

# Substructure Lensing

**Shude Mao**

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# Outline

## **I. Introduction**

## **II. Flux ratio Anomalies**

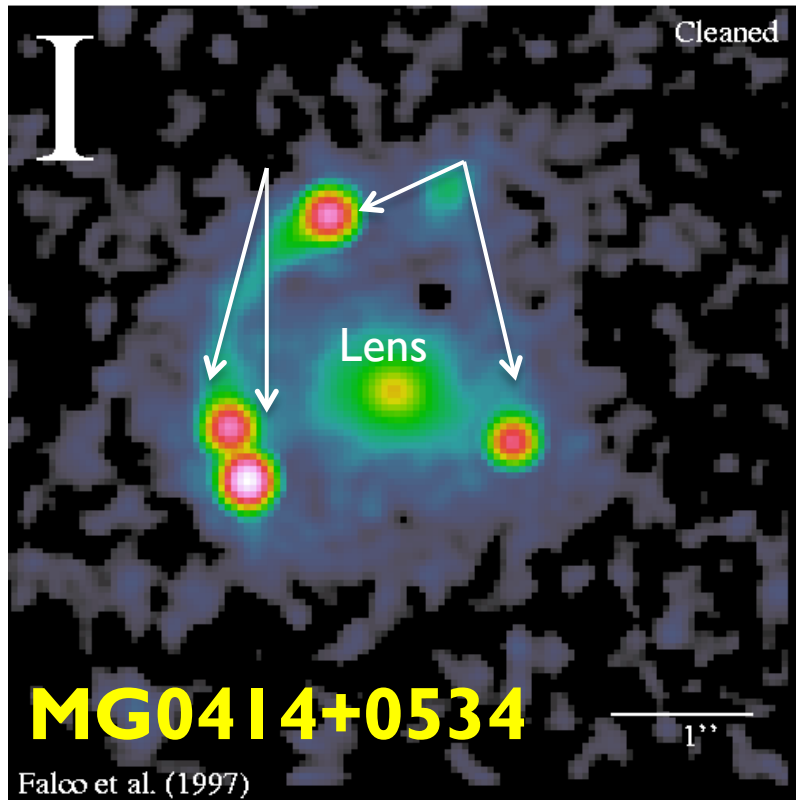
- Statistical predictions**

- Modelling observed lenses**

## **III. Discussion & future outlook**

# I. Introduction:

## Difficulty in modelling lenses

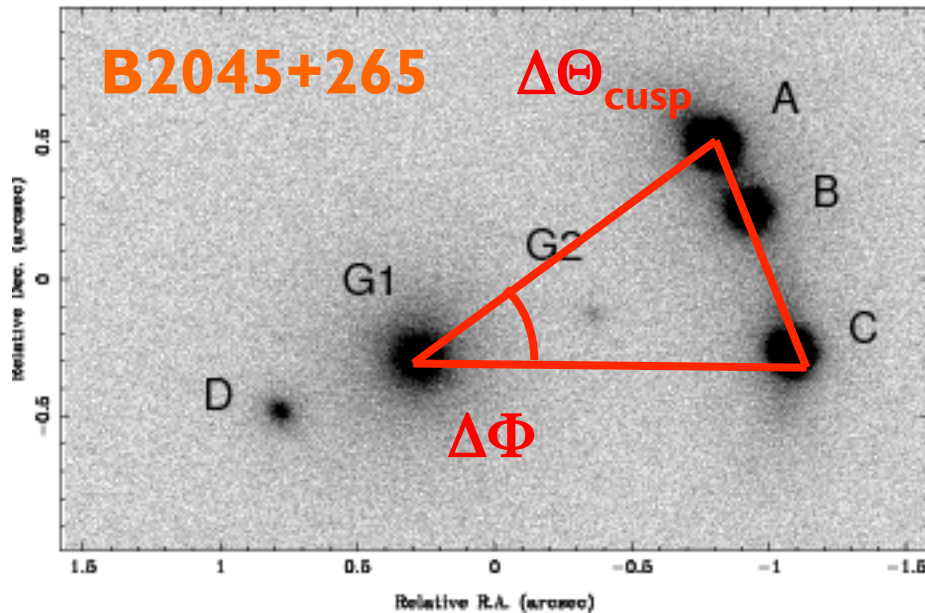


5 quads, 4 doubles, 3 rings

**Kochanek (1991)**  
modelled 12 lenses and  
pointed out:

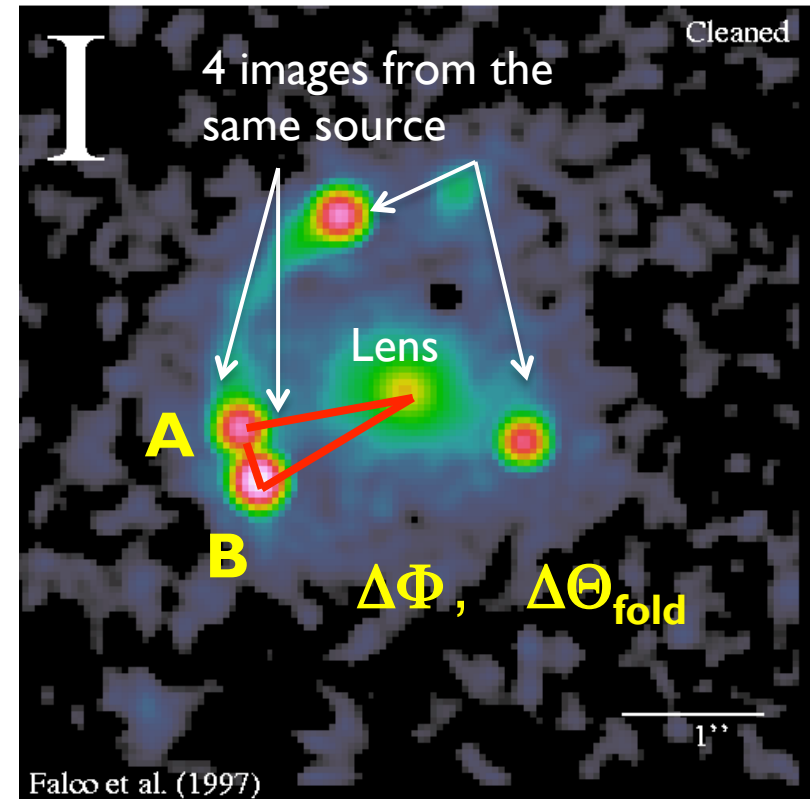
- It is pretty easy to reproduce the image positions
- But often more difficult to fit the magnifications (“wild fluctuations”)

# Asymptotic relations for folds & cusps



$$R_{\text{cusp}} = \frac{\text{Middle-Outer}}{|\text{Middle+Outer}|} \rightarrow 0$$

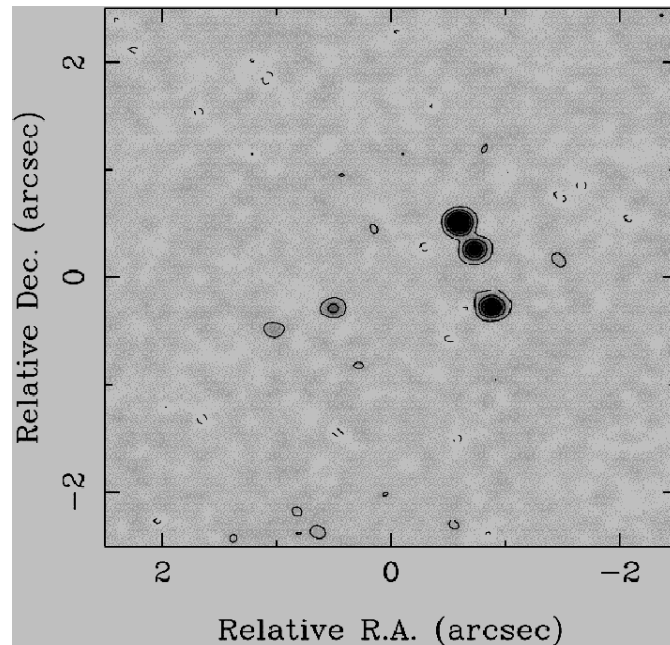
when  $\Delta\Phi \rightarrow 0$



$$R_{\text{fold}} = \frac{A-B}{|A+B|} \rightarrow 0$$

**Valid for any smooth models!**

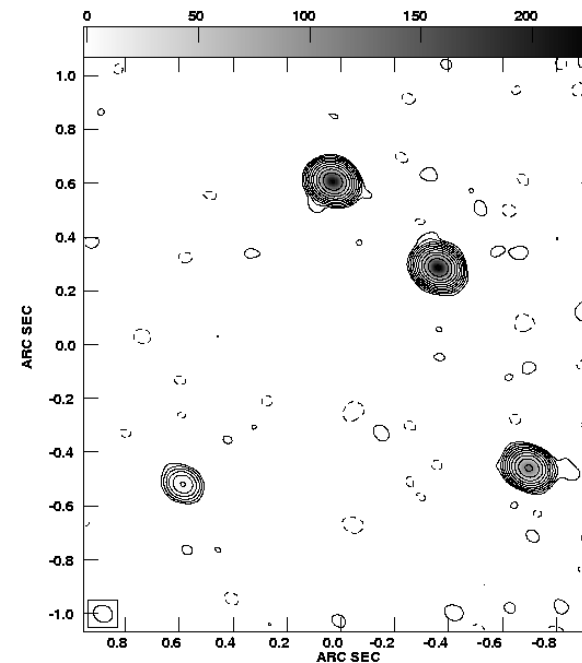
# Observed flux ratios in cusp triples



**B2045+265**

(Fassnacht et al. 1999)

$$R_{\text{cusp}} = 0.5$$



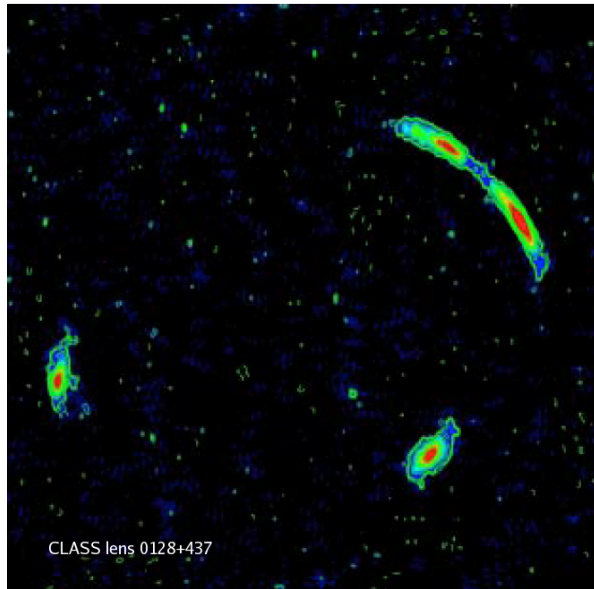
**B1422+231**

(Patnaik et al. 1992)

$$R_{\text{cusp}} = 0.2$$

- Substantial deviations from the predicted value (0)
- Saddle images are fainter than expected

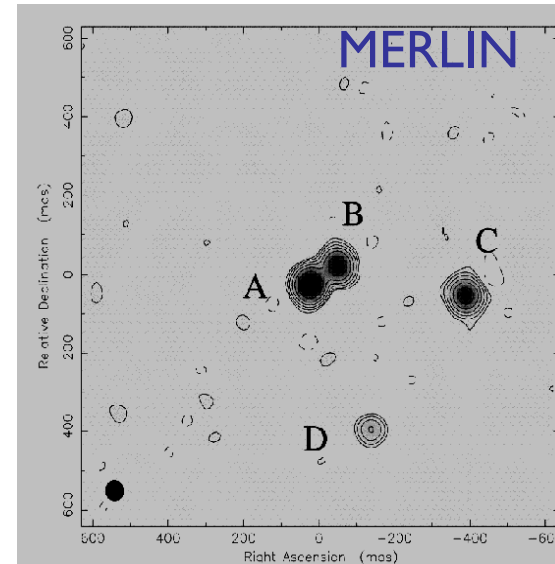
# Observed flux ratios in fold pairs



B0128+437

(Phillips et al. 2001)

$$R_{\text{fold}} = 0.33$$



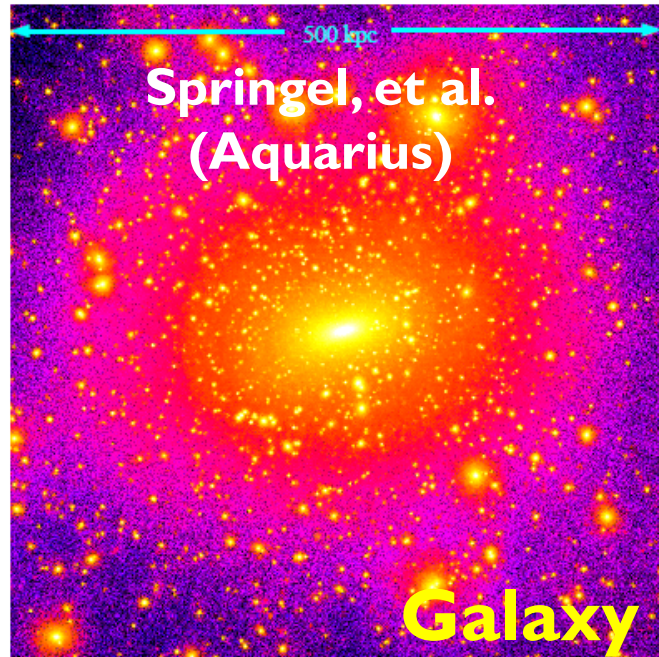
B1555+375

(Marlow et al. 1999)

$$R_{\text{fold}} = 0.27$$

- Substantial deviations from predicted ratio ( $\sim 0$ )
- Saddle images are fainter than expected

# Substructures in CDM models

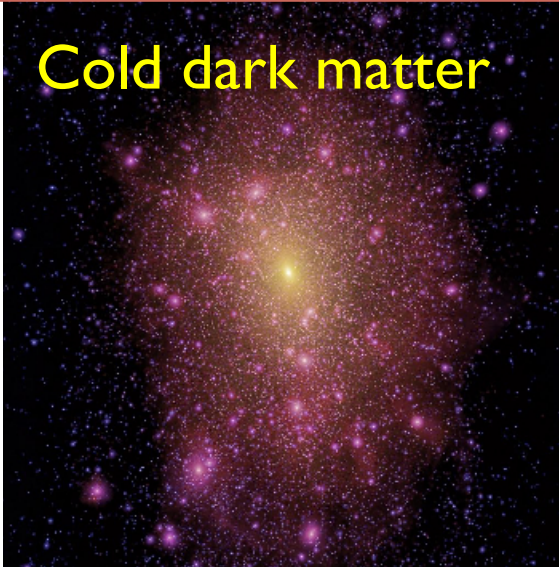


[e.g. Moore et al. 1999; Kauffmann et al. 1993; Klypin et al. 1999; Springel et al.; Kravstov et al.]

- 5-10% of halo mass is in substructures
- Mass function follows  $n(m)dm \sim m^{-1.9} dm$
- $\sim 10^4$  subhaloes are predicted with  $V_c > 3$  km/s

# Nature of dark matter & Small scale power-spectrum

Cold dark matter



Warm dark matter



If we can determine the mass function and spatial distribution of subhaloes

- **Constrain the nature of dark matter (cold or warm?)**
- **Small-scale matter power spectrum on  $\sim$ kpc scale**



# Lensing anomalies due to substructures

➤ **Magnification:**  $\Phi_{xx}, \Phi_{yy}, \Phi_{xy}$  [ $\Phi$ : lens potential]

(Mao & Schneider 1998; Metcalf & Madau 2001; Chiba 2002; Keeton 2001

Bradač et al. 2001; Moustakas & Metcalf 2003; Moller, Hewitt, & Blain 2003; Metcalf & Zhao 2002; Keeton, Gaudi & Petters 2002; Evans & Witt 2003;

Dalal & Kochanek 2002; Chen, Kravtsov, & Keeton 2003; Metcalf & Amara 2012)

➤ **Position (astrometry):**  $\Phi_x, \Phi_y$

(e.g., Chen et al. 2007; Vegetti et al. 2010, 2012, 2014)

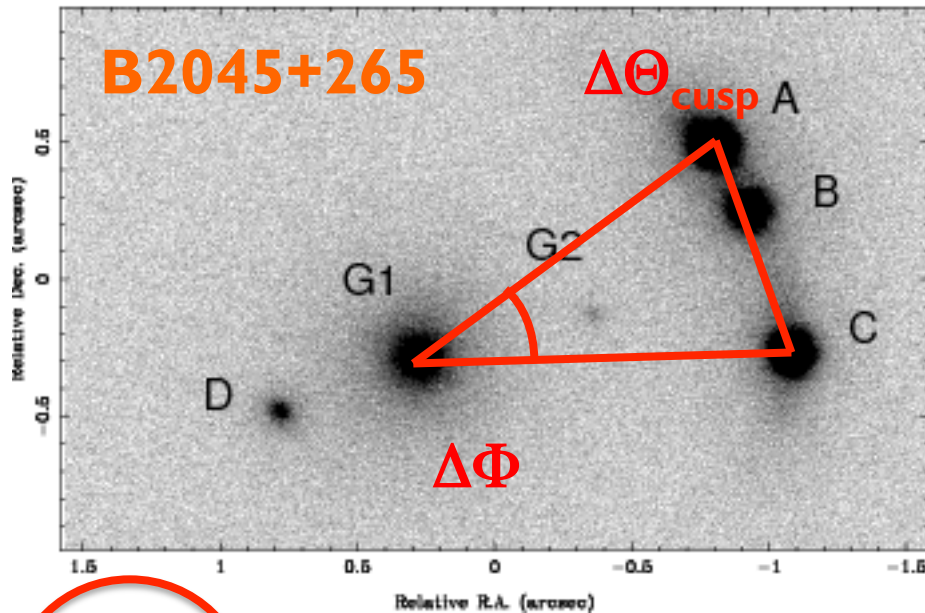
➤ **Time delay:**  $\Phi, \Phi_x, \Phi_y$

(e.g., Keeton & Moustakas 2009)

➤ **Flexions: third order derivative**

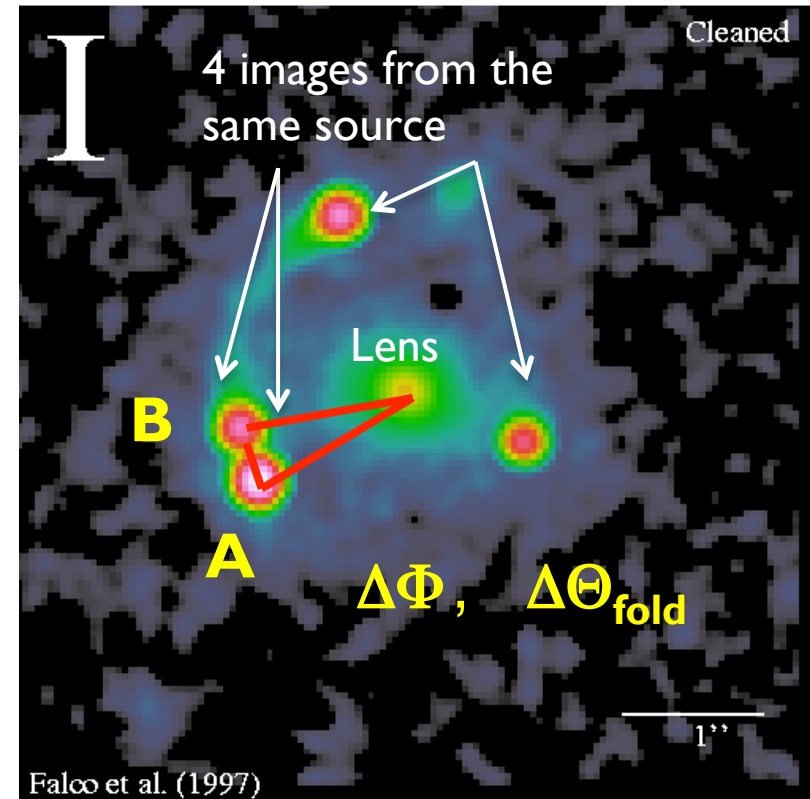
(e.g., Bacon et al. 2006; Er, Ismael, Mao 2012)

# Asymptotic relations for folds & cusps



$$R_{\text{cusp}} = \frac{|\text{Middle-Outer}|}{|\text{Middle+Outer}|} \rightarrow 0$$

when  $\Delta\Phi \rightarrow 0$



$$R_{\text{fold}} = \frac{|A-B|}{|A+B|} \rightarrow 0$$

## II. $R_{\text{cusp}}$ & $R_{\text{pair}}$ : statistical predictions

- **Singular isothermal ellipsoid (SIE)**
  - + **higher-order multipole amplitudes ( $a_3, a_4$ ):**
    - ✓ Einstein radius = 1 arcsecond
    - ✓ shape parameters are drawn from SDSS galaxies (Hao, Mao et al. 2006)
- **Adopt randomly orientated external shear**
  - ✓ amplitude is assumed to be a lognormal distribution
- **Adding in a subhalo population**

# Rescaling the subhalo population

Aquarius

6 Milky Way-sized  
halos:  
 $\sigma \approx 150 \text{ km/s}$

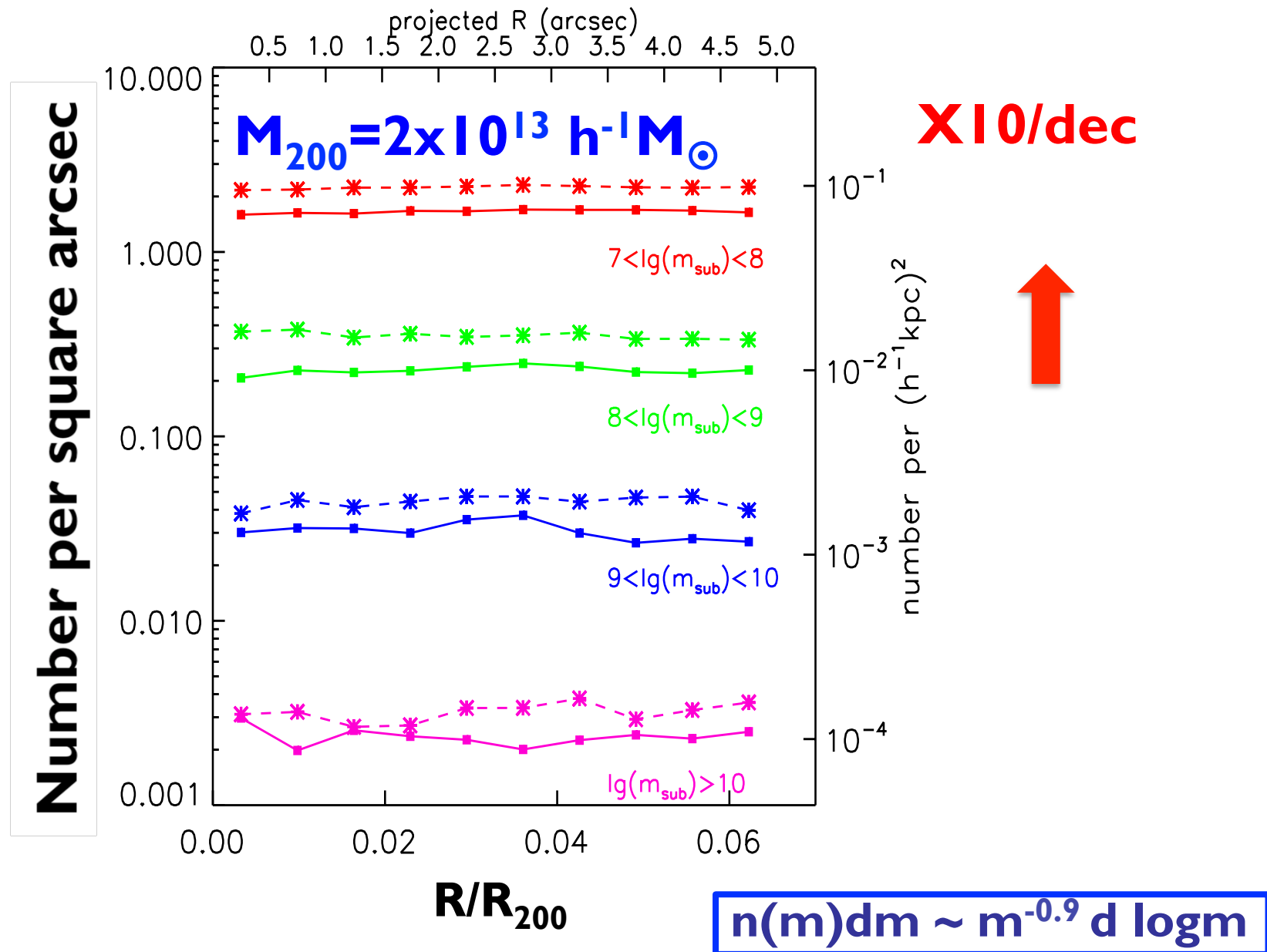
Observed  
lenses:  
 $\sigma \approx 300 \text{ km/s}$

Phoenix

9 Cluster-sized  
halos:  
 $\sigma \approx 900 \text{ km/s}$

- Rescale subhalo properties:  $r \sim M_{200}^{1/3}$ ,  $v \sim M_{200}^{1/3}$ ; characteristic over-density unchanged
- For each rescaled halo, we assume an Einasto density profile

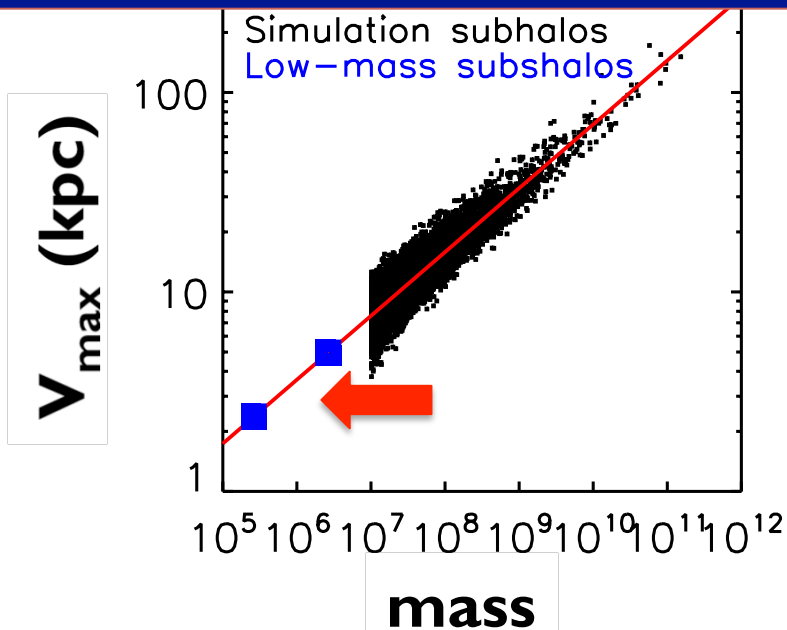
# Projected number of subhaloes



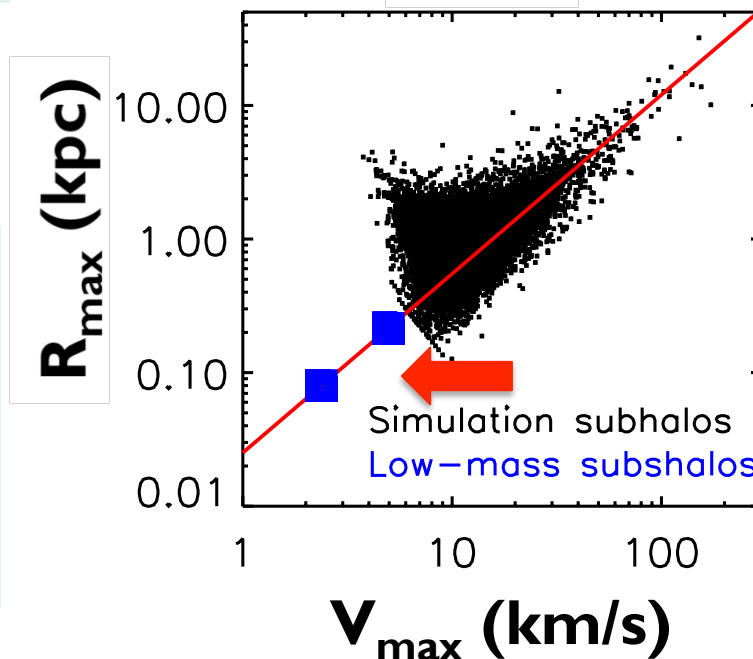
# Treating unresolved subhaloes

subhaloes below  $10^7 M_{\odot}$  are unresolved

Substructures contribute  $\llsim 1\%$  around typical Einstein radius



$$V_{\max} \sim m^{1/3}$$



$$R_{\max} \sim v_{\max}^{4/3}$$

# Statistical predictions

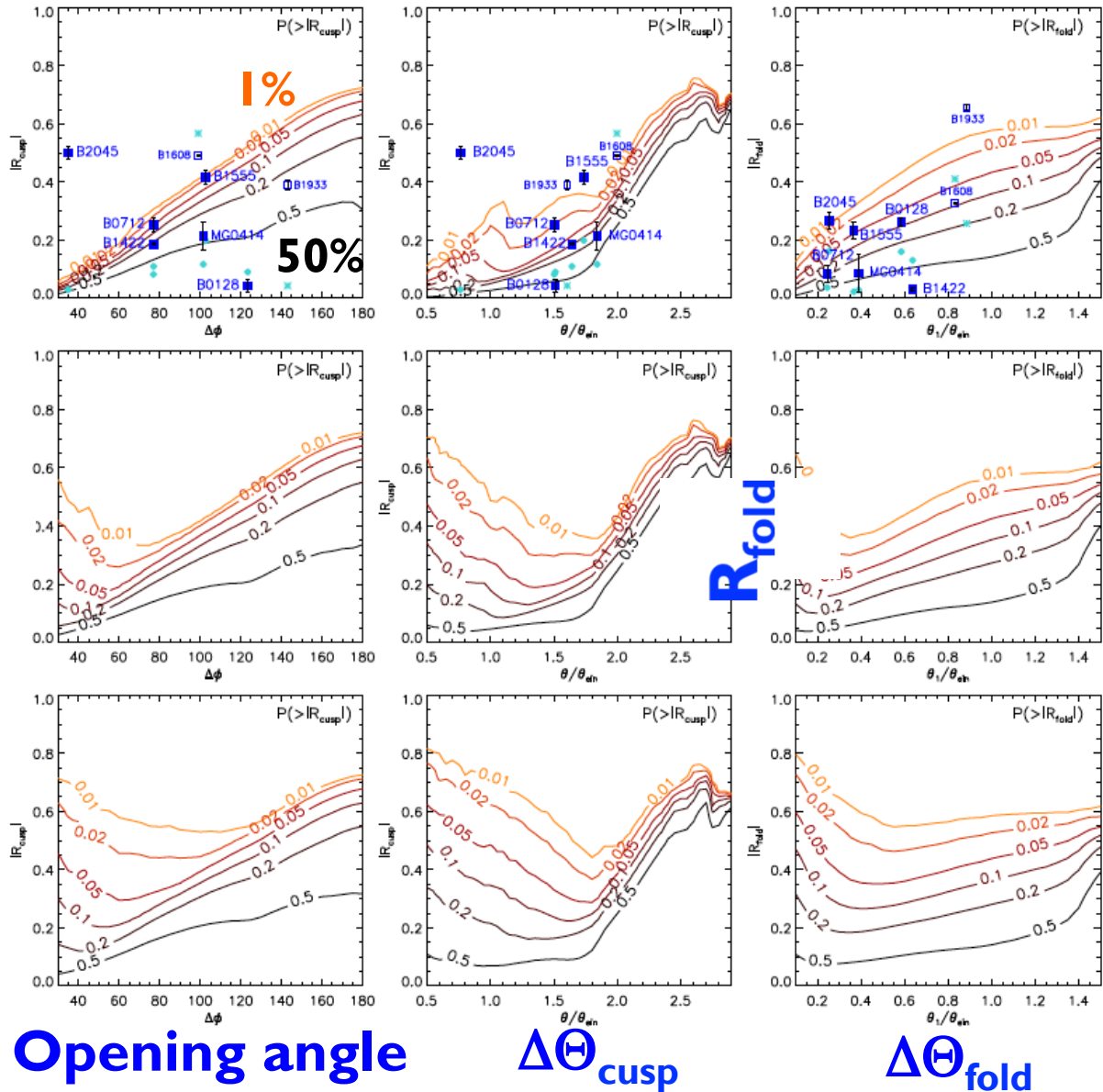
Smooth  
haloes

+ subhaloes  
in Milky  
Way-sized  
haloes

+ subhaloes in  
group-sized  
haloes  
( $5 \times 10^{13} M_{\odot}$ )

$R_{\text{cusp}}$

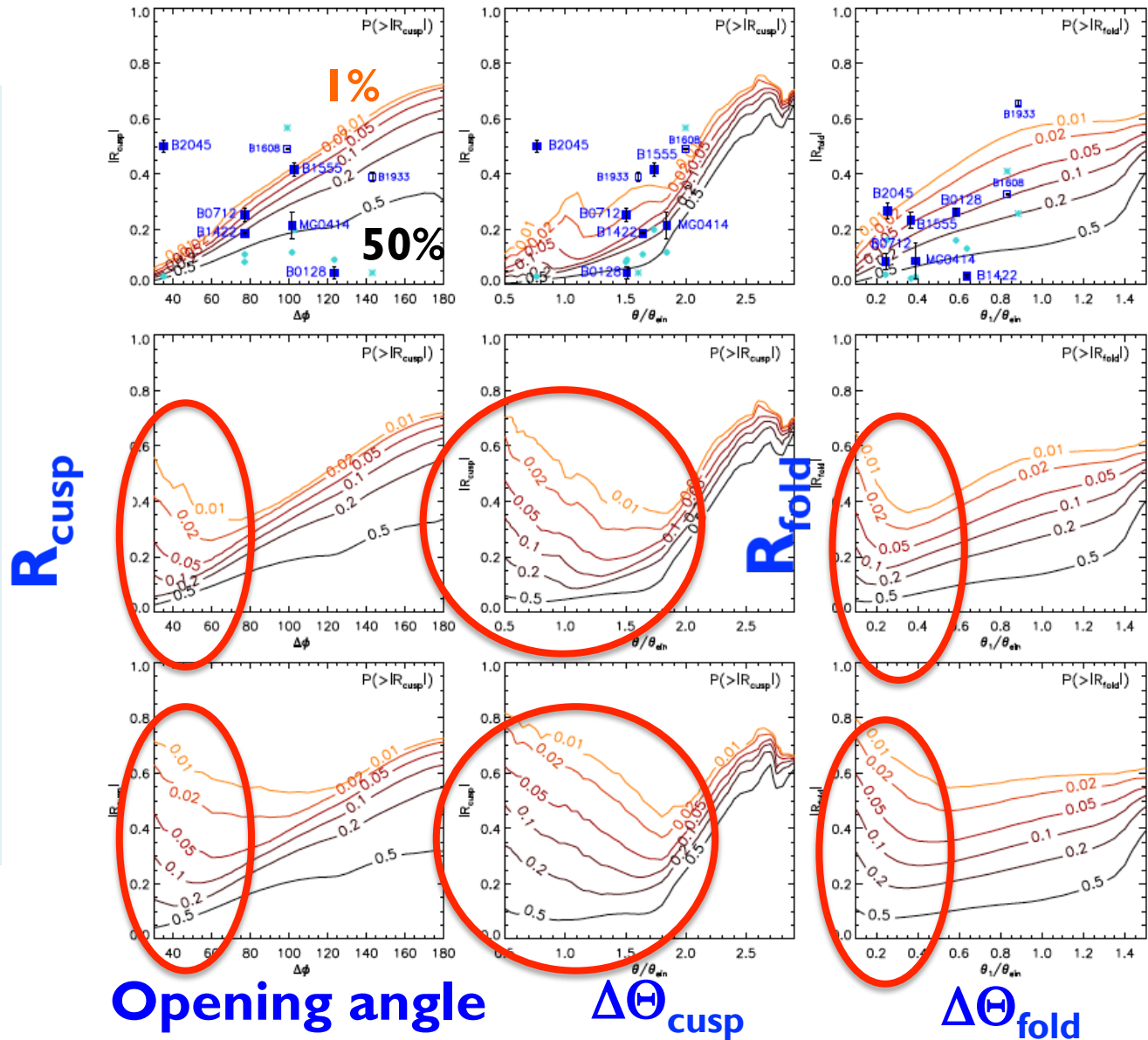
$R_{\text{fold}}$



# Statistical trends

➤ Subhaloes induce much larger scatters.

➤ close pairs and triples show more deviations





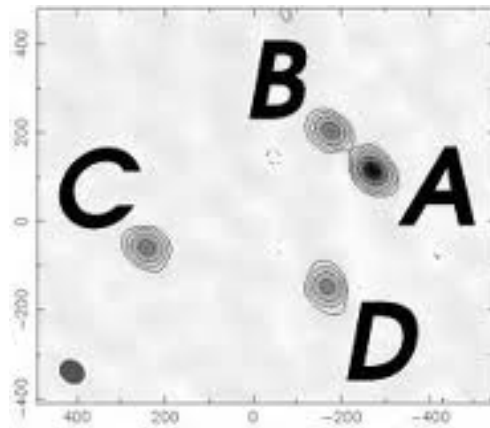
# Limitations

**The above statistical study does not take into account**

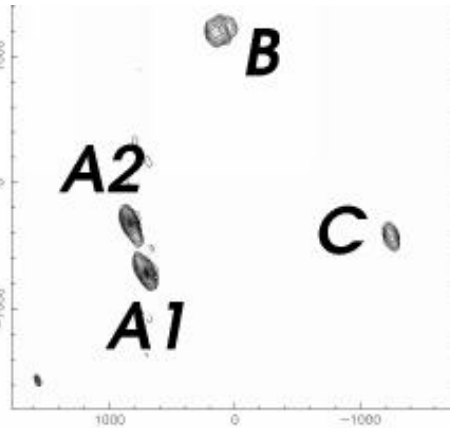
- Lens populations
- Environmental effects
- Selection effects
- Magnification bias

**An alternative is to add substructures directly into observed radio lenses**

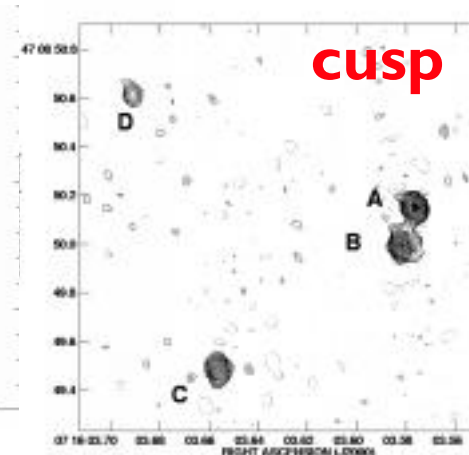
# Fold and cusp radio lenses



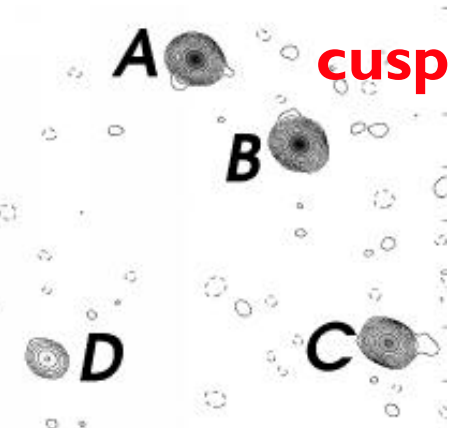
B0128+437



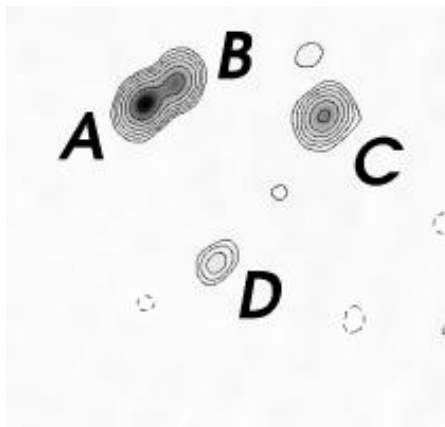
MG0414+053



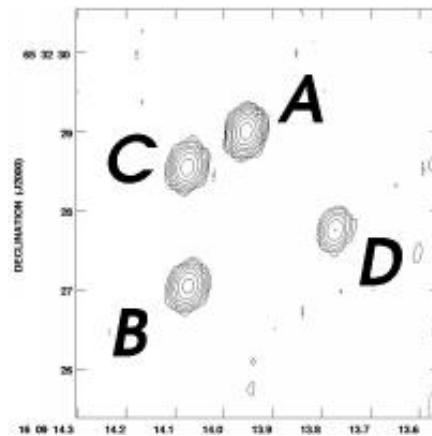
B0712+231



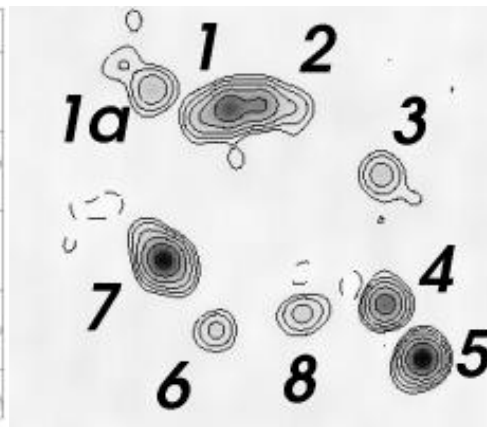
B1422+231



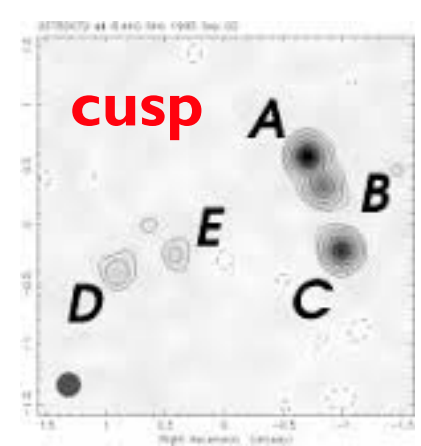
B1555+375



B1608+656



B1933+503



B2045+265

# Studying substructure effects in observed radio lenses

## ➤ **Macro-model for each observed lens:**

SIE (main lens) +  $\gamma_{\text{ext}}$  + SIS (secondary lens)

## ➤ **Adopt rescaled subhalo population from Aquarius and Phoenix simulations**

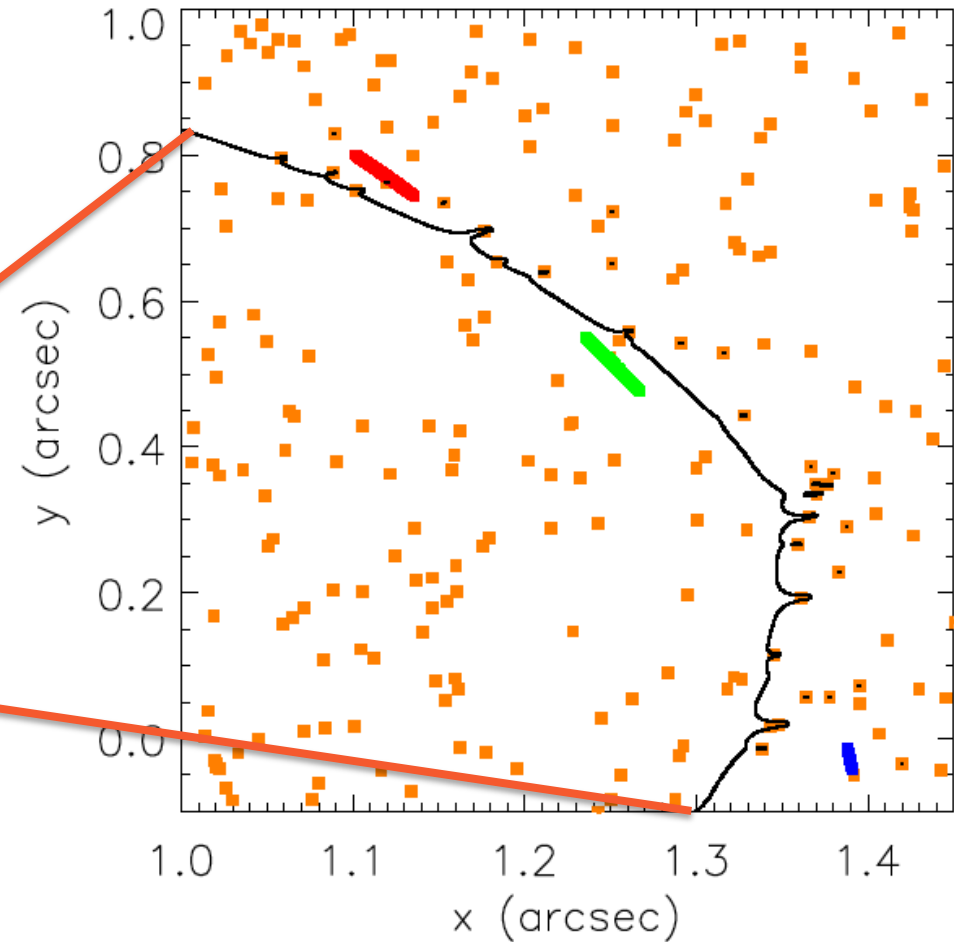
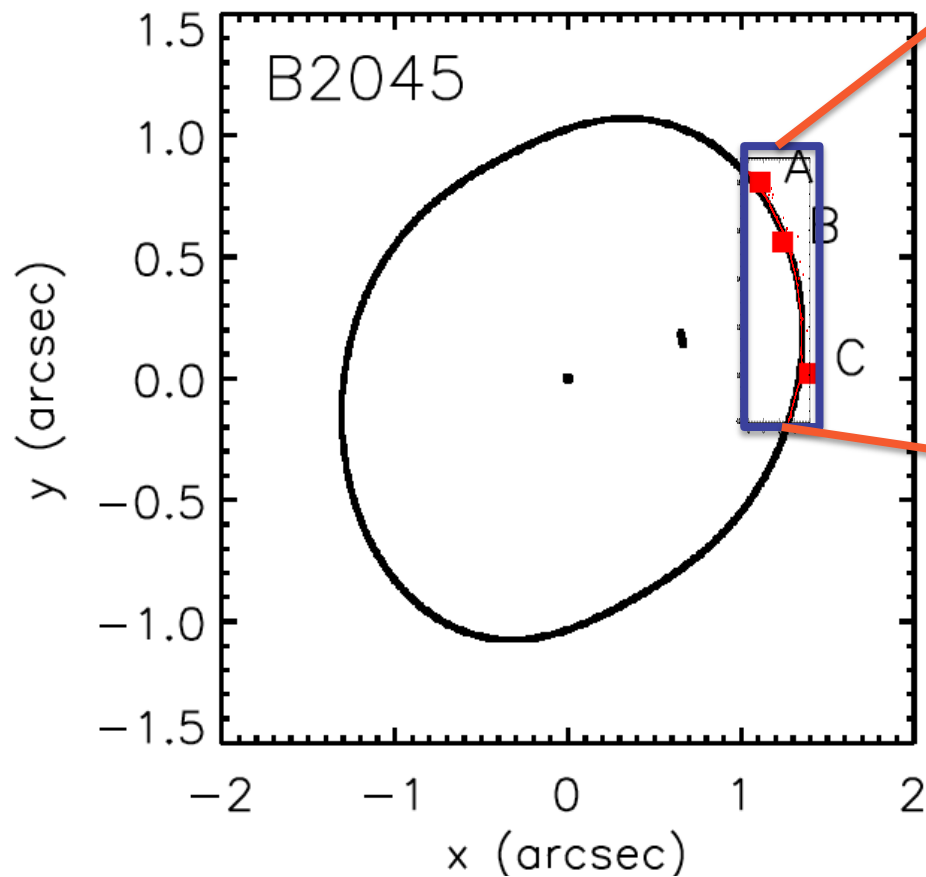
observed velocity dispersion  $\rightarrow$  halo  $v_{\text{max}} = 2^{1/2} \sigma$

## ➤ **Selecting similar lens systems**

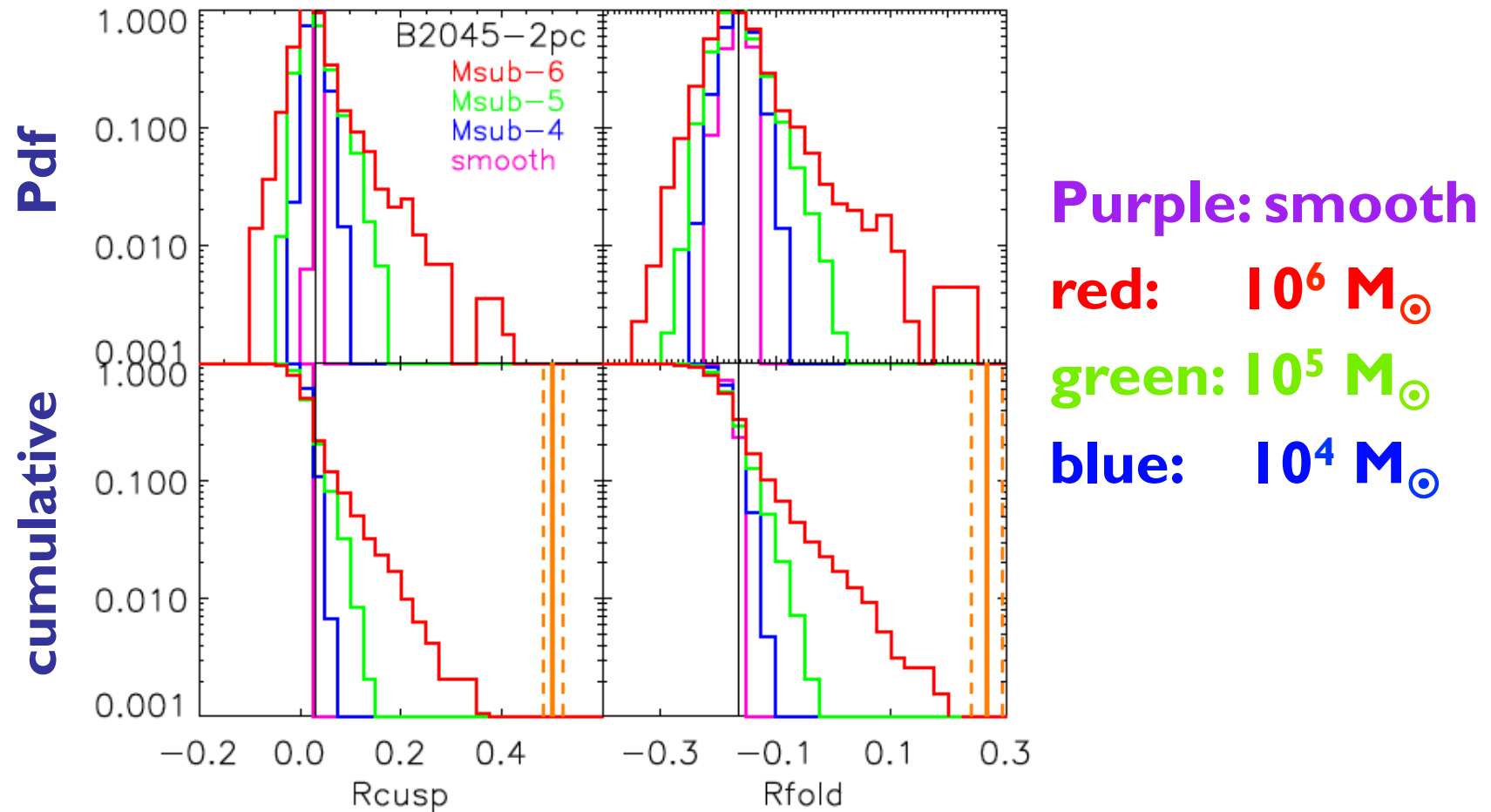
- Opening angle, Einstein radius and close pair (triple) image separations are within 10% of the observed values

# Critical curves with substructures

## case study B2045

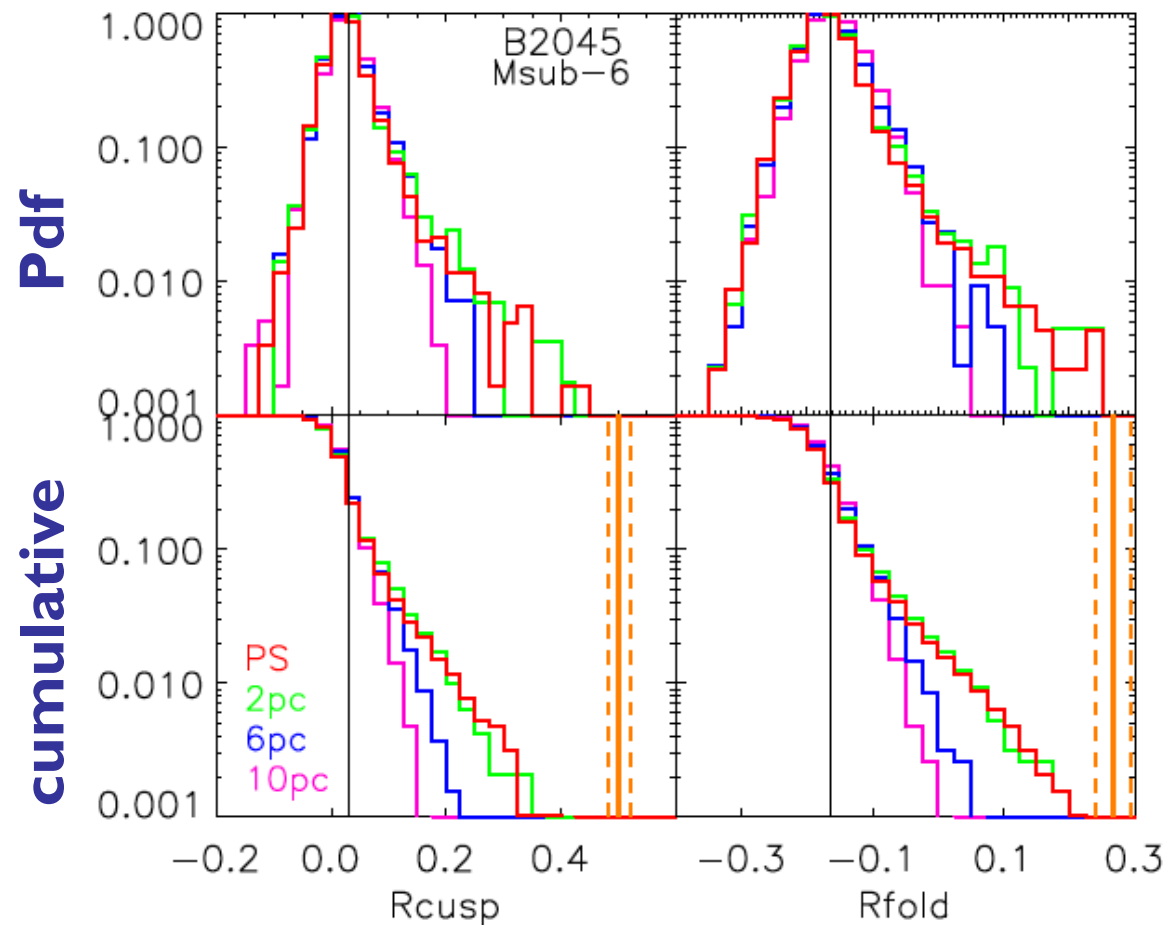


# Contributions from low mass subhaloes



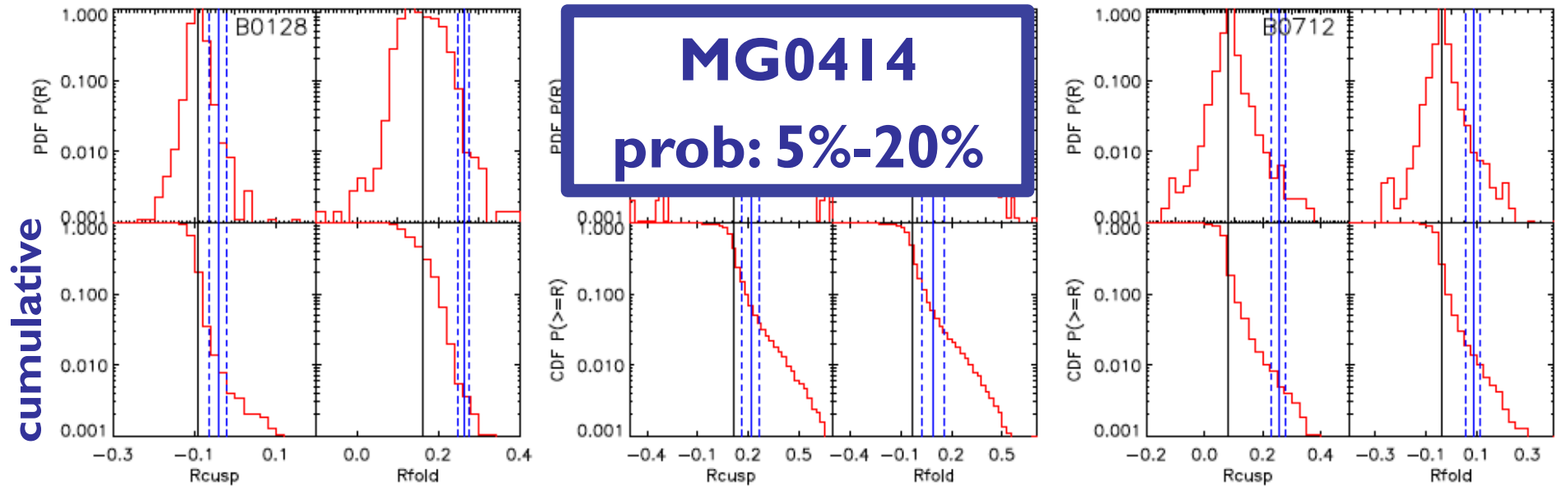
- Large subhaloes cause most deviations
- We can ignore subhalos below  $10^5$  solar masses

# Dependence on source size

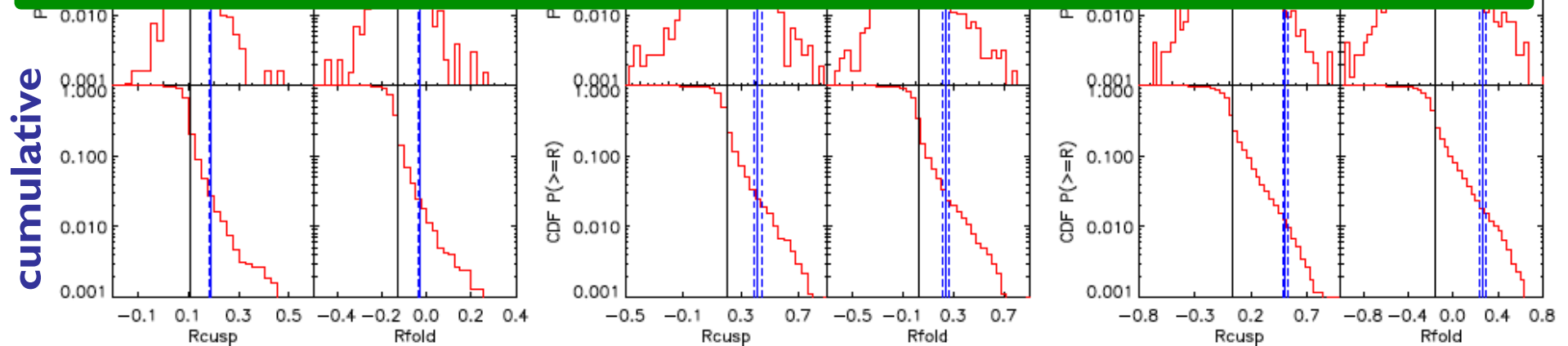


- The smaller the source size, the more significant the deviations
- Assume point source for maximum deviations

# Main results



**For other lenses, prob: 1%-4% → It seems difficult for substructures to explain observed flux ratio anomalies**



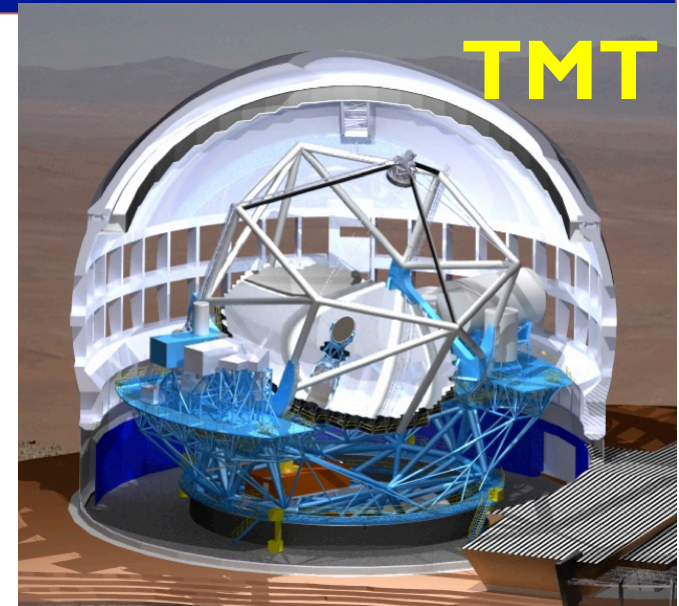
# Discussions

- **CDM substructures appear to under-predict the observed radio flux ratio anomalies**
- **Line of sight effects? (Xu et al. 2012; see talks by Inoue & Takahashi)**
- **Oversimplifications in the macro-model**
  - ✓ **Environment?**
  - ✓ **SIE too simplistic (isophotal twists, deviations from ellipses, non-concentricity)?**
- **Why are saddle images always fainter than expected?**



# Future outlook: observations

- **higher resolution and deeper images and more kinematical data**
- **Astrometric lensing (Vegetti's talk): more promising? More codes?**
- **Time delay anomaly?**
- **Narrow-line flux ratios (talk by Nierenberg)**
- **TMT can probe substructures with mass lower by a factor of 100**



# Future outlook: theory

- **Theoretically, better hydro-simulations are needed**
  - ✓ **to assess subhalo abundance and spatial distribution: importance of baryons**
  - ✓ **High-resolution group simulations needed?**
  - ✓ **particle number sufficient?**
- **better smoothing algorithm (Augulo et al. 2013)? Tracking?**
- **We need to better assess the line of sight effect**

