Polar angle requirements for CMB B-mode experiments

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Summary

- Presentation based on the work by Vielva, M-G and Casas 2020, in preparation
- Development of a formalism to determine the polar angle requirements for CMB B-mode experiments
- Uncertainties in the detectors polar angle propagate to the tensor-to-scalar ratio r as a bias
- The specific component separation method used to extract the cosmological signal plays a key role
- Possible correlations among detectors are included in the formalism
- The polar angle requirements are quantified for a given bias on r
- As an illustrative example, results are provided for the LiteBIRD space mission

Formalism

Given an experiment with n frequency channels, the CMB polarization signal is estimated as a (linear) combination of the form

$$\begin{pmatrix} Q \\ U \end{pmatrix}(p)=\sum_{\nu=1}^{n} w_\nu \begin{pmatrix} Q_\nu \\ U_\nu \end{pmatrix}(p) \quad , \qquad \sum_{\nu=1}^{n} w_\nu = 1$$

Estimated CMB at position p

Weight at frequency ν

Data at frequency ν and position p

or equivalently for the spherical harmonic coefficients

$$\begin{pmatrix} e_{lm} \\ b_{lm} \end{pmatrix}=\sum_{\nu=1}^{n} w_\nu \begin{pmatrix} e_{lm}^{\nu} \\ b_{lm}^{\nu} \end{pmatrix}$$

This linear combination is typical of the ILC method (e.g. Fernandez-Cobos et al. 2016). Also an optimal parametric fitting method, where all foreground components are recovered up to the noise limit, is expected to provide inverse noise weighting (as it is the case, e.g., for the default method used for LiteBIRD, see Errar & Stompor (2018)).

- The rotation of the polarization axes by an angle α transforms the intrinsic polarization pseudo-vector (Q,U) in a rotated one (Q^{rot},U^{rot}) as follows:

$$\begin{pmatrix} Q^{rot} \\ U^{rot} \end{pmatrix}(p)=\begin{pmatrix} \cos(2\alpha) & -\sin(2\alpha) \\ \sin(2\alpha) & \cos(2\alpha) \end{pmatrix}\begin{pmatrix} Q \\ U \end{pmatrix}(p)$$

or equivalently for the polarization modes e_{lm} and b_{lm} (assuming a uniform rotation over the sky):

$$e_{lm}^{rot} = \cos(2\alpha)\, e_{lm} - \sin(2\alpha)\, b_{lm}$$

$$b_{lm}^{rot} = \sin(2\alpha)\, e_{lm} + \cos(2\alpha)\, b_{lm}$$

- Let us consider how the CMB polarization signal, estimated from the combination of the n frequency channels, changes when the polarization axes of each channel ν are rotated by an angle α_ν:

$$e_{lm}^{o} = e_{lm}\sum_{\nu=1}^{n} w_\nu \cos(2\alpha_\nu) - b_{lm}\sum_{\nu=1}^{n} w_\nu \sin(2\alpha_\nu)$$

$$b_{lm}^{o} = e_{lm}\sum_{\nu=1}^{n} w_\nu \sin(2\alpha_\nu) + b_{lm}\sum_{\nu=1}^{n} w_\nu \cos(2\alpha_\nu)$$

and the change in the BB power spectrum would be (assuming a null primordial EB):

$$B_l^{o} = E_l\left(\sum_{\nu=1}^{n} w_\nu \sin(2\alpha_\nu)\right)^2 + B_l\left(\sum_{\nu=1}^{n} w_\nu \cos(2\alpha_\nu)\right)^2$$

- Assuming a Gaussian likelihood approximation for the BB spectrum, the bias induced on the r parameter is given by the following expression:

$$\delta_r = \frac{\sum_{l=2}^{l_{max}} \Delta B_l B_l^{fiducial} \Big/ Var(B_l)}{\sum_{l=2}^{l_{max}} \left(B_l^{fiducial}\right)^2 \Big/ Var(B_l)}$$

- Considering $l_{max} \approx 200$ is sufficient to include all the bias effect.

- $B_l^{fiducial}$ is the BB spectrum corresponding to the fiducial ΛCDM model for $r = 1$.

- ΔB_l is the biased BB spectrum after subtracting the known contributions to the observed BB signal B_l^o: the fiducial spectra for BB and lensing and the effective noise. B_l^o is given by:

$$B_l^o = \left(r\, B_l^{fiducial} + L_l\right)\sum_{cos} + E_l \sum_{sin} + N_l^{eff}$$

with E_l, L_l and N_l^{eff} are the fiducial EE, lensing and effective noise spectra resulting from the linear combination. The $\sum_{cos}$ and $\sum_{sin}$ terms account for the impact of the polarization angle offsets of each frequency channel and will be given below.

- The effective noise power spectrum is given in terms of the noise spectra of the n channels, N_l^{ν}, and the weights used in the linear combination of the frequency channels w_{ν}:

$$N_l^{eff} = \sum_{\nu=1}^{n} N_l^{\nu} w_{\nu}^2$$

- The biased BB spectrum ΔB_l is given by the subtraction of the known contributions to the observed B_l^o:

$$\Delta B_l = \left(r\, B_l^{fiducial} + L_l\right)\left(\Sigma_{cos} - 1\right) + E_l \Sigma_{sin}$$

Obviously these contributions can be removed at the power spectrum level but not from the cosmic variance (here we do not attemp to do delensing at map level):

$$Var(B_l) = \frac{2}{f_{sky}(2l+1)} B_l^o$$

with f_{sky} accounting for the sampling variance.

- As commented above, the $\sum_{cos}$ and $\sum_{sin}$ terms account for the impact of the polarization angle offsets of each frequency channel

$$\Sigma_{cos} = \left(\sum_{\nu=1}^{n} \cos(2\alpha_\nu)\, w_\nu\right)^2 \qquad \Sigma_{sin} = \left(\sum_{\nu=1}^{n} \sin(2\alpha_\nu)\, w_\nu\right)^2$$

where the sum is over all the channels and α_ν is the polarization angle offset of channel ν.

- From the above expressions, it is clear that in the limit of very small angle offsets then $\sum_{cos}$=1 and $\sum_{sin}$=0, and therefore ΔB_l=0 and also $\delta_r = 0$ as one would expect.

- Since typical instrumental offsets are expected to be at the degree level at most, it is worth considering the small angle approximation. In this case the previous expression for $\sum_{cos}$ and $\sum_{sin}$ take the following form up to first order:

$$\sum\nolimits_{cos} \approx 1 - 4\sum_{\nu=1}^{n} \alpha_\nu^2 w_\nu^2$$

$$\sum\nolimits_{sin} \approx 4\left(\sum_{\nu=1}^{n} \alpha_\nu w_\nu\right)^2$$

and the bias in the r parameter, δ_r, is given by (also considering that $r\, B_l^{fiducial} + L_l \ll E_l$):

$$\delta_r \approx \frac{4\left(\sum_{l=2}^{l_{max}} \frac{E_l B_l^{GW}}{Var(B_l)}\right)}{\sum_{l=2}^{l_{max}} \frac{(B_l^{GW})^2}{Var(B_l)}} \left(\sum_{\nu=1}^{n} \alpha_\nu w_\nu\right)^2$$

This is a general expression that only depends on the polarization angle mismatch per channel, α_ν, and the weight that each channel has to build the final CMB map, w_ν.

Requirements on the detector offsets

For a given experiment, a certain budget of the systematics may be assigned to the r bias, δ_r, coming from the miscalibration of the detectors' polarization angle. The problem is that there are many possibilities to translate that budget in δ_r to the requirements for each channel or, even worse, for each single detector.

Another complication is that the detector offsets can be correlated among themselves. These correlations may come form different levels of the experimental configuration: S/C, focal plane, wafer or detector. They can originate in e.g. the fabrication process of the wafers, cooling system, optical elements (telescope, HWP, …) or the S/C orientation.

As commented above, the component separation method used to separate the CMB signal from the contaminating Galactic and extragalactic emissions plays also an important role in the determination of the requirements. If this process can be approximated as a linear combination of the different channels, then its effect is completely included in the weights of that combination.
An additional assumption is that the foreground residuals are $R_l \leq L_l$, as expected for future experiments.

Weighting schemes

Different weighting schemes can be considered, in particular the following three:

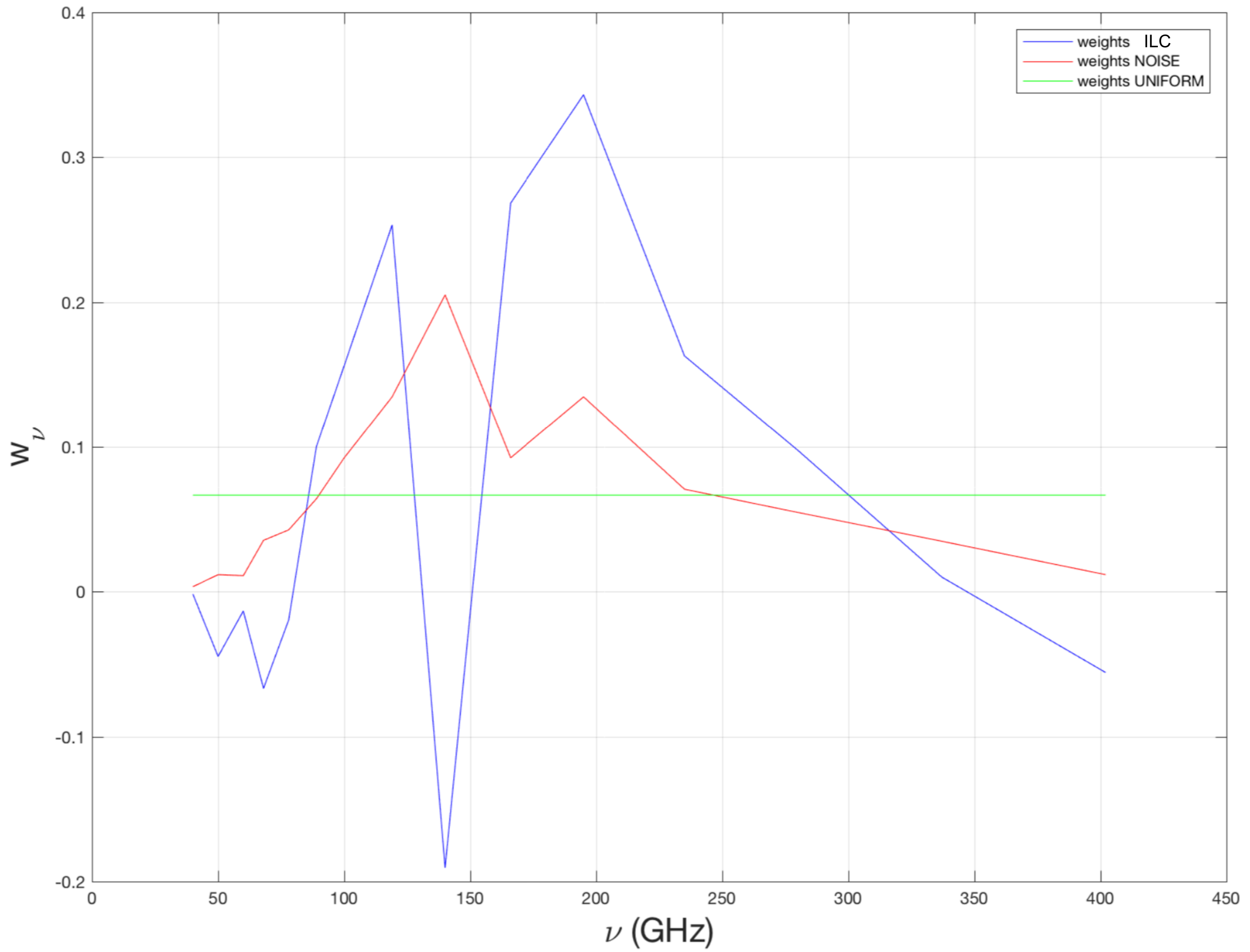
- Internal Linear Combination (ILC): weights estimated by imposing minimum variance to the CMB polarization signal derived from a linear combination of the frequency maps, following the formalism of Fernández-Cobos et al. 2016 for polarization data.

- Inverse noise:

$$w_\nu^{in} = \frac{\sigma_\nu^{-2}}{\sum_{\nu=1}^{n} \sigma_\nu^{-2}}$$

- Uniform:

$$w_\nu^{uni} = \frac{1}{n}$$



Example of weights computed from simulations with constant spectral indices across the sky of the LiteBIRD frequencies.

Correlations among offsets

The previous expression allows us to perform simulations, where random numbers of the α_ν parameters can be drawn from a Gaussian distribution with a given dispersion σ_{α_ν}. However, it is reasonable to think that the offset in one channel could be correlated with the one in another channel. In particular, channels belonging to the same wafer or the same instrument could suffer from such correlations due to optics, cooling system, etc. But also, there could be systematics that affect globally to all the channels, independently of the instrument.

Denoting by $\boldsymbol{C}$ the matrix that defines such correlations among the polarization angle offsets, α_ν, then we have:

$$\langle \alpha_{\nu_1} \alpha_{\nu_2} \rangle \equiv C_{\nu_1 \nu_2} = \rho_{\nu_1 \nu_2} \sigma_{\nu_1} \sigma_{\nu_2}$$

where $\rho_{\nu_1 \nu_2}$ is the correlation coefficient between channel ν_1 and channel ν_2.

Let us now consider the expected value of the previous expression for δ_r. It is easy to show that:

$$\langle\delta_r\rangle \approx \frac{4\left(\sum_{l=2}^{l_{max}} \frac{E_l B_l^{GW}}{Var(B_l)}\right)}{\sum_{l=2}^{l_{max}} \frac{(B_l^{GW})^2}{Var(B_l)}} \sum_{\nu_1,\nu_2=1}^{n} C_{\nu_1\nu_2} w_{\nu_1} w_{\nu_2}$$

As discussed above, the choices to establish the n values of σ_{α_ν} for a given $\langle\delta_r\rangle$ requirement can be very large. A natural approach to this problem is to consider that, in the sense of the ensemble average, all the terms in the last sum of the expression for δ_r add evenly, i.e. that $\sigma_{\alpha_\nu} = c w_\nu^{-1}$, where c is a constant. With this assumption, an expression can be obtained to provide the requirements for α_ν:

$$\langle\delta_r\rangle \approx c^2 \frac{4\left(\sum_{l=2}^{l_{max}} \frac{E_l B_l^{GW}}{Var(B_l)}\right)}{\sum_{l=2}^{l_{max}} \frac{(B_l^{GW})^2}{Var(B_l)}} \sum_{\nu_1,\nu_2=1}^{n} \rho_{\nu_1\nu_2}$$

In the case in which all channels are fully correlated, we obtain the following expression:

$$\langle \delta_r \rangle \approx n^2 c^2 \frac{4\left(\sum_{l=2}^{l_{max}} \frac{E_l B_l^{GW}}{Var(B_l)}\right)}{\sum_{l=2}^{l_{max}} \frac{(B_l^{GW})^2}{Var(B_l)}}$$

As expected, any alternative correlation scheme (i.e. with some $\rho_{\nu_1\nu_2} < 1$) will imply a larger value of c (for a fixed requirement on δ_r) and, therefore, will provide more relaxed values for the requirements on the different polarization angles.

In the limit where all the n elements are uncorrelated, then the requirements on the accuracy needed for the knowledge of the polarization angles becomes the weakest:

$$\langle \delta_r \rangle \approx n c^2 \frac{4\left(\sum_{l=2}^{l_{max}} \frac{E_l B_l^{GW}}{Var(B_l)}\right)}{\sum_{l=2}^{l_{max}} \frac{(B_l^{GW})^2}{Var(B_l)}}$$

Application to LiteBIRD

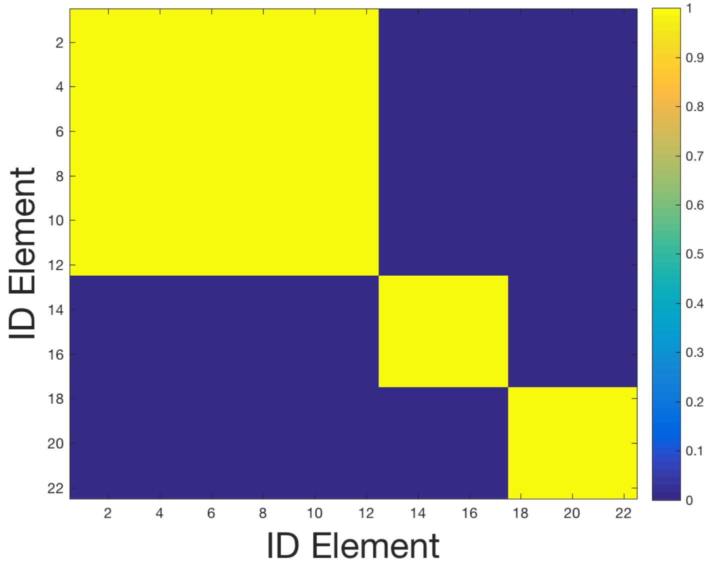
The polarization angle accuracy requirements can be divided in relative and absolute ones. The absolute ones consist in a global offset, which accounts for a possible mismatch between the SVM and the PLM, and three additional ones that account for the mismatch between the PLM and each of the three focal planes, LFT, MFT and HFT. For the relative ones we may consider different sets of detectors: 22 frequency channels, 70 wafers+frequencies, or several thousands of individual detectors. In what follows, we will refer to the $n = 22$ frequency channels that are included in the tree focal planes of the three telescopes.

The requirements corresponding to the relative angles, for a tolerated bias of 1% of the systematic budget of the r parameter (0.57×10^{-3}), are estimated with the previous formula for $\langle\delta_r\rangle$. We consider the following correlation cases:

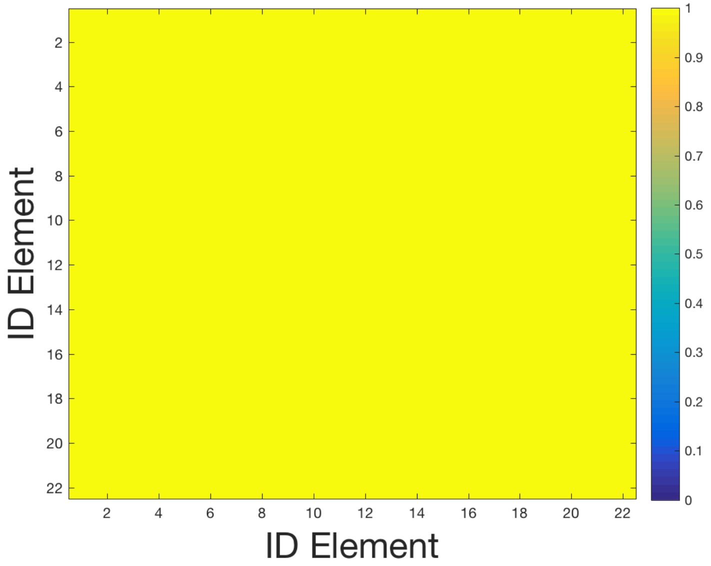
- Case 0: All the n elements are uncorrelated, except for those in the same telescope which are fully correlated.
- Case 1: All the n elements are fully correlated (strongest constraints).
- Case 2: All the n elements are partially correlated: $\rho_{\nu_1\nu_2} = 0.5$ (for any $\nu_1 \neq \nu_2$), except those within the same telescope which are fully correlated.
- Case 3: All the n elements are uncorrelated (weakest constraints).

Correlation cases considered

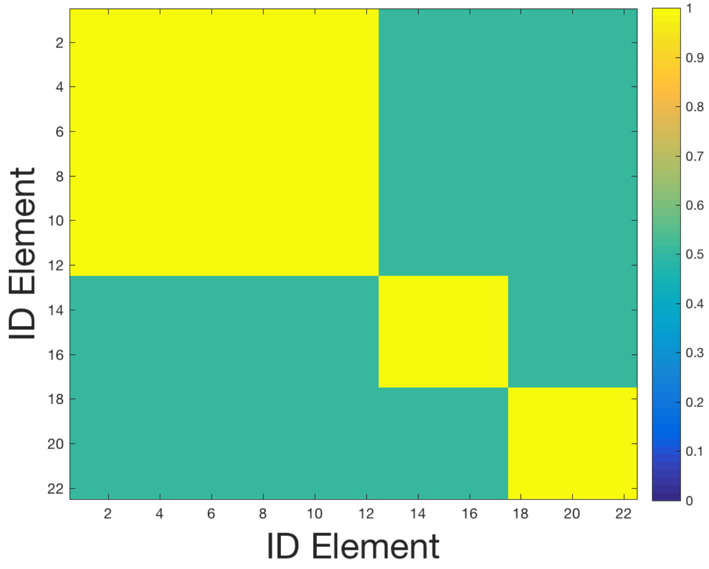
Case 0



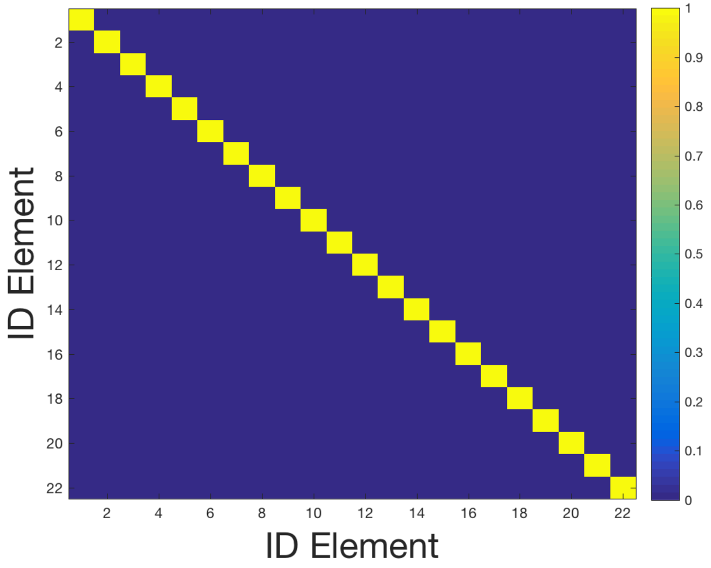
Case 1



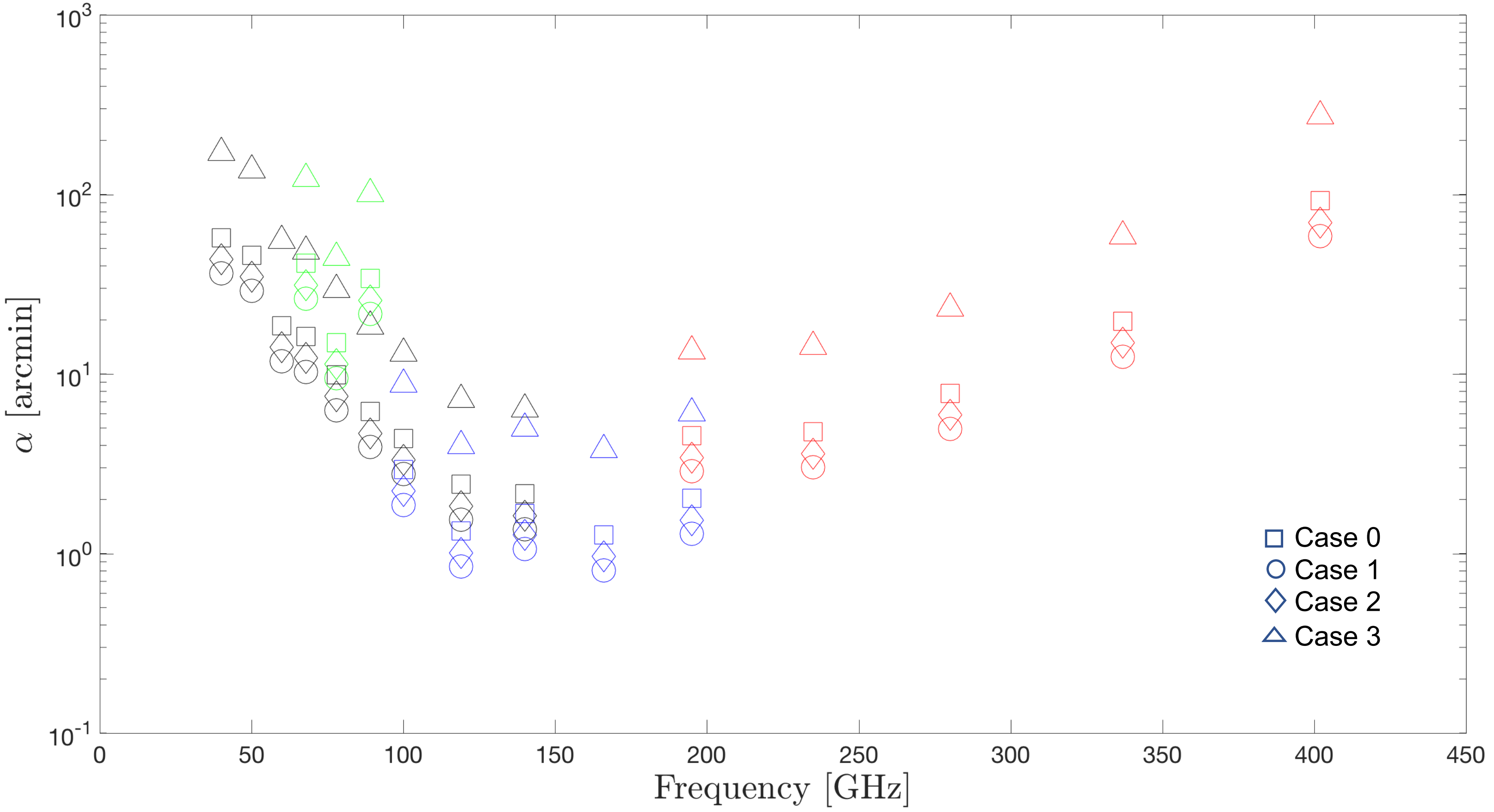
Case 2



Case 3



Requirements for relative angles



The requirements corresponding the absolute angles can be derived following similar steps as the ones used for the relatives angles. Following the equation for δ_r in the small angle approximation, the contribution to the bias of the global offset, α_g, and the three additional offsets corresponding to each of the focal plane, LFT, α_L, MFT, α_M, and HFT, α_H, is given by:

$$\delta_r \approx \frac{4\left(\sum_{l=2}^{l_{max}} \frac{E_l B_l^{GW}}{Var(B_l)}\right)}{\sum_{l=2}^{l_{max}} \frac{(B_l^{GW})^2}{Var(B_l)}} \left[\sum_{i=1}^{n_L} (\alpha_g + \alpha_L) w_i + \sum_{i=n_L+1}^{n_L+n_M} (\alpha_g + \alpha_M) w_i + \sum_{i=n_L+n_M+1}^{n} (\alpha_g + \alpha_H) w_i\right]^2$$

where n_L, n_M, n_H are the number of channels of the LFT, MFT and HFT, respectively, and $n = n_L + n_M + n_H$ is the total number of channels. Taking the average of the previous equation, it is obtained:

$$\langle\delta_r\rangle \approx \frac{4\left(\sum_{l=2}^{l_{max}} \frac{E_l B_l^{GW}}{Var(B_l)}\right)}{\sum_{l=2}^{l_{max}} \frac{(B_l^{GW})^2}{Var(B_l)}} \left(\begin{array}{r} \sigma_g^2 + w_L^2\sigma_L^2 + w_M^2\sigma_M^2 + w_H^2\sigma_H^2 + 2w_L C_{gL} + 2w_M C_{gM} + 2w_H C_{gH} \\ +2w_L w_M C_{LM} + 2w_L w_H C_{LH} + 2w_M w_H C_{MH} \end{array}\right)$$

where $w_L = \sum_{i=1}^{n_L} w_i$, $w_M = \sum_{i=n_L+1}^{n_M} w_i$, $w_H = \sum_{i=n_M+1}^{n_H} w_i$ (note that $w_g = \sum_{i=1}^{n} w_i = 1$).

As we did for the relative angles, we now make the same assumption that of the terms in the sum of δ_r add evenly on average. Then, it follows that $\sigma_g = c$, $\sigma_L = c w_L^{-1}$, $\sigma_M = c w_M^{-1}$, $\sigma_H = c w_H^{-1}$. Considering these relations and replacing the correlations by their corresponding correlation coefficients, we finally have:

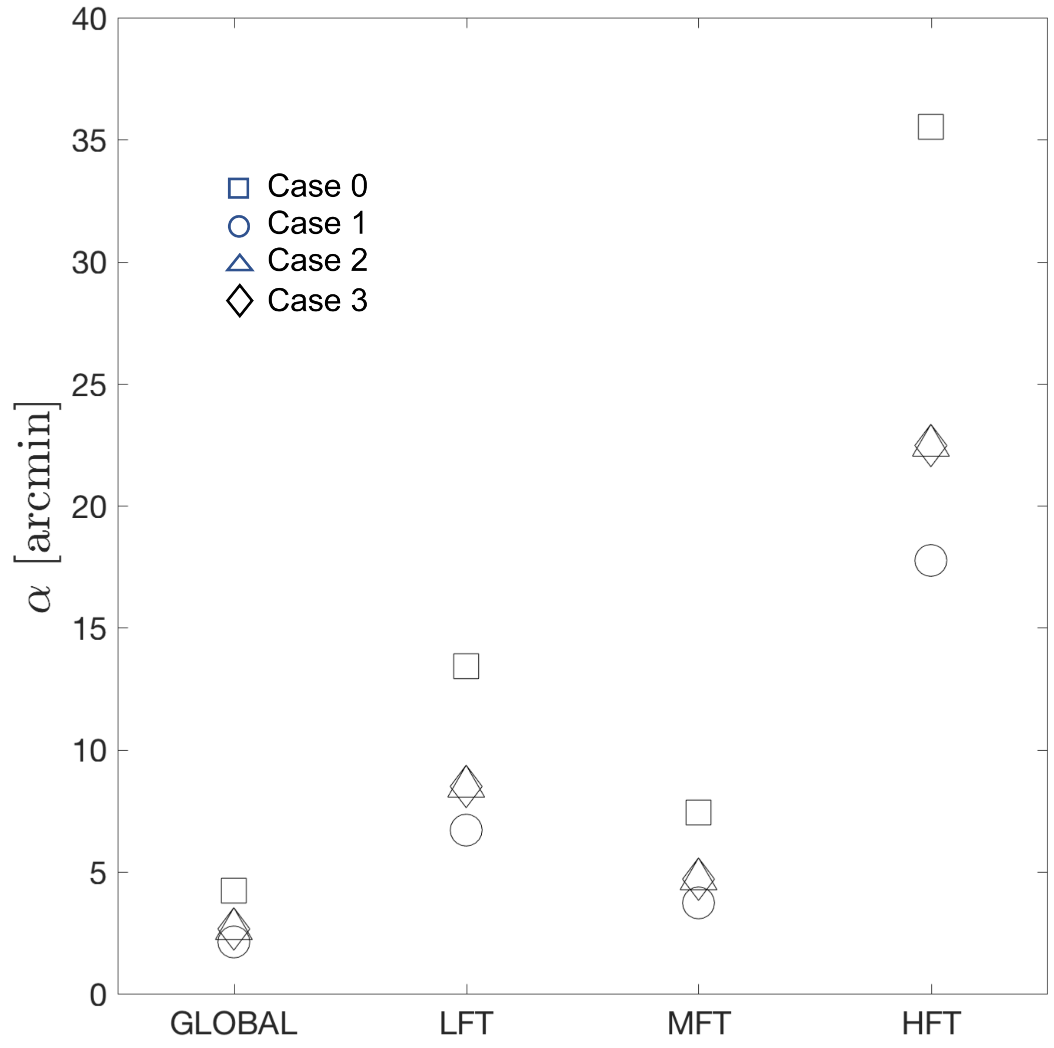
$$\langle \delta_r \rangle \approx 2c^2 \frac{4\left(\sum_{l=2}^{l_{max}} \frac{E_l B_l^{GW}}{Var(B_l)}\right)}{\sum_{l=2}^{l_{max}} \frac{(B_l^{GW})^2}{Var(B_l)}} \left(2 + \rho_{gL} + \rho_{gM} + \rho_{gH} + \rho_{LM} + \rho_{LH} + \rho_{MH}\right)$$

Requirements on the accuracy of those four offset angles can be determined by assigning e.g. 1% of the systematics budget to the bias produced on r.

As an example, we obtain the requirements for the four absolute angles in the following three correlation cases:

- Case 0: No correlations.
- Case 1: The four offsets are fully correlated.
- Case 2: The global offset is uncorrelated with any of the three focal plane ones, and the latter ones are fully correlated.
- Case 3: The global offset is fully correlated with any of the three focal plane ones, and the latter are uncorrelated. It happens that Case 3 provides the same requirements than case 2.

Requirements for absolute angles



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