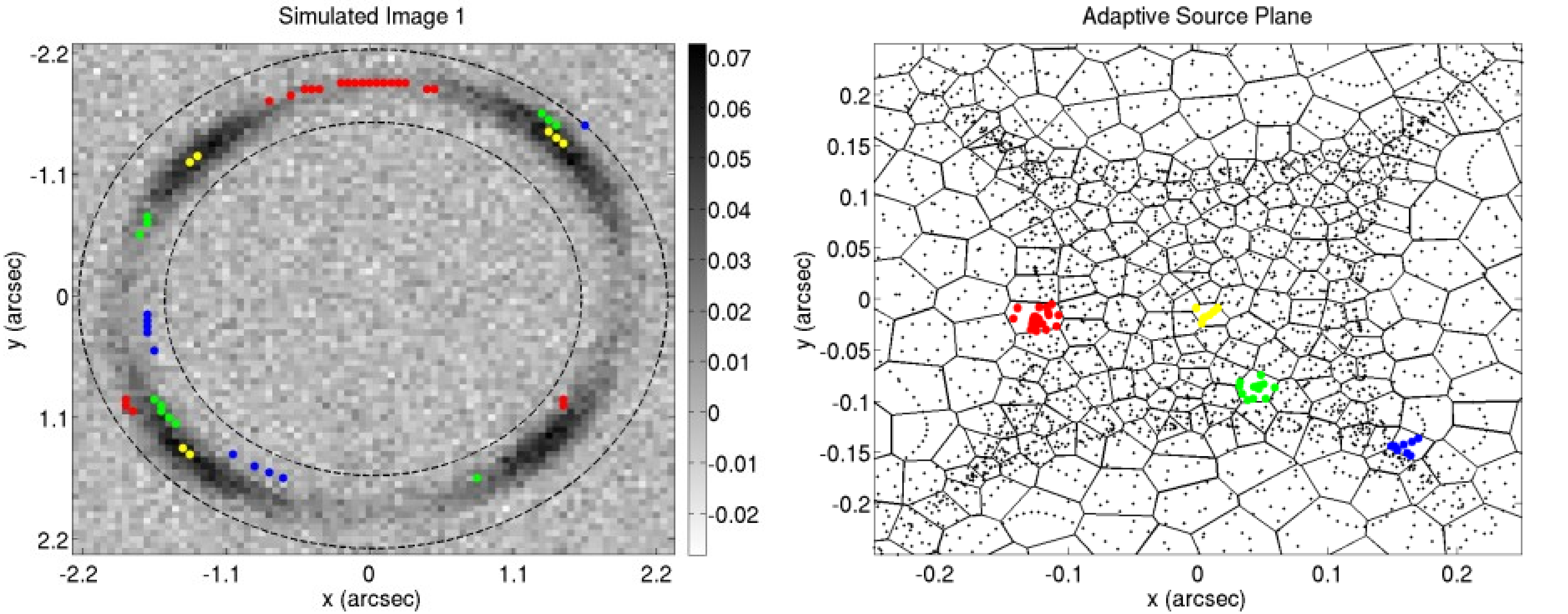

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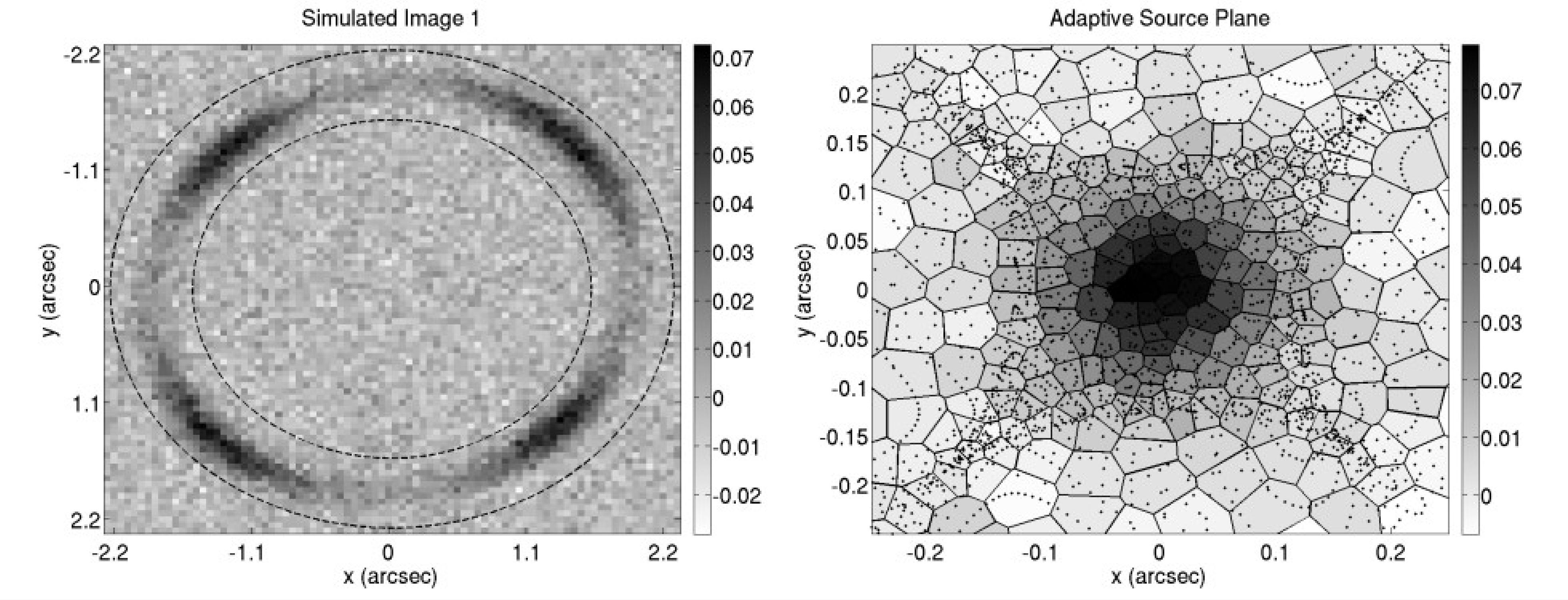

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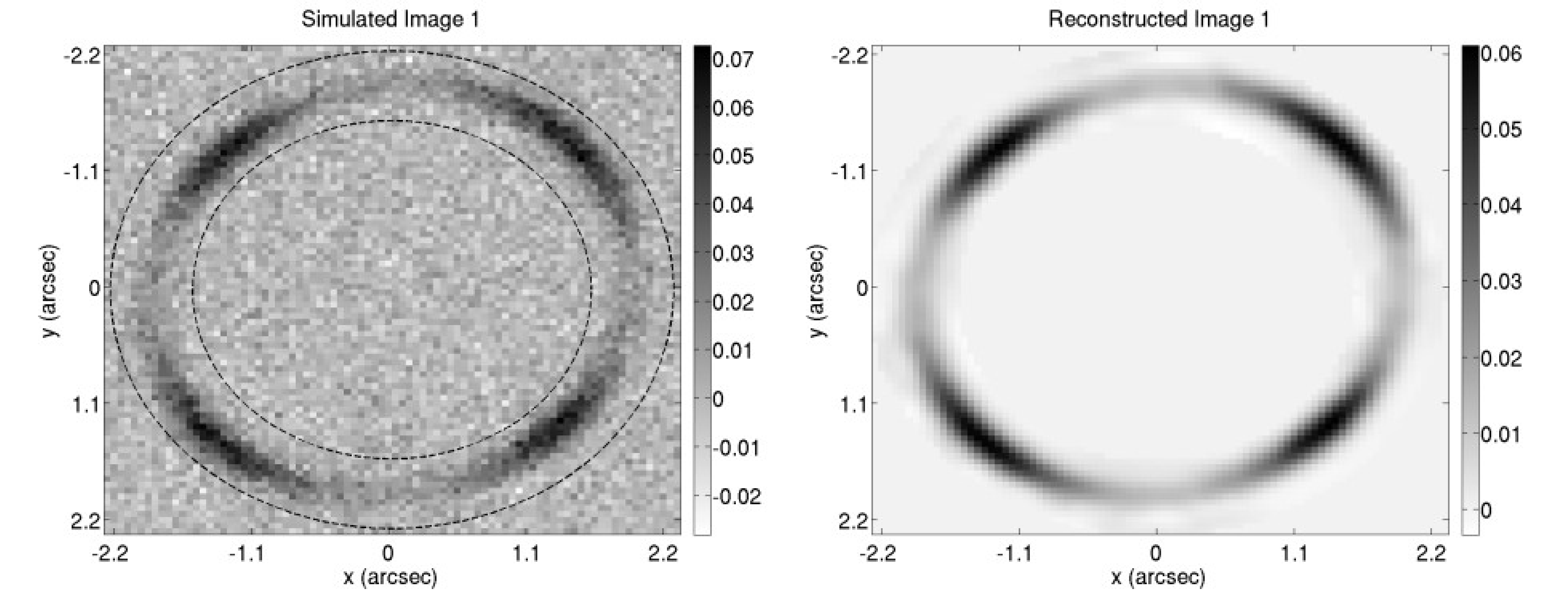

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  - Forces a physical source reconstruction.

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- **Maximize Bayesian Evidence (Suyu et al. 2006) to set level of Regularization for every lens model.**

Determining the Lens Model

- **Every trial lens model returns a reconstructed image**

  - Observed and reconstructed image residuals – want to minimize goodness of fit χ^2.

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- **Standard non-linear search problem.**

  - Find combinations of lens parameters (x, y, σ, q, Φ, α) which maximize ε.

  - MultiNest (Feroz et al. 2009) – based on nested sampling.

Benefits of Adaptive SLI

- **Pixelization derived from lens model magnification**

  - Fewer source pixels required -> faster inversion.

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- **Discretization the source plane in a *Unique* way for every lens model.**

  - Data discretization free of any geometric prescription.

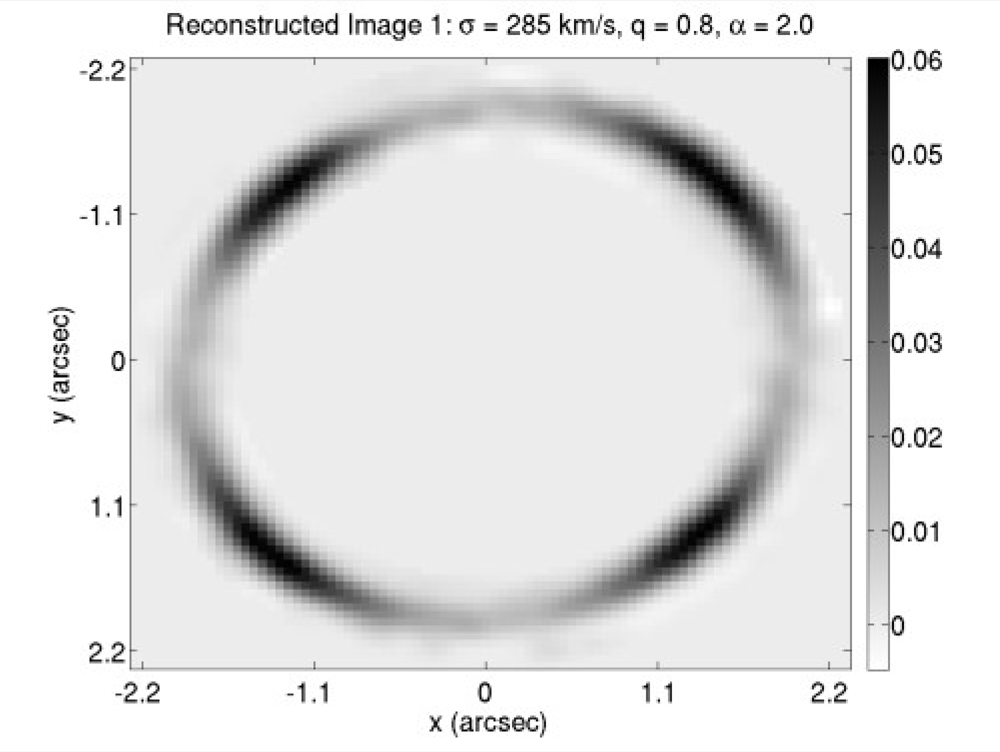
  - Every Pixelization is derived from a **random initailization** → **Unrelated** data discretization.

  - Data discretization is different for every single inversion.

Fitting a Power-Law (PL) Lens Model

- **Heavy degeneracy between σ and α.**

Input model: α = 2.0



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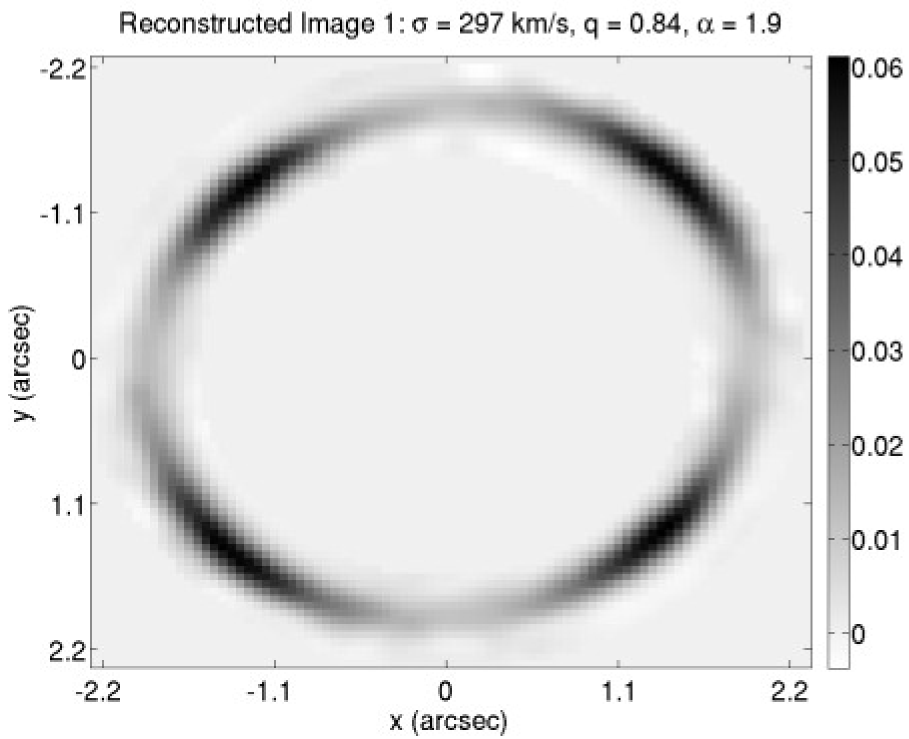
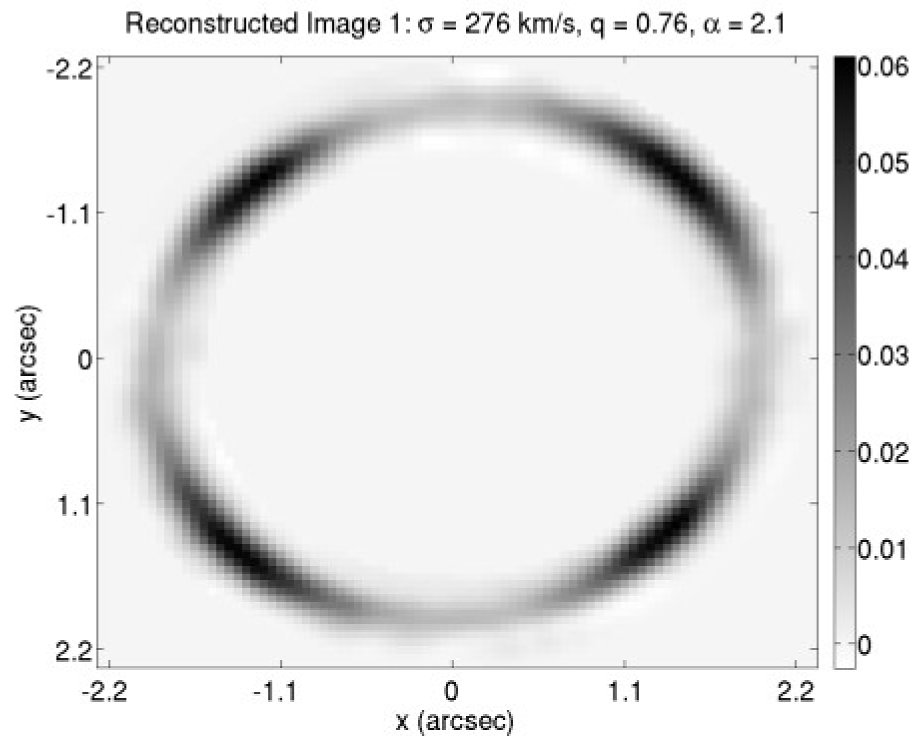
α = 1.9 **α = 2.1**





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- **With the NDOF fixed, Square SLI could not accurately fit a PL lens model.**
  - Different lens model calculated if inversion setup is changed.
  - Results varied with source plane resolution, size and position.
- **Investigation -> why do results change with setup?**
  - Fit 9 PL lens models to simulated image shown previously.
  - All 9 inversions identical, except source plane position.
  - Phase shift each grid by a fractional pixel interval.

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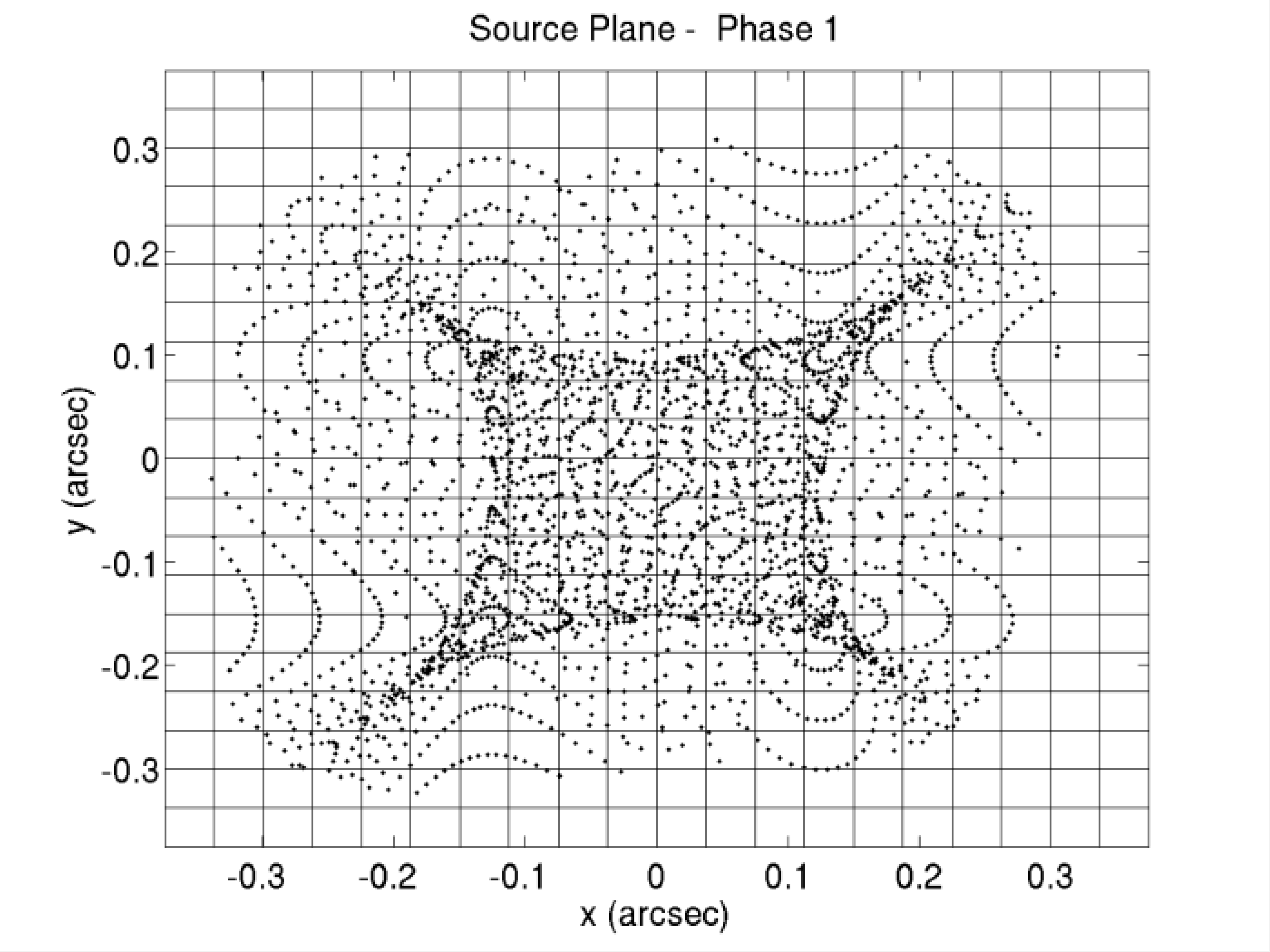
Phase Shifts



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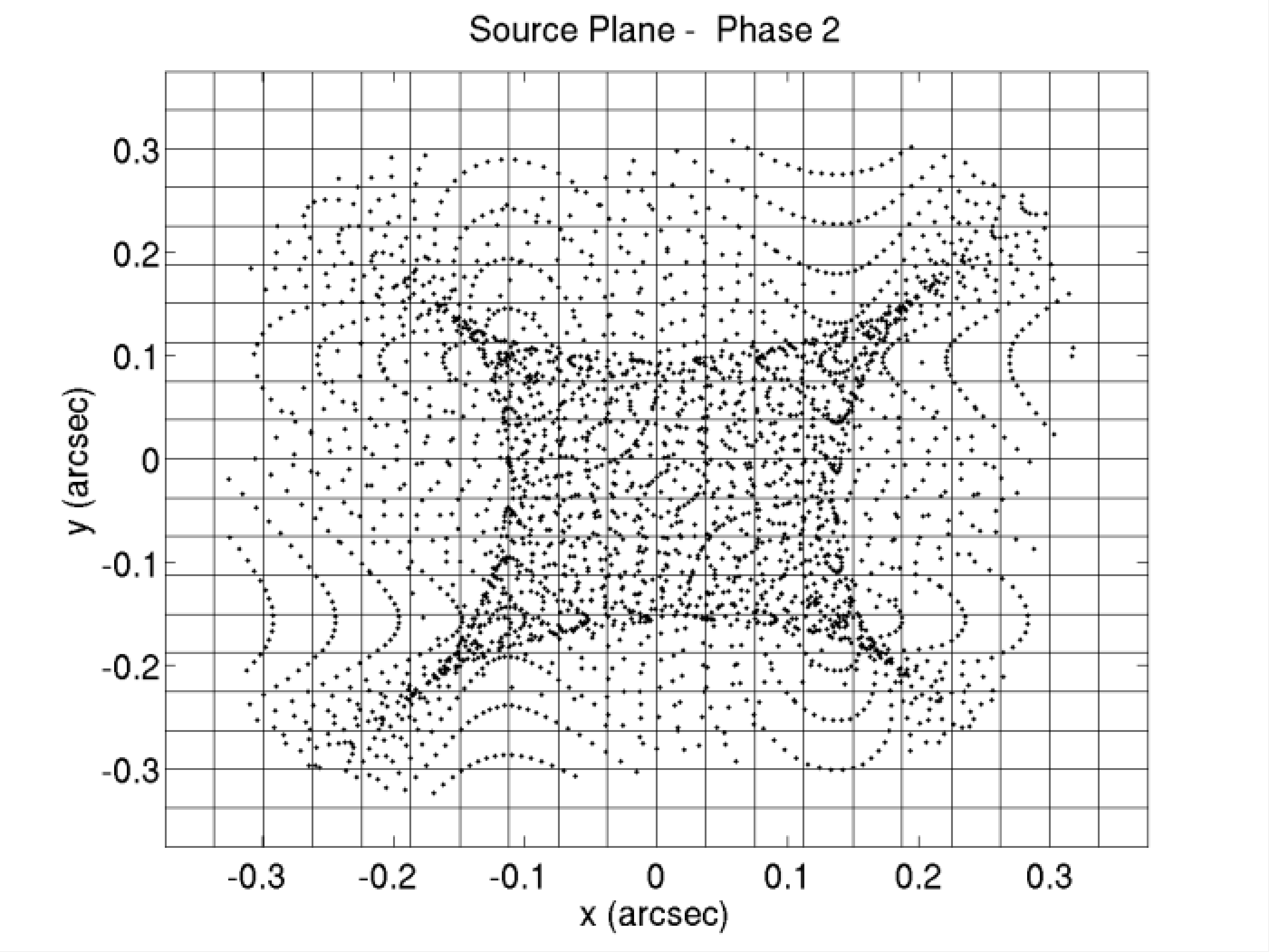

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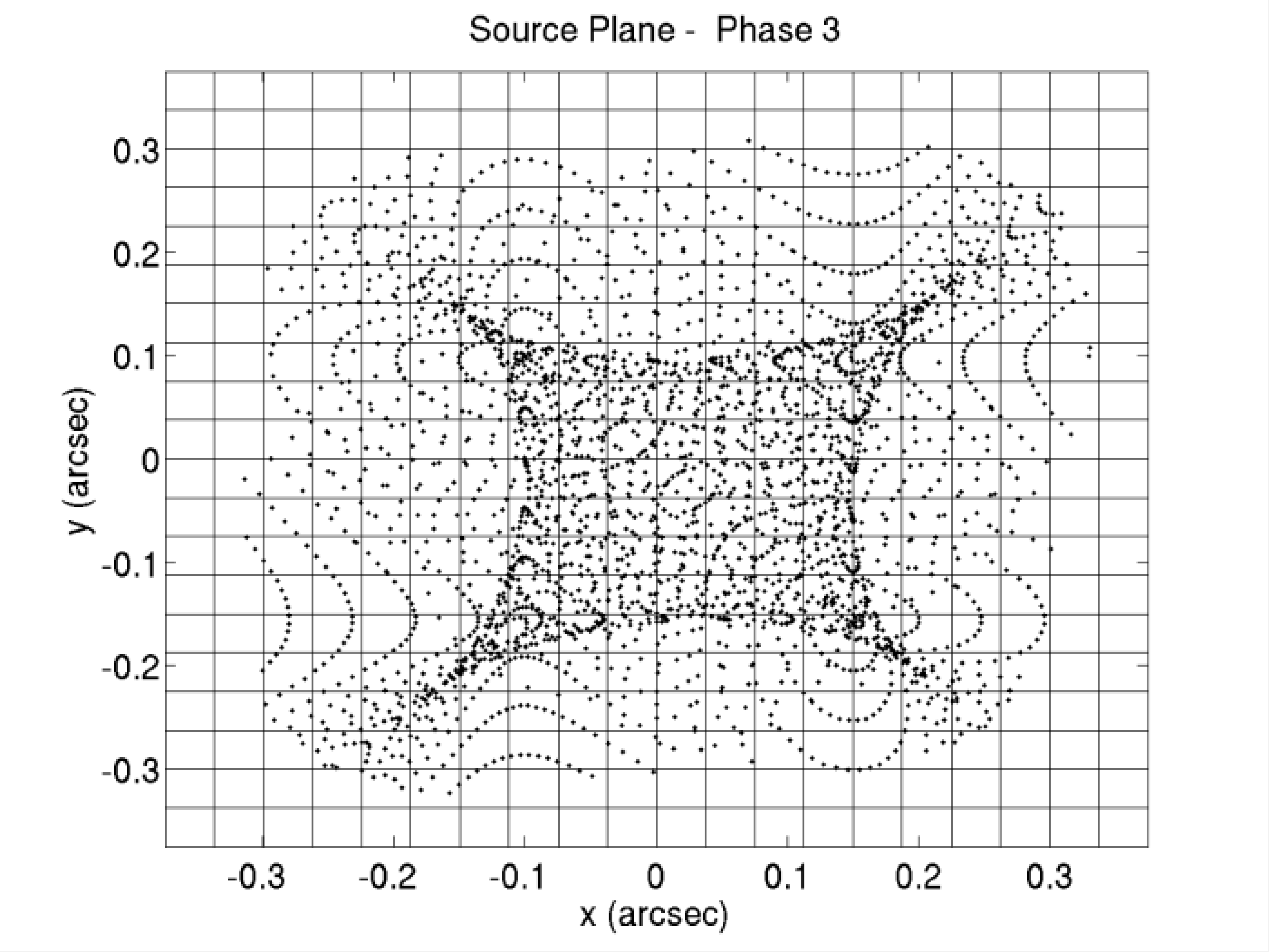

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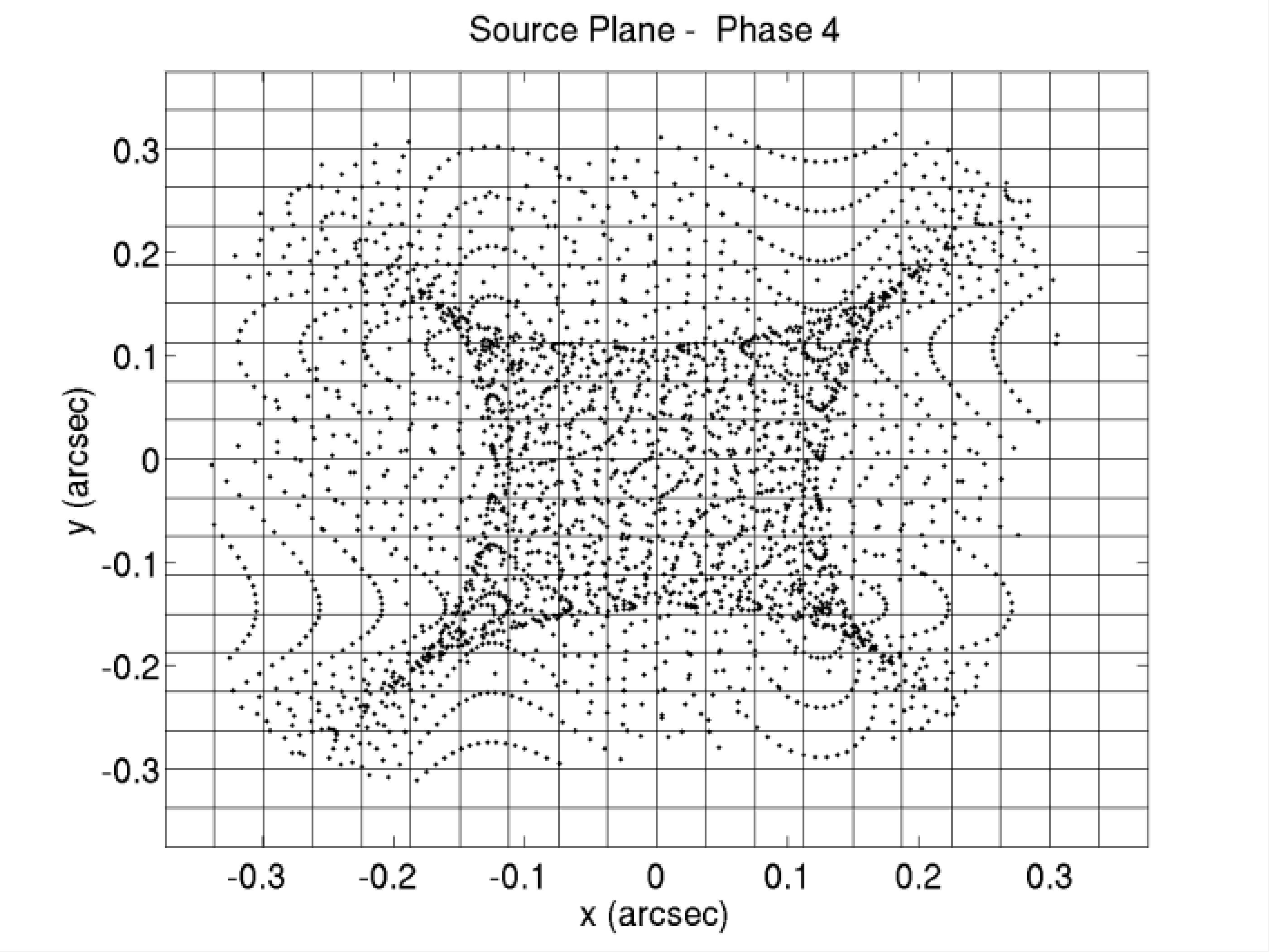

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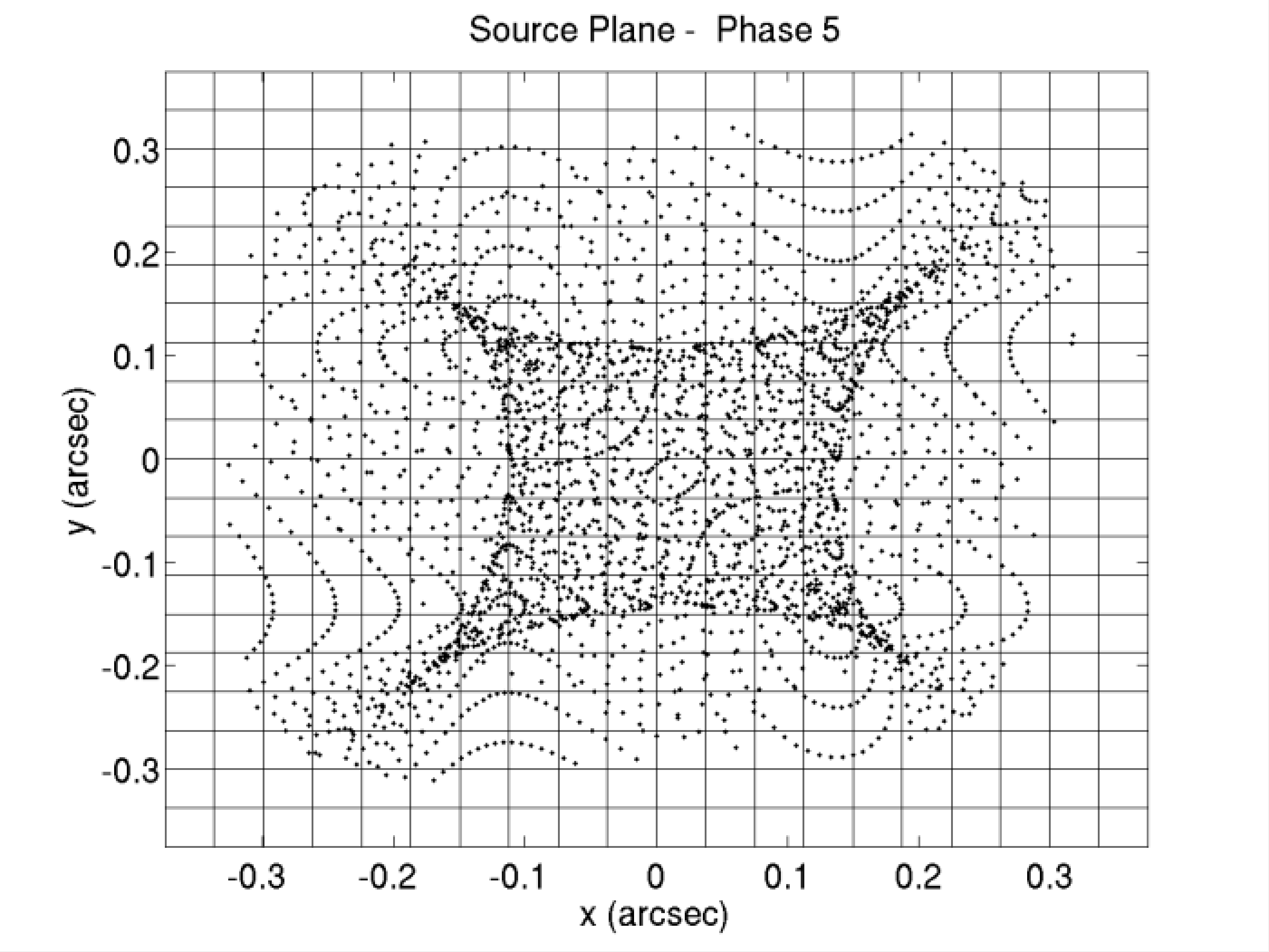

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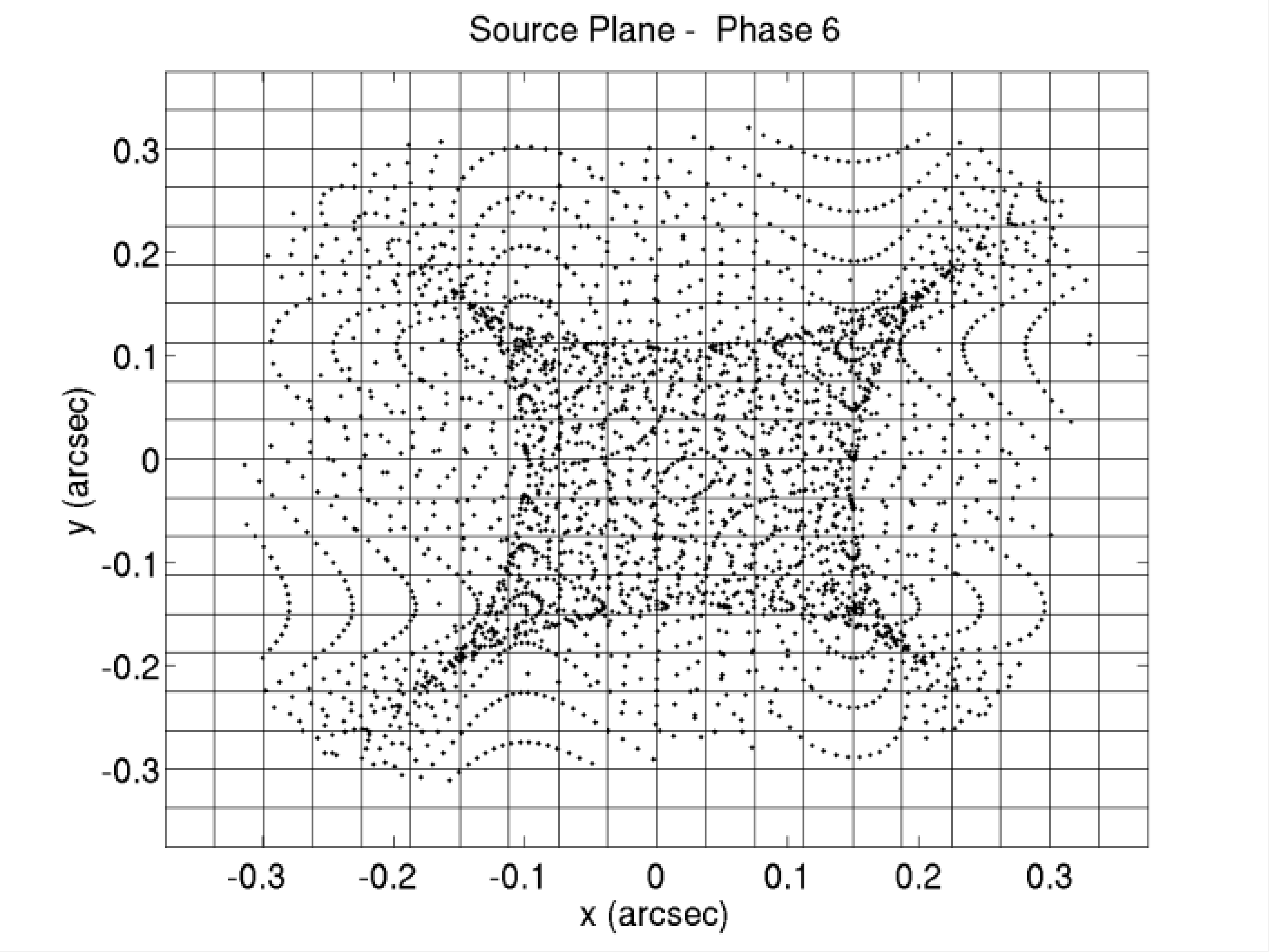

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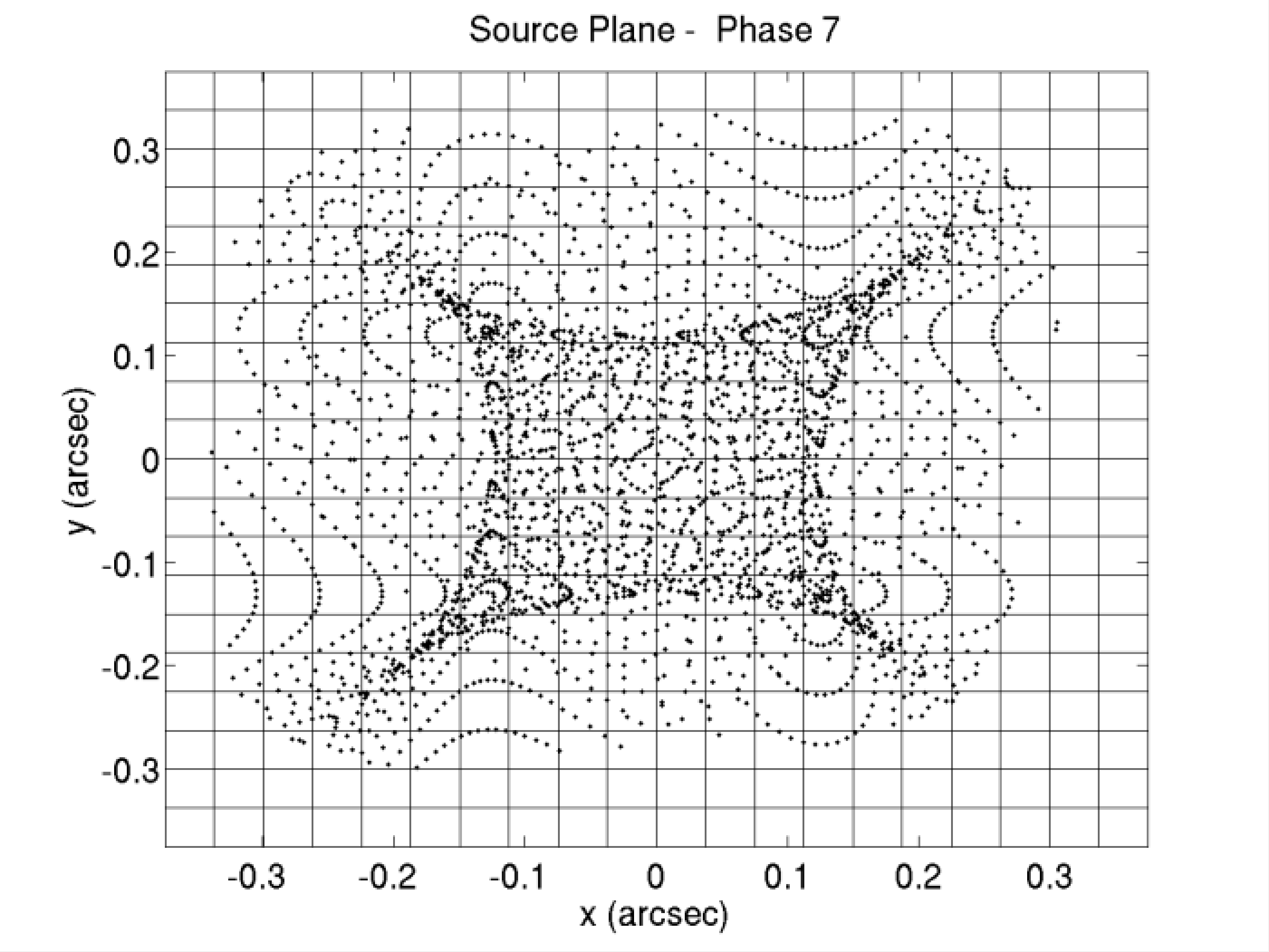

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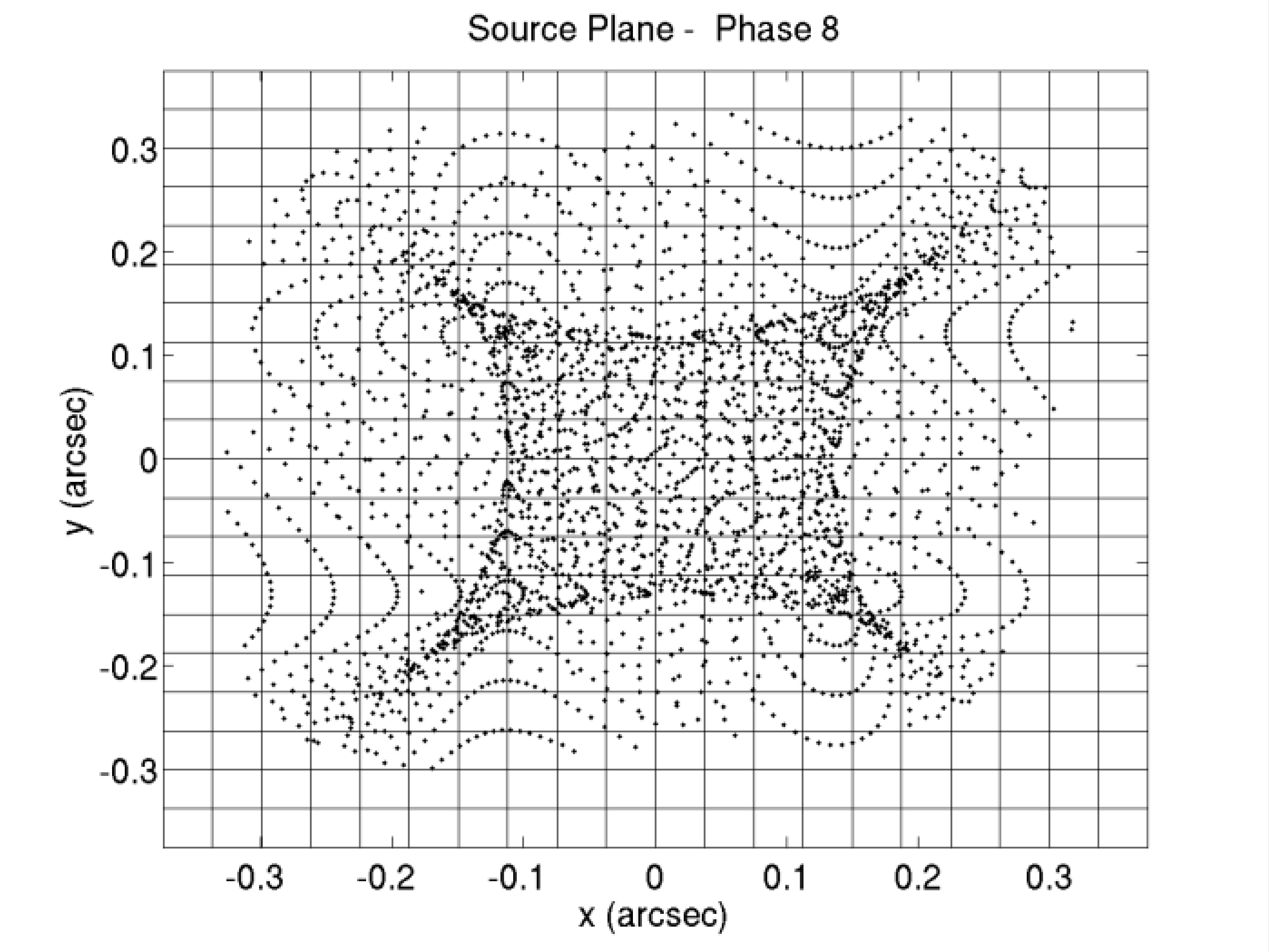

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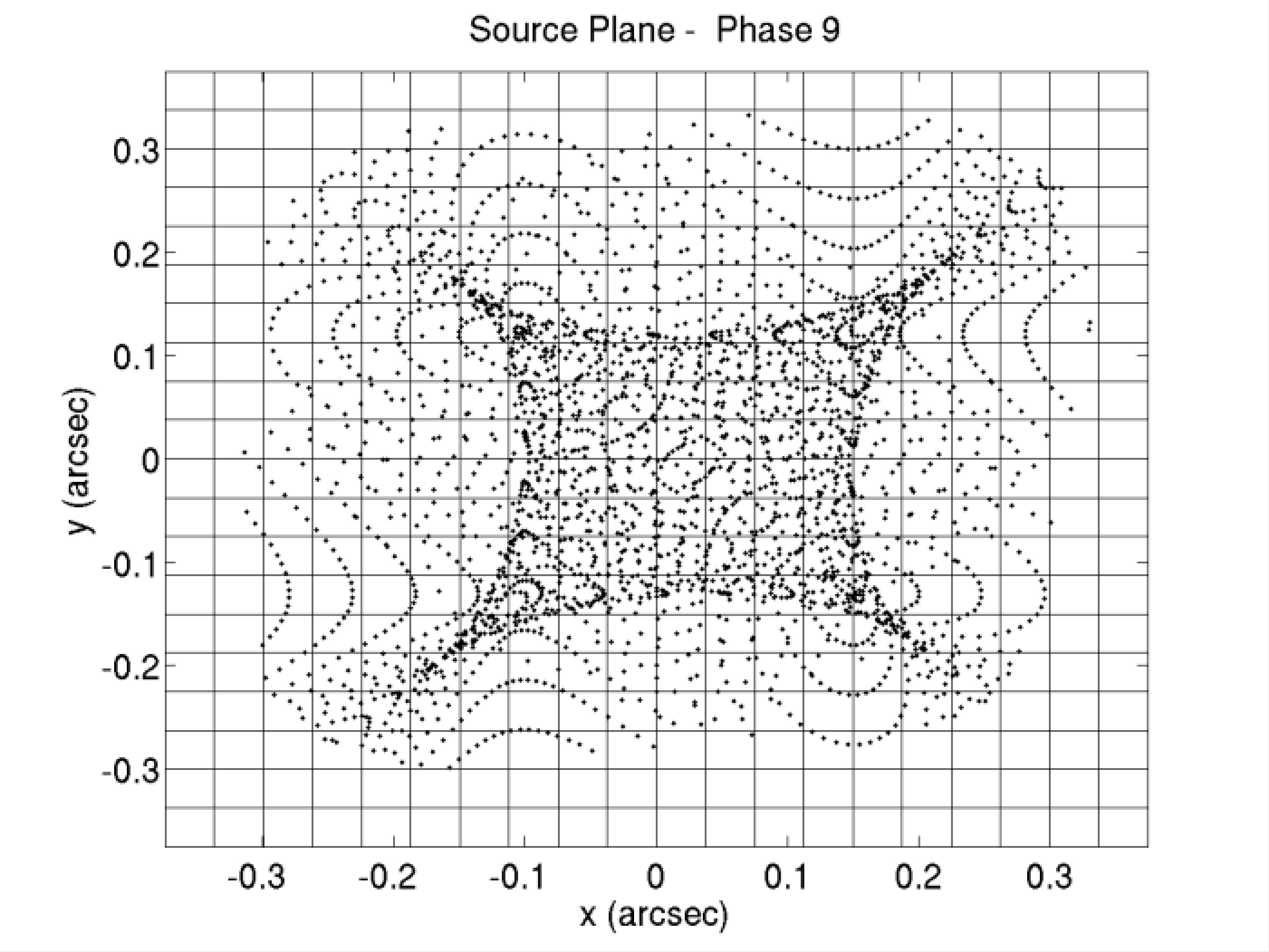
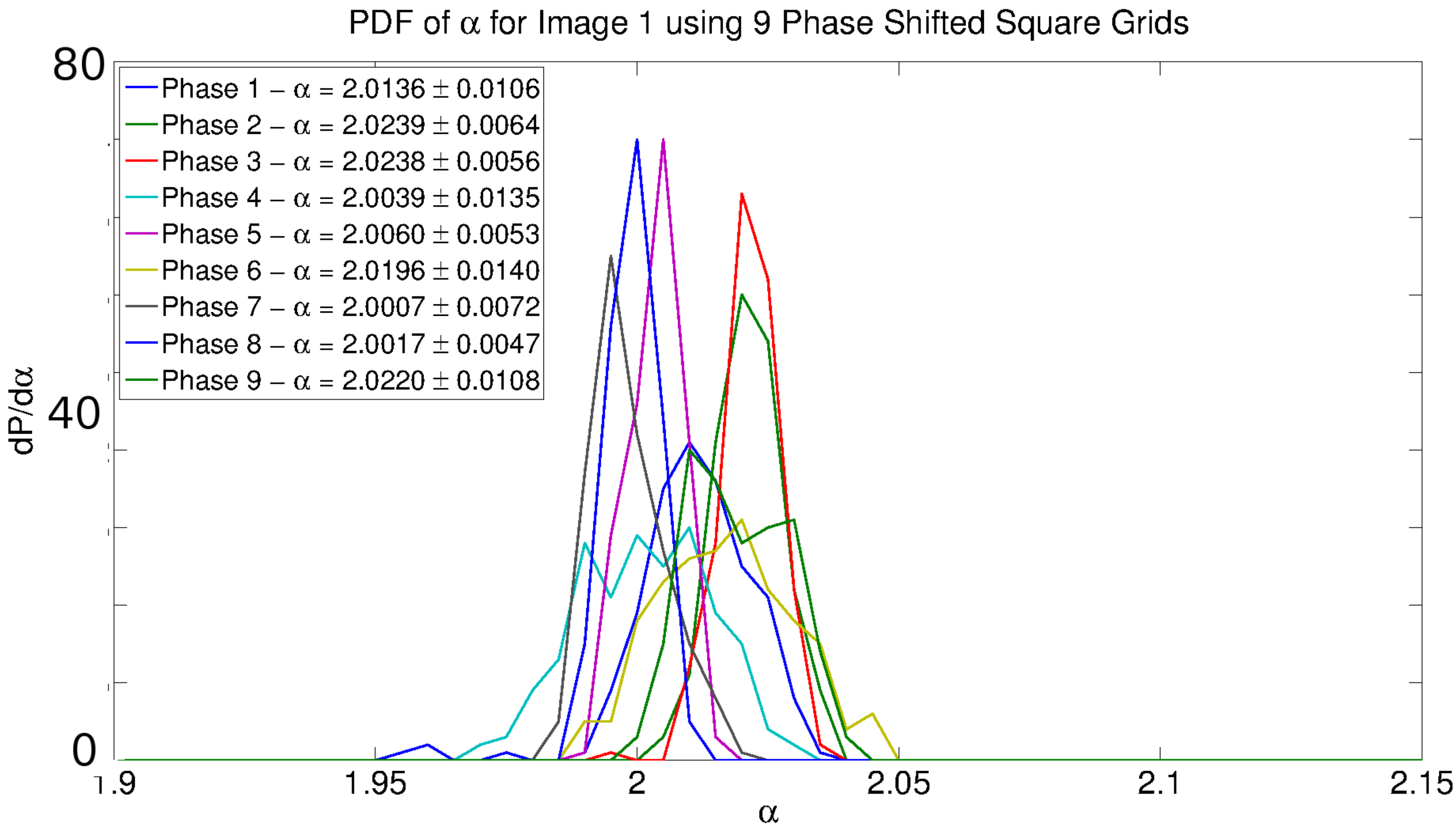


Phase Shifts – α Estimation



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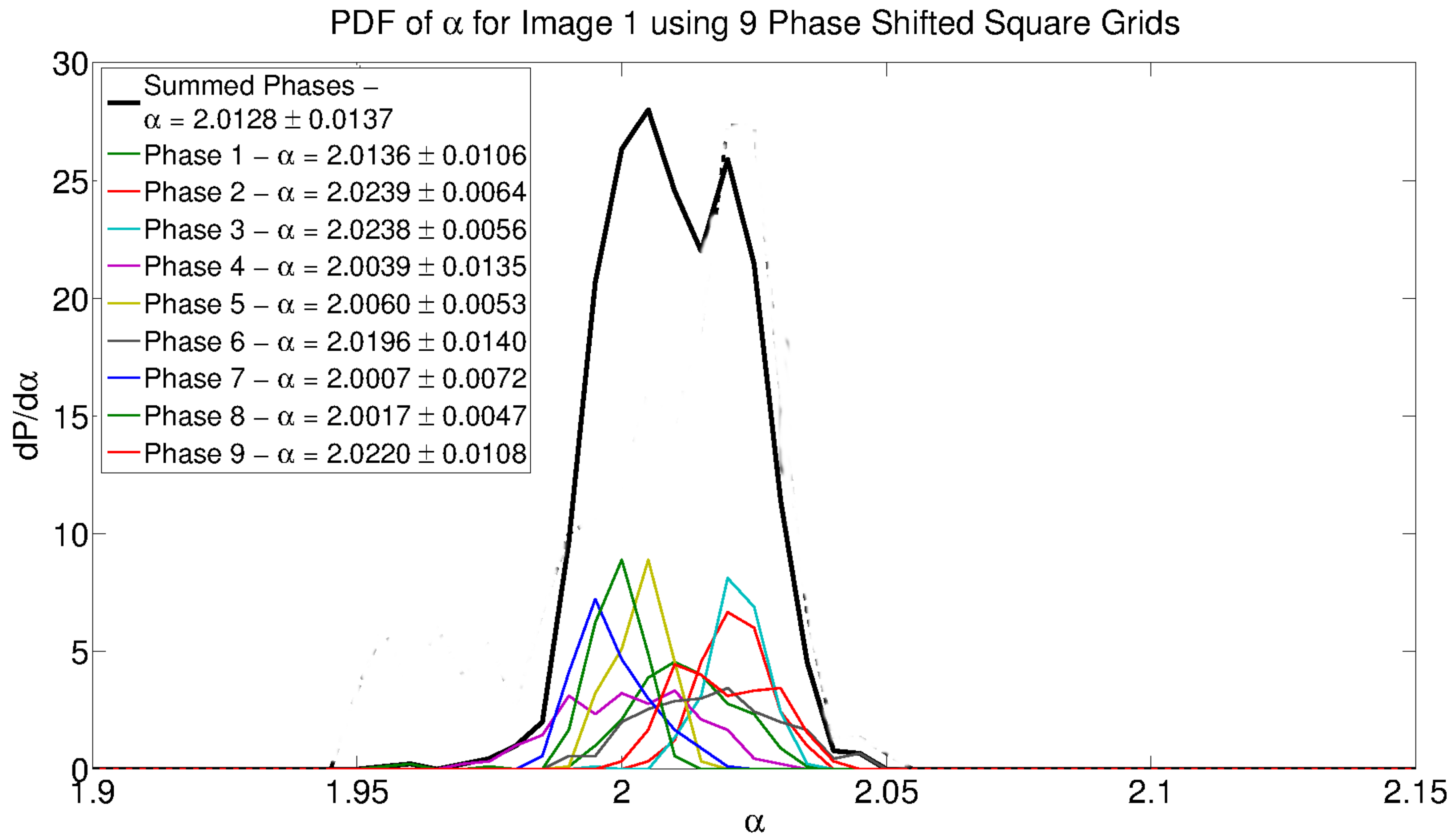
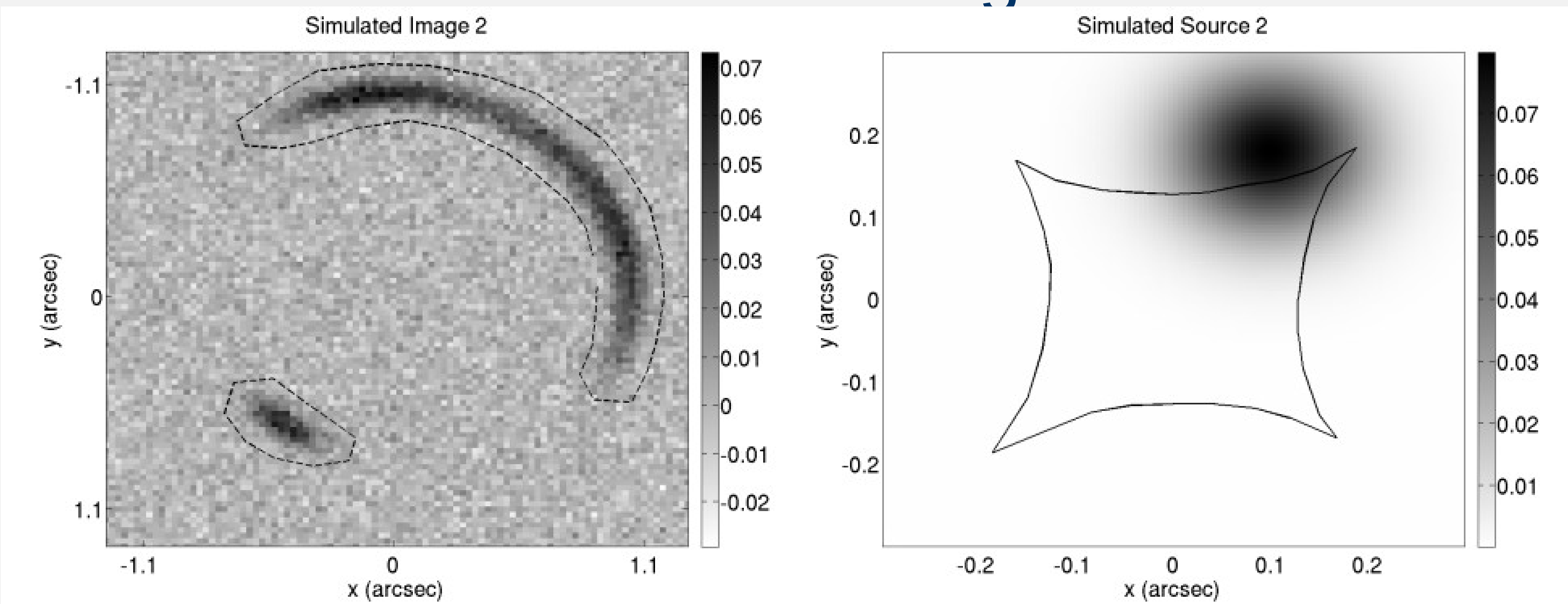


Simulated Image 2



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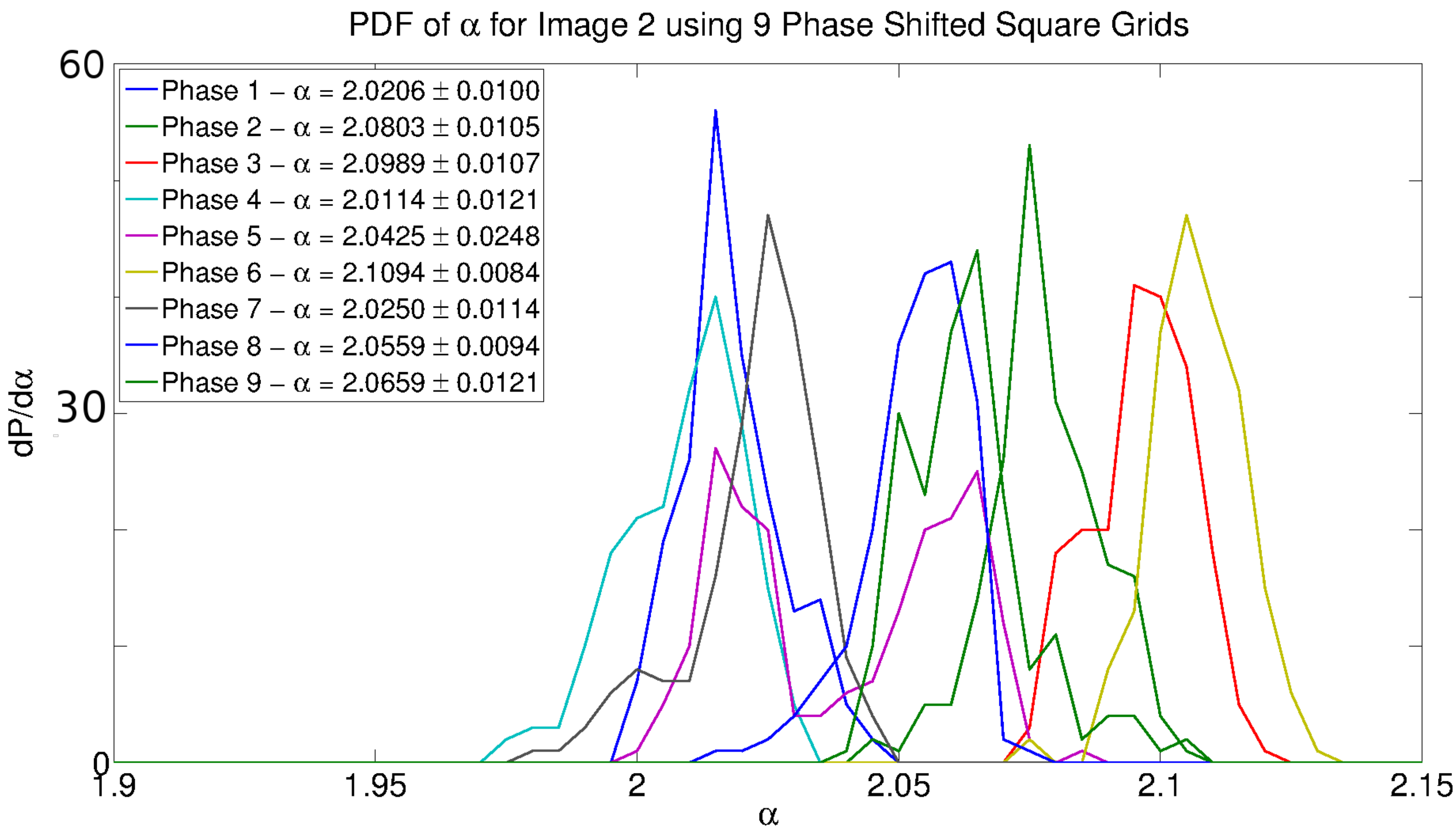

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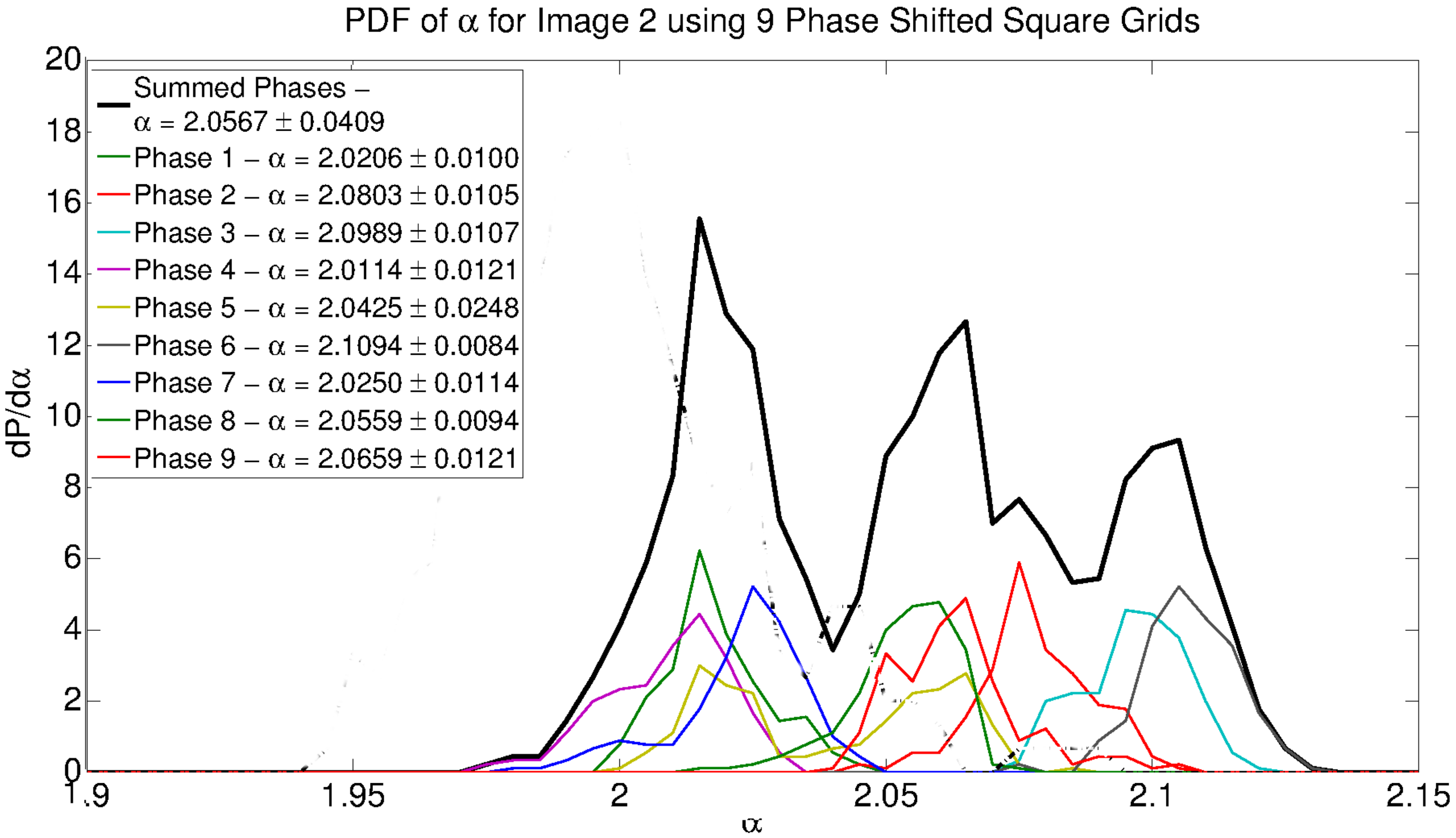

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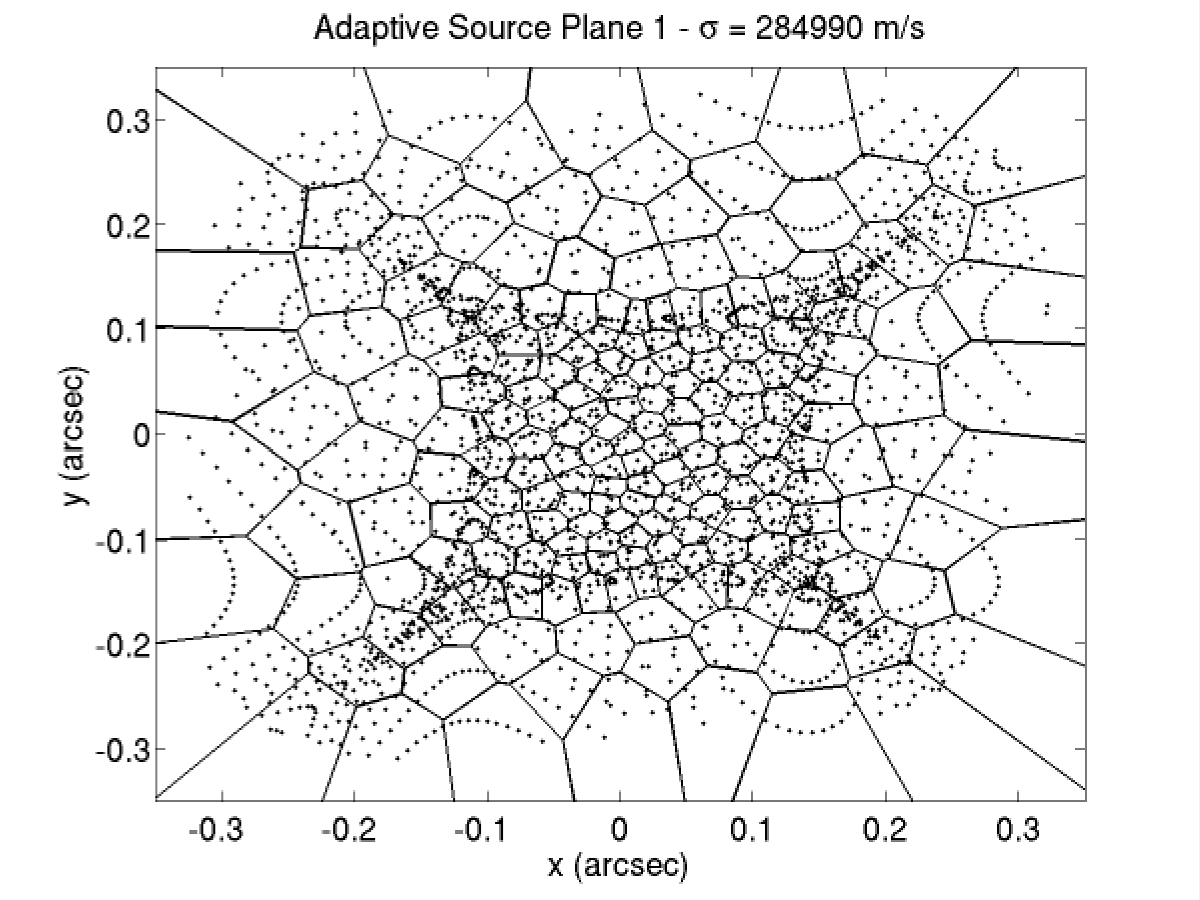
  - Each phase shifts changes the accessible allocations of image pixels to source pixels.

Adaptive SLI – Unique Grids



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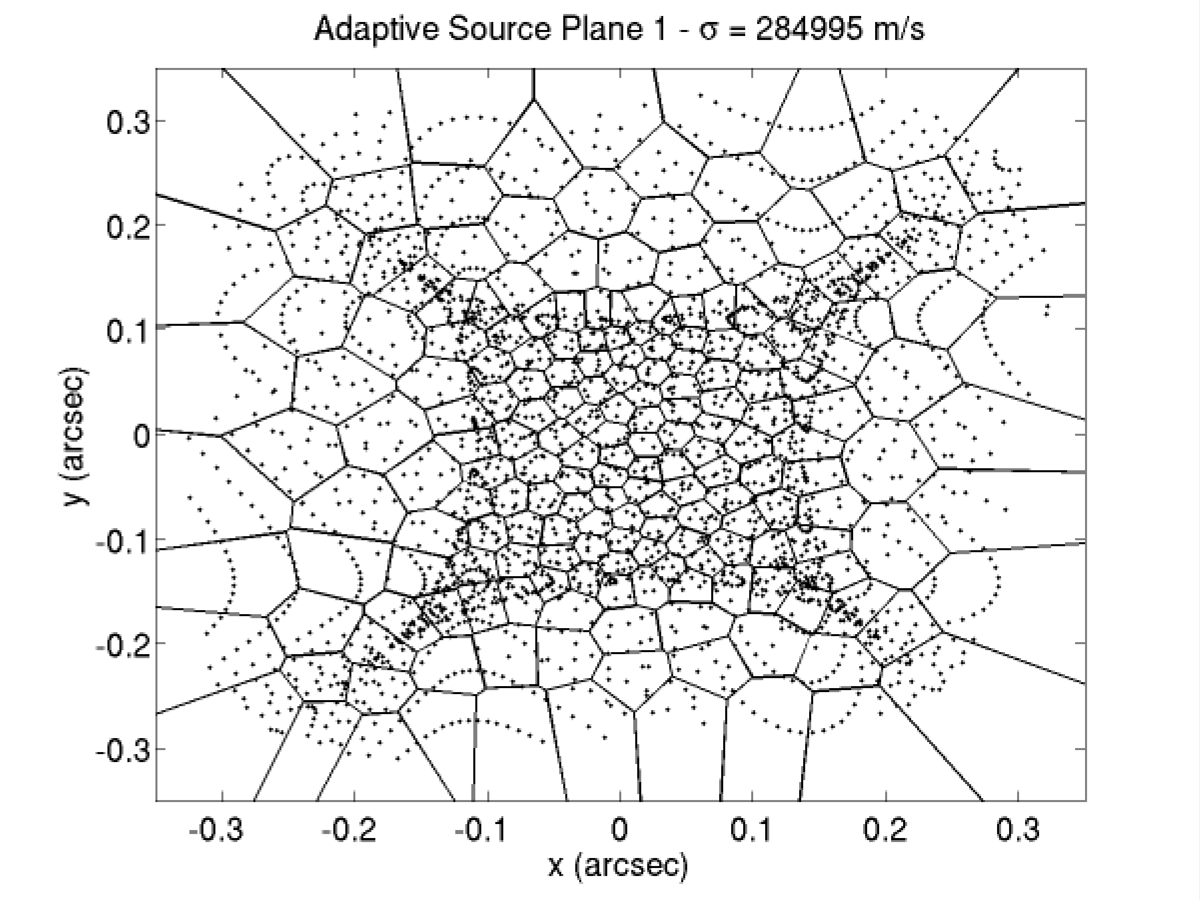

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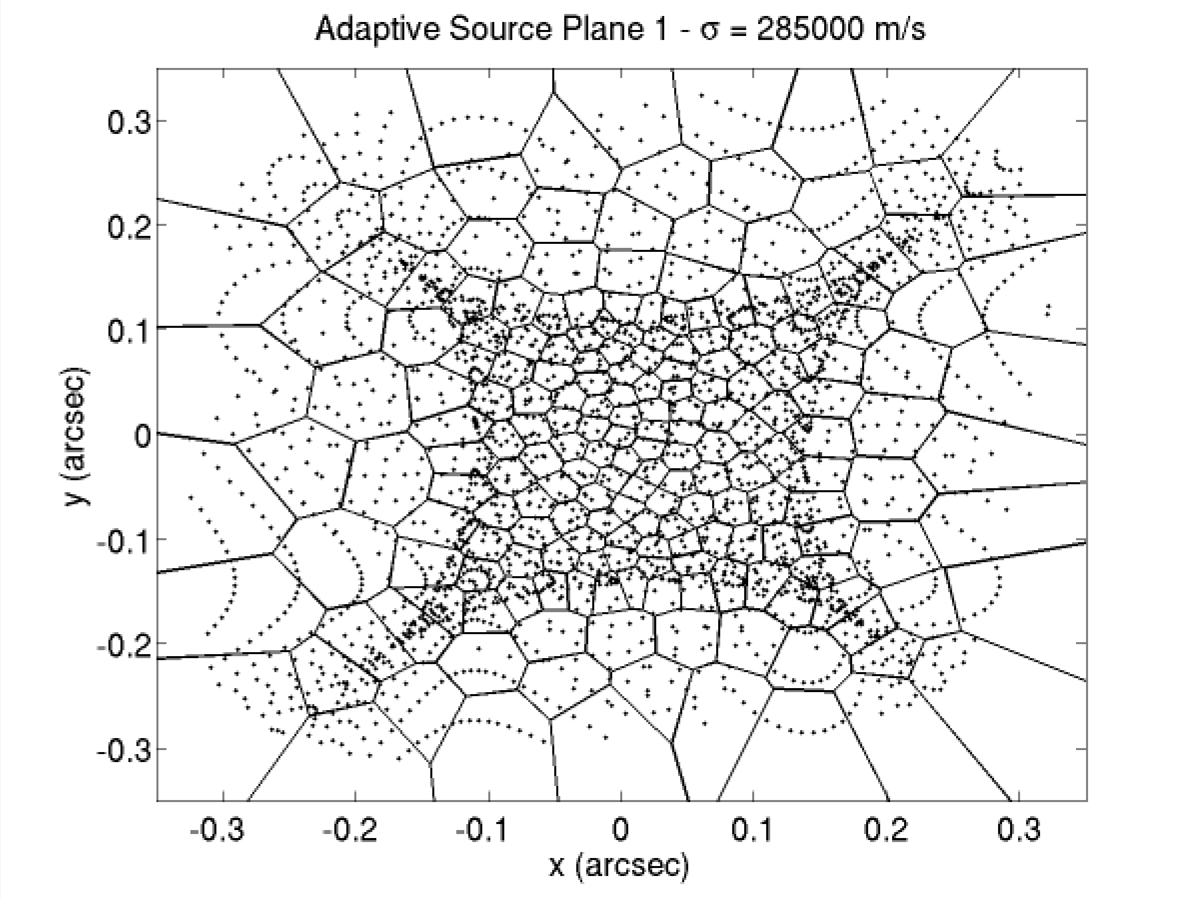

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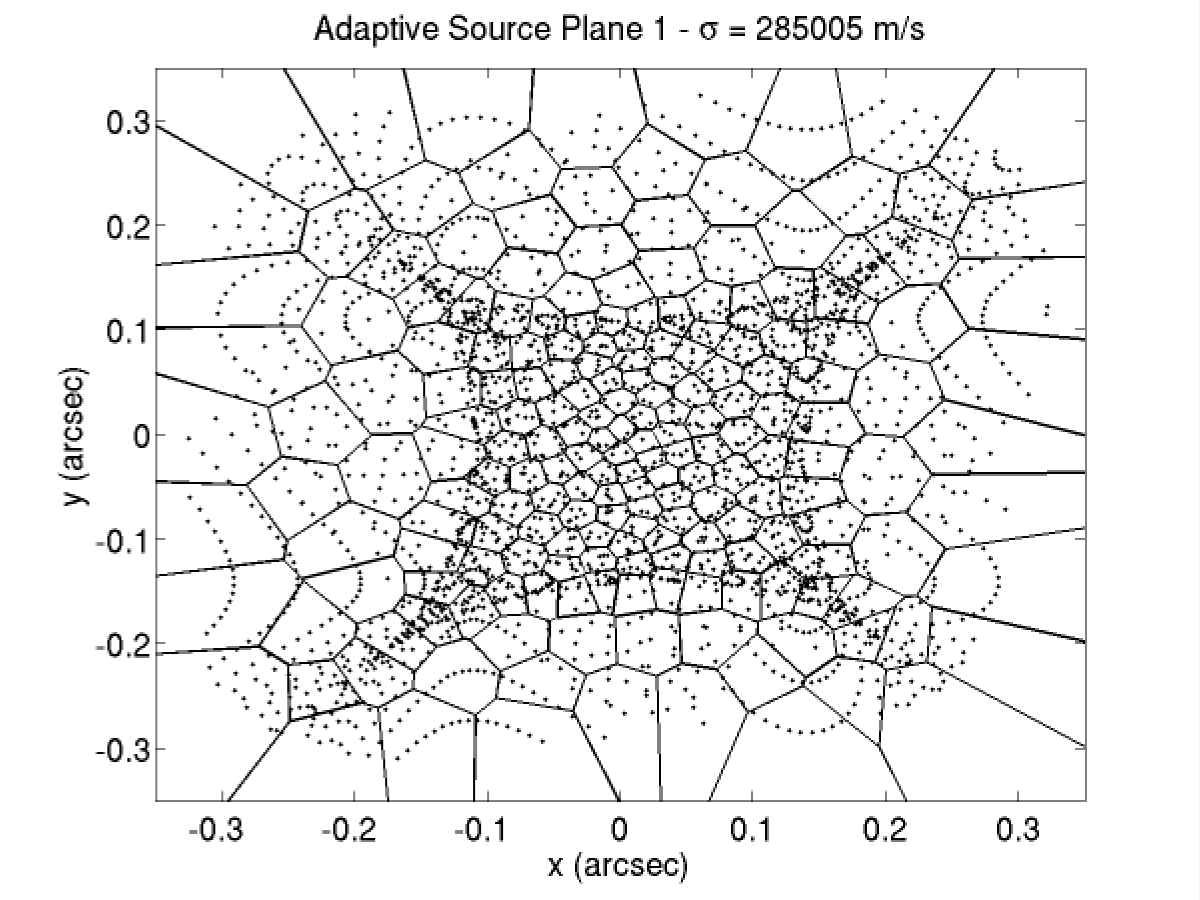

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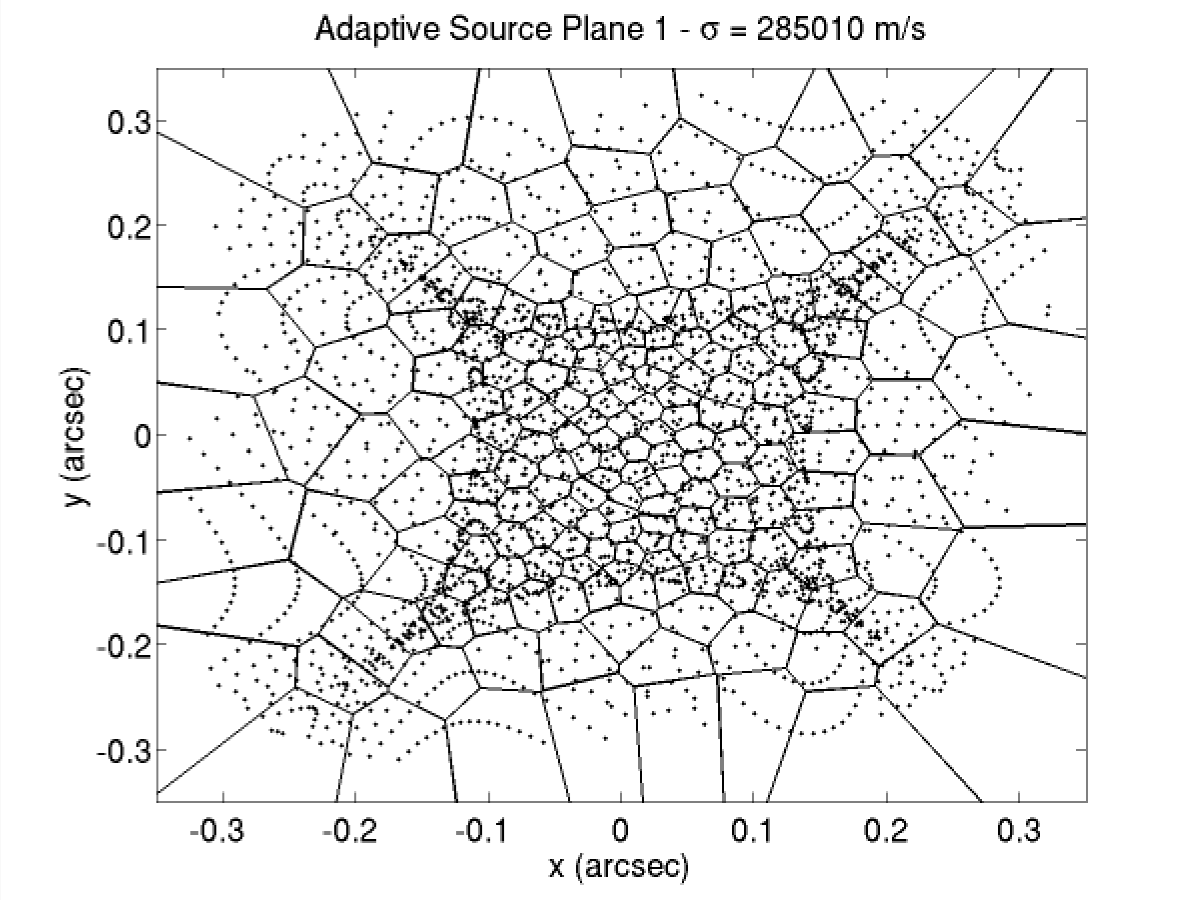

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- **Adaptive SLI naturally explores the underlying systematics.**

  - More natural error sampling.

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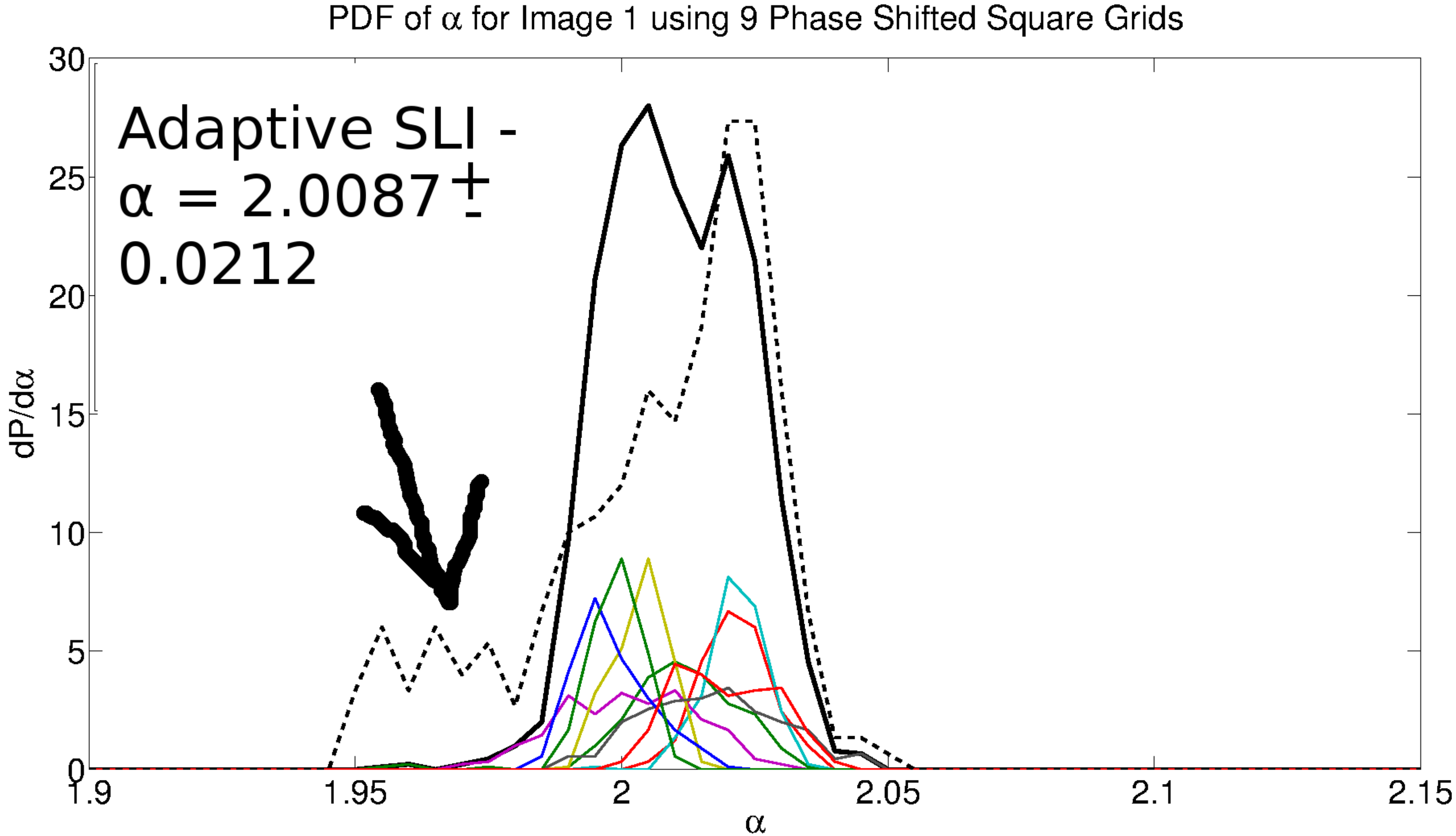

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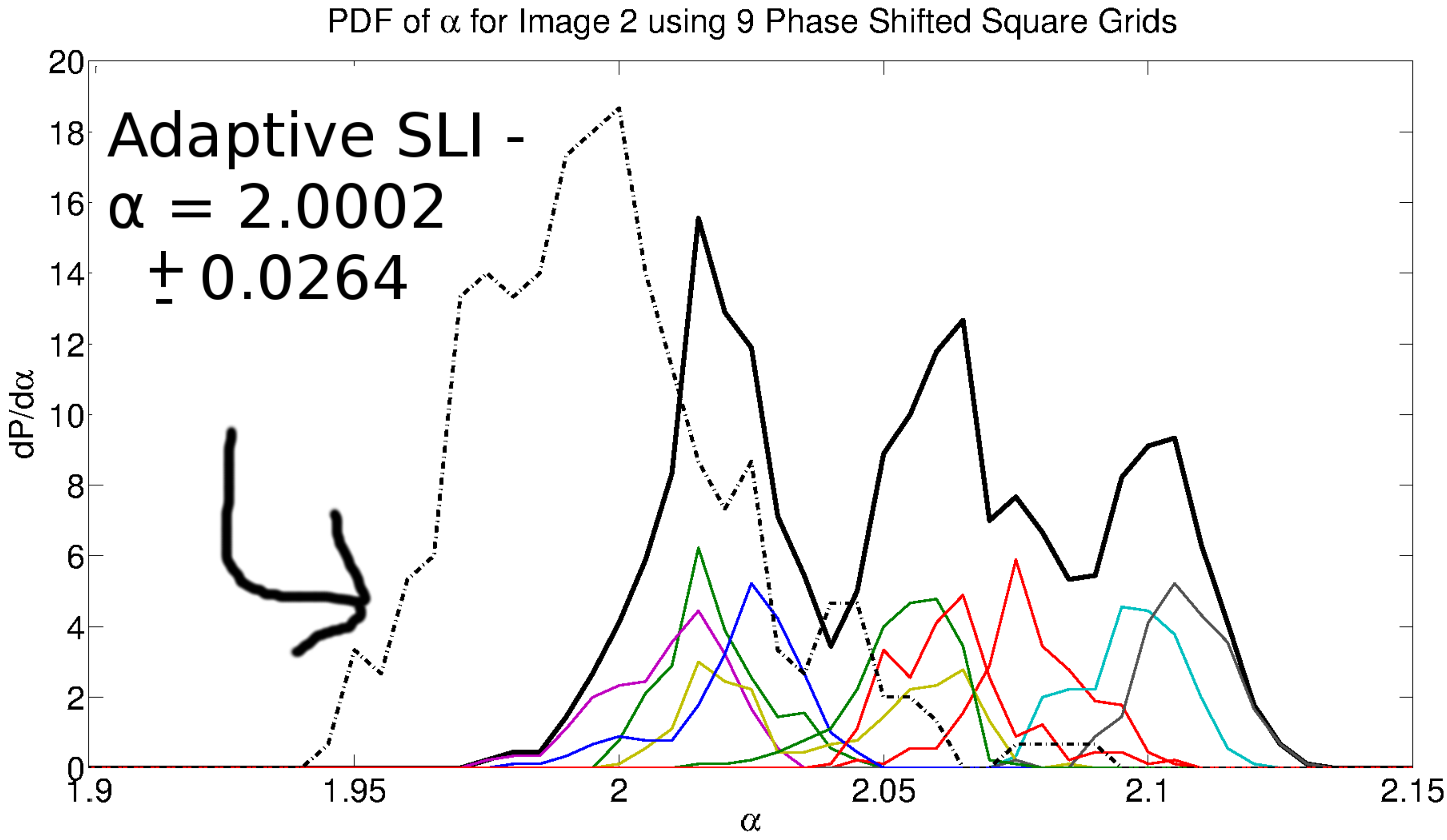


Strong Lensing Analysis Pipeline (Preliminary Work)

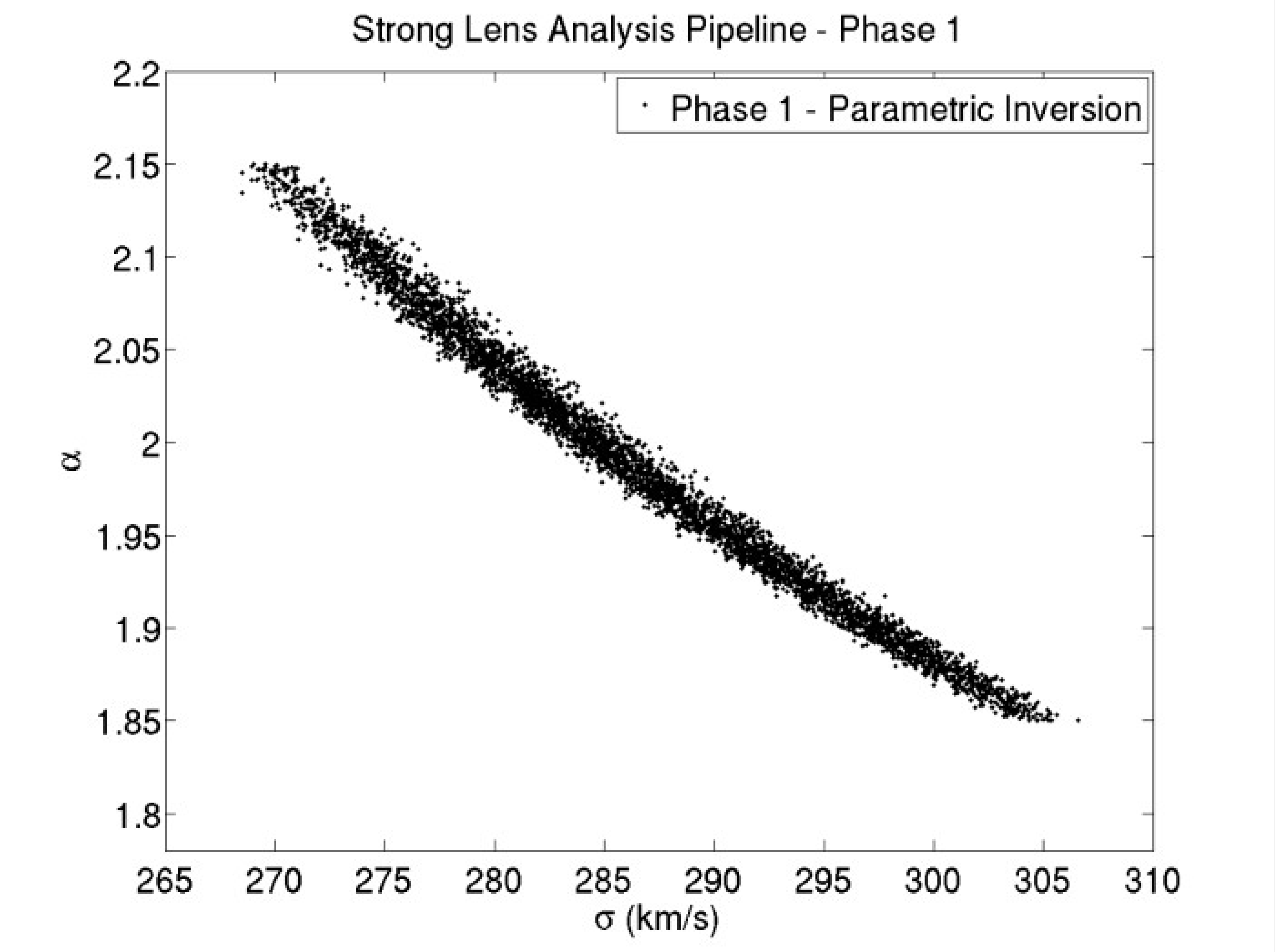
Pipeline Development – 3 Phase Inversion

- **Phase 1 – Fast inversion used to explore parameter space and map out degeneracy.**

  - Use parametric source inversion.

  - Maps out lens model and degeneracies -> Initializes phase 2.

Pipeline Development – Phase 1



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  - Lower Source resolution.

  - Lower Image Subgridding.

  - Faster evaluation of Bayesian evidence.

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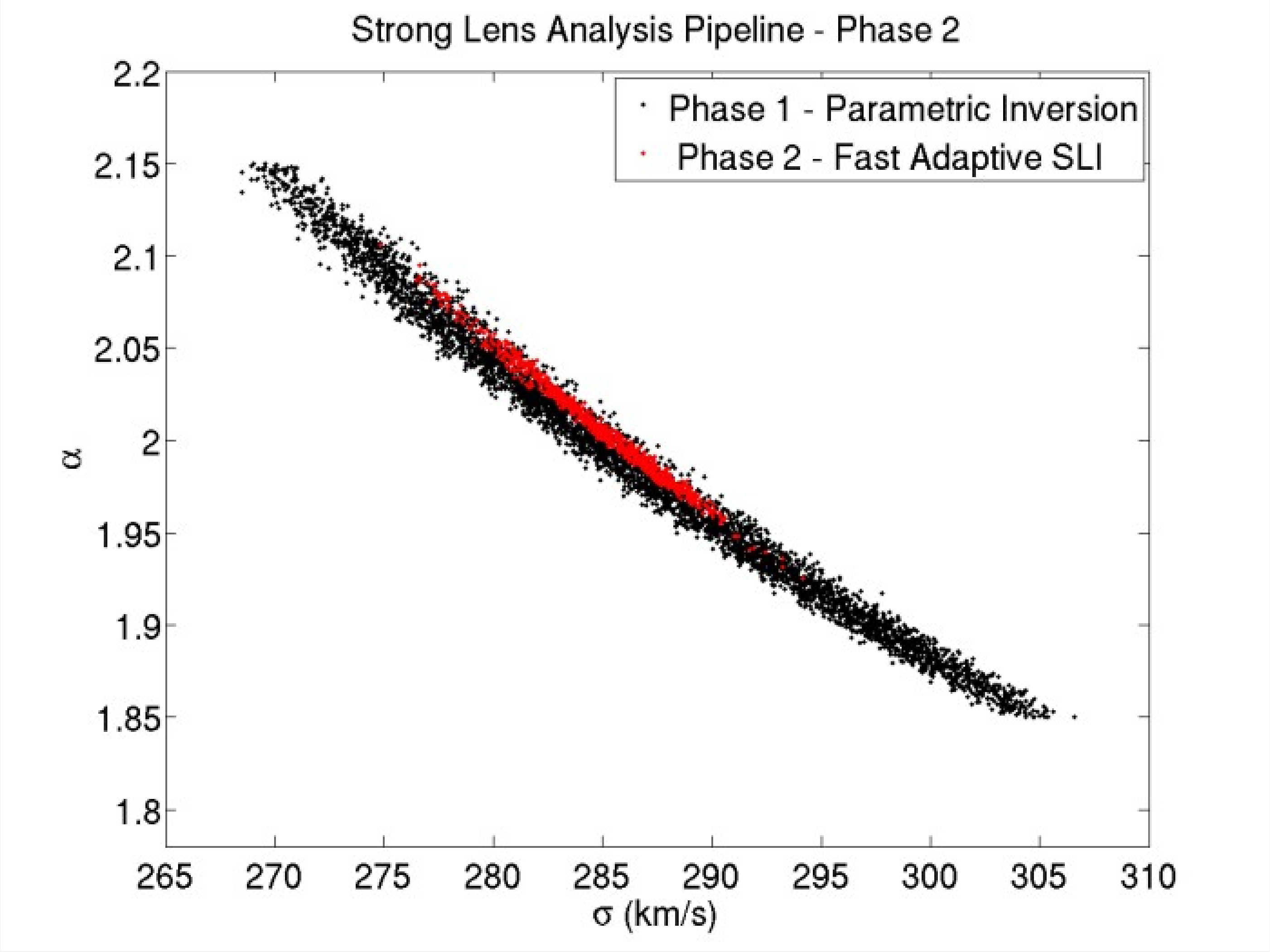
Pipeline Development – Phase 2



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- **Phase 3 – Full Adaptive SLI**

  - Use Adaptive SLI as presented here.

  - Accurate lens model and fully sampled errors.

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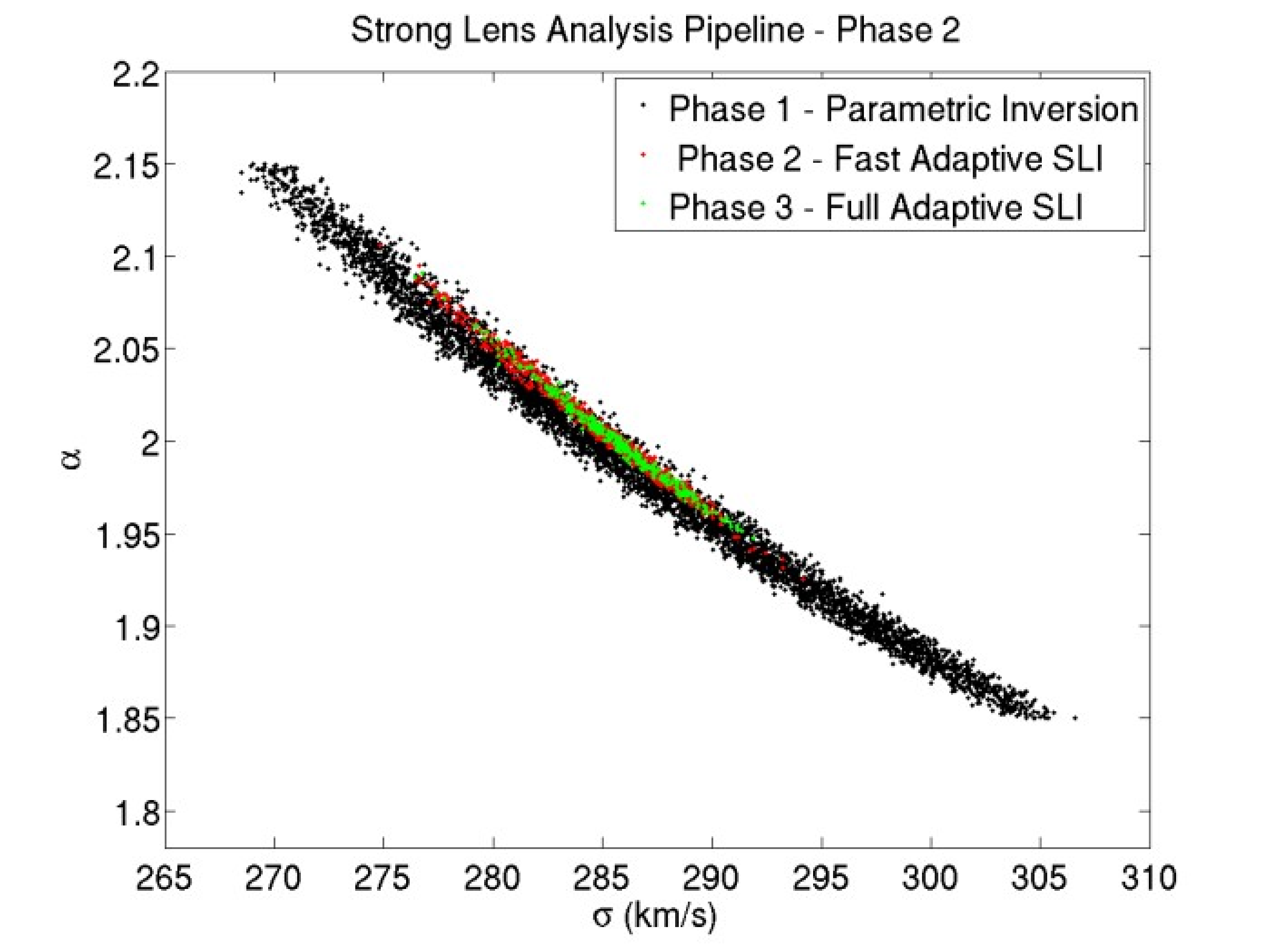
Pipeline Development – Phase 3



Pipeline Development

- **Must rigorously test on both observational and simulated data.**

  - Huge number of potential 'leaks' → must know of them.

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Pipeline Development

- **Must rigorously test on both observational and simulated data.**
  - Huge number of potential 'leaks' → must know of them.
  - Compare with other modeling → learn more about systematics.
- **How streamed lined can we go – Depends on what you want?**
  - Reliable source reconstructions?
  - Accurate fitting of degenerate lens models?
  - Extremely precise modeling for cosmology or substructure?
- **When will modeler input be required?**

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Summary

- **Adaptive SLI – A new method for the modeling of strong lens data.**

  - Creates an **unrelated** pixelization in a **unique** way for every lens model.

  - Removes data discretization biases.

  - More natural error sampling.

- **Strong lens analysis pipeline.**

  - Modeling process can be streamlined...

  - But how streamlined and for what purpose?

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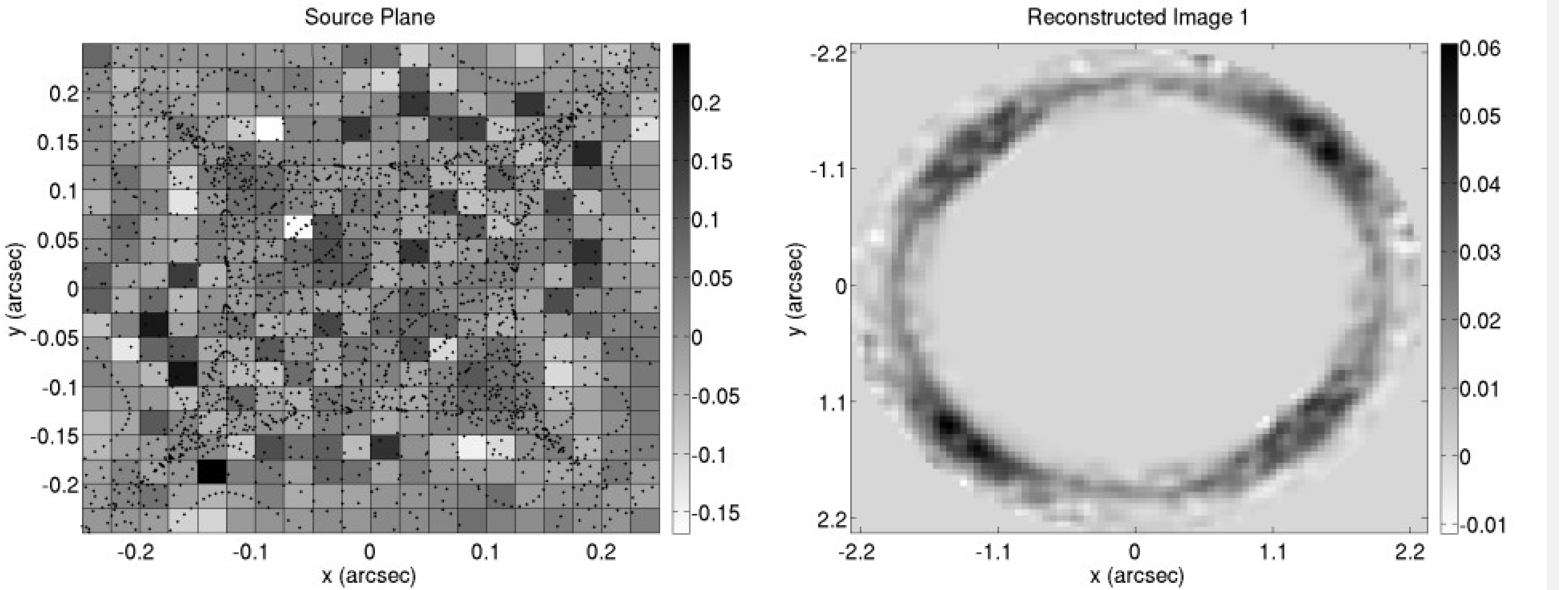
Additional Slides

Example – Poor Lens Model



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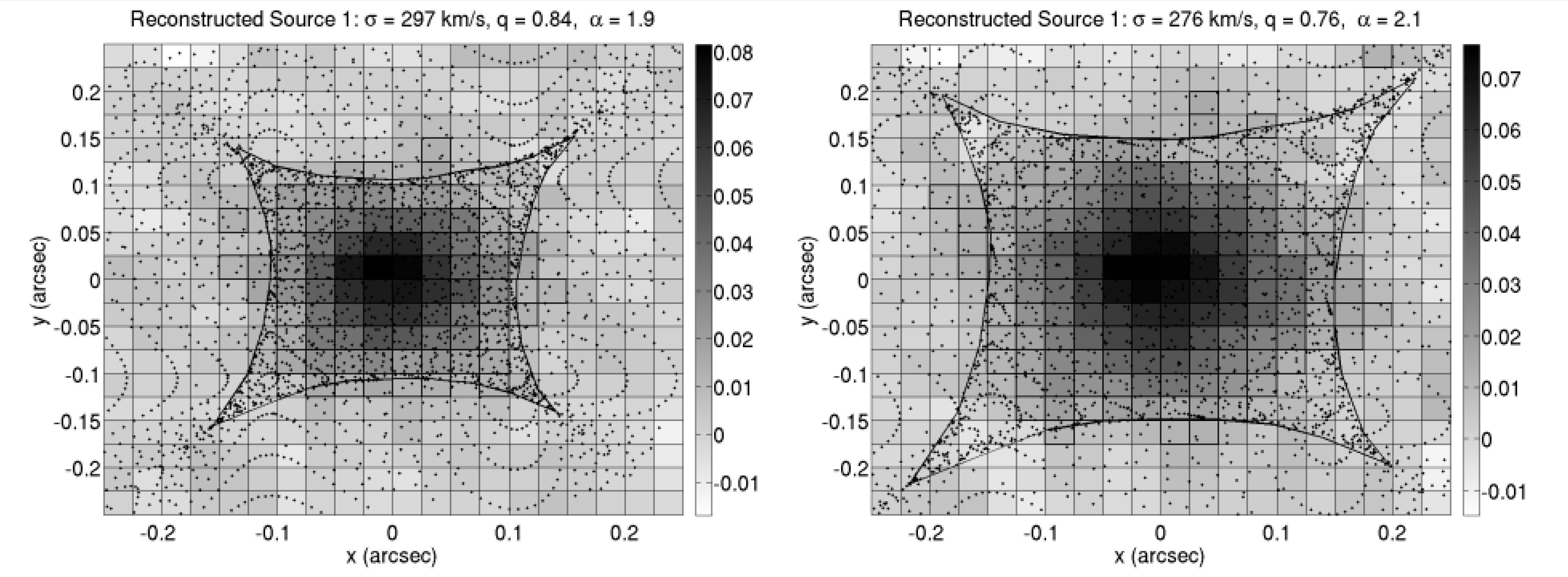
Fixing NDOF

Bias 1 – Varying NDOF

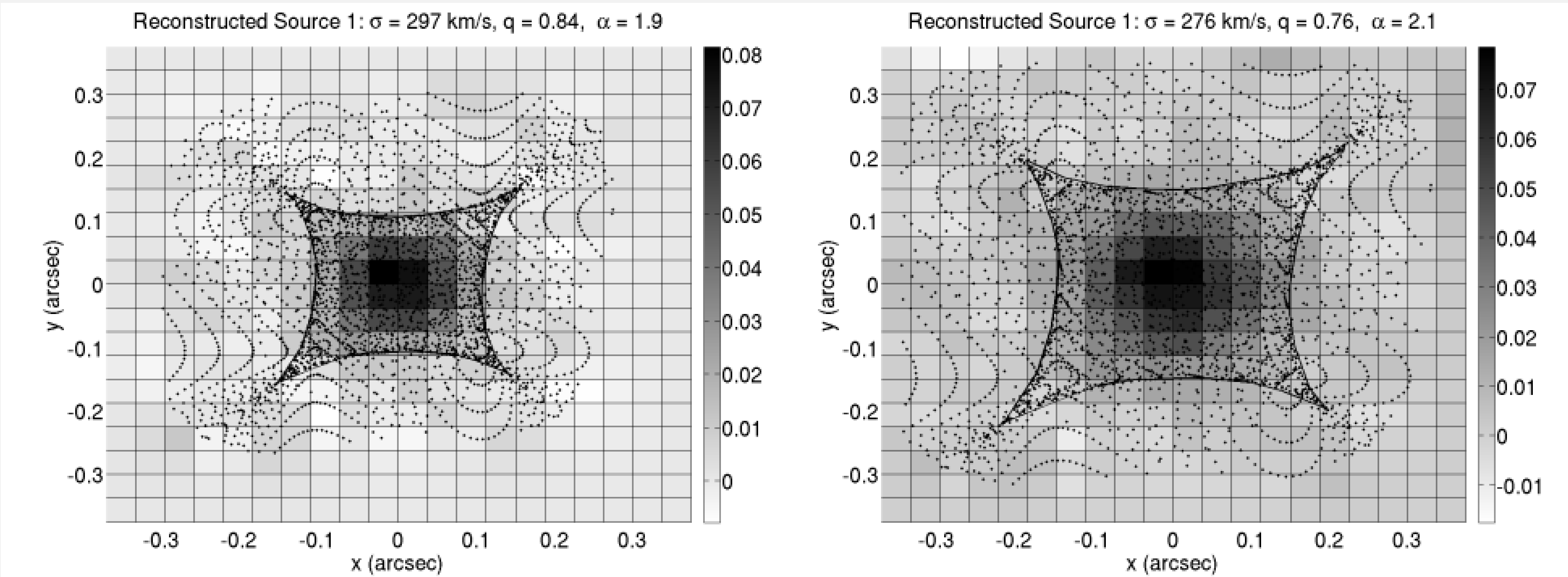
- **Degeneracy – Gives isotropic source scaling**



- **Source expands -> Fewer image pixels trace into source plane.**
- **Solutions biased to those which minimize the number of traced image pixels -> Must fix NDOF.**

Fix NDOF

- **NDOF must be fixed for every possible lens model.**

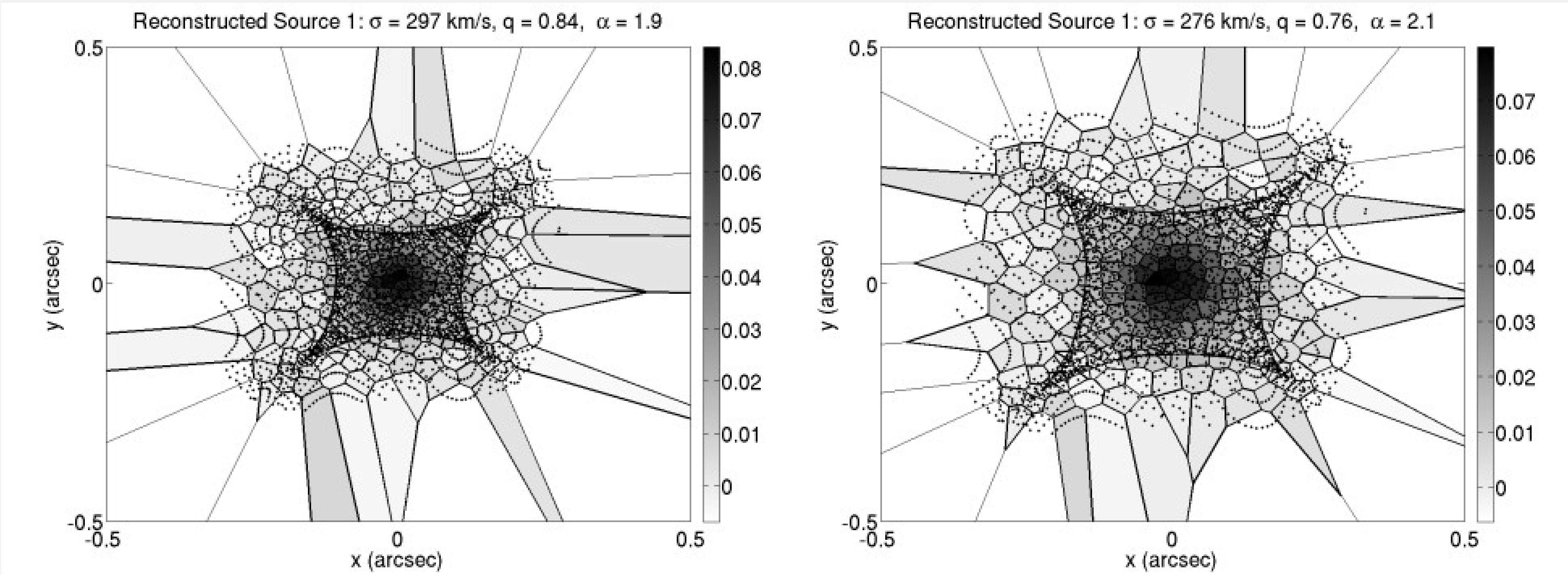


- Large source plane, narrow priors and tightened image mask.

- inefficient resolution, pixels constrained by just regularization and varying effective resolution.

Fix NDOF

- **NDOF must be fixed for every possible lens model.**

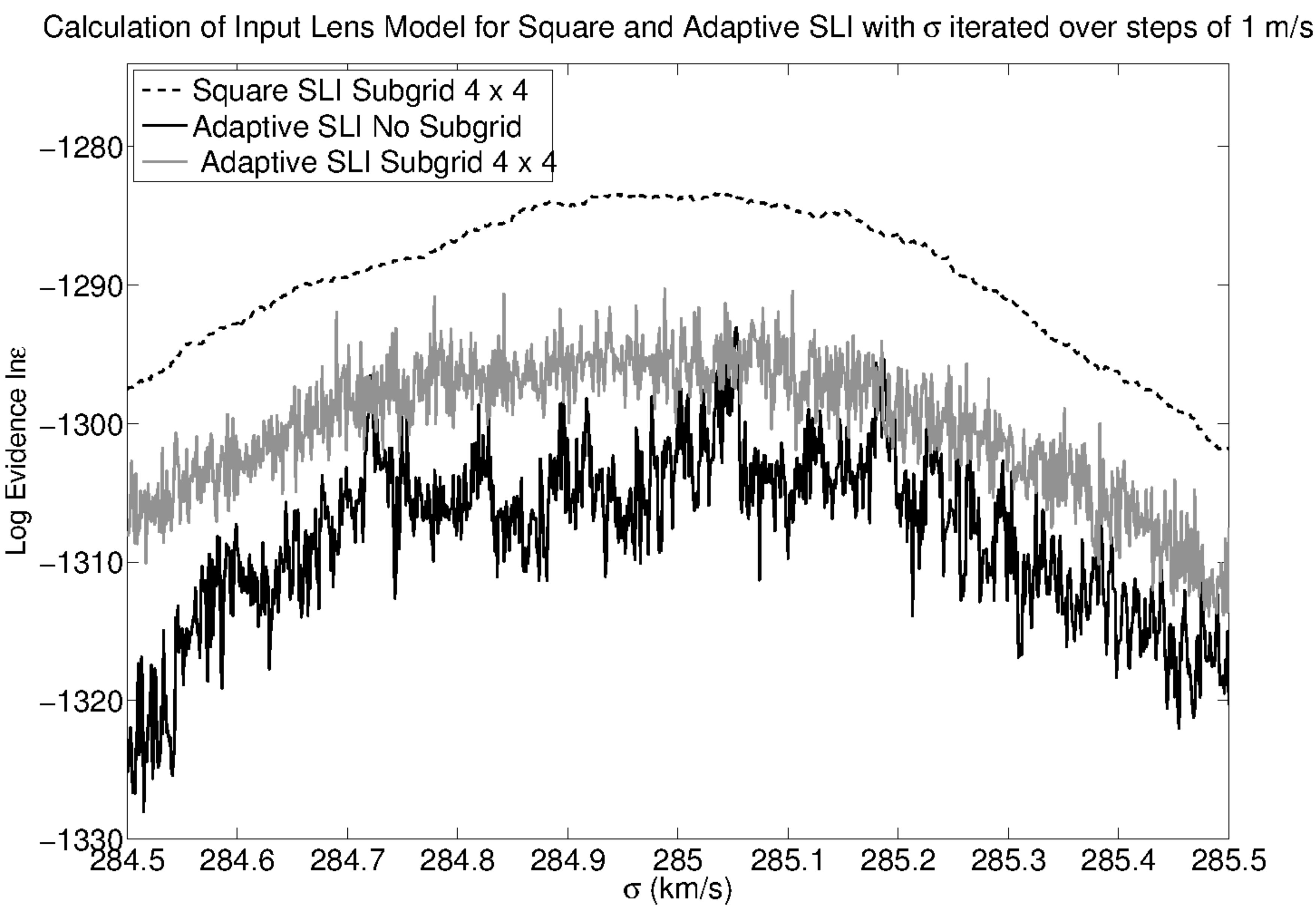


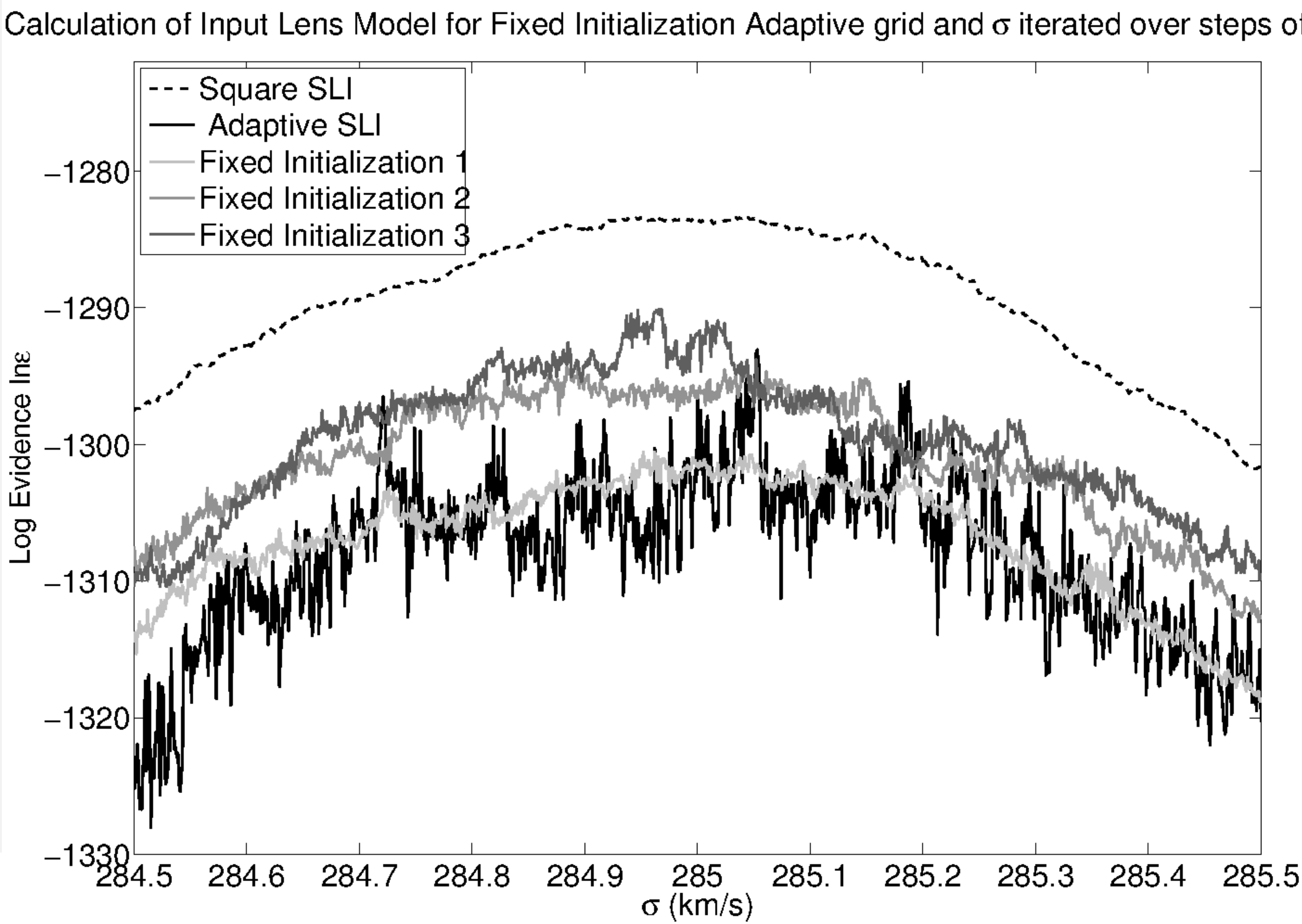
- Can set Adaptive SLI source plane to arbitrarily large size.

- Circumvents all previous problems

Noisey / Fluctuating Parameter Space

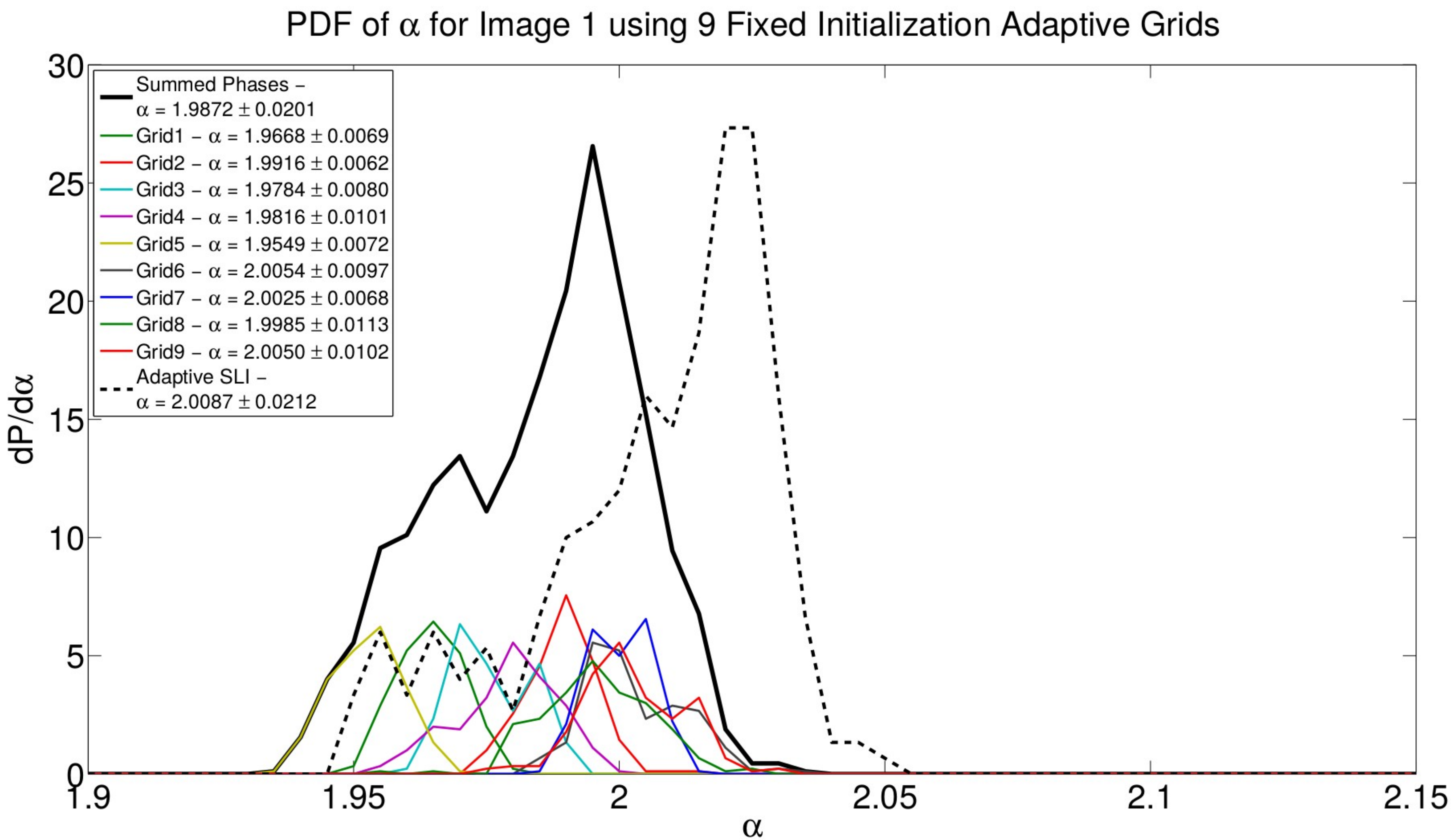
The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

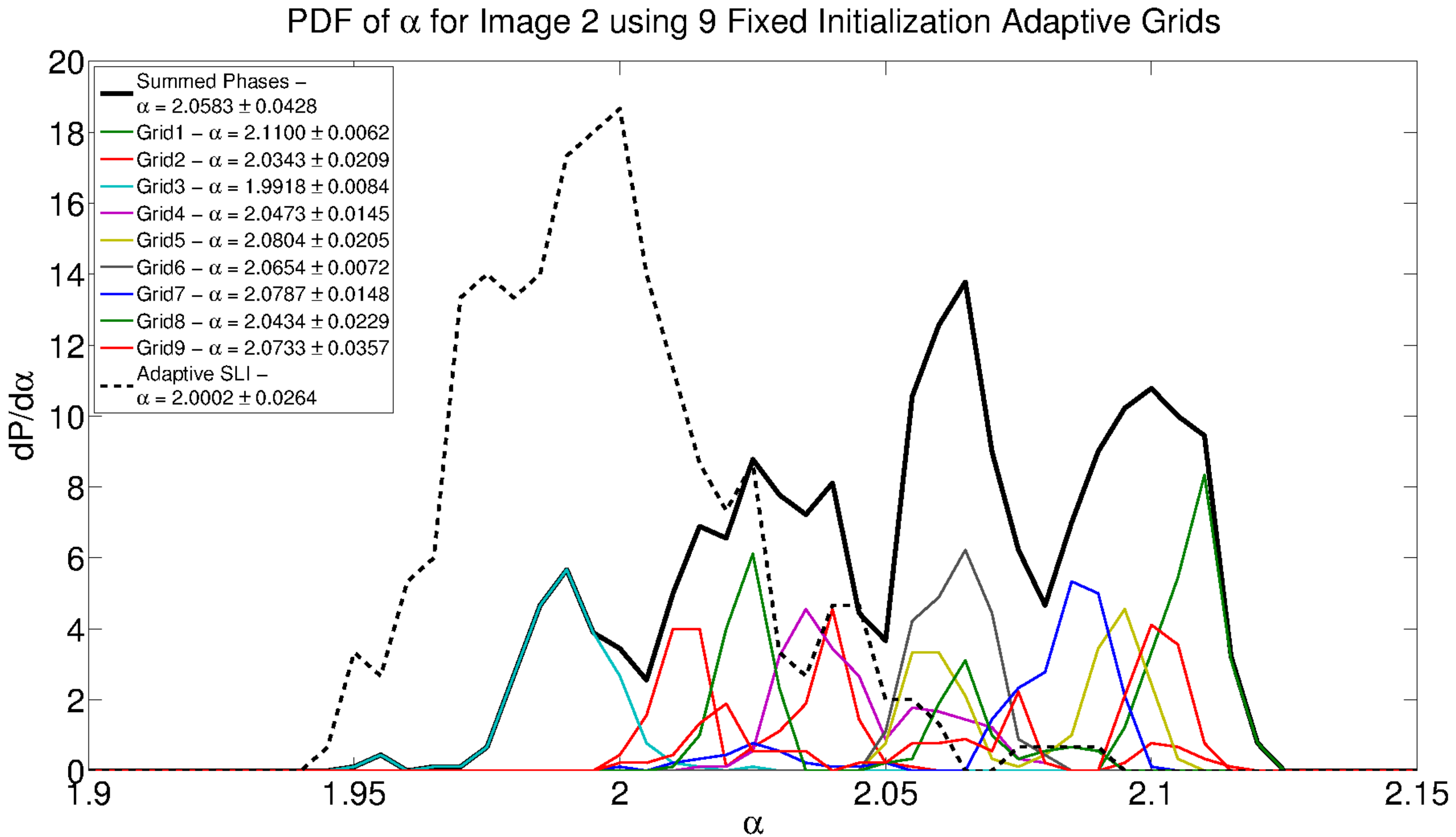



Calculation of Input Lens Model for Fixed Initialization Adaptive grid and σ iterated over steps of 1 m/s
Square SLI
Adaptive SLI
Fixed Initialization 1
Fixed Initialization 2
Fixed Initialization 3
Log Evidence lnε
−1280
−1290
−1300
−1310
−1320
−1330
284.5
284.6
284.7
284.8
284.9
285
285.1
285.2
285.3
285.4
285.5
σ (km/s)

Discretization Bias With Related Pixelization

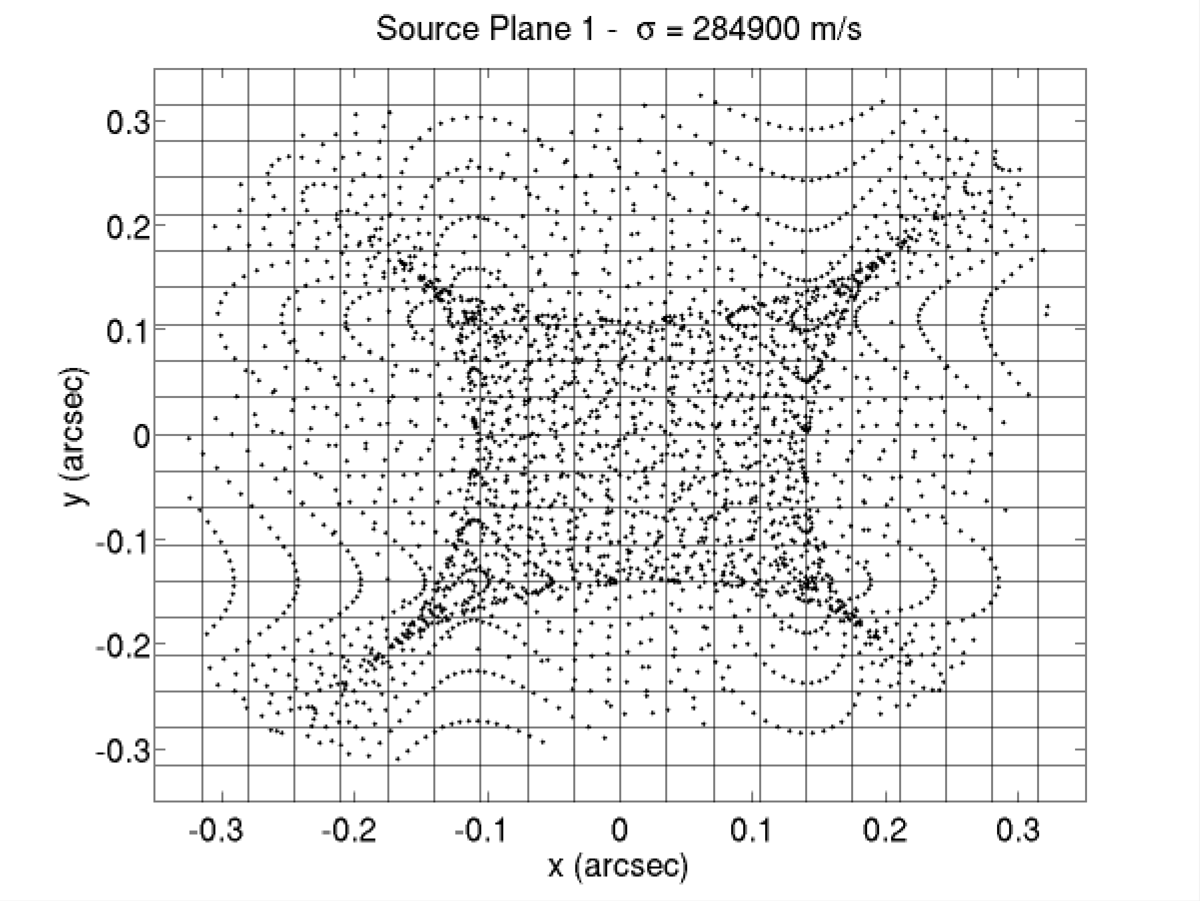

PDF of α for Image 2 using 9 Phase Shifted High Resolution (36 x 36) Square Grids
Summed Phases – α = 2.0480 ± 0.0260
Phase 1 – α = 2.0527 ± 0.0194
Phase 2 – α = 2.0657 ± 0.0182
Phase 3 – α = 2.0452 ± 0.0292
Phase 4 – α = 2.0303 ± 0.0151
Phase 5 – α = 2.0469 ± 0.0398
Phase 6 – α = 2.0289 ± 0.0154
Phase 7 – α = 2.0511 ± 0.0178
Phase 8 – α = 2.0650 ± 0.0158
Phase 9 – α = 2.0462 ± 0.0222
Adaptive SLI – α = 2.0002 ± 0.0264
dP/dα
α
0
2
4
6
8
10
12
14
16
18
20
1.9
1.95
2
2.05
2.1
2.15


PDF of α for Image 1 using 9 Fixed Initialization Adaptive Grids
Summed Phases – α = 1.9872 ± 0.0201
Grid1 – α = 1.9668 ± 0.0069
Grid2 – α = 1.9916 ± 0.0062
Grid3 – α = 1.9784 ± 0.0080
Grid4 – α = 1.9816 ± 0.0101
Grid5 – α = 1.9549 ± 0.0072
Grid6 – α = 2.0054 ± 0.0097
Grid7 – α = 2.0025 ± 0.0068
Grid8 – α = 1.9985 ± 0.0113
Grid9 – α = 2.0050 ± 0.0102
Adaptive SLI – α = 2.0087 ± 0.0212
dP/dα
α
30
25
20
15
10
5
0
1.9
1.95
2
2.05
2.1
2.15


PDF of α for Image 2 using 9 Fixed Initialization Adaptive Grids
Summed Phases – α = 2.0583 ± 0.0428
Grid1 – α = 2.1100 ± 0.0062
Grid2 – α = 2.0343 ± 0.0209
Grid3 – α = 1.9918 ± 0.0084
Grid4 – α = 2.0473 ± 0.0145
Grid5 – α = 2.0804 ± 0.0205
Grid6 – α = 2.0654 ± 0.0072
Grid7 – α = 2.0787 ± 0.0148
Grid8 – α = 2.0434 ± 0.0229
Grid9 – α = 2.0733 ± 0.0357
Adaptive SLI – α = 2.0002 ± 0.0264
dP/dα
α

Square / Adaptive Discretization Comparison

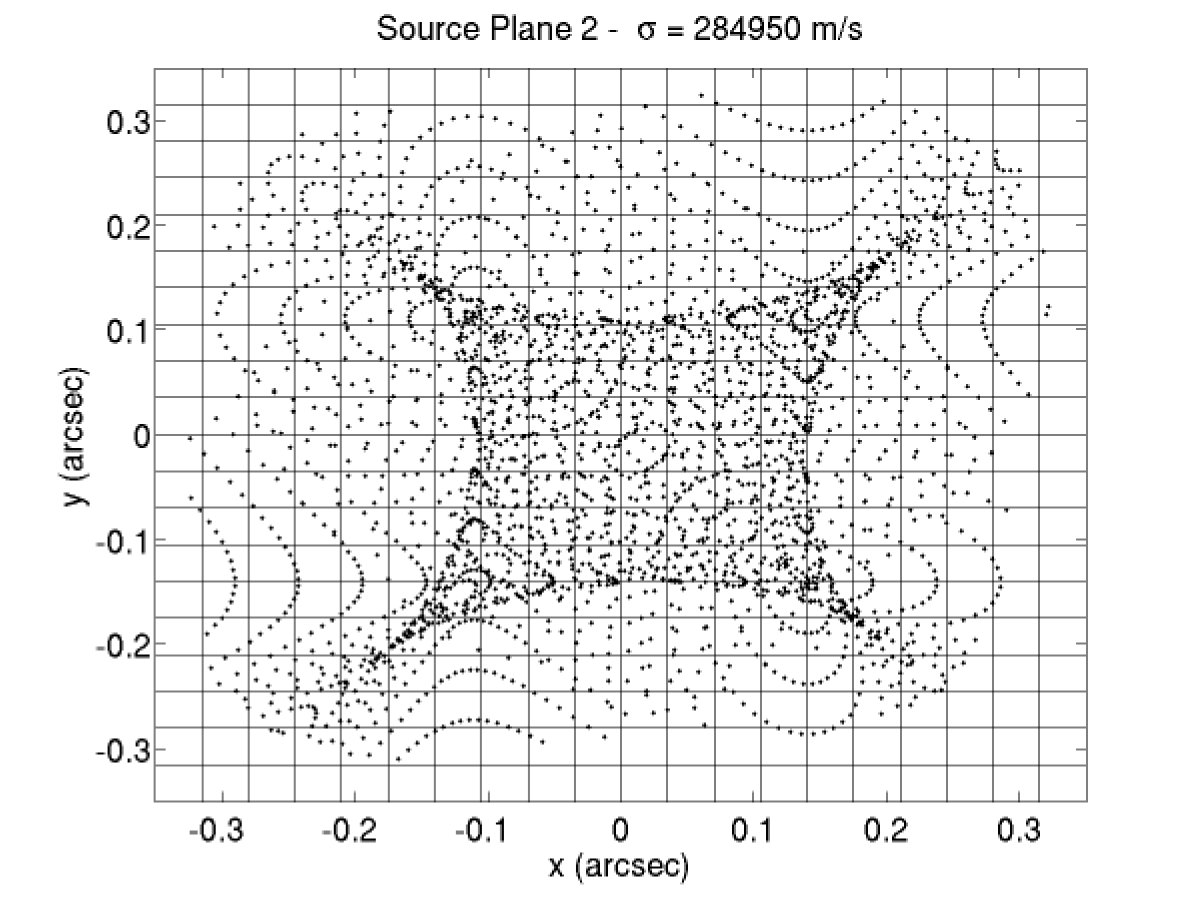
Square SLI – Fixed Grid



- **5 lens model – all parameters fixed except σ which is changed by 50 m/s in each.**

  - $\varepsilon_1 = -1599.877$

Square SLI – Fixed Grid

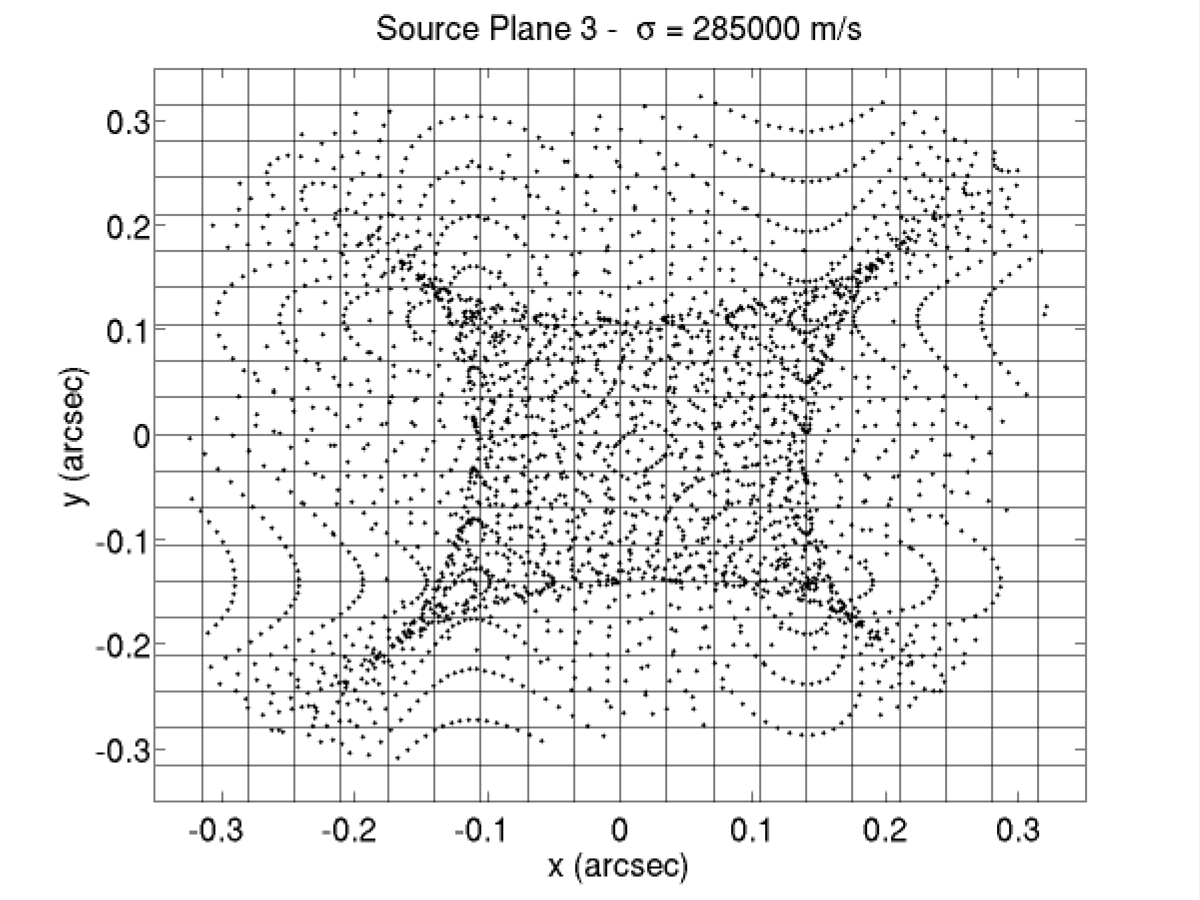


- **5 lens model – all parameters fixed except σ which is changed by 50 m/s in each.**

  - $\varepsilon_1 = -1599.877$

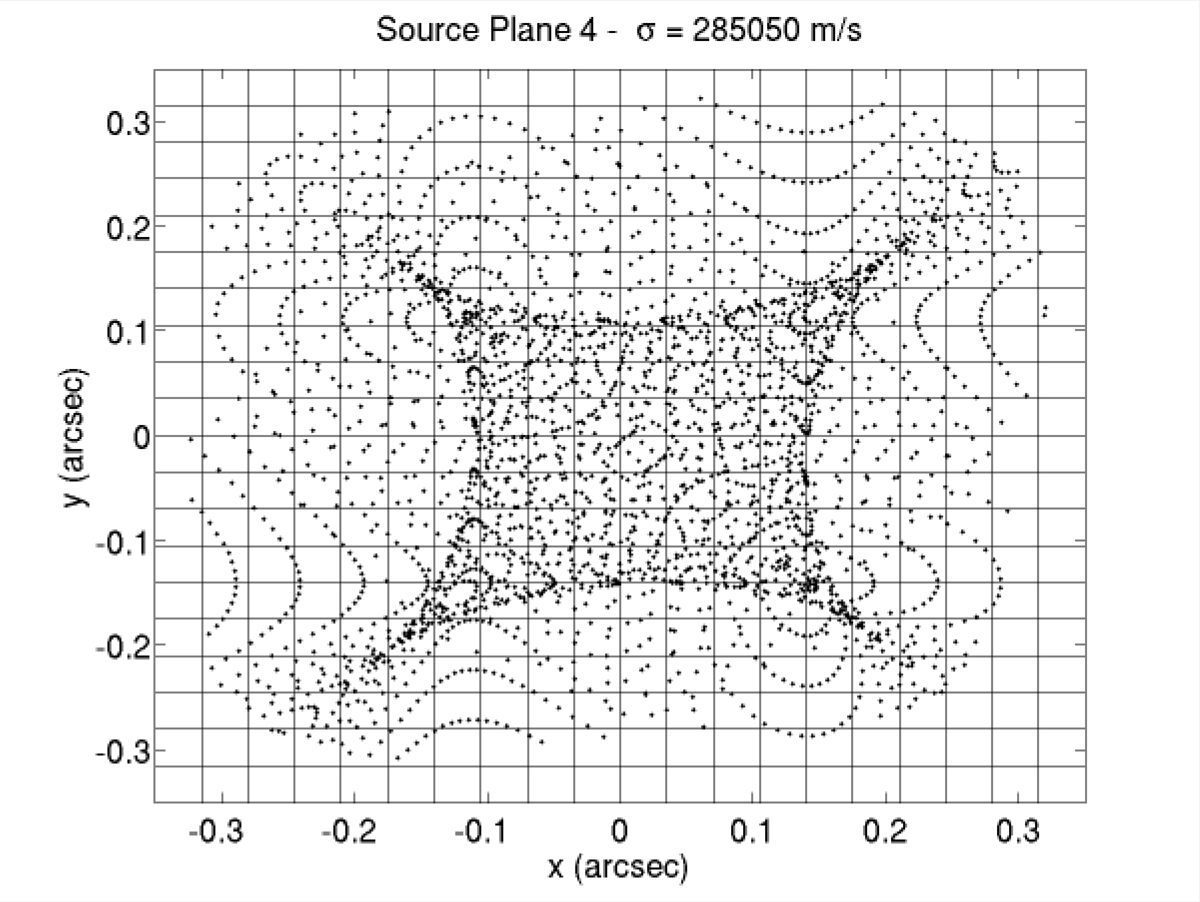
  - $\varepsilon_2 = -1599.026$

Square SLI – Fixed Grid



- **5 lens model – all parameters fixed except σ which is changed by 50 m/s in each.**
  - $\varepsilon_1 = -1599.877$
  - $\varepsilon_2 = -1599.026$
  - $\varepsilon_3 = -1599.000$

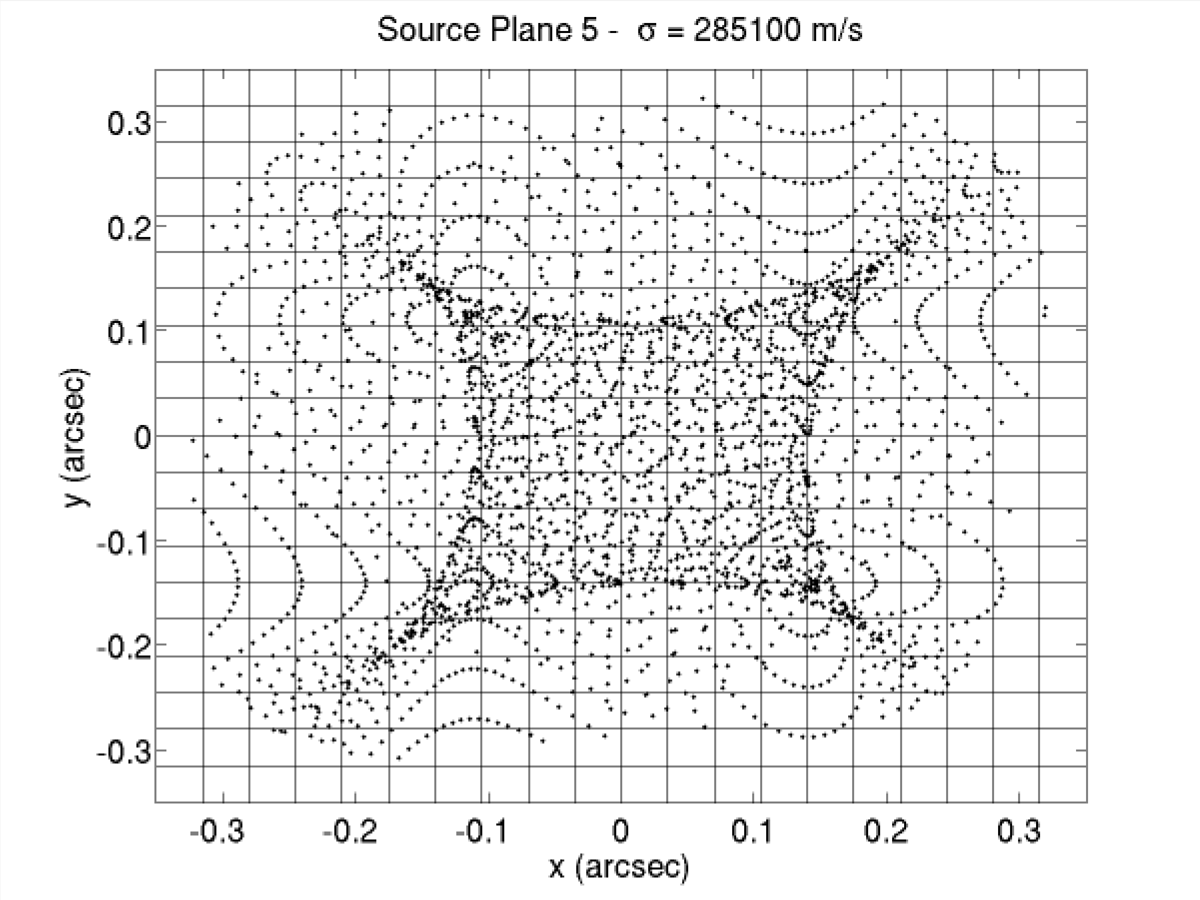
Square SLI – Fixed Grid



- **5 lens model – all parameters fixed except σ which is changed by 50 m/s in each.**

  - ε_1 = -1599.877
  - ε_2 = -1599.026
  - ε_3 = -1599.000
  - ε_4 = -1599.390

Square SLI – Fixed Grid



- **5 lens model – all parameters fixed except σ which is changed by 50 m/s in each.**
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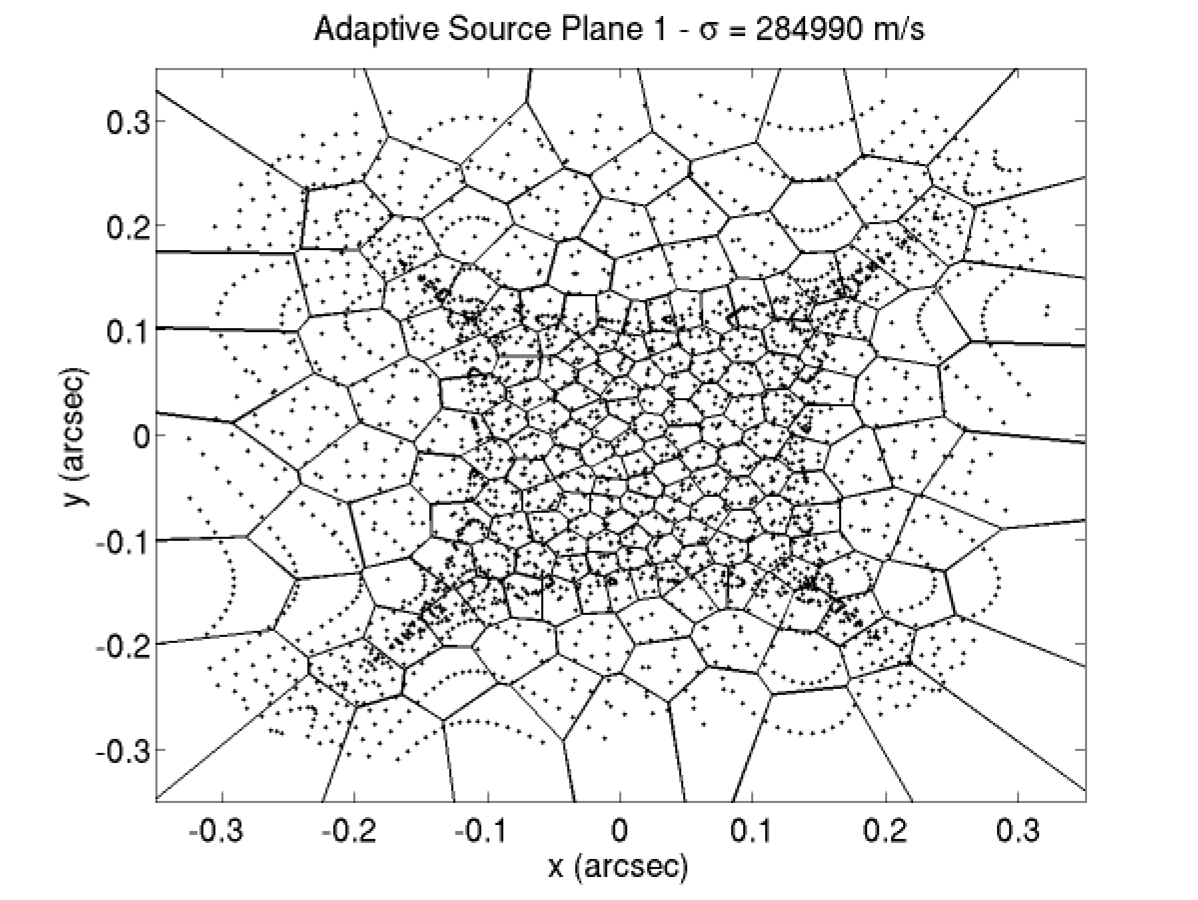
  - $\varepsilon_3 = -1599.000$
  - $\varepsilon_4 = -1599.390$
  - $\varepsilon_5 = -1599.705$

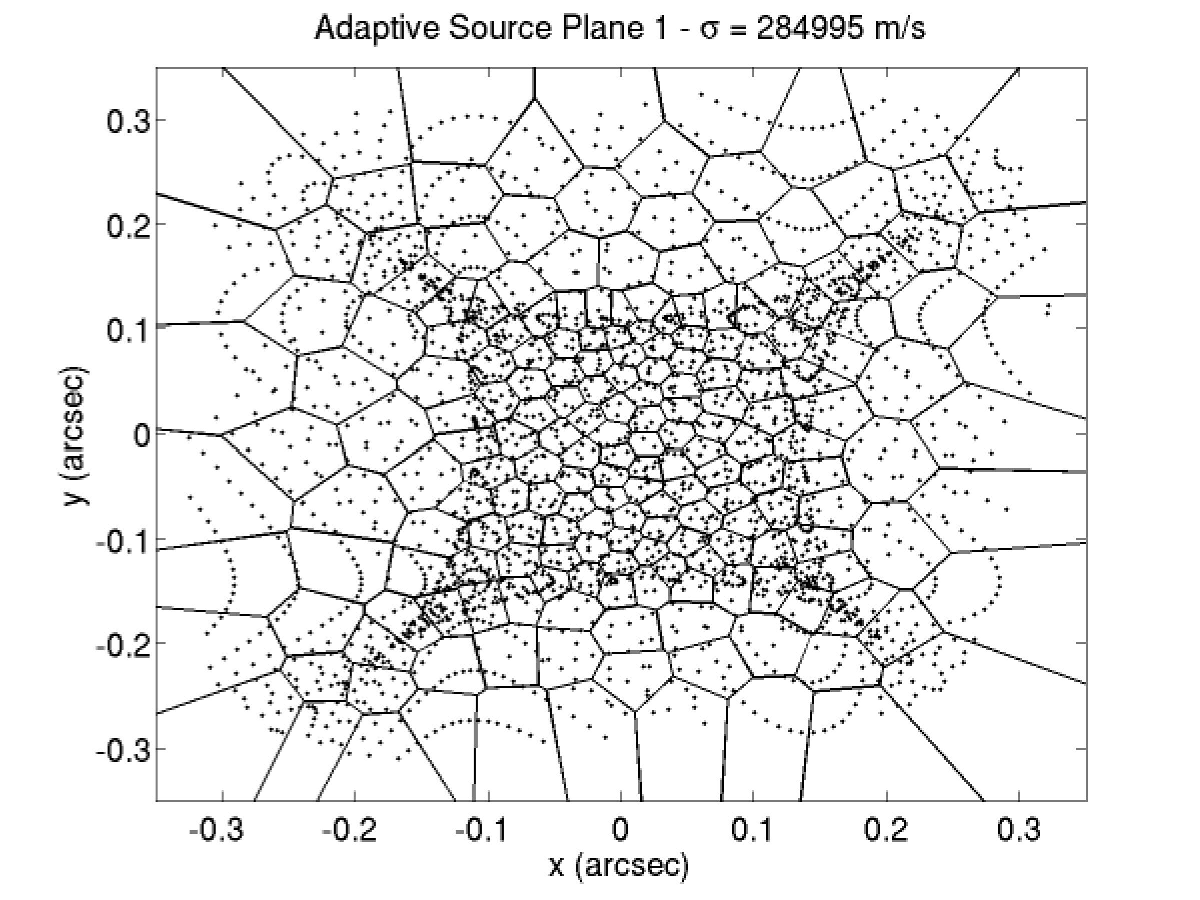
Adaptive SLI – Unique Grids



- **5 lens model – all parameters fixed except σ which is changed by 5 m/s in each.**

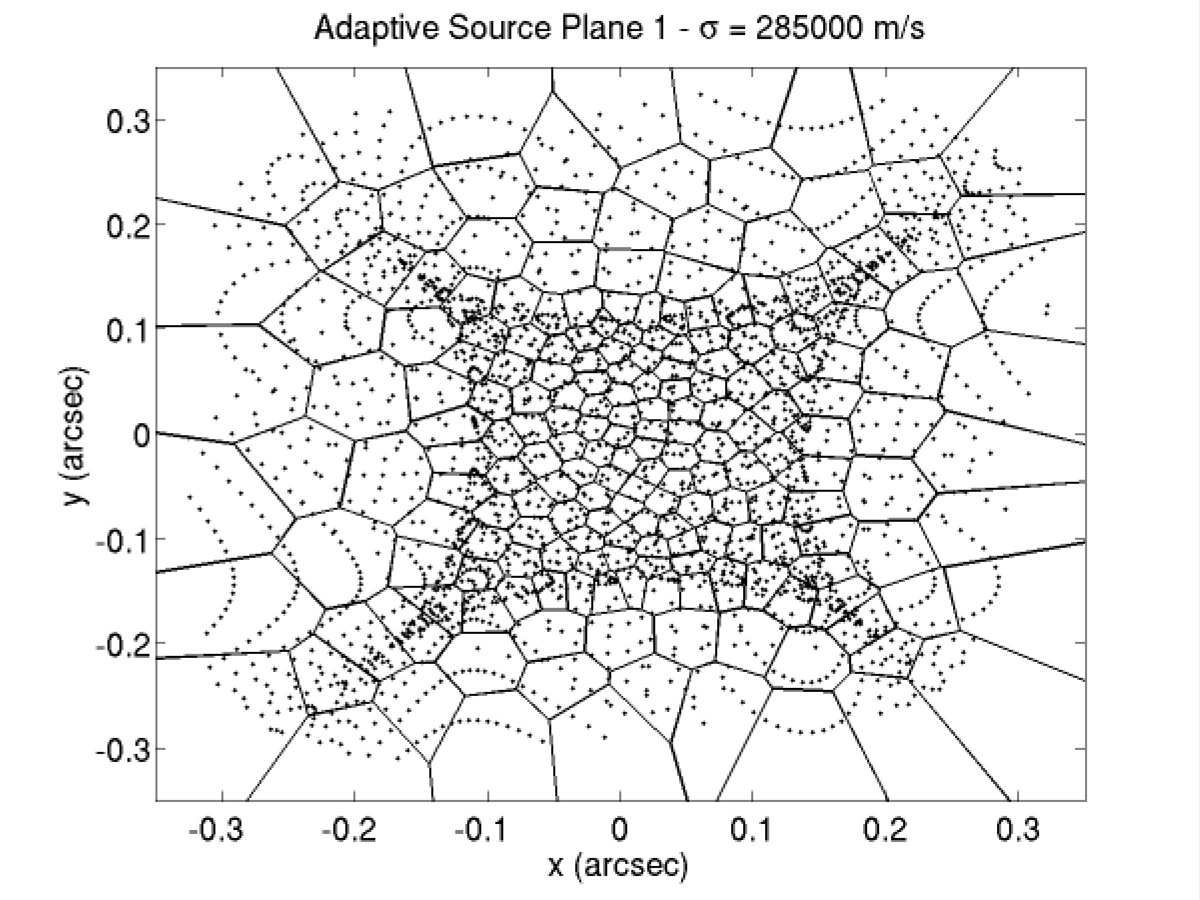
  - $\varepsilon_1 = -1599.773$

Adaptive SLI – Unique Grids



- **5 lens model – all parameters fixed except σ which is changed by 5 m/s in each.**
  - $\varepsilon_1 = -1599.773$
  - $\varepsilon_2 = -1606.381$

Adaptive SLI – Unique Grids



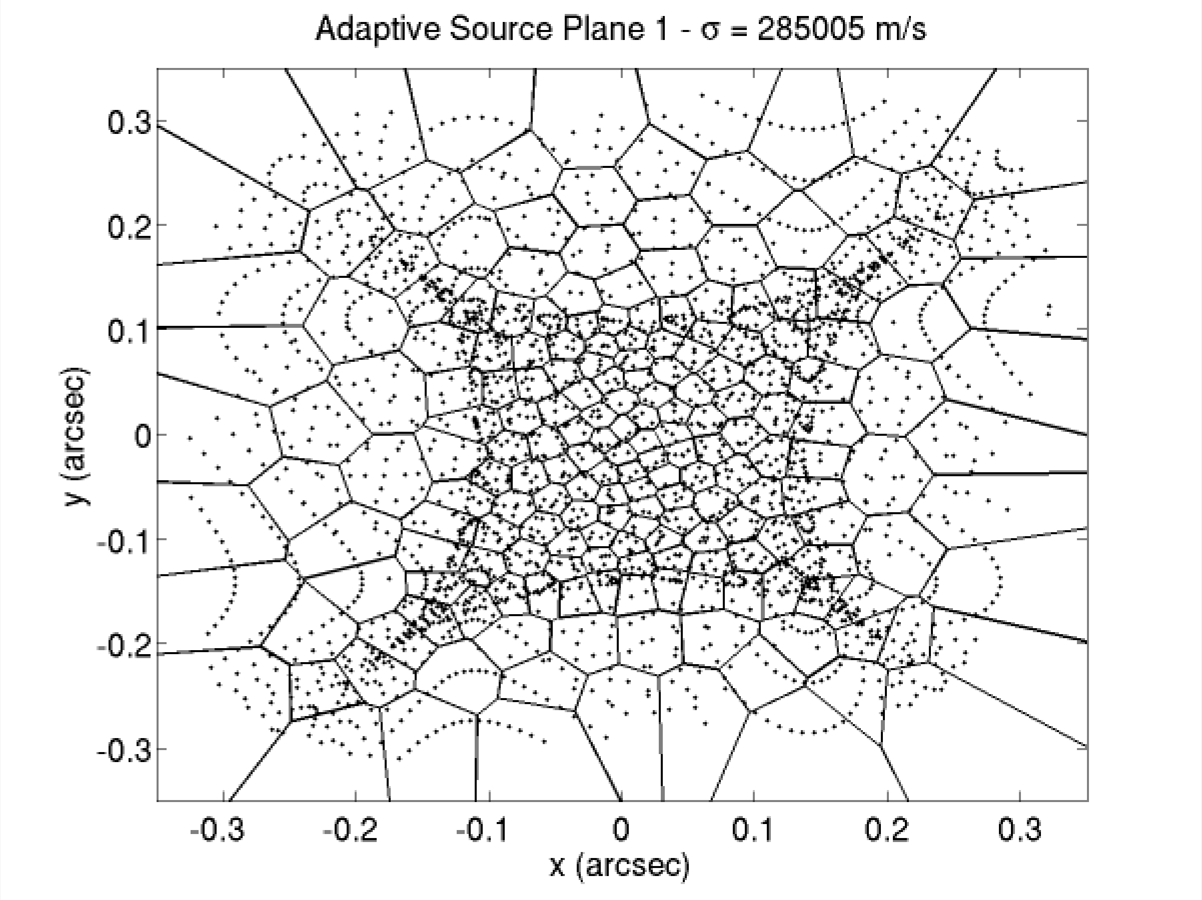
- **5 lens model – all parameters fixed except σ which is changed by 5 m/s in each.**

  - $\varepsilon_1 = -1599.773$

  - $\varepsilon_2 = -1606.381$

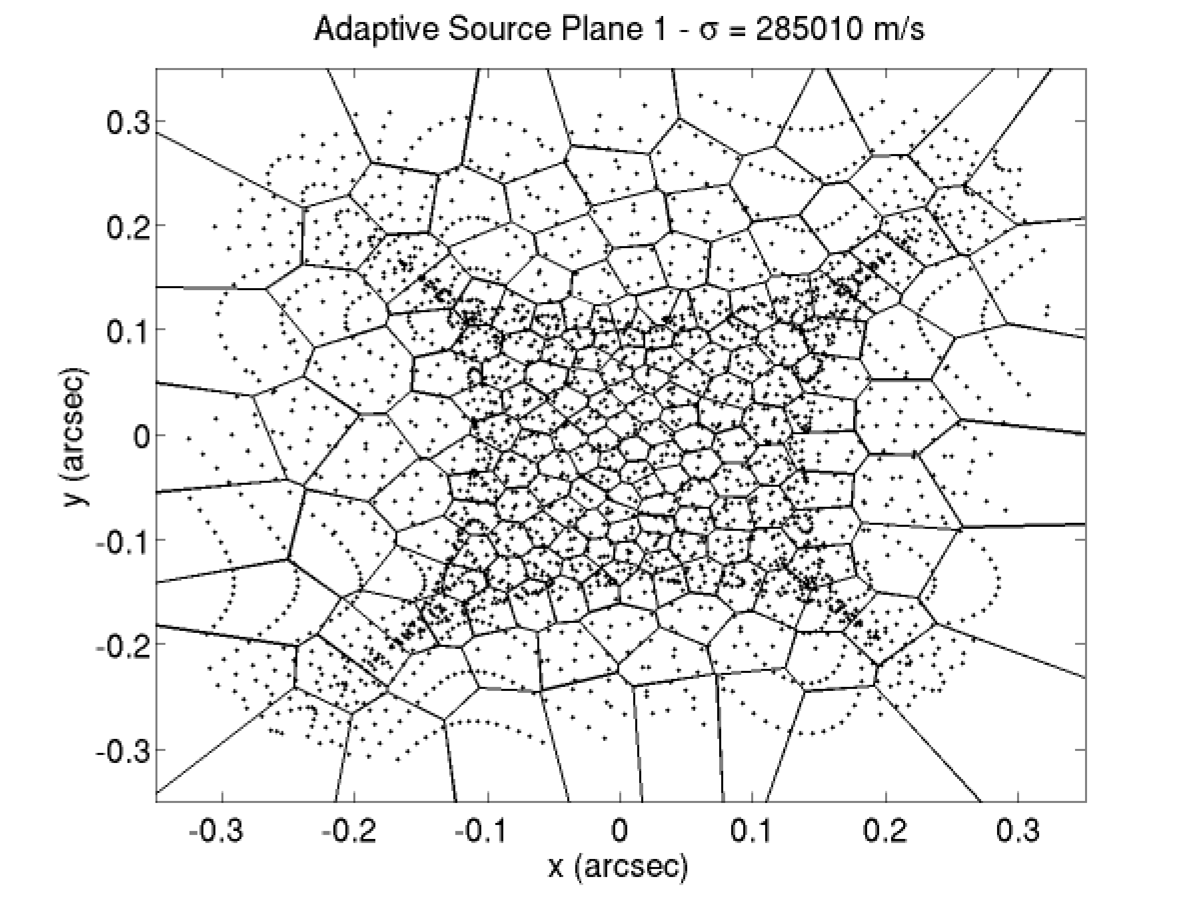
  - $\varepsilon_3 = -1598.001$

Adaptive SLI – Unique Grids



- **5 lens model – all parameters fixed except σ which is changed by 5 m/s in each.**
  - $\varepsilon_1 = -1599.773$
  - $\varepsilon_2 = -1606.381$
  - $\varepsilon_3 = -1598.001$
  - $\varepsilon_4 = -1604.852$

Adaptive SLI – Unique Grids



- **5 lens model – all parameters fixed except σ which is changed by 5 m/s in each.**
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  - $\varepsilon_2 = -1606.381$
  - $\varepsilon_3 = -1598.001$
  - $\varepsilon_4 = -1604.852$
  - $\varepsilon_5 = -1599.654$

Why Use Adaptive SLI For a Pipeline?

- **Pixelization adapts to lens magnification.**

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  - More robust, accurate inversions process.

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- **Naturally changes data discretization, removing alignment biases.**

  - Accurately fits degenerate lens models like a PL.

  - Errors determined purely by fit of model and observation.