Strong but not quite:
Gravitational flexion in galaxy clusters

Matteo Maturi

Center for Astronomy, Heidelberg University

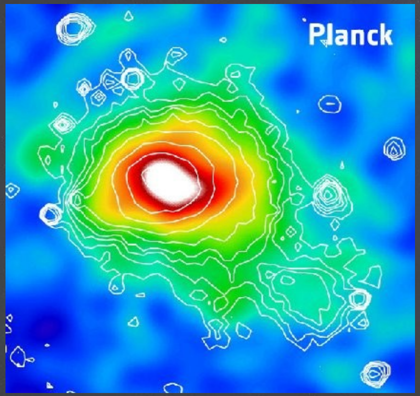
The many faces of a galaxy cluster

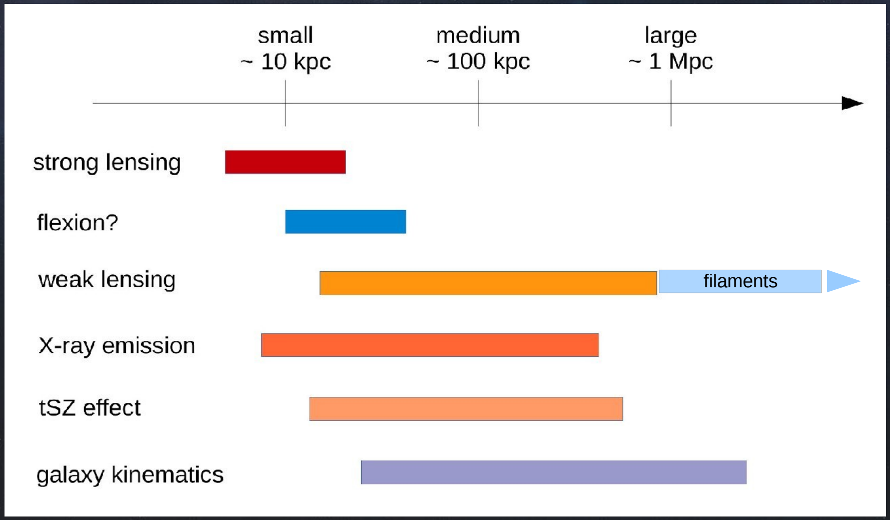
N-Body simulations

Gravitational lensing(s)

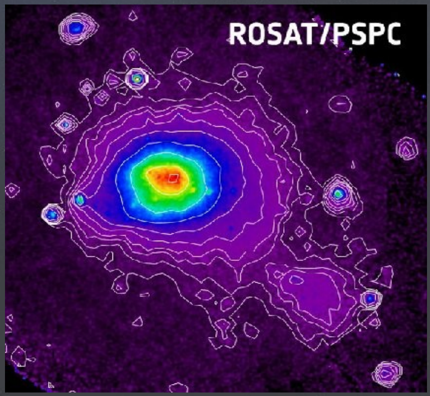
Galaxy overdensity

SZ

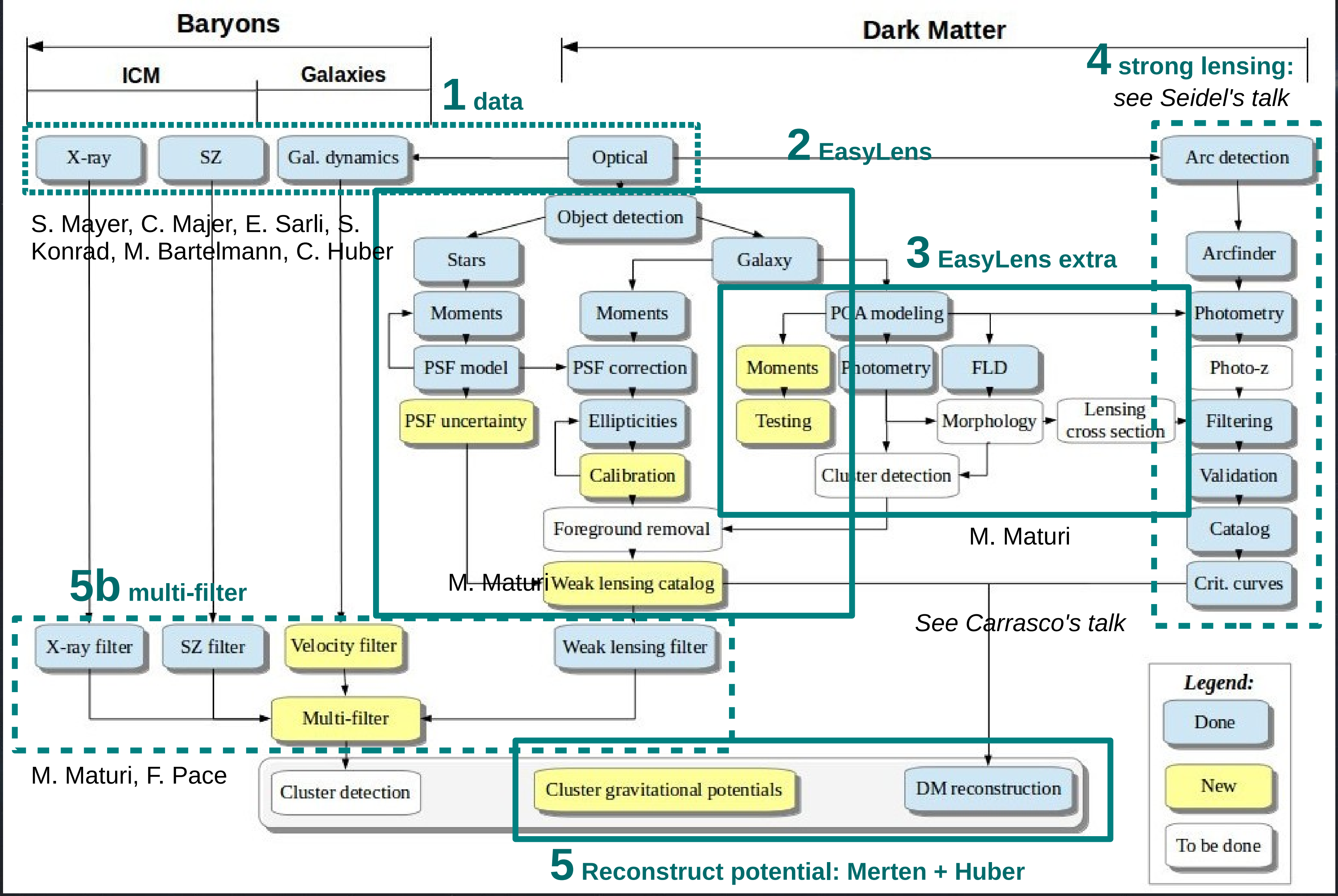




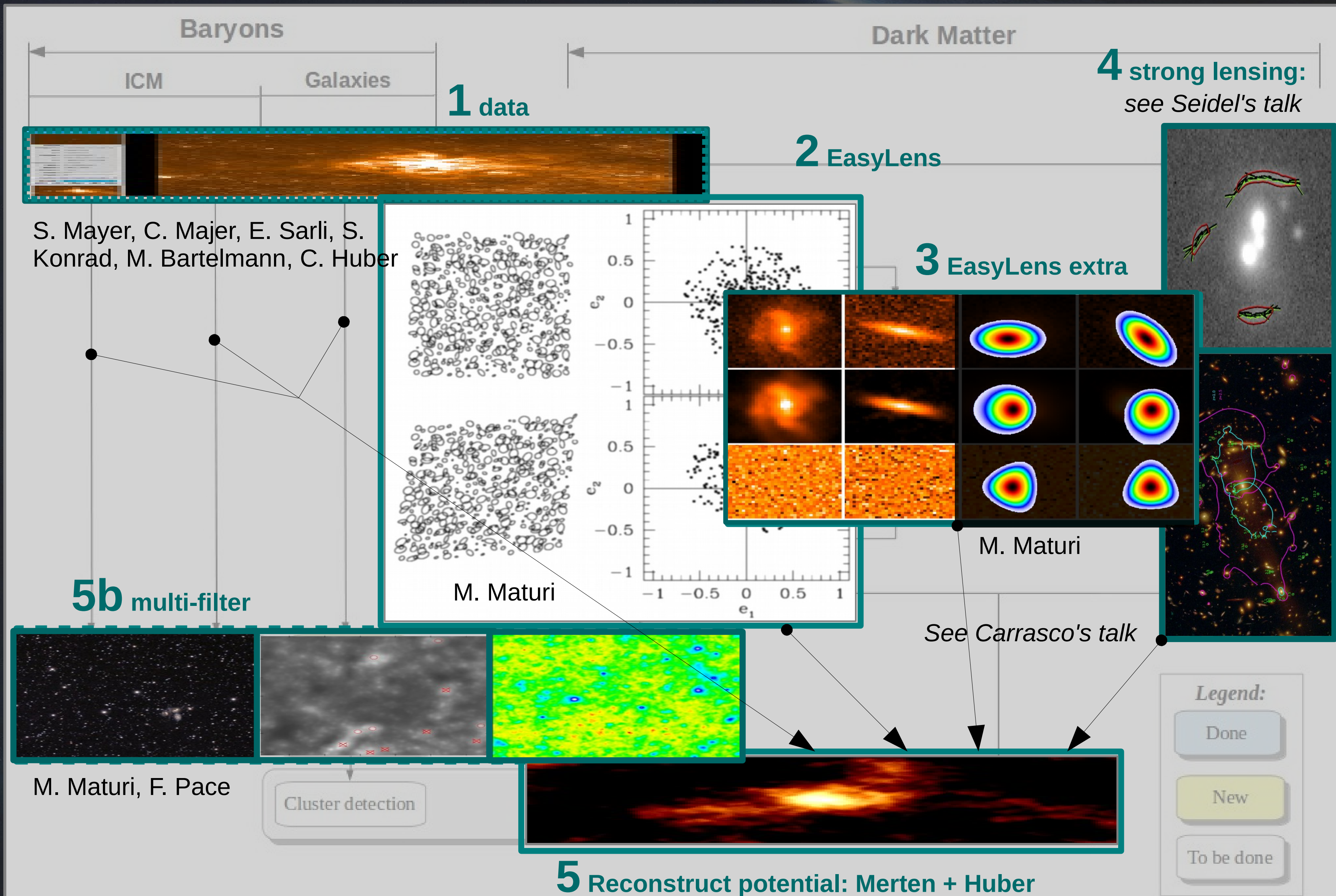
X-ray emission



Galaxy clusters pipeline



Galaxy clusters pipeline



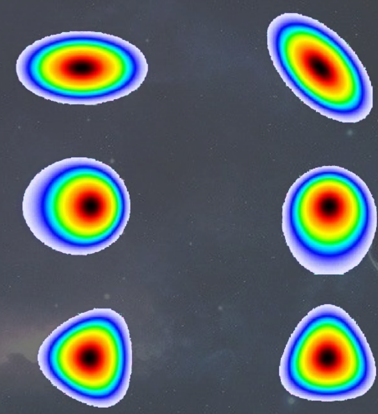
Gravitational lensing in short

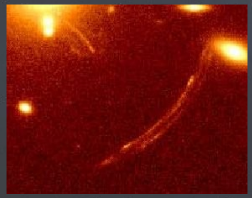
All lensing effects given by effective lensing potential:

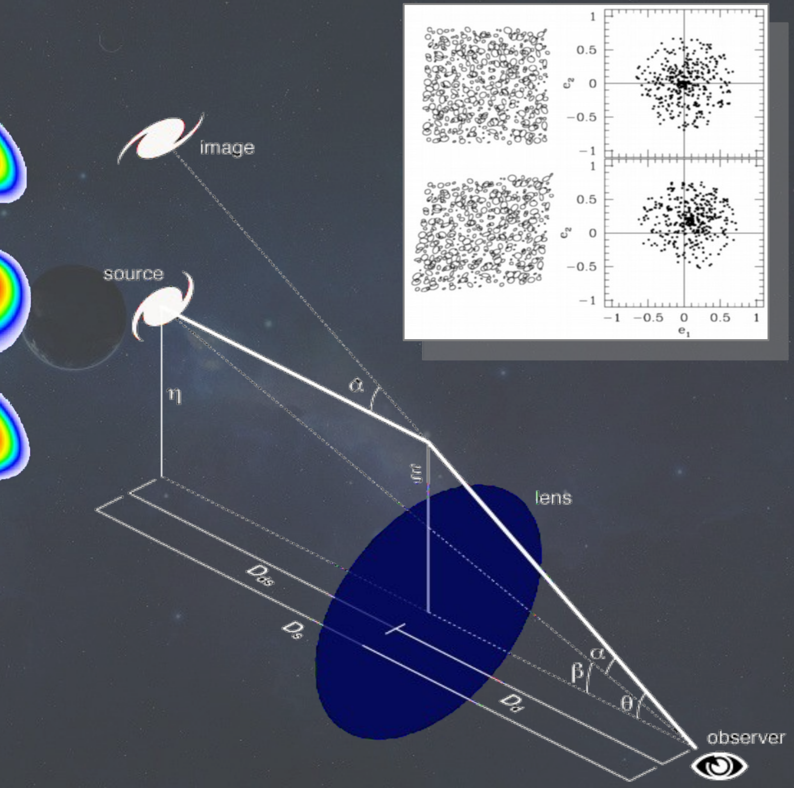
$$\psi = \frac{2}{c^2}\frac{D_{\rm ds}}{D_{\rm d}D_{\rm s}}\int \Phi {\rm d}z$$

(Potential) observables:

- Surface-mass density: $\kappa = \partial^\dagger \partial \psi$
- Shear: $\gamma = \partial^2 \psi$
- Critical curves: $\det\left(\delta_{ij} - \partial_i \partial_j \psi\right) = 0$
- Flexion: $\mathcal{F} = \partial\kappa$, $\mathcal{G} = \partial\gamma$

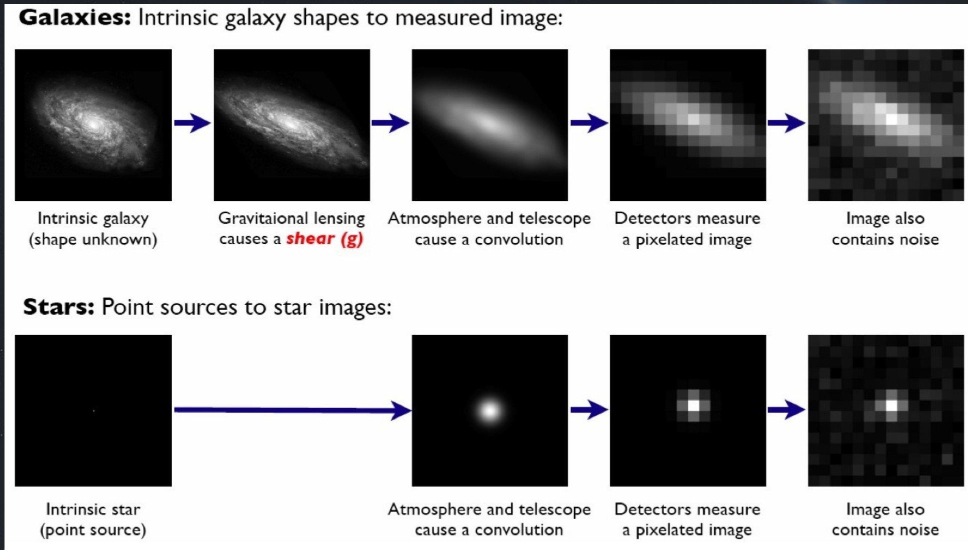


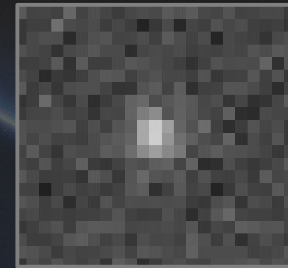




"Strong but not quite: gravitational flexion in galaxy clusters"

Gravitational lensing is very successful in recovering information both on large scales (weak lensing) and on small scales (strong lensing) but many difficulties still have to be faced in recovering the intermediate regime, i.e. gravitational flexion. Being very sensitive to substructures, flexion would be of great help in completing our understanding of galaxy clusters. Although it is based on very clean physics, its actual measure is complicated by intervening spurious contributions and observational effects. **In this talk I am going to discuss the main aspects that have to be confronted to recover this challenging and hopefully powerful signal.**





Measuring lensing

Ellipticity and flexion

Euclide collaboration

W(x)	$W^{-1}(x)$

Moment of brightness

$$\{G\}_{i,j} \equiv \int \mathrm{d}^2x \; G(\boldsymbol{x}) \, x_1^i x_2^j$$

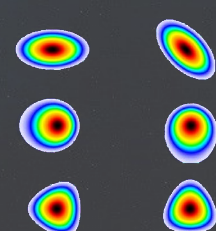
$$\chi \equiv \frac{\{G\}_{2,0} - \{G\}_{0,2} + 2\mathrm{i}\{G\}_{1,1}}{\{G\}_{2,0} + \{G\}_{0,2}}$$

Correct for the telescope PSF

$$\{G^{\star}\}_{i,j} = \sum_{k}^{i} \sum_{l}^{j} \binom{i}{k} \binom{j}{l} \{G\}_{k,l} \{P\}_{i-k,j-l}$$

- - - - - - - - - - ! GREAT, BUT ! - - - - - - - - -

Cope with noise: apply a weight



$$I(\boldsymbol{x}) = G(\boldsymbol{x}) + N(\boldsymbol{x})$$

$$I_{\mathrm{W}}(\boldsymbol{x}) \equiv W(\boldsymbol{x})\, I(\boldsymbol{x})$$

De-weight:

$$W^{-1}(\boldsymbol{x}) \approx W^{-1}(\boldsymbol{0}) - W'(\boldsymbol{0}) \left[\sum_{k=1}^{2} c_k x_k^2 + 4\epsilon_2 x_1 x_2 \right] + \ldots$$

Weighting fails with flexion

Let's try with something else

Data and model

$$d(\boldsymbol{x}) = g(\boldsymbol{x}) + n(\boldsymbol{x})$$

$$\tilde{g}(x) = \sum_{k=1}^{M} a_k w_k(x) \,,$$

Split signal and noise

$$d(x) = \sum_{k=1}^{M} a_k w_k(x) + \sum_{k=1}^{n-M} a_k w_k(x) = \tilde{g}(x) + \tilde{n}(x) \,,$$

$$a_k =< d(\boldsymbol{x}) w_k(\boldsymbol{x}) >= \sum_{i=1}^{n} d_i w_{ki} \,,$$

Search for the optimal basis

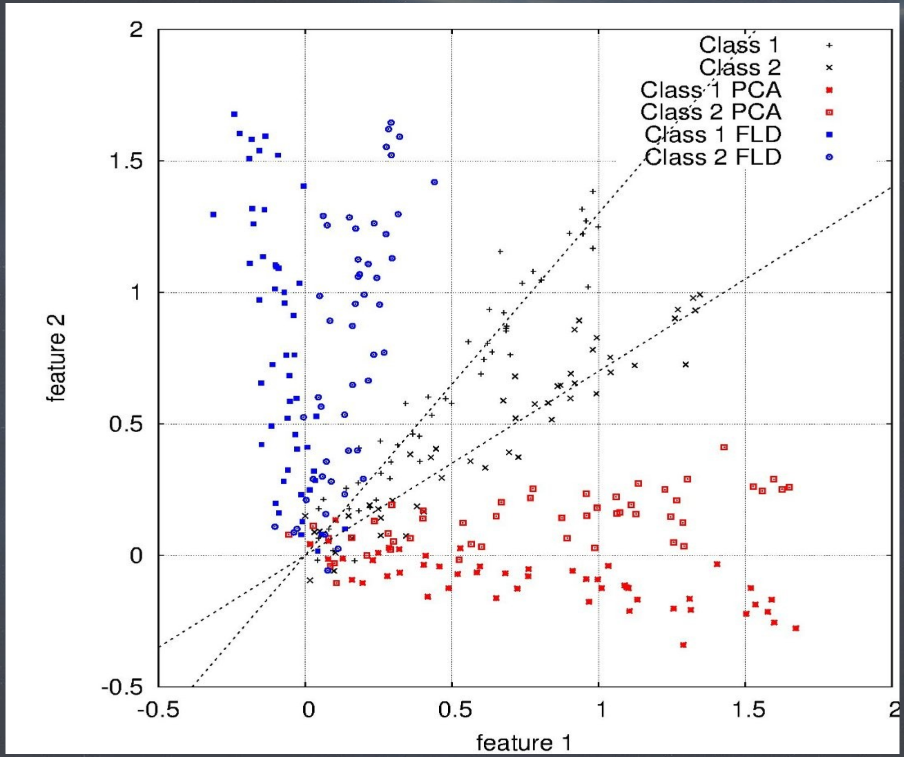
$\{\boldsymbol{d}_i \in \boldsymbol{D} \mid i = 1, \ldots, m\}$	$\in \Re^{n \times m}$	Ensamble of all m galaxies image of size n
$\boldsymbol{\mu} = \sum_{i=1}^{m} \boldsymbol{d}_i / m$	$\in \Re^{n}$	Mean of the galaxies image
$\boldsymbol{S} = \sum_{i=1}^{m} \frac{1}{m} (\boldsymbol{d}_i - \boldsymbol{\mu})(\boldsymbol{d}_i - \boldsymbol{\mu})^T$	$\in \Re^{n \times n}$	Covariance matrix
$\{\boldsymbol{w}_k \in \boldsymbol{W} \mid k = 1, \ldots, n\}$	$\in \Re^{n \times n}$	The n principal components of size n: $\boldsymbol{S}\boldsymbol{w}_k = \lambda_k$

$$L = \boldsymbol{w}_k^T \boldsymbol{S} \boldsymbol{w}_k + \lambda_k \left(\boldsymbol{w}_k^T \boldsymbol{w}_k - 1 \right) \,,$$

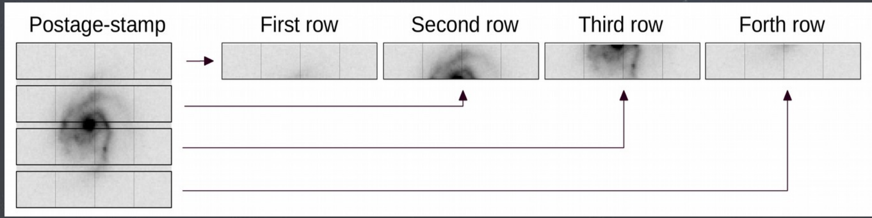
$$\delta L / \delta \boldsymbol{w}^T = 0$$

Use statistics of galaxies to model them all

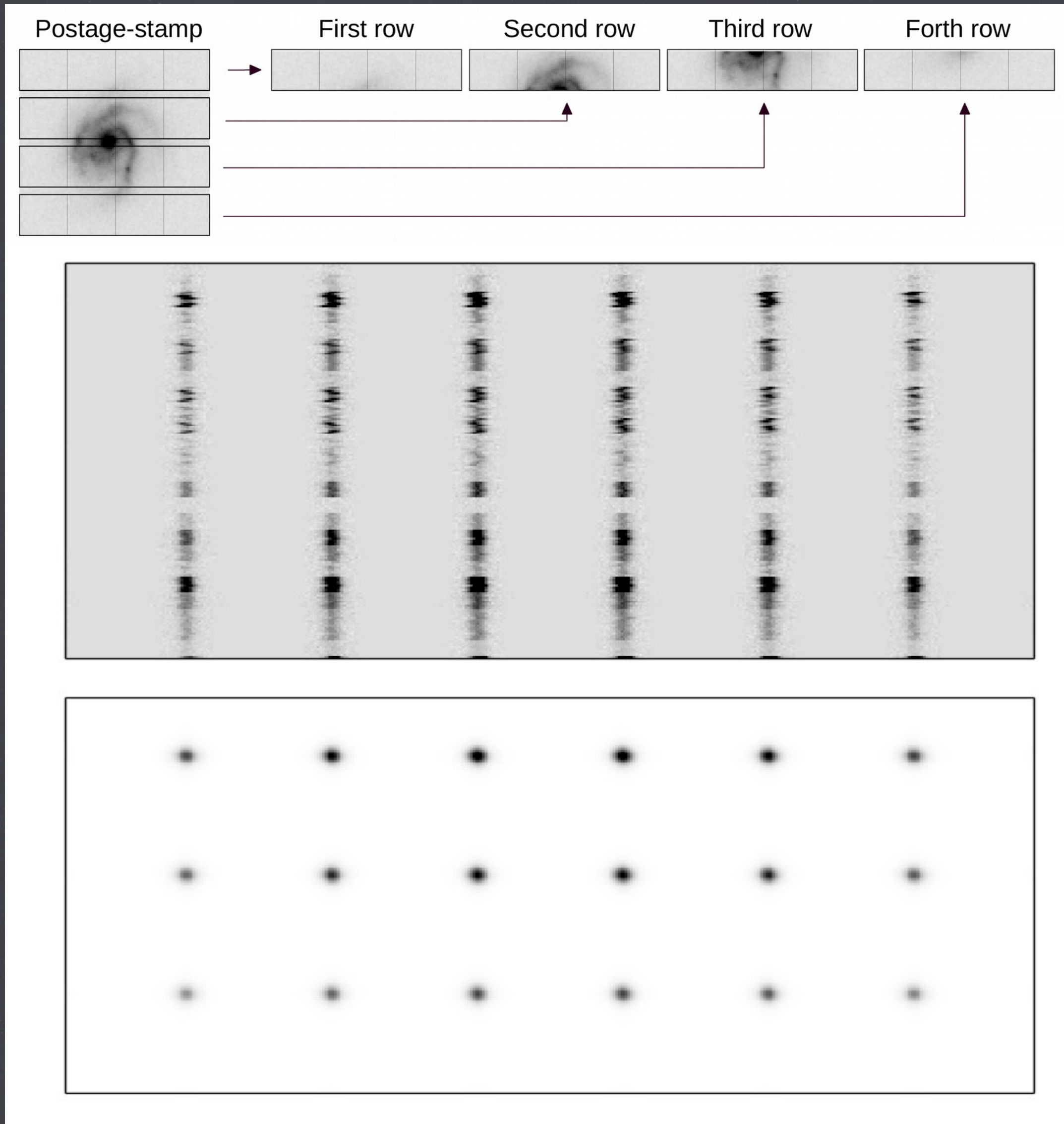
An image with 2 pixels

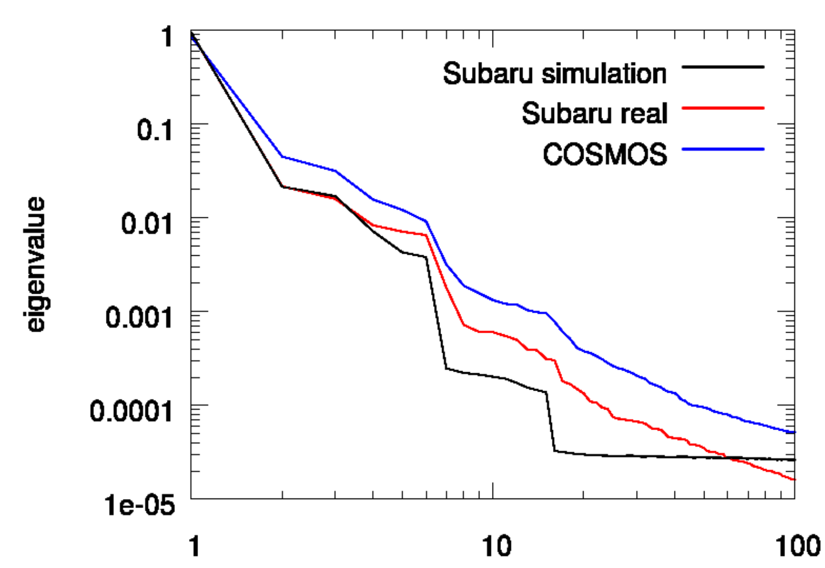


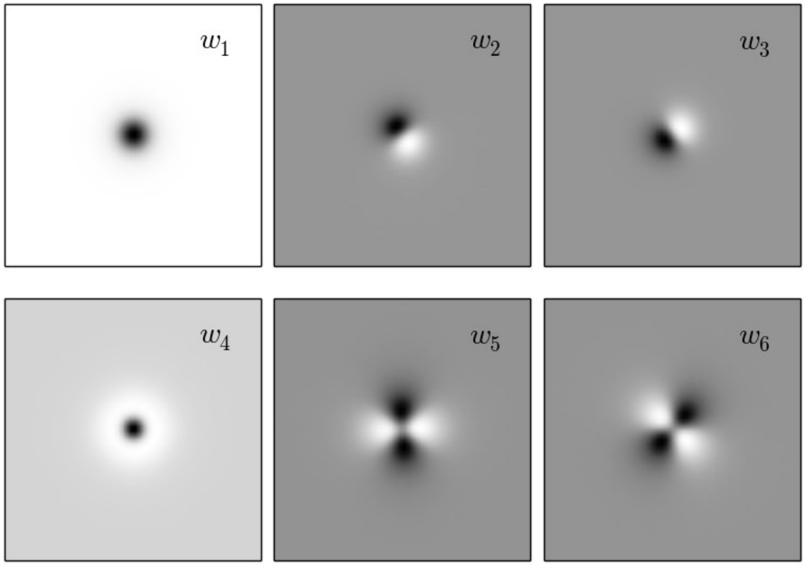
From image... to... vector

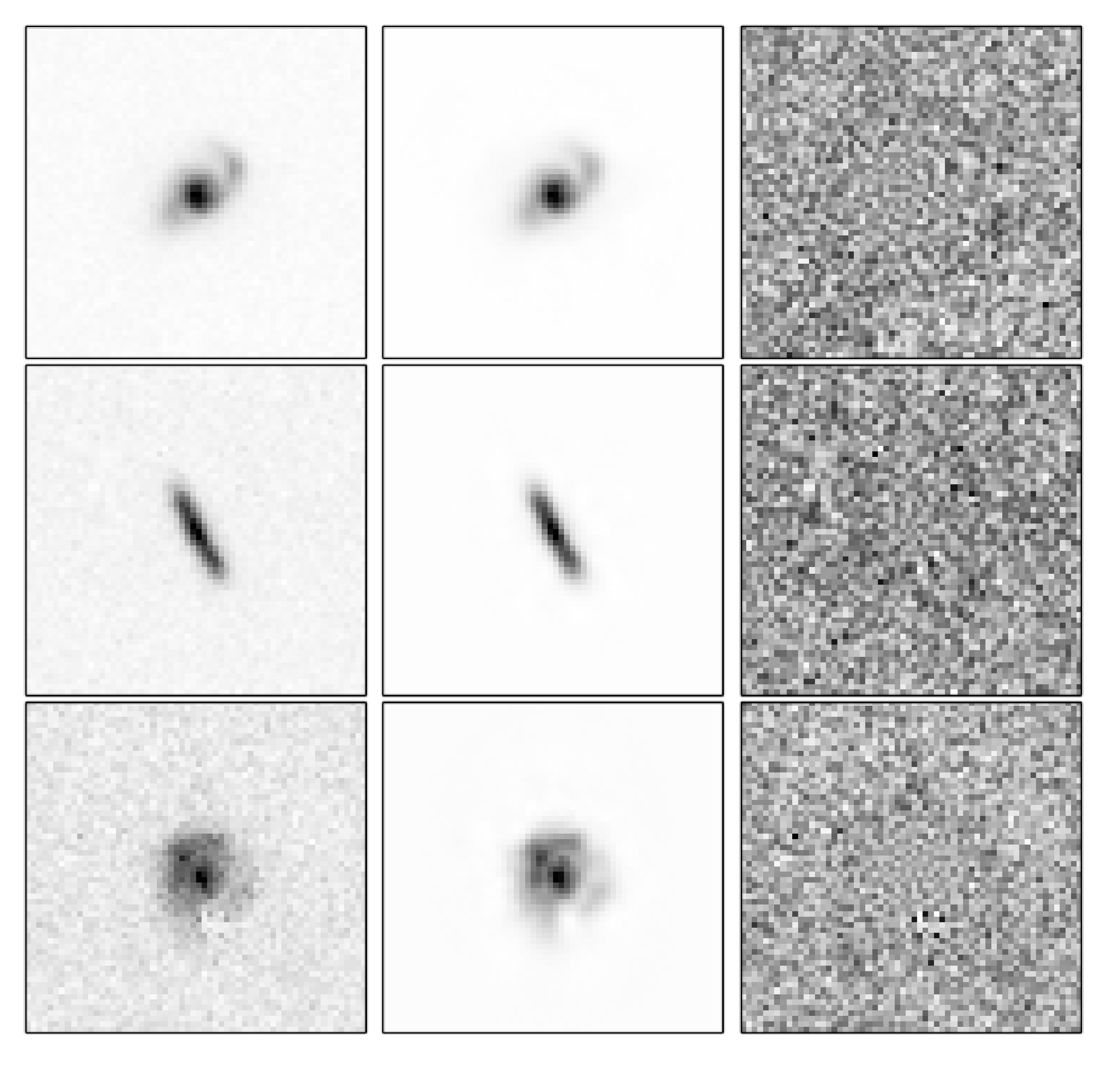


$\{d_i \in D \mid i = 1,\dots,m\}$	$\in \Re^{n\times m}$	Ensamble of all m galaxies image of size n
$\mu = \sum_{i=1}^{m} d_i/m$	$\in \Re^{n}$	Mean of the galaxies image
$S = \sum_{i=1}^{m} \frac{1}{m}(d_i-\mu)(d_i-\mu)^T$	$\in \Re^{n\times n}$	Covariance matrix
$\{w_k \in W \mid k = 1,\dots,n\}$	$\in \Re^{n\times n}$	The n principal components of size n: $Sw_k = \lambda_k$

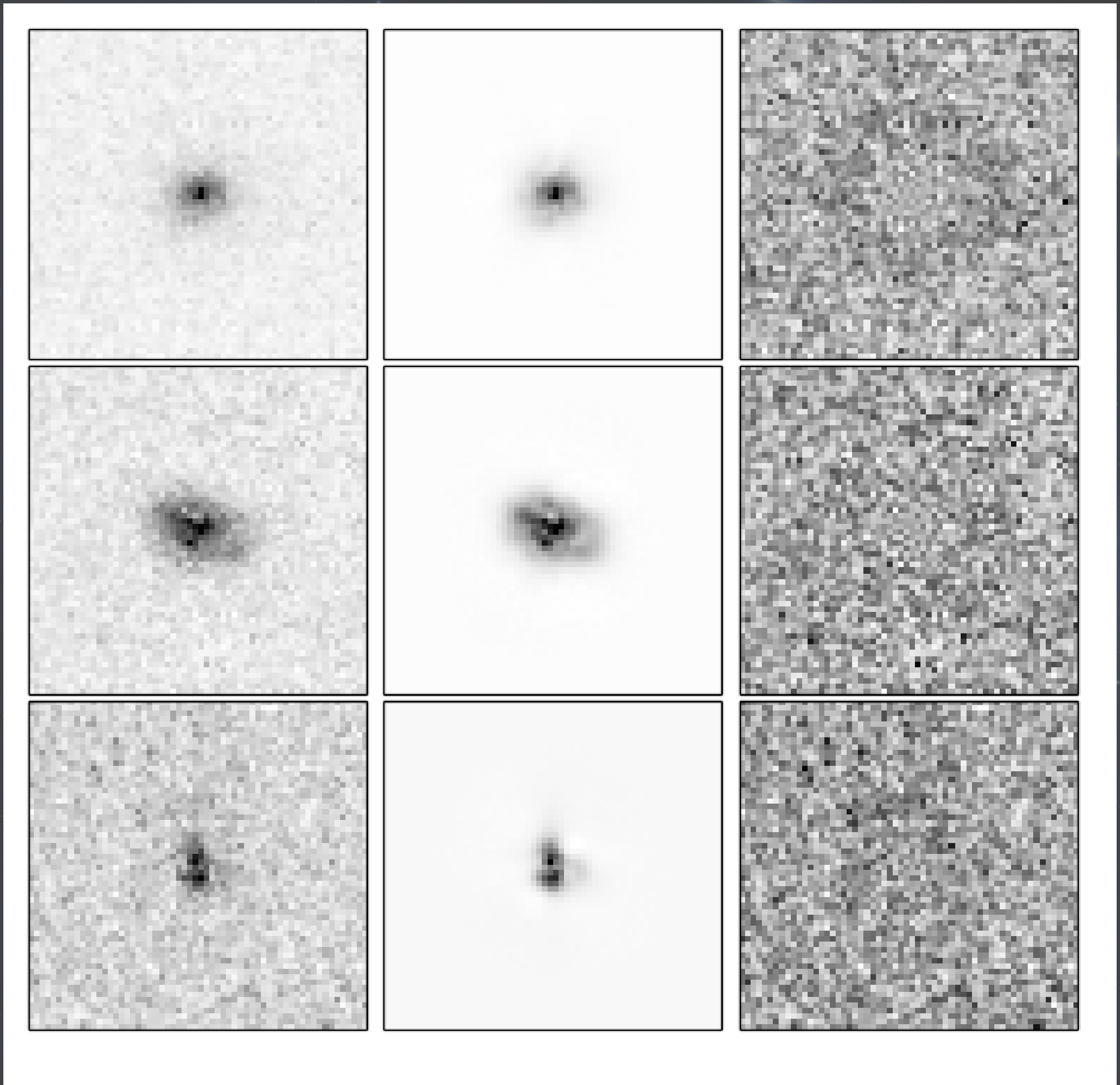






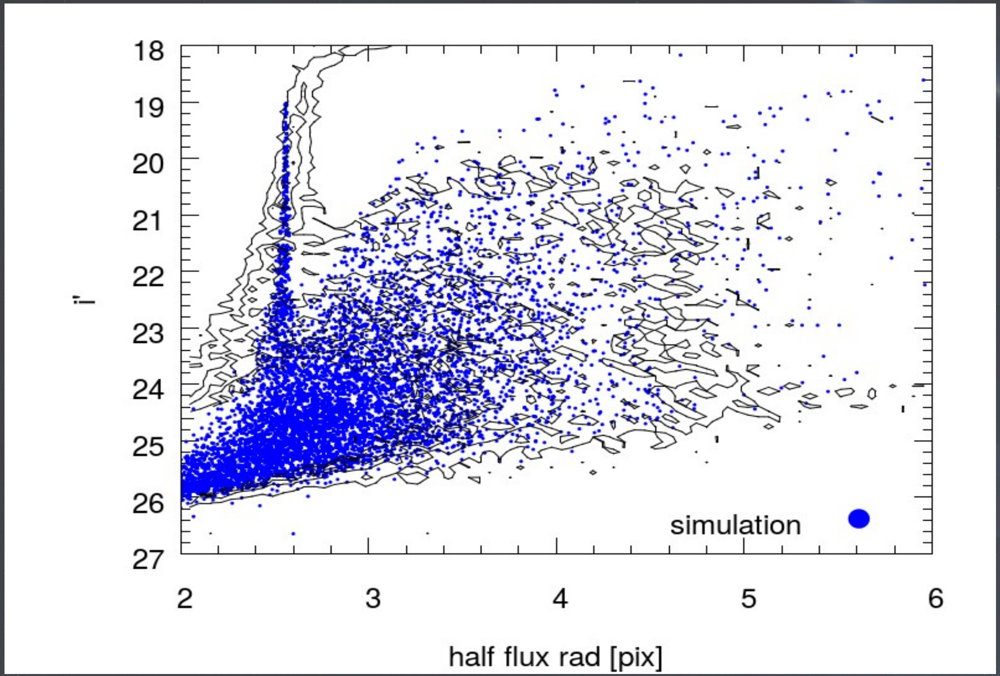


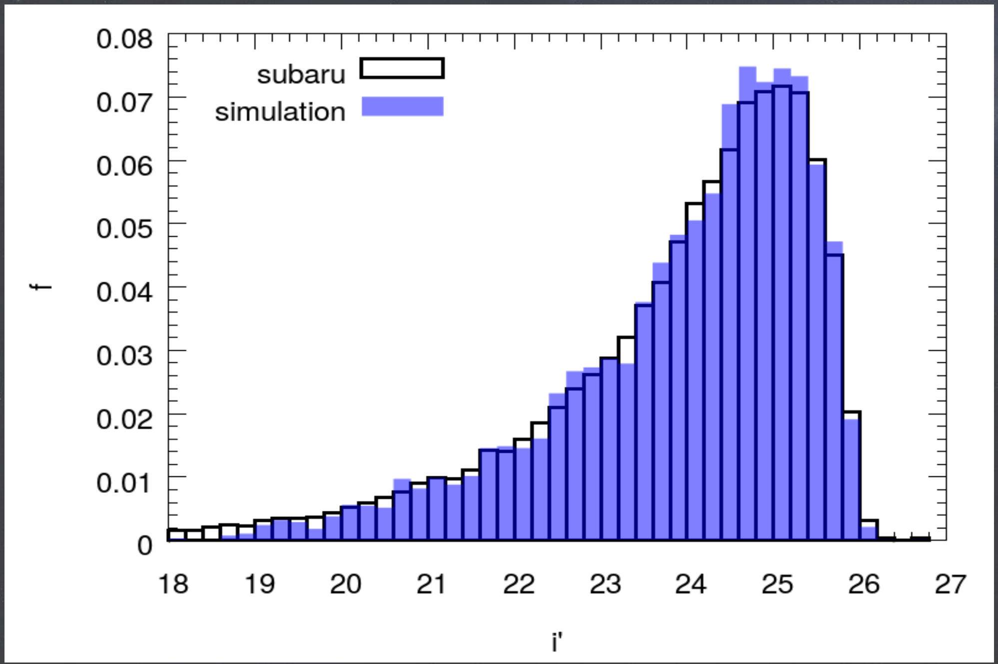
Subaru CLASH
psf=0.54”, 0.2”/pixel

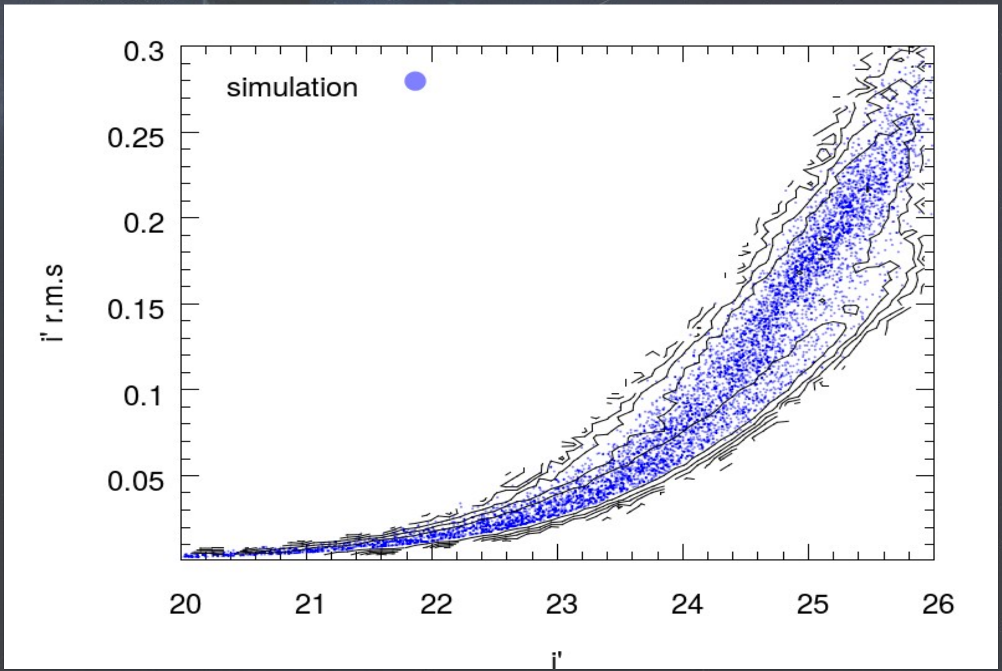


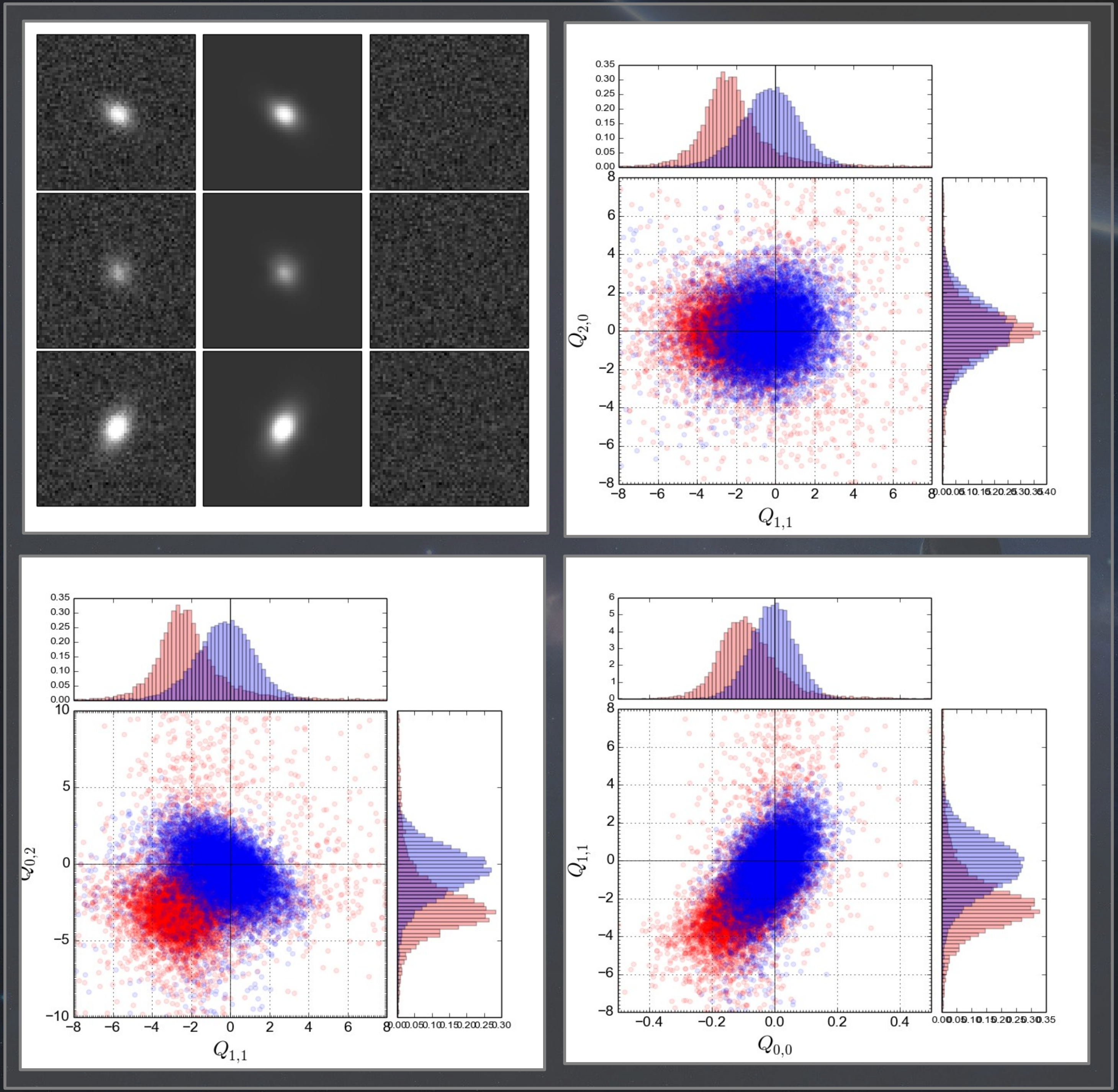
COSMOS
ACS/HST 0.05"/pixel

Test with simulations (Subaru)



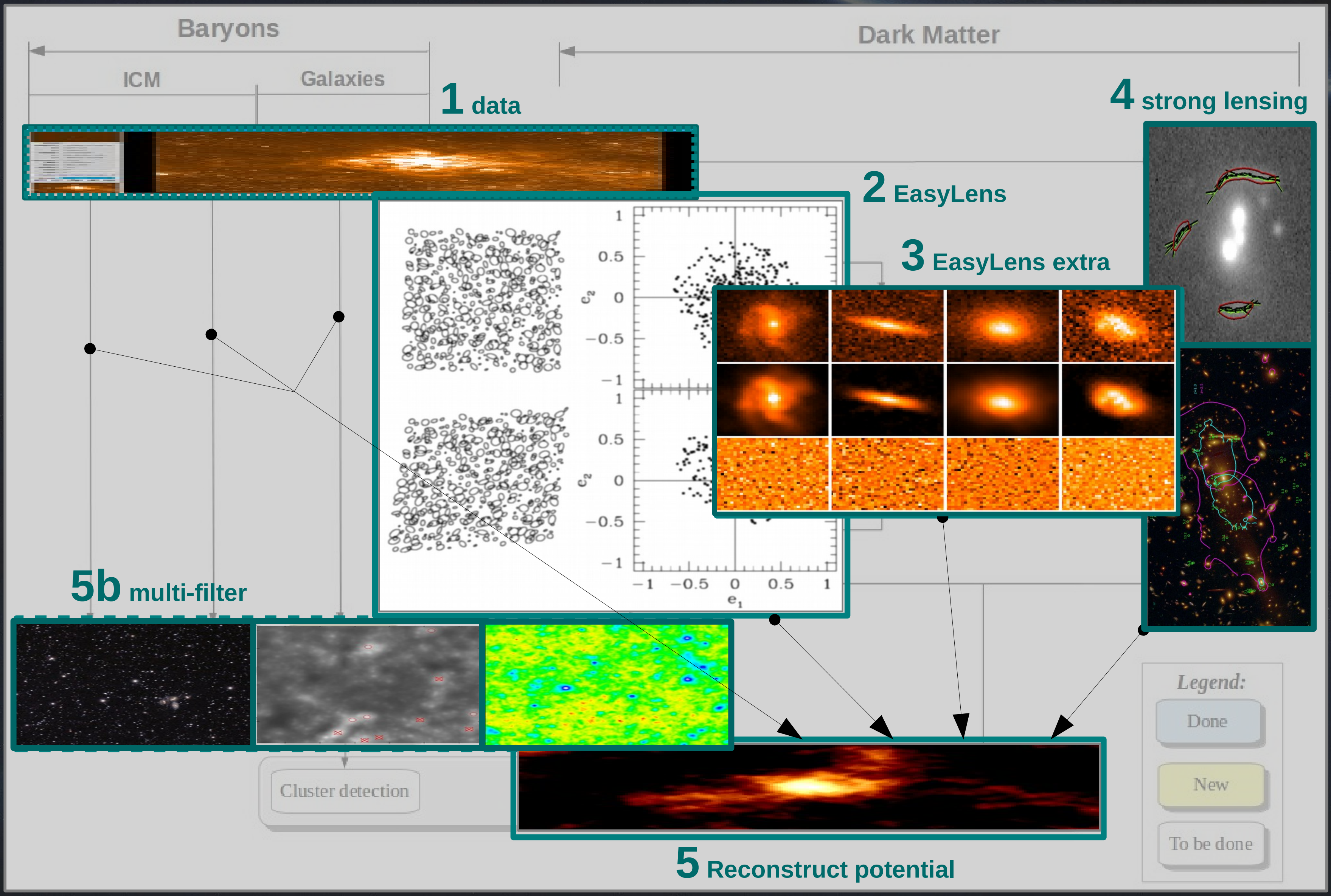




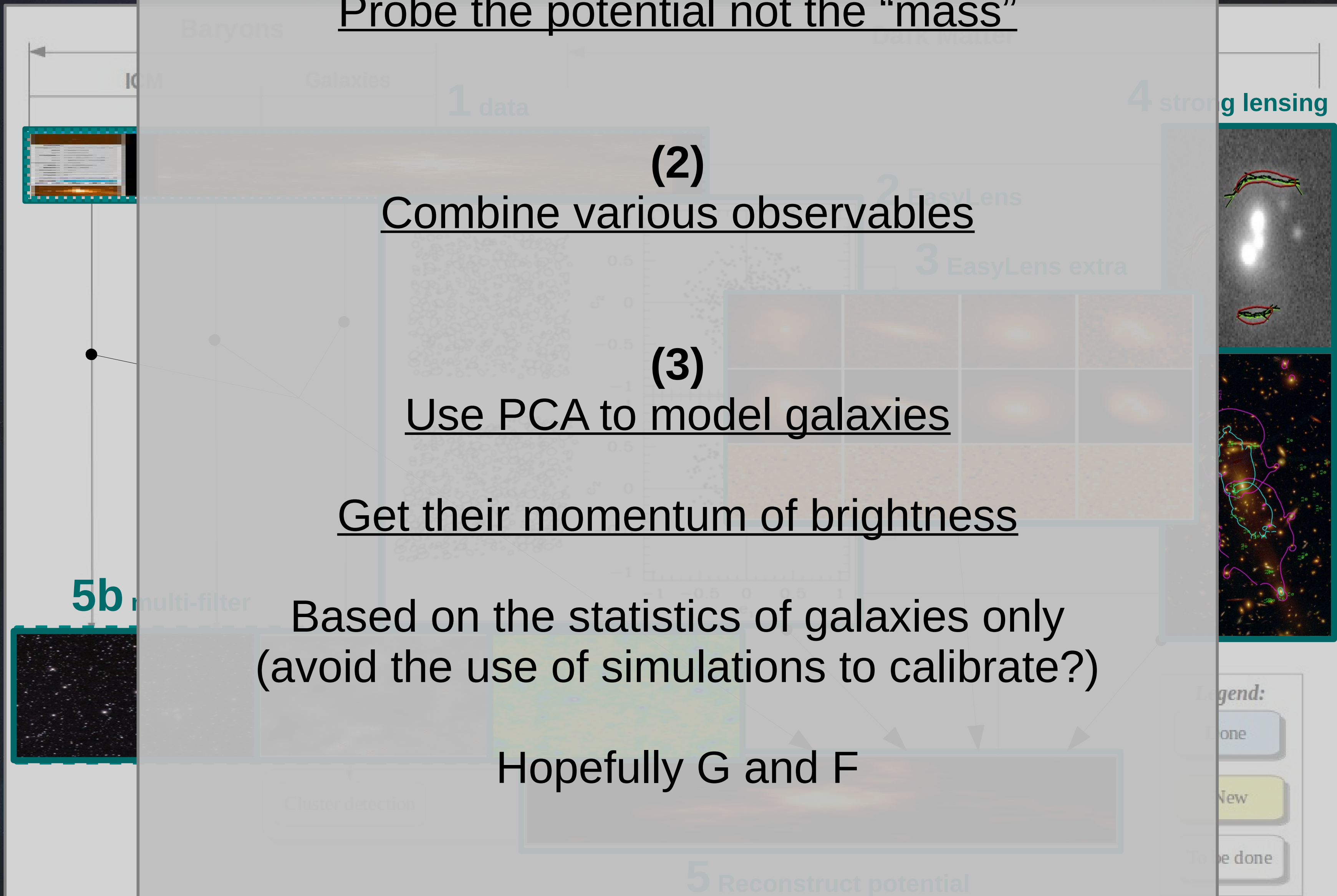


Moments of brightness

Galaxy clusters pipeline



Galaxy clusters pipeline



(1)
<u>Probe the potential not the "mass"</u>

(2)
<u>Combine various observables</u>

(3)
<u>Use PCA to model galaxies</u>

<u>Get their momentum of brightness</u>

Based on the statistics of galaxies only
(avoid the use of simulations to calibrate?)

Hopefully G and F