Gravitational lensing in presence of plasma: chromatic lensing indeed

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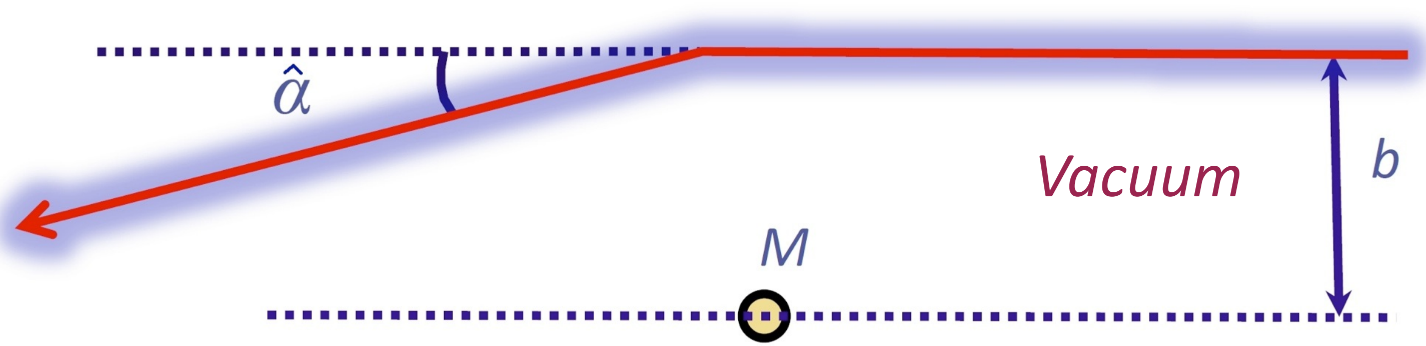
Vacuum:

Einstein angle:

$$\hat{\alpha} = \frac{4GM}{c^2 b} = \frac{2R_S}{b}$$

$$b \gg R_S = \frac{2GM}{c^2}$$



Deflection angle of the photon in vacuum does not depend on the photon frequency (or energy). Deflection in vacuum is **achromatic**.

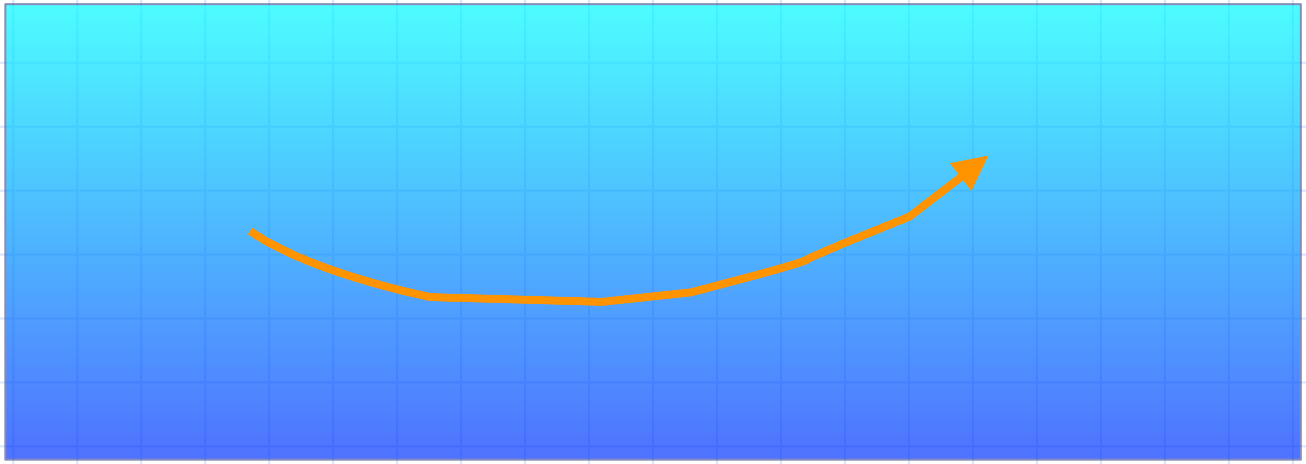
Gravitational lensing in vacuum is achromatic.

Plasma:

How is this situation changed in presence of *plasma*?

In plasma two effects should be taken into account:

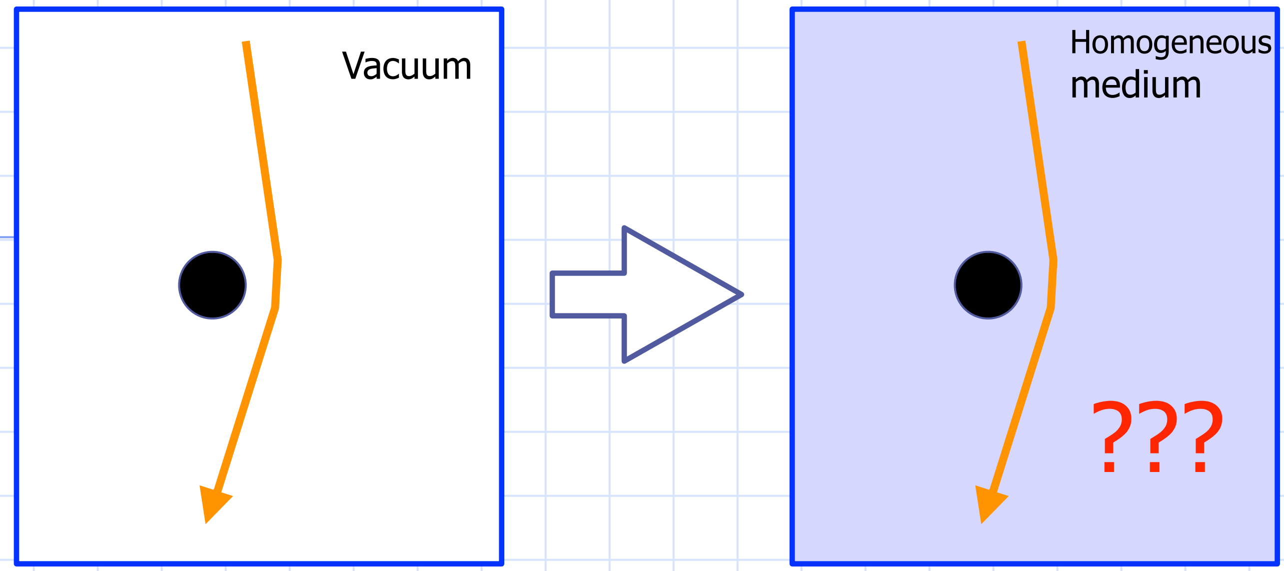
1) Effect of refractive deflection in non-homogeneous medium



$n=n(x)$

'Non-homogeneous' means here that the refractive index depends explicitly on space coordinates

But what if we consider a homogeneous medium?



2) A rigorous treatment of the light bending in gravity and plasma requires an answer to the question:

is the gravitational deflection of the light rays in the medium the same as in vacuum?

We mean here the homogeneous medium, so there is no refraction.

We need general theory for geometrical optics in arbitrary medium (dispersive or not) in curved space-time (in presence of gravity).

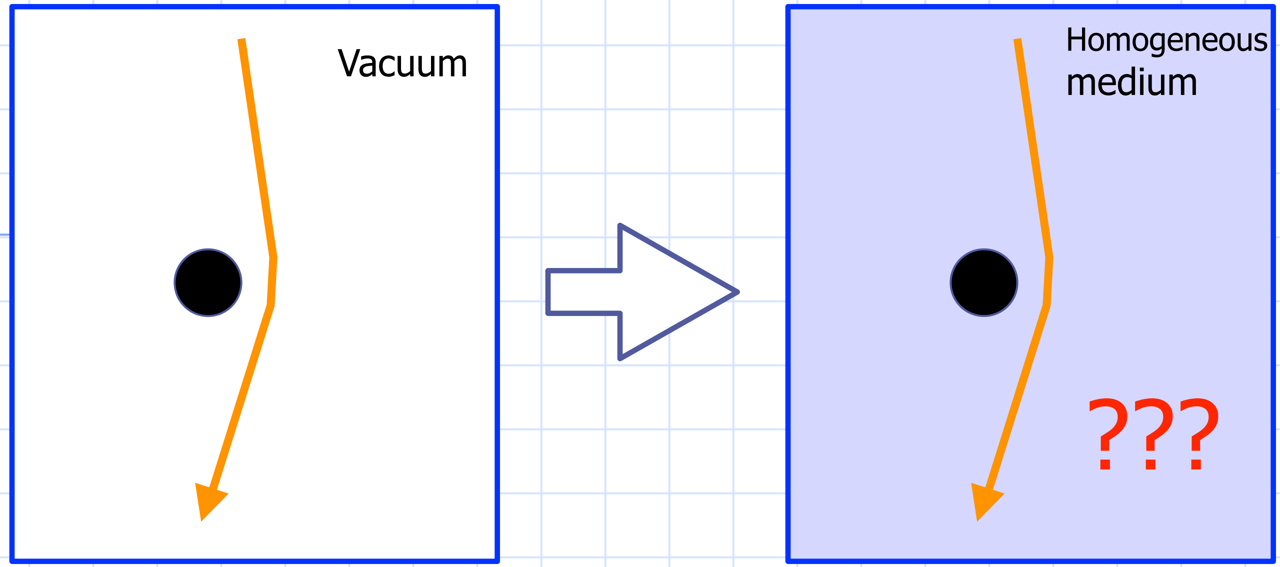
J.L. Synge, Relativity: The General Theory (North Holland Publishing Company, Amsterdam, 1960)
The first self-consistent approach for geometrical optics in medium in presence of gravity

See also:
J. Bicˇa´k and P. Hadrava, Astron. Astrophys. 44, 389 (1975)

The first monograph completely devoted to review of general-relativistic ray optics in medium:
V. Perlick, Ray Optics, Fermat's Principle, and Applications to General Relativity (Springer-Verlag, Berlin, 2000)

On language of gravitational lensing the problem of plasma influence is considered for the first time:
P.V. Bliokh and A.A. Minakov, Gravitational Lenses [Russian] (Naukova Dumka, Kiev, 1989).

We use general approach of Synge and obtain:



is the gravitational deflection of the light rays in the medium the same as in vacuum?

We mean here the *homogeneous* medium, so there is no refraction.

Answer is:

yes, deflection is the same as in vacuum, if medium is non-dispersive (n=const)

No, it differs, if medium is dispersive! (n=n(omega))

The physical reason is a dependence of the wave frequency on space coordinates in presence of gravity (gravitational redshift)

Bisnovatyi-Kogan and Tsupko (2009, 2010)

Gravitational deflection of light rays in presence of plasma

Refraction index of plasma:

$$n^2 = 1 - \frac{\omega_e^2}{[\omega(x^\alpha)]^2}, \quad \omega_e^2 = \frac{4\pi e^2 N(x^\alpha)}{m}.$$

We denote $\omega(\infty) = \omega$, e is the charge of the electron, m is the mass of the electron, $N(x^\alpha)$ is the electron concentration in the inhomogeneous plasma, ω_e is the electron plasma frequency in the plasma.

It is shown for the first time, that the gravitational deflection in homogeneous plasma differs from the vacuum deflection angle, and depends on frequency of the photon:

$$\hat{\alpha} = \frac{2R_S}{b} = \frac{4GM}{c^2 b}$$

in vacuum

$$\hat{\alpha} = \frac{R_S}{b}\left(1 + \frac{1}{1 - (\omega_e^2/\omega^2)}\right)$$

in homogeneous plasma

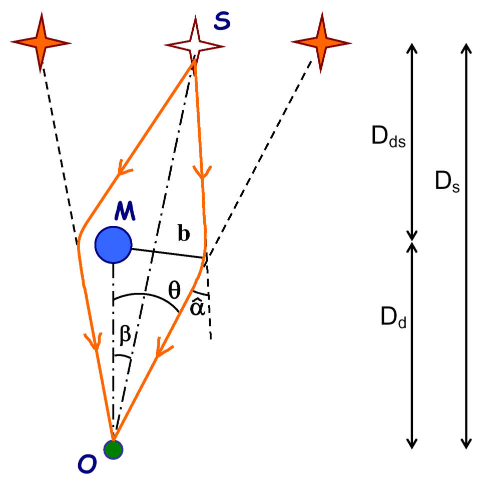
Chromatic gravitational deflection!

Bisnovatyi-Kogan and Tsupko (2009, 2010)

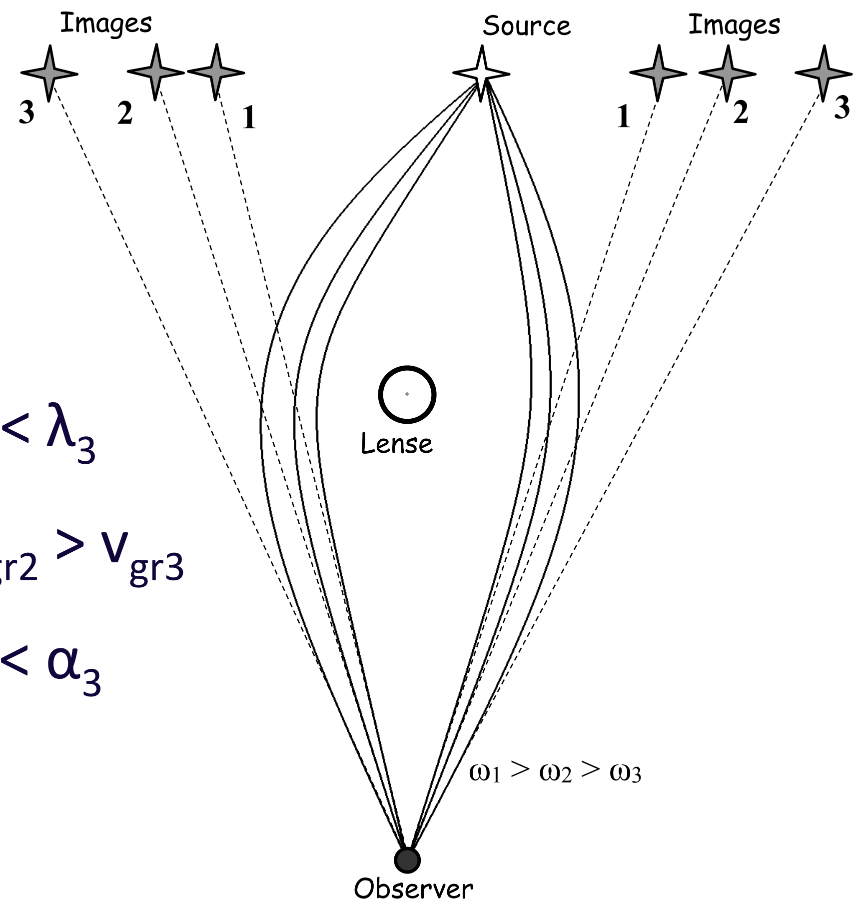
Effect of 'Gravitational radiospectrometer'

Instead of two concentrated images with complicated spectra, we will have two 'rainbow' images, formed by the photons with different frequencies, which are deflected by different angles.

vacuum



homogeneous plasma



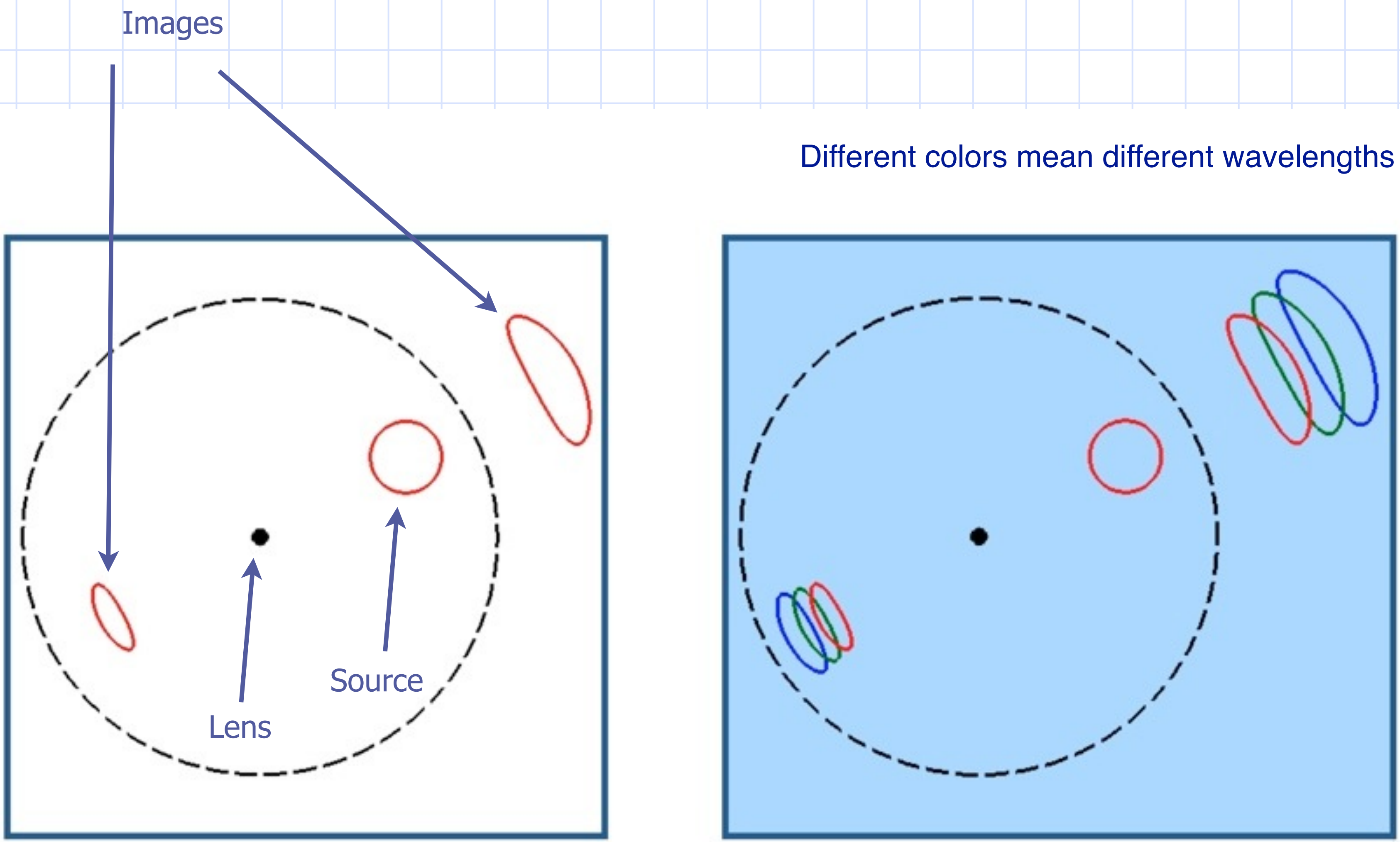
$\lambda_1 < \lambda_2 < \lambda_3$

$v_{gr1} > v_{gr2} > v_{gr3}$

$\alpha_1 < \alpha_2 < \alpha_3$

Point-mass gravitational lens
in homogeneous plasma:
it acts like spectrometer!

Effect is significant only for radiowaves



vacuum

plasma

Gravitational lensing in non-homogeneous plasma:

In non-homogeneous plasma two effects should be taken into account:

1) Difference of gravitational deflection from vacuum case due to plasma presence

2) Refraction

Both effects are chromatic in plasma

Total deflection angle

Bisnovatyi-Kogan and Tsupko (2009, 2010)

gravitational deflection in plasma

$$\hat{\alpha} = \underbrace{\alpha_{einst} + \alpha_{add}} + \alpha_{refr}$$

$$= \frac{2R_S}{b} \qquad \propto \frac{R_S}{b}\frac{\omega_e^2}{\omega^2} \qquad \propto \nabla \frac{\omega_e^2}{\omega^2}$$

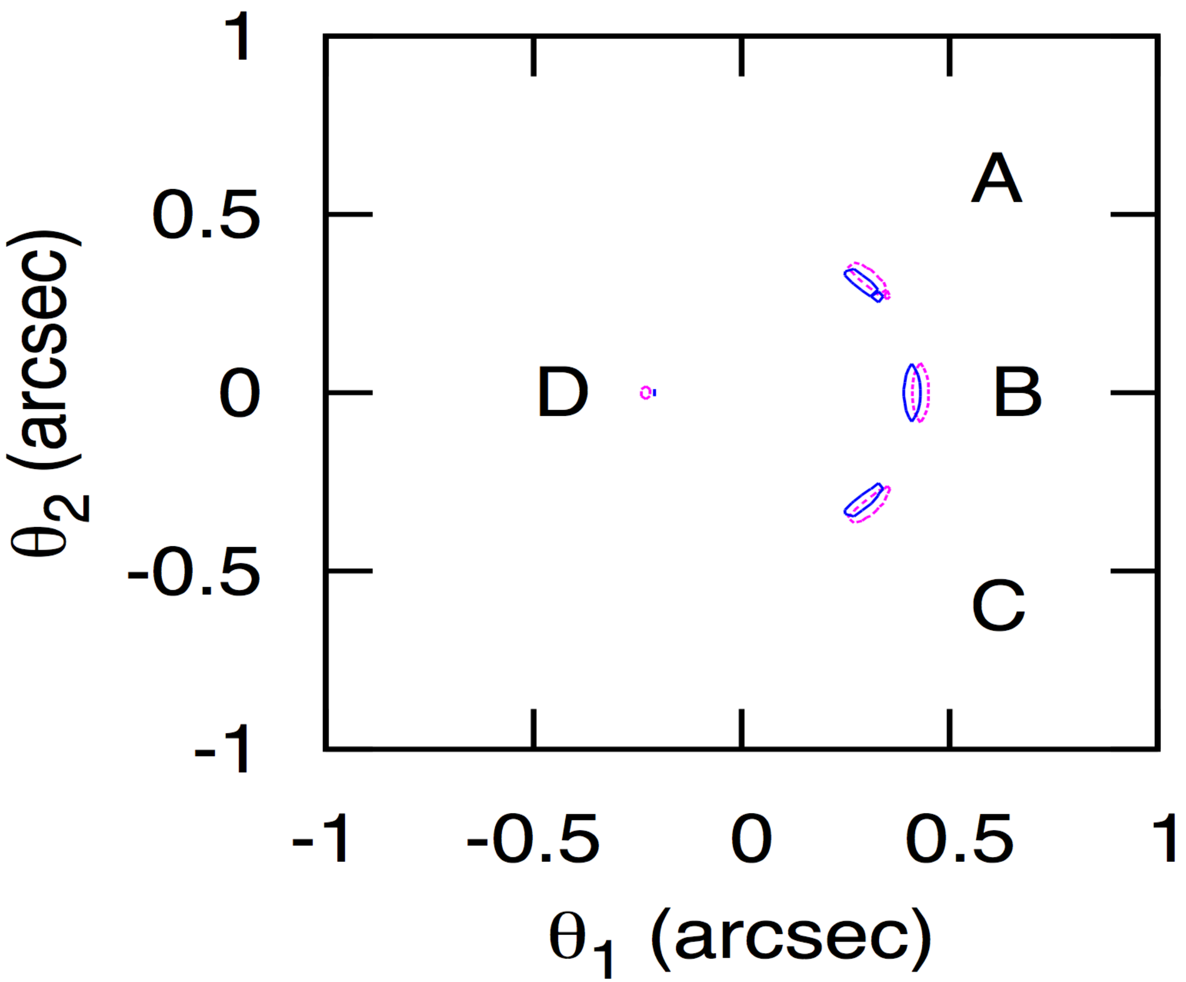
Vacuum gravitational deflection (Einstein)

Additional correction to the gravitational deflection due to plasma presence. It depends on the photon frequency. It takes place both in homogeneous and inhomogeneous plasma

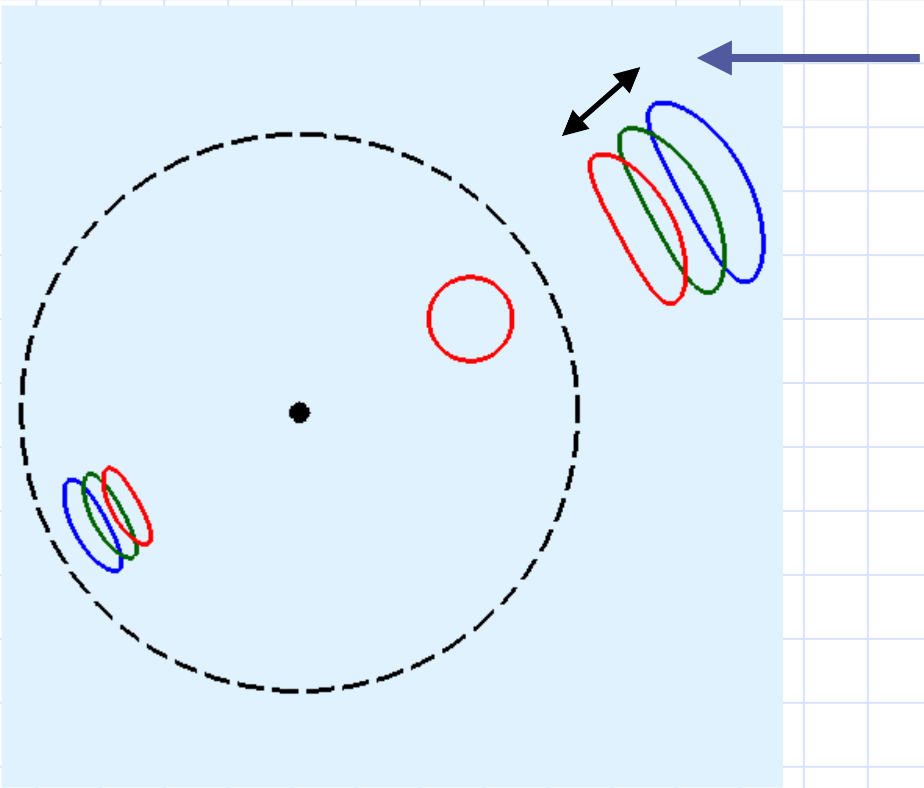
The refraction connected with the plasma inhomogeneity. It depends on the photon frequency because the plasma is dispersive medium. This angle equals to zero if the plasma is homogeneous.

Observational predictions:

- Xinzhong Er, Shude Mao, 2014
- Angular difference between optical and radio images is up to 10^(-3) arcsec due to presence of inhomogeneous plasma



What we propose for observations:



1) Compare observations of strong lens system with multiple images in optical and radio band, or compare observations in two radio bands

2) Shift of angular position of every image can be observed

3) As a result: investigation of plasma properties in vicinity of lens

Exact formula for the photon deflection in spherically distributed plasma and Schwarzschild metric

$$\hat{\alpha} = 2\int_R^\infty \frac{dr}{\sqrt{r(r-2M)}\sqrt{\frac{h^2(r)}{h^2(R)} - 1}} - \pi.$$

$$h(r) = r\sqrt{\frac{1}{A(r)} - \frac{\omega_e^2(r)}{\omega^2}} = r\sqrt{\frac{r}{r-2M} - \frac{\omega_e^2(r)}{\omega^2}},$$

V. Perlick (2000)
O.Yu. Tsupko and G.S. Bisnovatyi-Kogan (2013)

Conclusions:

1. In presence of both gravity and plasma the deflection angle is physically defined by mutual combination of different phenomena: gravity, dispersion, refraction.

2. In weak deflection approximation two effects should be taken into account:
- difference of gravitational deflection from vacuum case
- refractive deflection (usually bigger)

Presence of plasma always makes gravitational lensing chromatic

3. It leads to difference in angular position of the same image at different (radio) wavelengths.

Publications:

1. G.S. Bisnovatyi-Kogan, O.Yu. Tsupko, Gravitation and Cosmology, 15(1), 20-27 (2009).

2. G.S. Bisnovatyi-Kogan and O.Yu. Tsupko, Monthly Notices of the Royal Astronomical Society 404, 1790–1800 (2010)

3. O.Yu. Tsupko and G.S. Bisnovatyi-Kogan, Physical Review D 87, 124009 (2013)