Measuring Angular Diameter Distances Using Time-delay Lenses

arXiv:1410.7770

Inh Jee (MPA)
Eiichiro Komatsu (MPA), Sherry Suyu (ASIAA)

2014/11/17

Measuring D_A using time-delay lenses

Paraficz & Hjorth (2009)

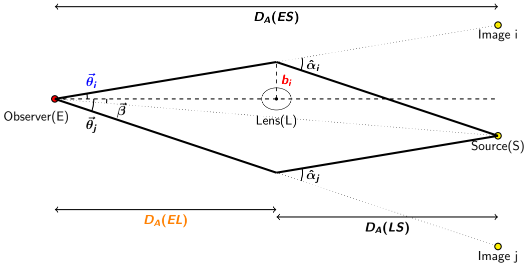
- Singular isothermal sphere (SIS) density profile
- Combine lensing dynamics (velocity dispersion) and the time-delay
- Measured the angular diameter distance using time delay $\Delta t_{i,j}$, velocity dispersion σ^2, lens redshift z_{L} and the image positions θ_i, θ_j

$$D_A(EL) = \frac{c^3 \Delta t_{i,j}}{4\pi\sigma^2(1+z_{\mathrm{L}})} \frac{1}{(\theta_j - \theta_i)} \quad (1)$$

Limitations

- Spherically symmetric mass distribution, isotropic velocity dispersion
- No study on the effect of the external convergence

Physical Intuition



When the mass distribution is known,

- Time delay → Mass estimate
- Velocity dispersion → Potential

⇒ Combine them to get the **physical size** (b) of the system

Observation of strong lensing arc gives the **angular size** (θ) of the system

⇒ The system can be used as a standard ruler to measure the **angular diameter distances** to the lens galaxy ($D_A(EL) = \frac{b}{\theta}$)

Model

Mass density model

- Spherically symmetric power-law density profile

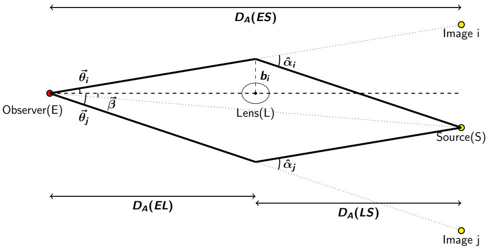
External convergence

- Mass-sheet transformation (MST)

Velocity dispersion model

- Anisotropic velocity dispersion
- Osipkov-Merritt anisotropy
- Aperture averaged vs. spatially resolved velocity dispersion

Power-law density profile and the deflection angle



Density profile

$$\rho = \rho_0 \left(\frac{r}{r_0}\right)^{-\gamma} \tag{2}$$

Deflection angle at the lens plane

$$\hat{\alpha} = \frac{2GM(b)}{c^2 b} \frac{\sqrt{\pi}\Gamma[0.5(-1+\gamma)]}{\Gamma(\gamma/2)} \propto \sigma_r^2(b) \tag{3}$$

Scaled deflection angle

$$\vec{\alpha} = \hat{\alpha}\frac{D_A(LS)}{D_A(ES)} \tag{4}$$

Power-law density profile and the time delay

Time delay

$$\begin{aligned}\Delta t_{i,j} =& \frac{(1+z_{\rm L})}{2c}\frac{D_A(EL)D_A(ES)}{D_A(LS)}\left[(\vec{\alpha}_i+\vec{\alpha}_j)\cdot(\vec{\theta}_i-\vec{\theta}_j)-\frac{2}{3-\gamma}(\vec{\alpha}_i\cdot\vec{\theta}_i-\vec{\alpha}_j\cdot\vec{\theta}_j)\right]\\ =& \frac{(1+z_{\rm L})}{2c}D_A(EL)\left[(\hat{\alpha}_i+\hat{\alpha}_j)\cdot(\vec{\theta}_i-\vec{\theta}_j)-\frac{2}{3-\gamma}(\hat{\alpha}_i\cdot\vec{\theta}_i-\hat{\alpha}_j\cdot\vec{\theta}_j)\right]\end{aligned} \tag{5}$$

$\Rightarrow$ The angular diameter distance becomes

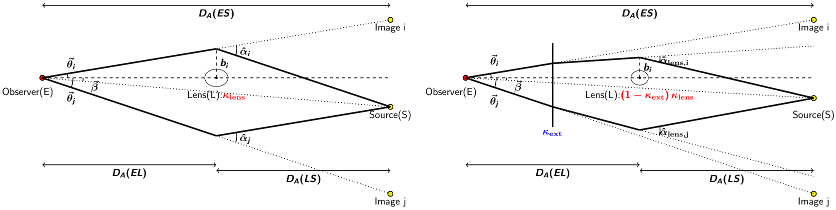
$$D_A(EL) = \frac{c^3\Delta t_{i,j}}{4\pi\sigma_r^2(r)(1+z_{\rm L})}(\Delta\tilde{\theta}_{i,j})^{-1} \tag{6}$$

where $\Delta\tilde{\theta}_{i,j}$ is a function of θ_i, θ_j and γ.

Mass-sheet transformation : properties

Convergence transformation via MST

$$\kappa_{\rm lens} \quad \rightarrow \quad \kappa_{\rm MST} = \kappa_{\rm ext} + (1-\kappa_{\rm ext})\kappa_{\rm lens} \tag{7}$$



Scaling lens convergence results in

$$\begin{aligned} \vec{\alpha}_{\rm model} \quad &\rightarrow \quad \vec{\alpha}_{\rm MST} = \kappa_{\rm ext}\,\vec{\theta} + (1-\kappa_{\rm ext})\,\vec{\alpha}_{\rm model} \\ \vec{\alpha}_{\rm lens} \quad &\rightarrow \quad \vec{\alpha}_{\rm MST,lens} \equiv (1-\kappa_{\rm ext})\,\vec{\alpha}_{\rm lens} \\ \hat{\alpha}_{\rm lens} \quad &\rightarrow \quad \hat{\alpha}_{\rm MST,lens} = (1-\kappa_{\rm ext})\,\hat{\alpha}_{\rm lens} \end{aligned} \tag{8}$$

Mass-sheet transformation : effect

Time-delay after the MST

$$\Delta t_{i,j,\mathrm{MST}} = (1 - \kappa_{\mathrm{ext}})\, \Delta t_{i,j} \tag{9}$$

Velocity dispersion after the MST

$$\sigma^2_{\mathrm{MST}} = (1 - \kappa_{\mathrm{ext}})\, \sigma^2 \tag{10}$$

$\Rightarrow D_A(EL) \propto \frac{\Delta t}{\sigma^2}$ invariant under the MST!

No need to model the external convergence!!

Uncertainty on D_A

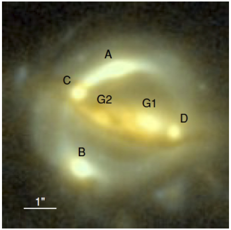


Figure: B1608+656

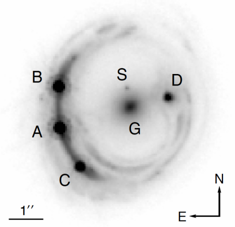


Figure: RXJ1131-1231

Tests on B1608+686 & RXJ1131−1231
(Data and figures from Suyu et al. 2010 & Suyu et al. 2013, and references therein, respectively)

- Uncertainties from γ and $\Delta t_{i,j}$ are negligible
- Velocity dispersion is the biggest source of uncertainty
- Uncertainty on D_A is $\sim 13 - 14\%$ with current data
- Potential estimation (velocity dispersion) seems to play an important role: How to take into account the anisotropic velocity dispersion?

Anisotropic velocity dispersion : modeling

Osipkov-Merritt anisotropy

$$\beta_{\rm ani}(r) = \frac{r^2}{r_{\rm a}^2 + r^2} = 1 - \frac{\sigma_T^2(r)}{\sigma_r^2(r)} \tag{11}$$

- Anisotropy parametrization : $r_{\rm a} = nR_{\rm eff}$
- Isotropic core & radial envelope

Jeans equation

$$\frac{1}{\rho_*}\frac{d(\sigma_r^2\rho_*)}{dr} + 2\beta_{\rm ani}\frac{\sigma_r^2}{r} = -\frac{GM(\leq r)}{r^2} \tag{12}$$

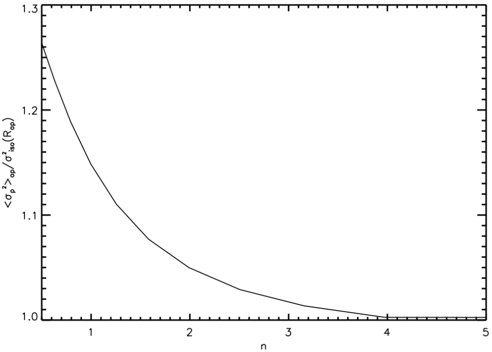
Projection & luminosity weighting (Hernquist profile)

$$\sigma_p^2(R) = I_H(R)\sigma_s^2(R) = 2\int_R^\infty (1 - \beta_{\rm ani}\frac{R^2}{r^2})\frac{\rho_*(r)\sigma_r^2(r)rdr}{\sqrt{r^2 - R^2}} \tag{13}$$

Aperture-averaged velocity dispersion

Measured velocity dispersion is luminosity-weighted, aperture-averaged

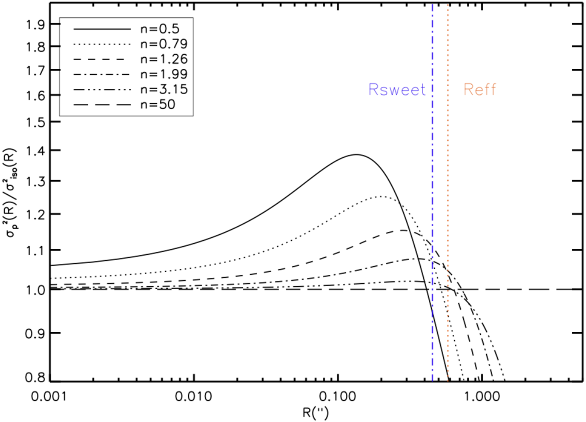
$$\langle \sigma_p^2 \rangle_{\rm ap} \equiv \frac{\int_{\rm ap} \sigma_s^2 I_H R \; dR \; d\theta}{\int_{\rm ap} I_H R \; dR \; d\theta} \tag{14}$$



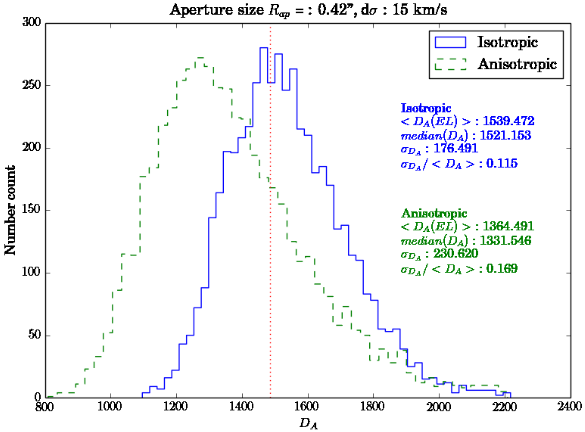
The velocity dispersion varies significantly due to the anisotropy!

Sweet-spot method

Radius where the scatter in anisotropic velocity dispersion is minimized

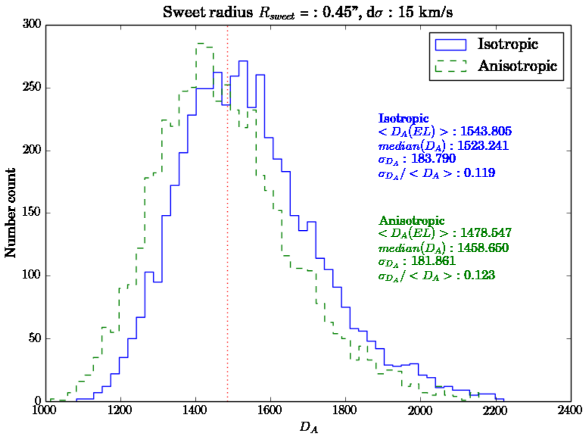


Monte-Carlo simulation 1: D_A measured at $\langle\sigma_p^2\rangle_{\rm ap}$



Anisotropic velocity dispersion biases the distribution, and the width of the distribution is increased

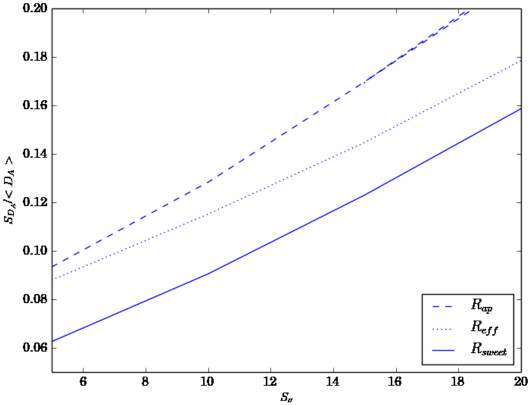
Monte-Carlo simulation 2: D_A measured at $\sigma_p^2(R_{\rm sweet})$



Anisotropic velocity dispersion does not bias the distribution, and the width of the distribution does not change significantly

Expectation

Uncertainties on measured velocity dispersion [km/s] vs. the fractional uncertainty on D_A inferred



$\sim$ 7% precision is achievable with 5% precision measurement on σ^2 from a single system!

Summary

- Strong lens with time delay can be used as a standard ruler to measure the angular diameter distances to the lens
- The external convergence cancels out : The main source of uncertainty in measuring the time-delay distances is not there
- The biggest uncertainty on D_A is from the velocity dispersion and its anisotropy
- Using spatially resolved velocity dispersion profile at the sweet spot radius will improve the precision
- More studies on anisotropy parametrization are required
- **arXiv:1410.7770** for more details!

Discussion

- Preciseness & accuracy of time-delay measurement (e.g. Liao et al. 2014)
- Possibility of spatially resolved velocity dispersion profile and its accuracy (e.g. Agnello & Suyu)
- Defining "effective" convergence
- Uncertainty caused by focusing at the lens plane due to the external convergence
- Study of anisotropic structure of velocity dispersion using nearby elliptical galaxies?