Requirements for future CMB satellite missions: photometric and band-pass response calibration

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CMB systematics and calibration focus workshop
Virtually @ Kavli IPMU 30/11-3/12/2020

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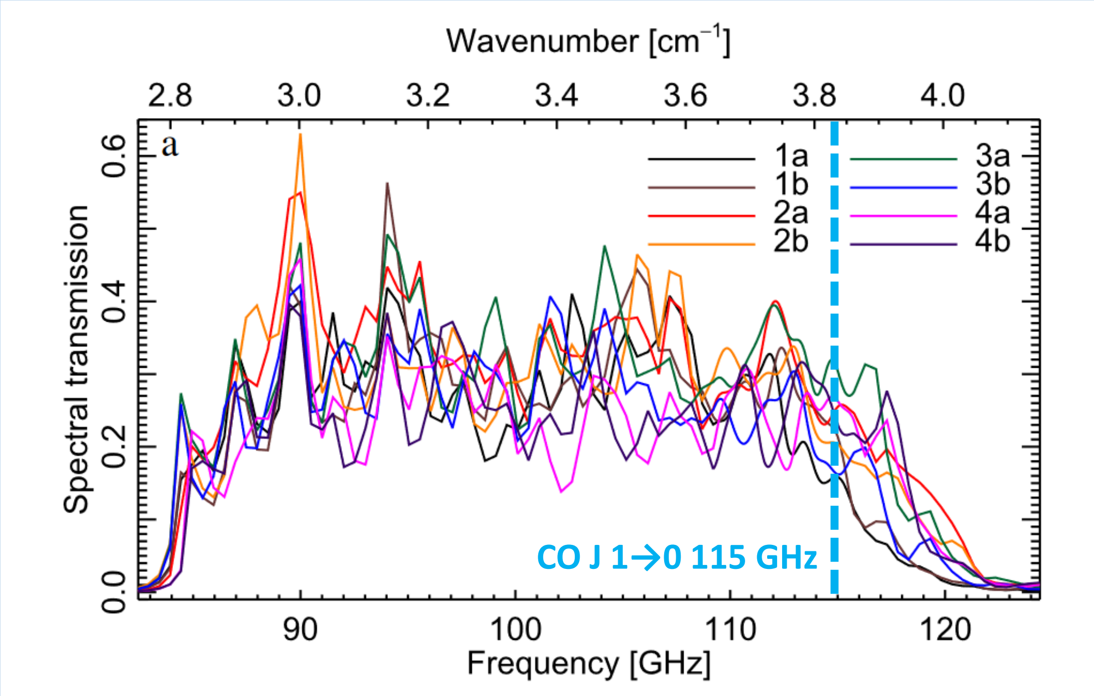
Outline/Summary:

- Photometric calibration and bandpass requirements for future satellite mission:
  - Based on the recently published T. Ghigna *et al* JCAP11(2020)030 (with T. Matsumura, G. Patanchon, H. Ishino and M. Hazumi)
  - Formalism w/ and w/o HWP
  - Example analysis applied to LiteBIRD:
    - **requirements driven by the high frequency channels** (see also Max's talk about SO)
    - **gain requirements below percent level**
    - **resolution requirement $\lesssim$ 1 GHz**

- CO line contamination:
  - Contamination level w/ and w/o HWP
  - Future missions: to notch or not to notch?

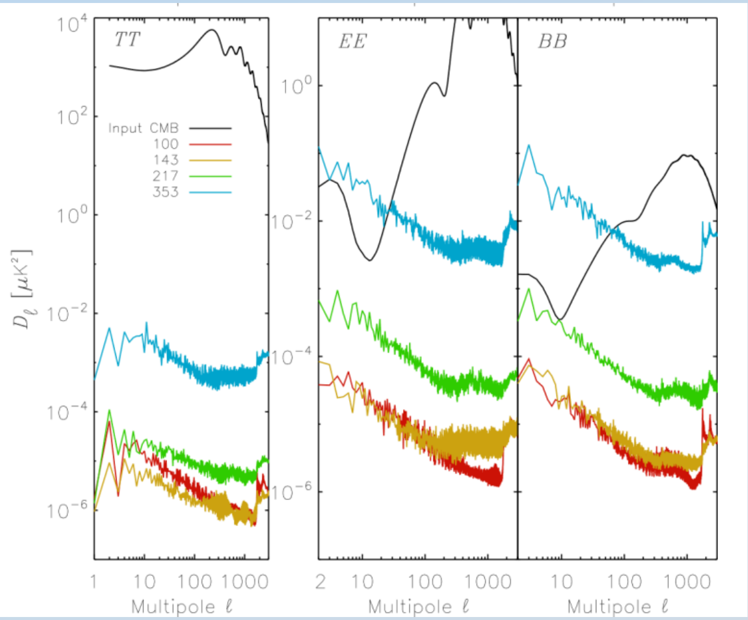
Why bandpass?

100 GHz channels

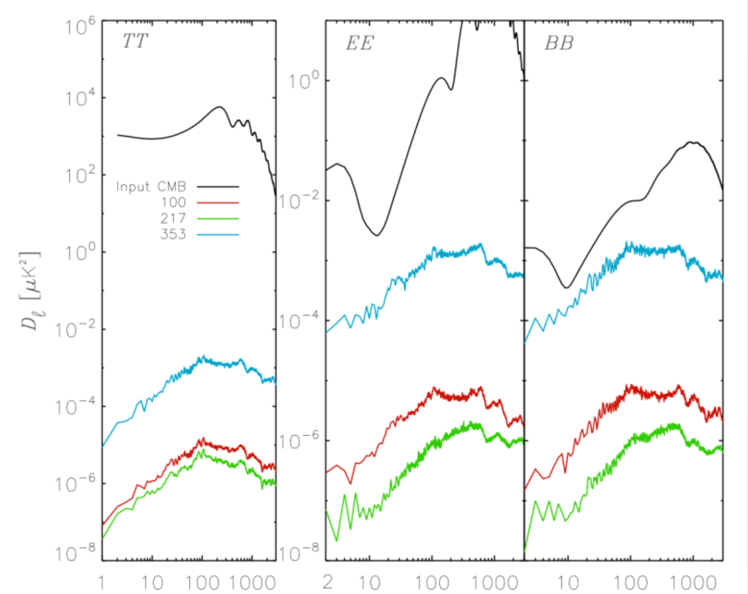


Planck 2013 results. IX. HFI spectral response

Dust leakage

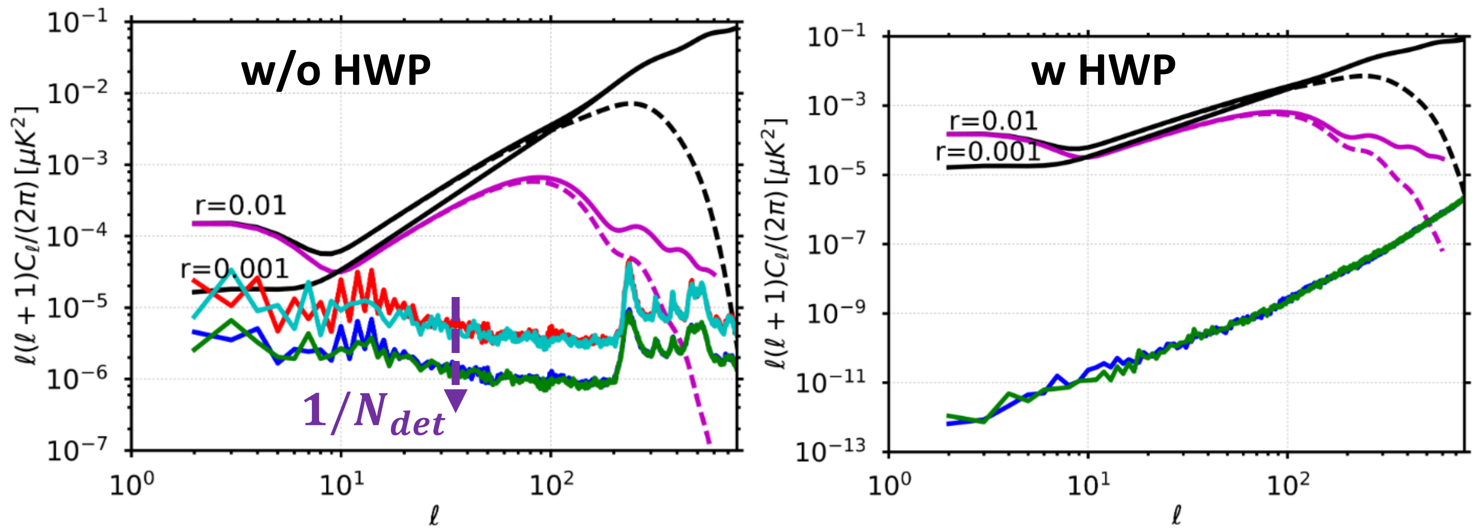


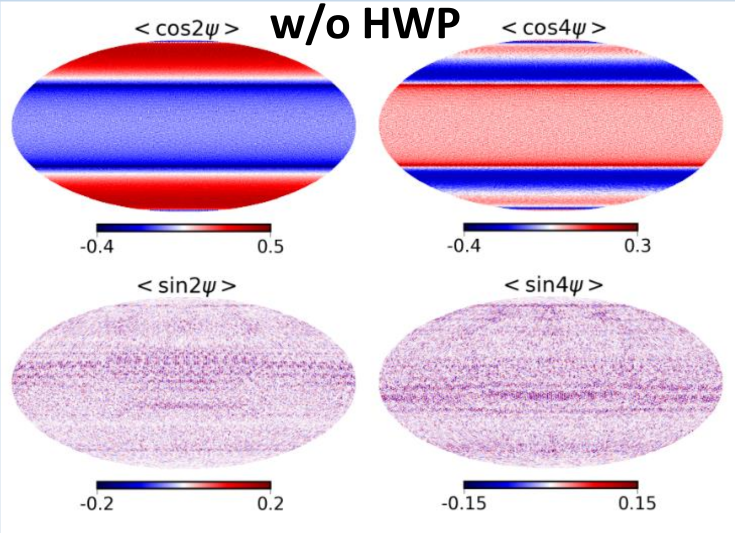
CO leakage



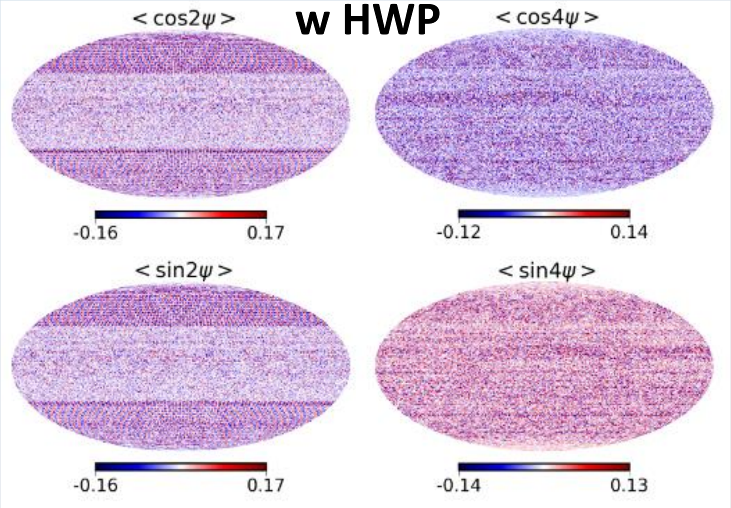
Planck 2018 results. III. High Frequency Instrument data processing and frequency maps

Bandpass - I→P





Hoang et al. 2017



- Galactic leakage reduced by the number of detectors as $\mathbf{1/N_{det}}$ in ℓ-space if we assume uncorrelated uncertainty of the transmission around a mean value.
- Galactic leakage suppressed by an ideal continuously rotating HWP.

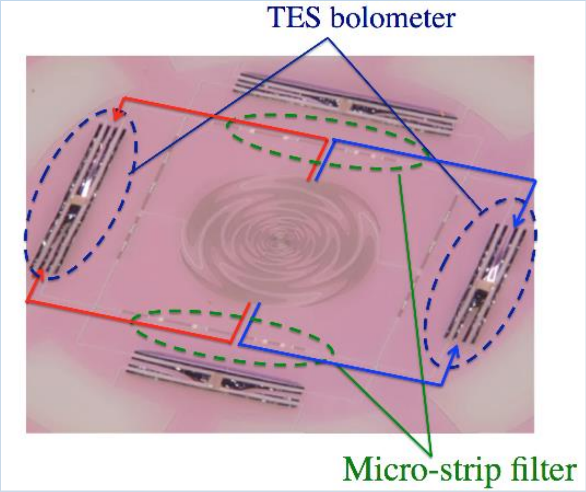
$$\begin{pmatrix}\delta Q\\ \delta U\end{pmatrix} = \delta I_{fg}\begin{pmatrix}\langle cos\ 2\varphi\rangle\\ \langle sin\ 2\varphi\rangle\end{pmatrix}$$

Differential systematics

Simple signal model for polarization sensitive detector:

$$d(t) = \frac{s(t)}{2} \int d\nu \frac{\lambda^2}{\Omega_b} \boldsymbol{G}(\boldsymbol{\nu}) \int d\Omega B(\nu, \Omega) \{ I(\nu) + \varepsilon(\nu) [Q(\nu) \cos 2\varphi + \dots$$

$$\dots + U(\nu) \sin 2\varphi] \}$$



Polarbear 2 - Inoue et al. 2016

Single sky pixel signal for each detector averaged over overall mission:

$$d_a = I + Q\langle \cos 2\varphi_a \rangle + U \langle \sin 2\varphi_a \rangle + S$$
$$d_b = I + Q\langle \cos 2\varphi_b \rangle + U \langle \sin 2\varphi_b \rangle$$

In case of a mismatch between two orthogonal detectors

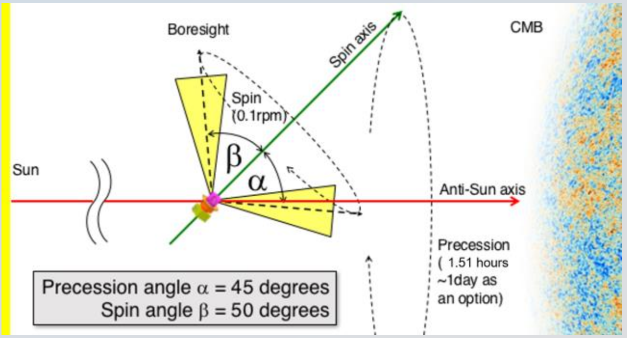
Demodulate to reconstruct sky signal ($\varphi_a = \varphi_b + {}^{\pi}/_{2}$):

$$\Delta d = \frac{1}{2}(d_a - d_b) = Q\langle \cos 2\varphi_a \rangle + U \langle \sin 2\varphi_a \rangle + S$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}^{-1}$$

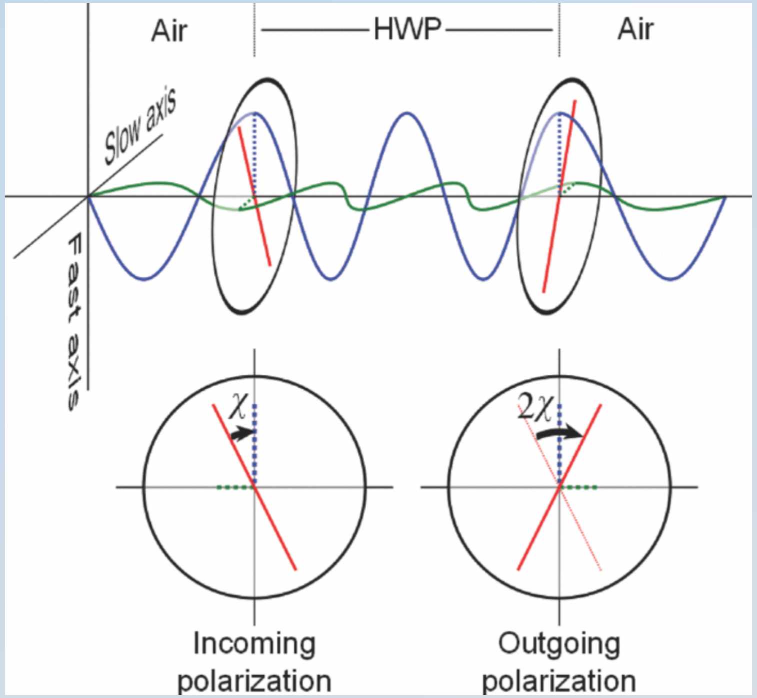
Optimal scanning strategy and focal plane design

Sky signal: $\begin{pmatrix} S \\ Q \\ U \end{pmatrix} = \begin{pmatrix} 1 & \langle cos\, 2\varphi \rangle & \langle sin\, 2\varphi \rangle \\ \langle cos\, 2\varphi \rangle & \langle cos^2\, 2\varphi \rangle & \langle sin\, 2\varphi \rangle \langle cos\, 2\varphi \rangle \\ \langle sin\, 2\varphi \rangle & \langle sin\, 2\varphi \rangle \langle cos\, 2\varphi \rangle & \langle sin^2\, 2\varphi \rangle \end{pmatrix}^{-1} \begin{pmatrix} \Delta d \\ \Delta d \langle cos\, 2\varphi \rangle \\ \Delta d \langle sin\, 2\varphi \rangle \end{pmatrix}$



Systematics – mitigation with HWP

Kusaka et al. 2014



- Half-Wave Plate is becoming a popular solution to mitigate systematics by modulating the sky polarized signal.
- Ground experiments: reduce 1/f noise due to atmosphere → possible to increase sensitivity at low ℓ.
- Instrumental polarization can be suppressed if HWP is first optical element.
- **No need to differentiate detectors to reconstruct Q and U → detector mismatch effects.**

Modulating function

Modulated signal:

$$d = I + \varepsilon \, Re\{(Q + iU)e^{-i4\chi}\}$$

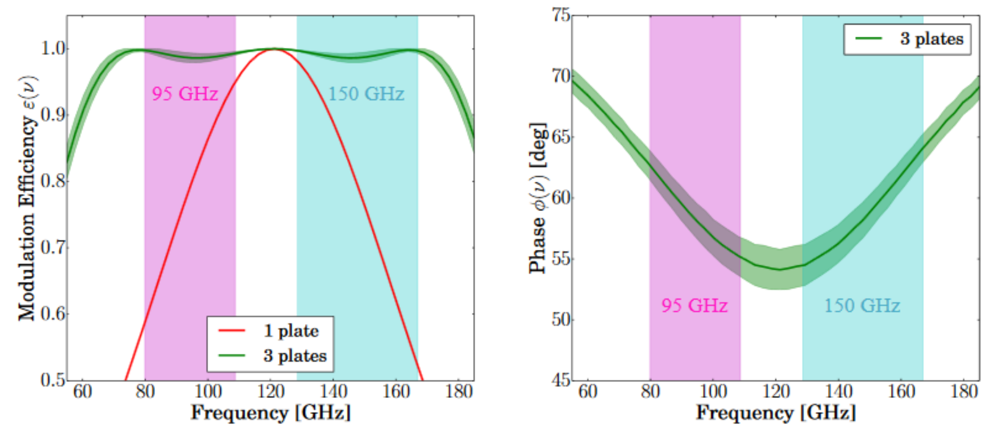
Need to track χ during the observation!

Demodulated signal obtained by multiplying the modulated signal by its complex conjugate.

Systematics – mitigation with HWP

Single sky pixel signal **with HWP** for each detector

averaged over overall mission:
$d = I + Q\langle\cos(4\chi - 2\varphi)\rangle + U\langle\sin(4\chi - 2\varphi)\rangle$



Hill et al. 2016

Demodulate to reconstruct sky signal:

$$\begin{cases} d = I + Q\langle\cos(4\chi - 2\varphi)\rangle + U\langle\sin(4\chi - 2\varphi)\rangle \\ d\langle\cos(4\chi - 2\varphi)\rangle = I\langle\cos(4\chi - 2\varphi)\rangle + Q\langle\cos^2(4\chi - 2\varphi)\rangle + U\langle\sin(4\chi - 2\varphi)\rangle\langle\cos(4\chi - 2\varphi)\rangle \\ d\langle\sin(4\chi - 2\varphi)\rangle = I\langle\sin(4\chi - 2\varphi)\rangle + Q\langle\sin(4\chi - 2\varphi)\rangle\langle\cos(4\chi - 2\varphi)\rangle + U\langle\sin^2(4\chi - 2\varphi)\rangle \end{cases}$$

Sky signal:

$$\begin{pmatrix} I \\ Q \\ U \end{pmatrix} = \begin{pmatrix} 1 & \langle cos(4\chi - 2\varphi)\rangle & \langle sin(4\chi - 2\varphi)\rangle \\ \langle cos(4\chi - 2\varphi)\rangle & \langle cos^2(4\chi - 2\varphi)\rangle & \langle sin(4\chi - 2\varphi)\rangle\langle cos(4\chi - 2\varphi)\rangle \\ \langle sin(4\chi - 2\varphi)\rangle & \langle sin(4\chi - 2\varphi)\rangle\langle cos(4\chi - 2\varphi)\rangle & \langle sin^2(4\chi - 2\varphi)\rangle \end{pmatrix}^{-1} \begin{pmatrix} d \\ d\langle cos(4\chi - 2\varphi)\rangle \\ d\langle sin(4\chi - 2\varphi)\rangle \end{pmatrix}$$

Systematics – bandpass

- Let's focus only on bandpass: $d = \int d\nu\, \boldsymbol{G}(\boldsymbol{\nu})\{I(\nu) + \varepsilon(\nu)[Q(\nu)cos2\varphi(\nu) + U(\nu)sin2\varphi(\nu)]\}$
- Sky signal: **CMB** + **fg** (**dust, synchrotron, …**) $S = (I, Q, U) \longmapsto S = S_{cmb} + S_d + S_s + \cdots$
- Integrating and writing sky components explicitly (photometric calibration):

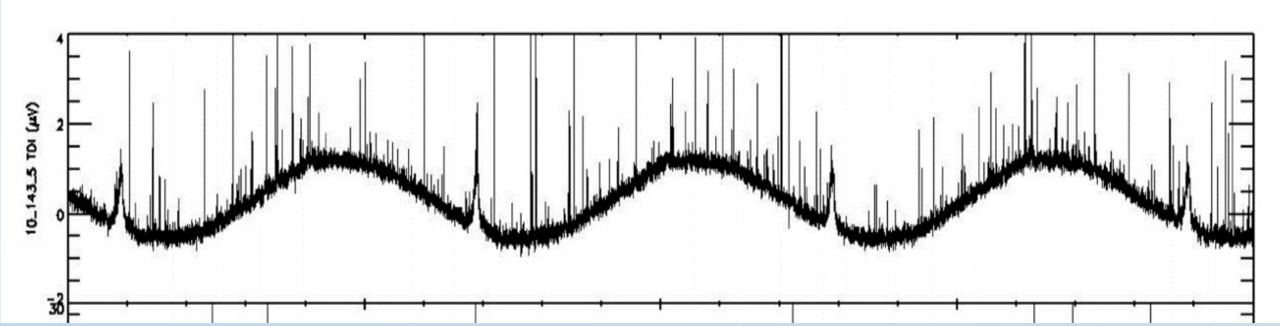
$$d(\nu_0) = \boldsymbol{g}(I_{cmb}(\nu_0) + \boldsymbol{\gamma_d} I_d(\nu_0) + \boldsymbol{\gamma_s} I_s(\nu_0) + G(\nu_{co})I_{co}) + \cdots$$

$$\ldots + \boldsymbol{g}\varepsilon[(Q_{cmb}(\nu_0) + \boldsymbol{\gamma_d} Q_d(\nu_0) + \boldsymbol{\gamma_s} Q_s(\nu_0) + G(\nu_{co})Q_{co})cos2\varphi + \cdots$$

$$\ldots + (U_{cmb}(\nu_0) + \boldsymbol{\gamma_d} U_d(\nu_0) + \boldsymbol{\gamma_s} U_s(\nu_0) + G(\nu_{co})U_{co})sin2\varphi]$$

ν_0 =effective central frequency of the given band

Calibration in space



Planck early results. VI. The High Frequency Instrument data processing

- Dipole calibration: ~3 mK signal due to motion of the Sun with respect to LSS.
  - We obtain **g** factor by fitting the data (it does not depend on bandpass knowledge).
  - Sensitive mostly to detector stability over dipole modulation period.

- Color correction (dust and synchrotron have different spectral shapes):

$$\boldsymbol{\gamma_d} = \frac{I_{cmb}(\nu_0)}{I_d(\nu_0)} \frac{\int d\nu\, G(\nu) I_d(\nu)}{\int d\nu\, G(\nu) I_{cmb}(\nu)};\ \boldsymbol{\gamma_s} = \frac{I_{cmb}(\nu_0)}{I_s(\nu_0)} \frac{\int d\nu\, G(\nu) I_s(\nu)}{\int d\nu\, G(\nu) I_{cmb}(\nu)} \Longrightarrow \delta\gamma = \frac{\gamma_{\Delta\nu} - \gamma_0}{\gamma_0}$$

Depends on the bandpass shape resolution $\Delta\nu$. **Bandpass measurement requirements.**

- If we don't calibrate on dipole we need to rethink this definition given the spectrum of the calibrator. If the calibrator is not known as well as the dipole the uncertainty of g will likely dominate the color correction uncertainty.

Bandpass Requirements

$$d(\nu_0) = \boldsymbol{g}(I_{cmb}(\nu_0) + \boldsymbol{\gamma_d} I_d(\nu_0) + \boldsymbol{\gamma_s} I_s(\nu_0) + G(\nu_{co}) I_{co}) + \cdots$$
$$\ldots + \boldsymbol{g}\varepsilon[(Q_{cmb}(\nu_0) + \boldsymbol{\gamma_d} Q_d(\nu_0) + \boldsymbol{\gamma_s} Q_s(\nu_0) + G(\nu_{co}) Q_{co}) cos2\varphi + \cdots$$
$$\ldots + (U_{cmb}(\nu_0) + \boldsymbol{\gamma_d} U_d(\nu_0) + \boldsymbol{\gamma_s} U_s(\nu_0) + G(\nu_{co}) U_{co}) sin2\varphi]$$

- $\boldsymbol{I \rightarrow P}$ **leakage** (studied by **Hoang et al. 2017** for CMB channels):
  - **Without HWP**. Foregrounds I to P leakage is non negligible. Bandpass measurement requirement to minimize the effect for channel 140 GHz: $\Delta\nu \sim 0.2$ GHz.
  - **With HWP**. Foregrounds I to P leakage is suppressed by efficiently choosing the scanning strategy and the presence of a continuously rotating polarization modulator.

- $\boldsymbol{P \rightarrow P}$ **leakage or Pol. miscalibration**:
  - **Without HWP**. Dominant term is I to P leakage, so requirement is driven by previous point.
  - **With HWP.** Next slides…

Bandpass Requirements

Sky model = CMB + Dust + Synchrotron (constant spectral parameters) + **noise**.

$$\text{Ideally: } \begin{pmatrix} I \\ Q \\ U \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}^{-1} \begin{pmatrix} I_{in} \\ 2Q_{in} \\ 2U_{in} \end{pmatrix} = \begin{pmatrix} I_{in} \\ Q_{in} \\ U_{in} \end{pmatrix}$$

$$\text{Calibration uncertainty: } \begin{pmatrix} I \\ Q \\ U \end{pmatrix}_i = [\mathcal{G}(\mu = 1, \sigma = \Delta_g)] \begin{pmatrix} I_{in} \\ Q_{in} \\ U_{in} \end{pmatrix}_i$$

#	ν_c (GHz)	Bandwidth (Frac.)	# Detectors	Pol. sensitivity ($\mu K amin$)
1	40	12 (30%)	64	39.76
2	50	15 (30%)	64	25.76
3	60	14 (23%)	64	20.69
4	68	16 (23%)	208	12.72
5	78	18 (23%)	208	10.39
6	89	20 (23%)	208	8.95
7	100	23 (23%)	530	6.43
8	119	36 (30%)	632	4.30
9	140	42 (30%)	530	4.43
10	166	50 (30%)	488	4.86
11	195	59 (30%)	640	5.44
12	235	71 (30%)	254	9.72
13	280	84 (30%)	254	12.91
14	337	101 (30%)	254	19.07
15	402	92 (23%)	338	43.53

Sugai et al 2020

What is the color correction accuracy we need to achieve in order to recover the tensor-to-scalar ratio with minimal bias?

- **Target**: $\delta_r \leq 5.7 \times 10^{-6}$ (Small compared to the target sensitivity $\sigma_r \sim 0.001$). **See Hirokazu's talk.**
- If there is no correlation among bandpass uncertainties we can then find the single detector requirement as $\delta_g = \Delta_g \sqrt{N_i}$, where N_i is the number of detectors in frequency channel *i*.

Bandpass Requirements

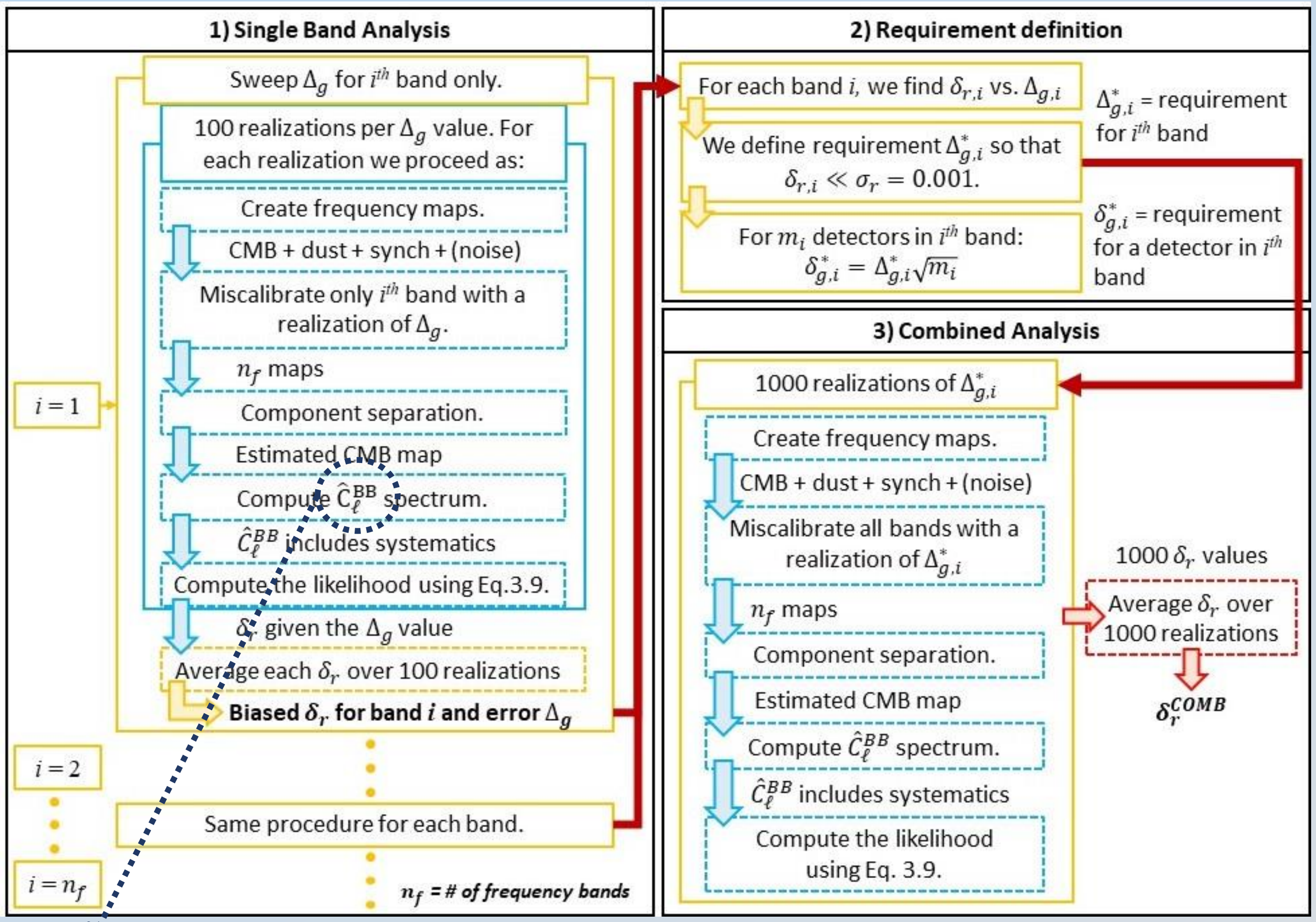
1

Mis-calibrate one frequency channel per time

Parametric component separation (FgBuster thanks to J. Errard and D. Poletti)

2

Find requirement per band and per detector



3

Produce mis-calibrated maps at all frequencies using the requirements found at step **2**

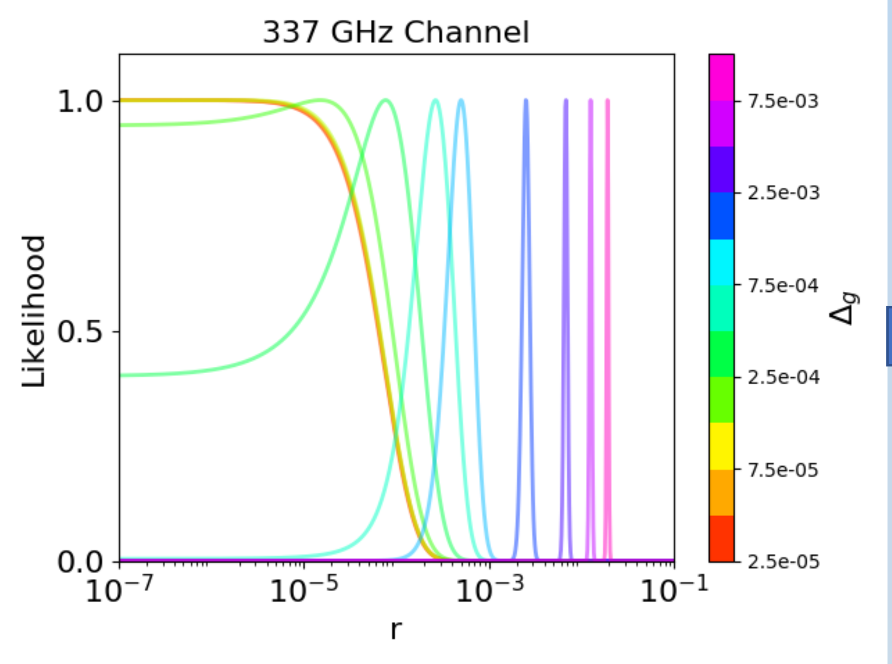
Re-run the analysis to determine the combined bias

Likelihood: $-2ln\mathcal{L}(r|\hat{C}_\ell^{BB}) = (2\ell+1)f_{sky}\left[\frac{\hat{C}_\ell^{BB}}{rC_\ell^{GW}+C_\ell^{L}+N_\ell^{BB}} + \ln\left(rC_\ell^{GW}+C_\ell^{L}+N_\ell^{BB}\right)\right] \Rightarrow ln\mathcal{L} = \sum_{\ell_{min}}^{\ell_{max}} ln\mathcal{L}(r|\hat{C}_\ell^{BB})$

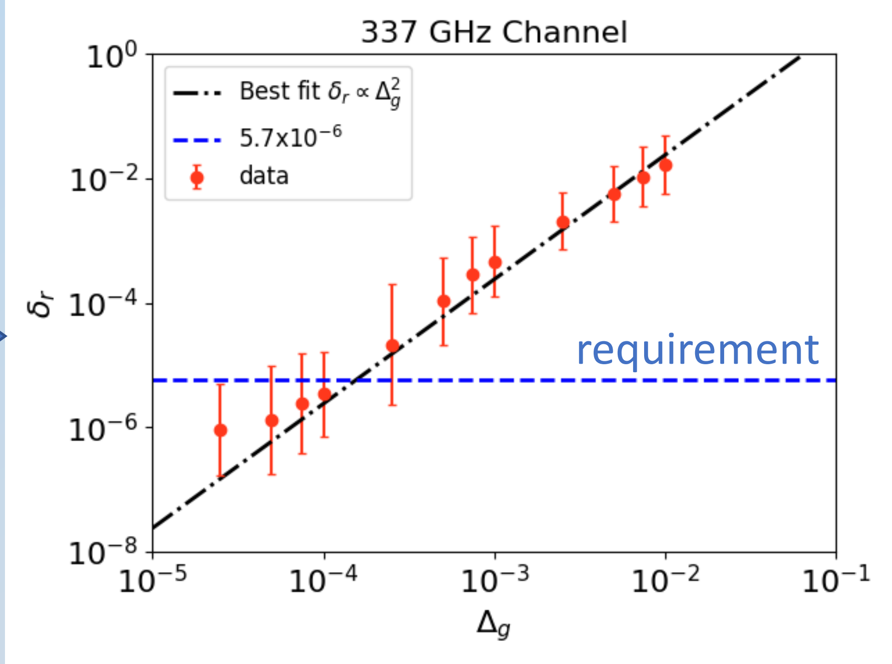
Hamimece & Lewis 2008

Results

100 realizations for a given Δ_g
(here showing only 1 to make it readable)



δ_r vs Δ_g



$\delta_r \leq 5.7 \times 10^{-6}$

ν_i (GHz)	Δ_g	δ_g	ν_i (GHz)	Δ_g	δ_g	ν_i (GHz)	Δ_g	δ_g
40	2.5×10^{-3}	$\mathbf{2.0 \times 10^{-2}}$	89	5.0×10^{-3}	$\mathbf{7.2 \times 10^{-2}}$	195	2.5×10^{-4}	$\mathbf{0.6 \times 10^{-2}}$
50	7.5×10^{-3}	$\mathbf{6.0 \times 10^{-2}}$	100	1.0×10^{-3}	$\mathbf{2.3 \times 10^{-2}}$	235	5.0×10^{-4}	$\mathbf{0.8 \times 10^{-2}}$
60	7.5×10^{-3}	$\mathbf{6.0 \times 10^{-2}}$	119	1.0×10^{-3}	$\mathbf{2.5 \times 10^{-2}}$	280	1.0×10^{-3}	$\mathbf{1.6 \times 10^{-2}}$
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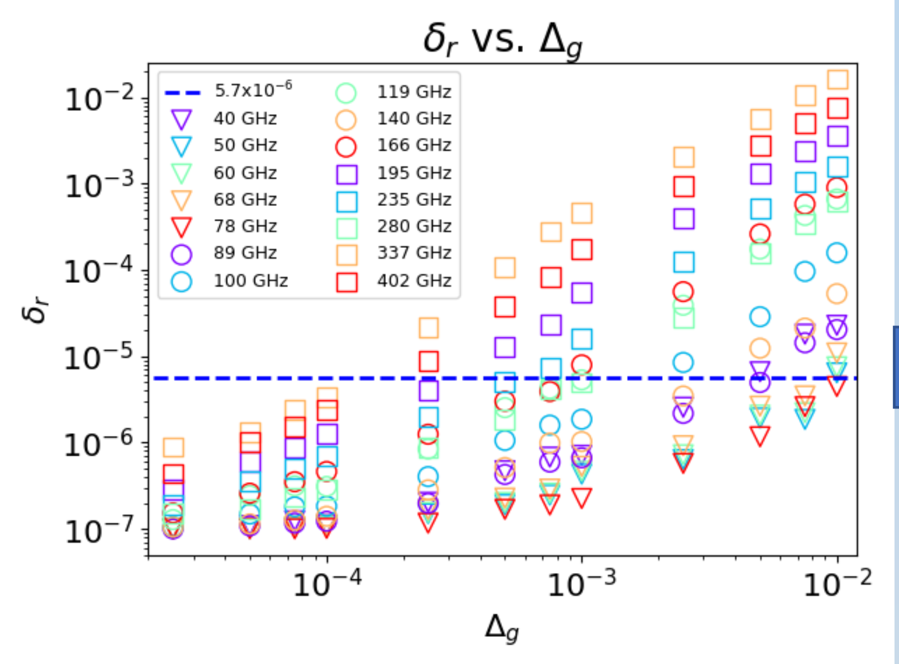
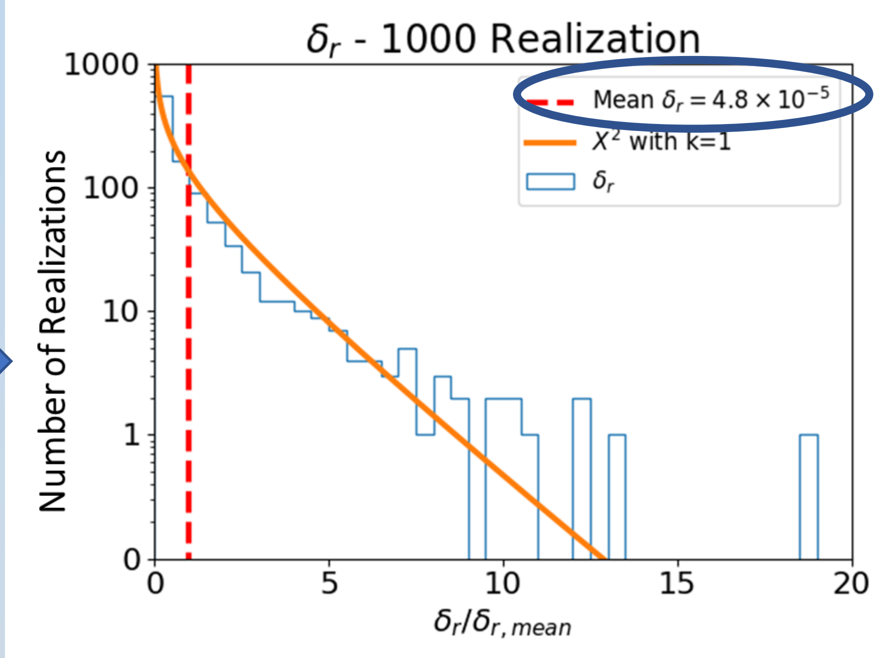
78	1.0×10^{-2}	$\mathbf{14.4 \times 10^{-2}}$	166	7.5×10^{-4}	$\mathbf{1.6 \times 10^{-2}}$	402	1.0×10^{-4}	$\mathbf{0.18 \times 10^{-2}}$

Results





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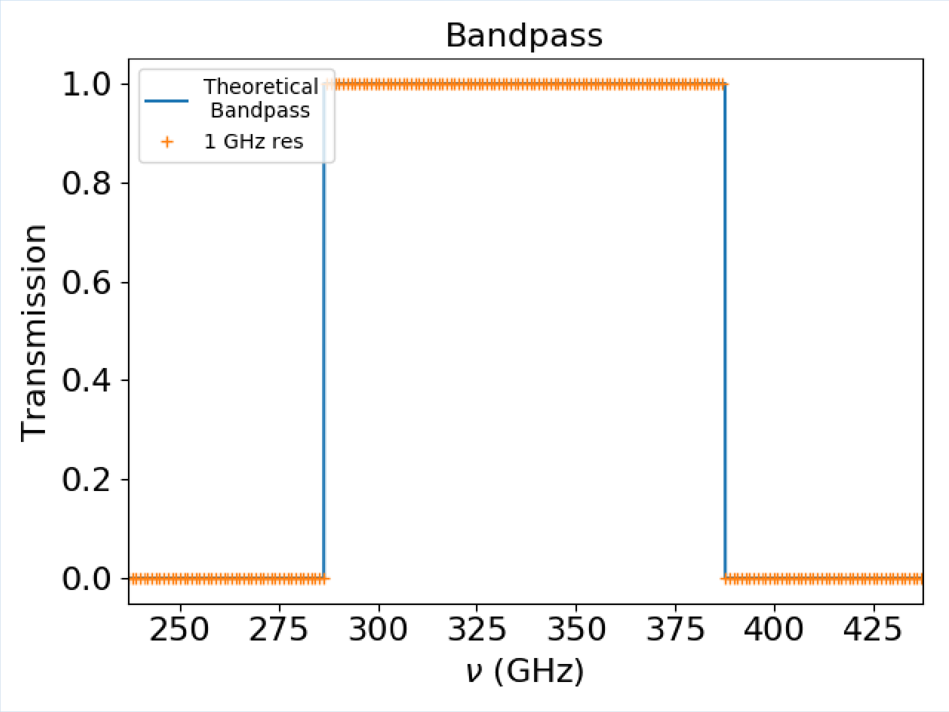
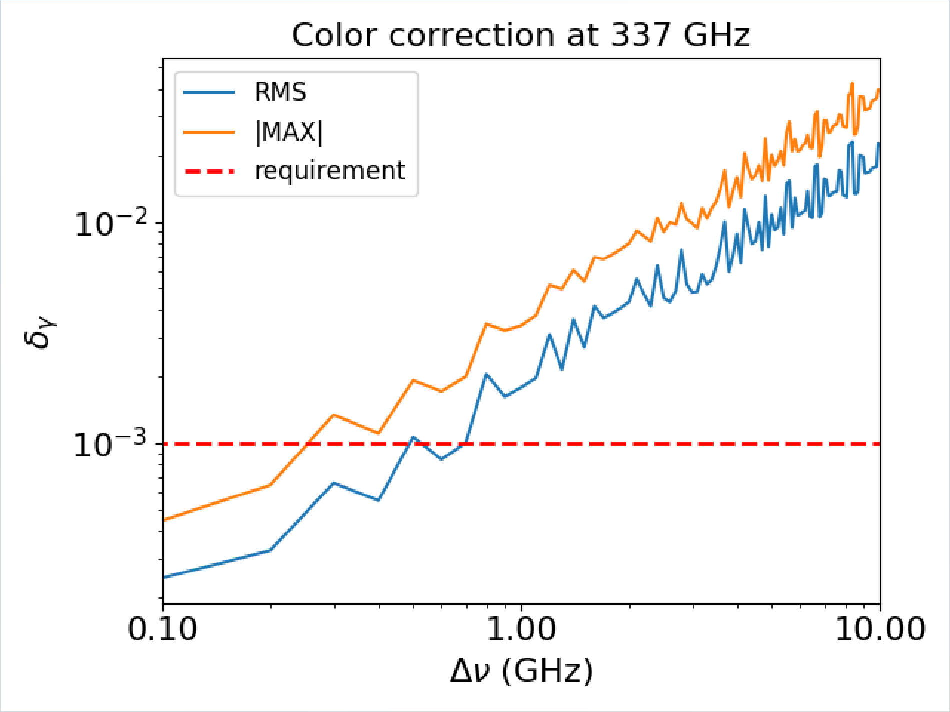
Results

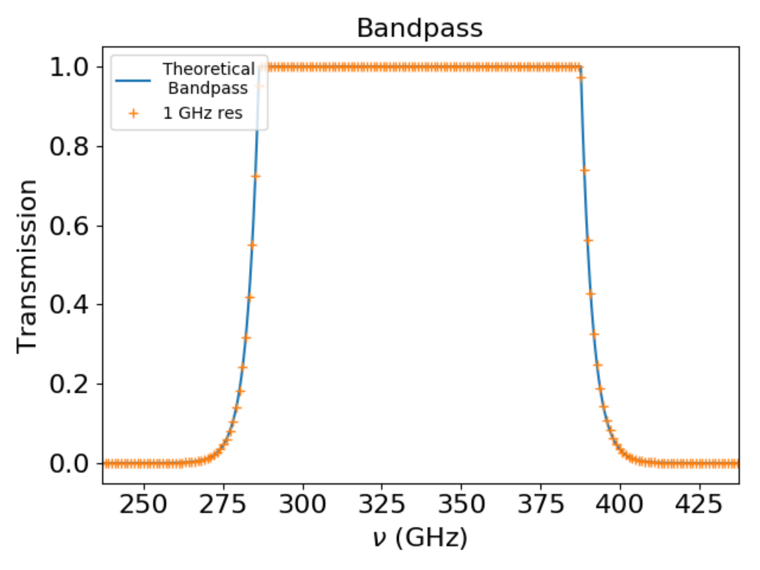
$$\gamma_d = \frac{I_{cmb}(\nu_0)}{I_d(\nu_0)} \frac{\int d\nu\, G(\nu) I_d(\nu)}{\int d\nu\, G(\nu) I_{cmb}(\nu)} \longrightarrow \delta\gamma = \frac{\gamma_{\Delta\nu} - \gamma_0}{\gamma_0}$$

Most stringent requirement is coming from channel 337 GHz (dust dominated): $\boldsymbol{\delta_g \sim 0.001}$.

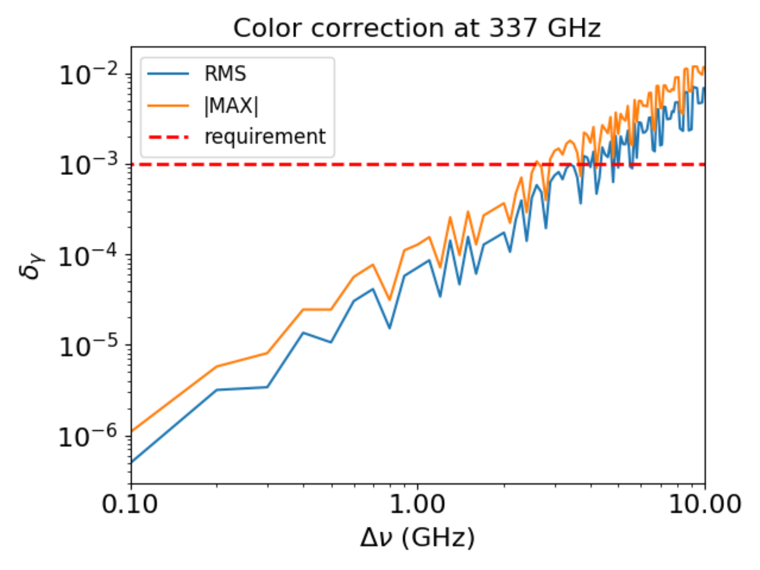




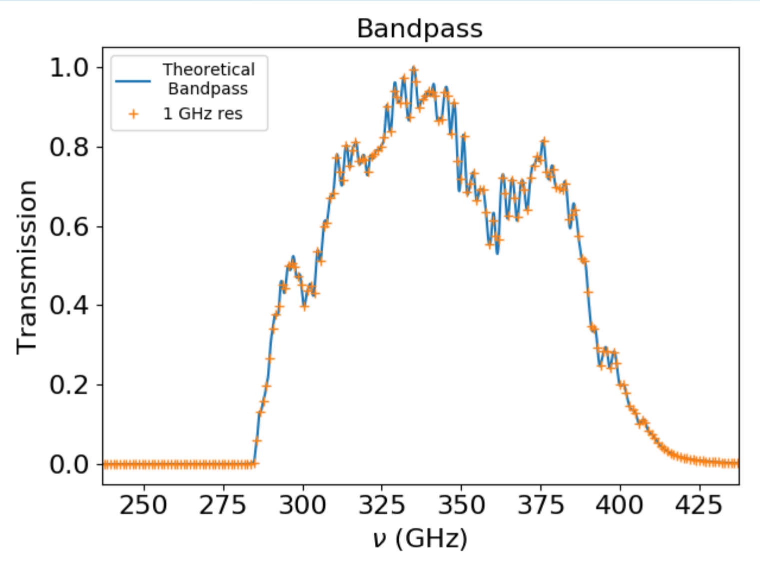
Results



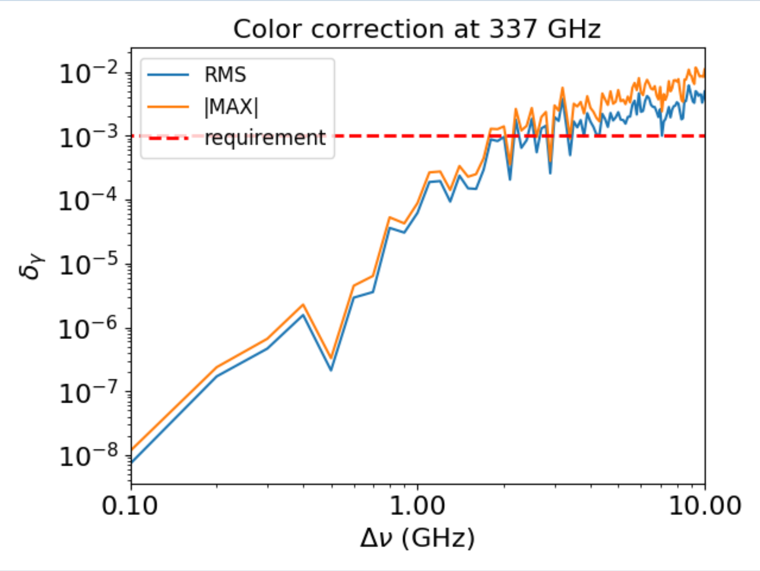
Bandpass
Theoretical Bandpass
1 GHz res
Transmission
ν (GHz)



Color correction at 337 GHz
RMS
|MAX|
requirement
δ_γ
Δν (GHz)



Bandpass
Theoretical Bandpass
1 GHz res
Transmission
ν (GHz)



Color correction at 337 GHz
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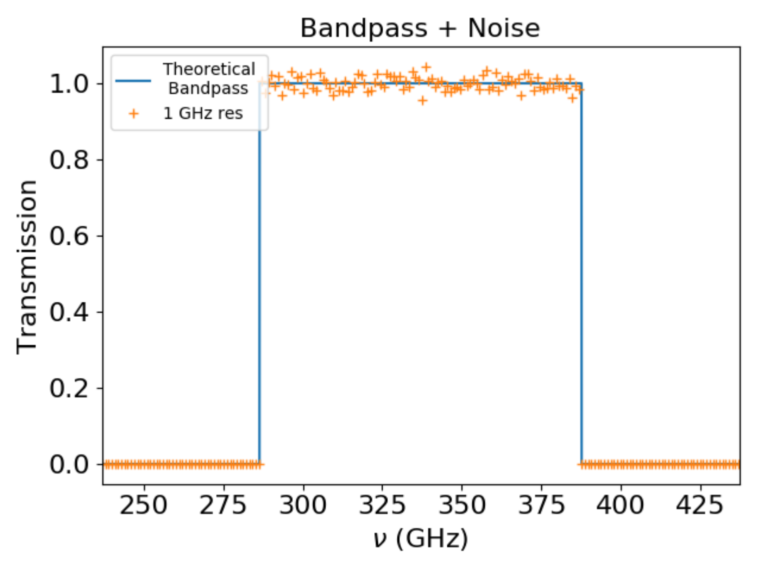
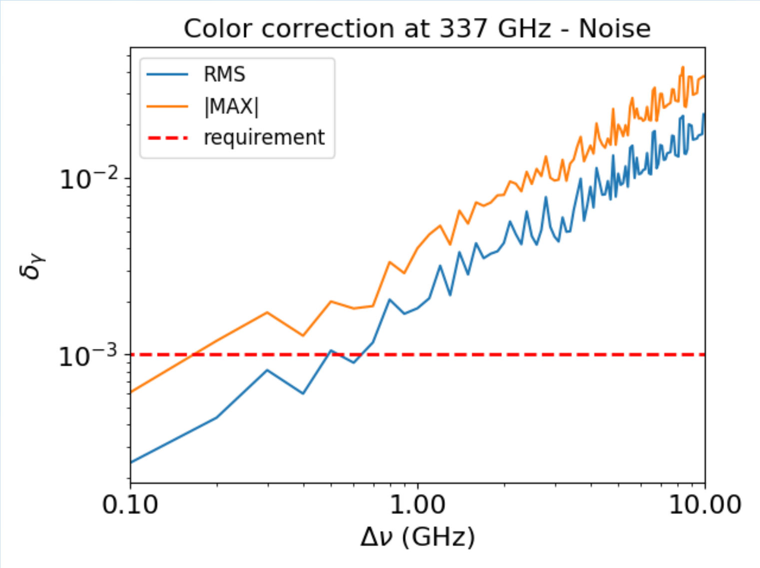
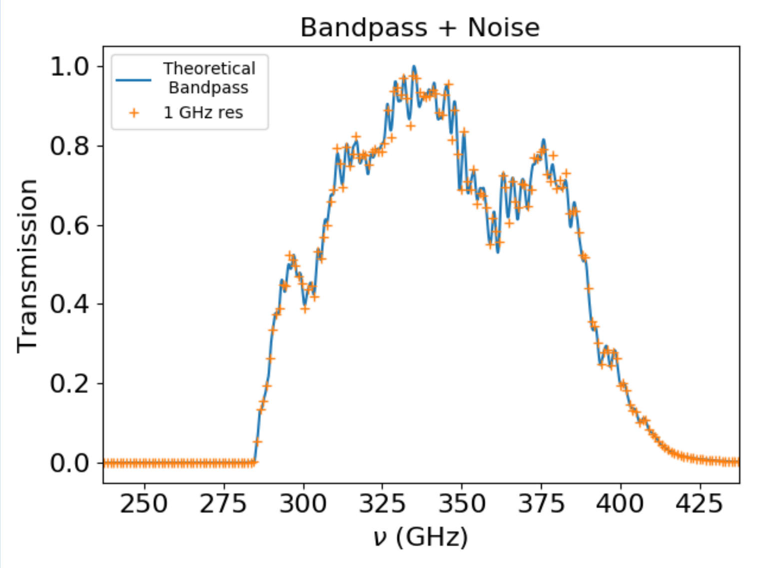
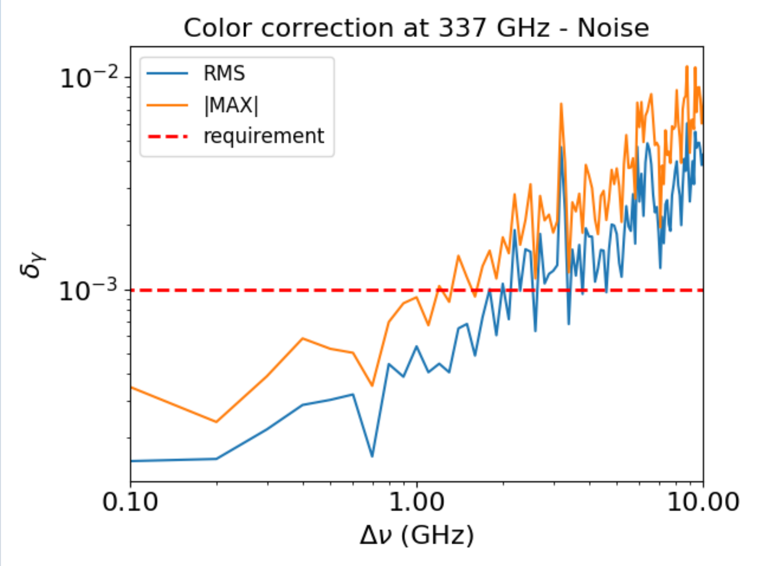
Results





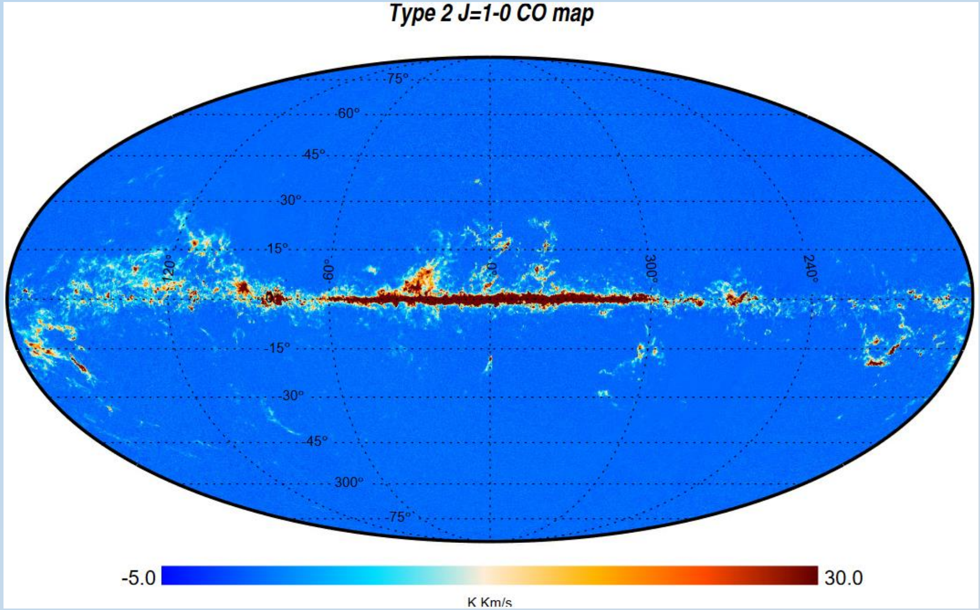
Max statistical uncertainty for FTS measurement from **F. Matsuda et al 2019** $\sim 2\%$





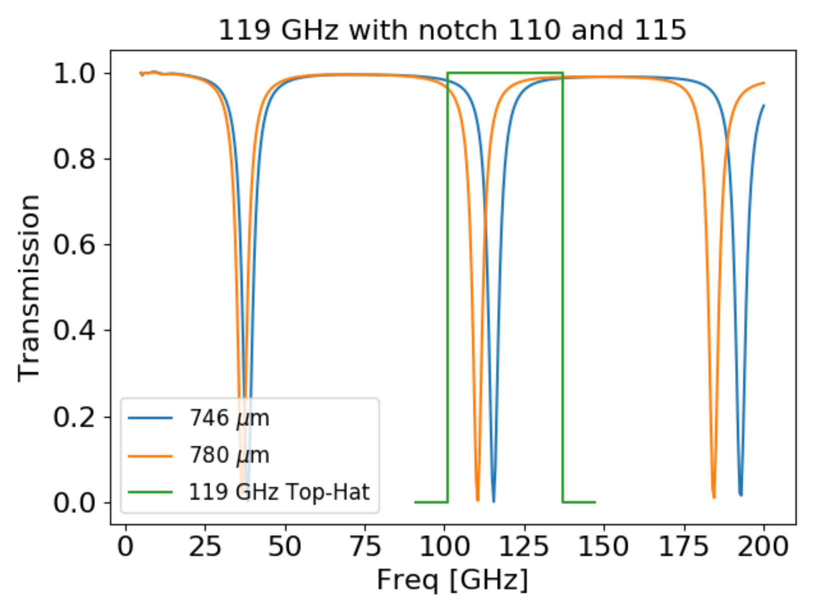
CO: should we notch?

- Carbon Monoxide line emission corresponding to rotational transitions: **J 1→0 at ∼115 GHz** (and ∼110 GHz), J 2→1 at ∼230 GHz (and ∼220 GHz), J 3→2 at ∼345 GHz (and ∼330 GHz) …



***Planck* 2013 results. XIII. Galactic CO emission**

Resonant stub to filter out the contaminated frequencies:



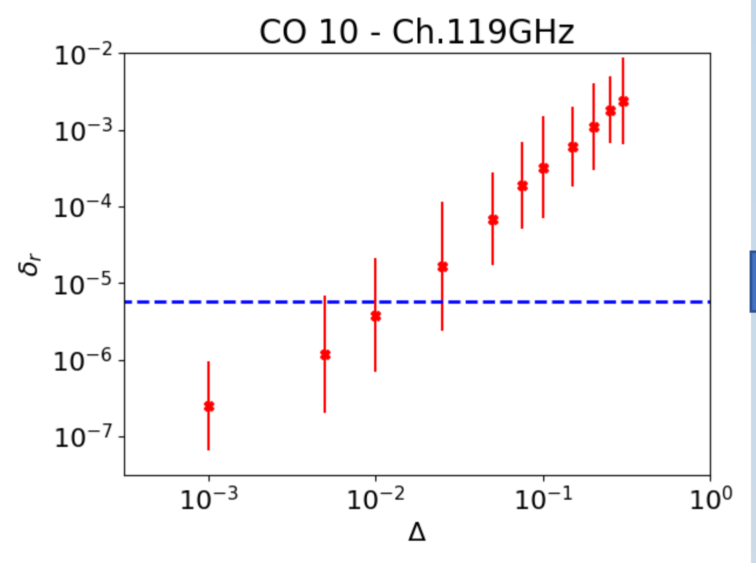
Data from **Aritoki Suzuki**

CO line notching (w/o HWP) – I to P

How much leakage can we tolerate? Do we need to notch?
I to P leakage due to bandpass mismatch, could be a big issue particularly in the case of no HWP:

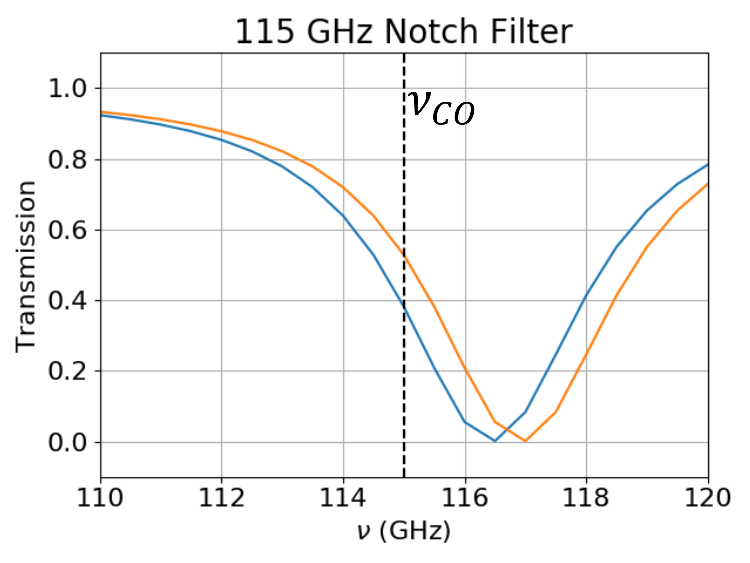
$$\Delta d = Q\cos(2\varphi) + U\sin(2\varphi) + \frac{1}{2} I\Delta \Rightarrow \begin{pmatrix} Q \\ U \end{pmatrix}_{leak} = I_{co}\Delta \begin{pmatrix} \langle\cos(2\varphi)\rangle \\ \langle\sin(2\varphi)\rangle \end{pmatrix}$$

$$\delta = \Delta \times \sqrt{N_{det}}$$



$\Delta < 1\%$

Notch Filter Requirement

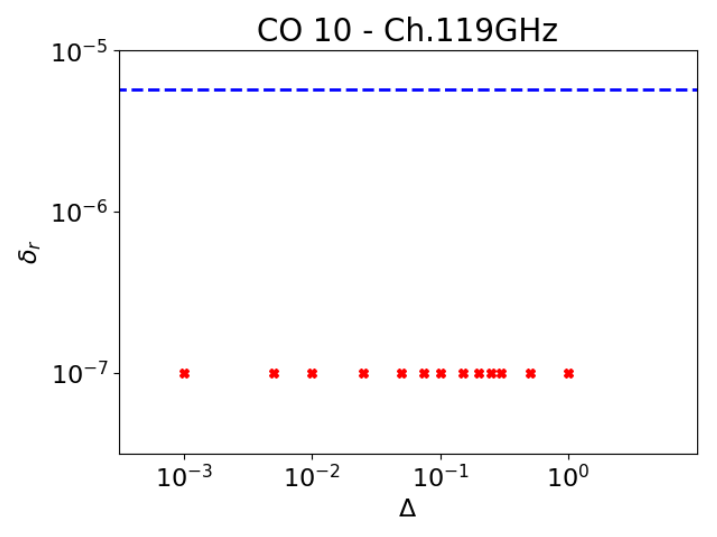


Data from **Aritoki Suzuki**

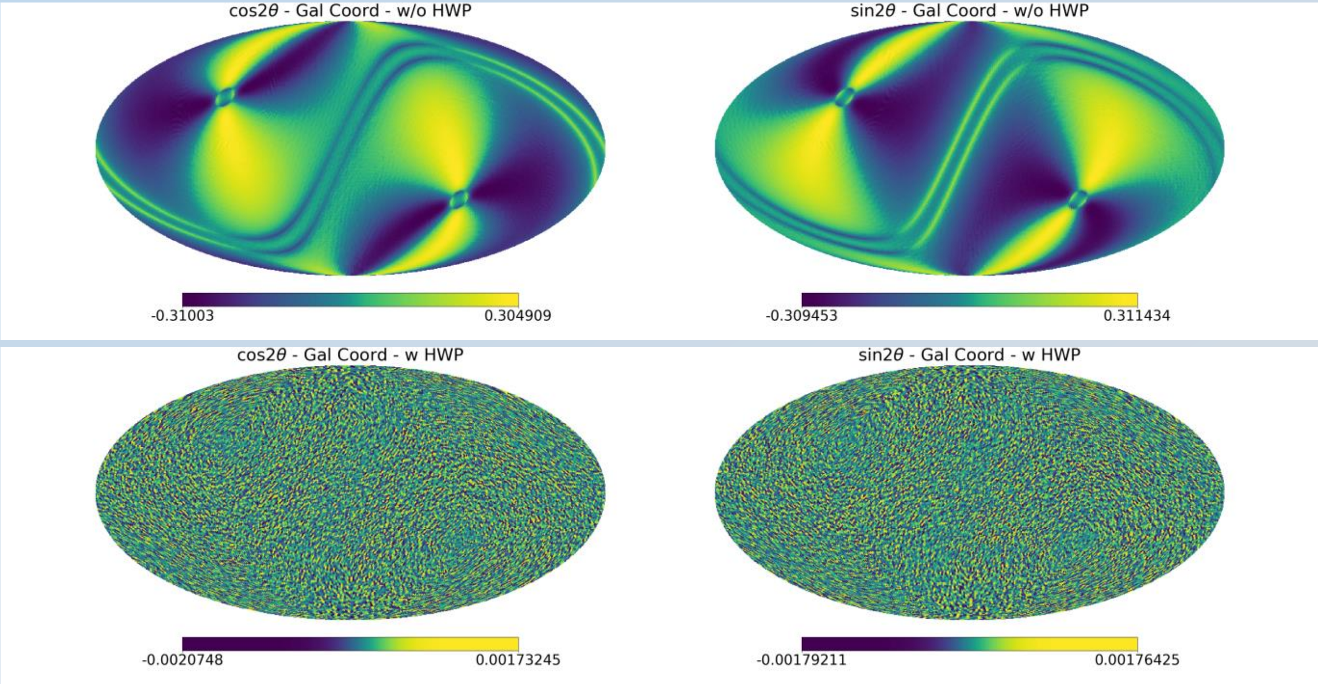
If notch filters distributed randomly around ν_{CO}.
Absolute requirement:
$\delta < 25\%$ translate to $\Delta\nu < 1.0$ GHz (half width at 25% level).
Relative requirement:
$\Delta\nu < 2.0$ GHz.

CO line notching (w HWP) – I to P

Continuously rotating HWP mitigates I to P leakage thanks to improved cross-linking.



- No effect detected up to 100% leakage.
- Requirement maybe coming from CO intrinsic polarization.



Cross linking maps in gal coordinated for LiteBIRD scan strategy with and without HWP.

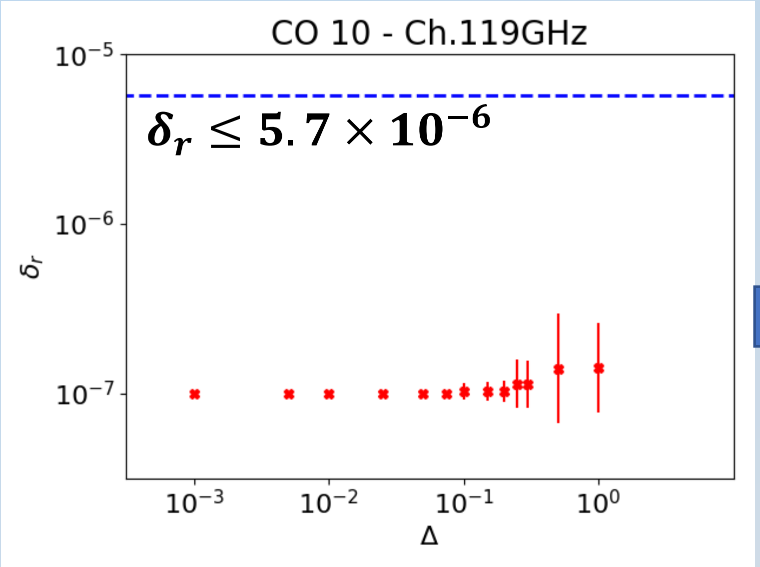
CO line notching – Intrinsic Polarization

How much leakage can we tolerate? Do we need to notch?

In first approximation no difference between wHWP and w/oHWP cases. w/oHWP case dominated by I to P leakage.

$$\begin{pmatrix} Q \\ U \end{pmatrix}_{leak} = \begin{pmatrix} \Delta Q_{CO} \langle \frac{1}{2} + \frac{1}{2}\cos 4\varphi \rangle^{-1} \langle \frac{1}{2} + \frac{1}{2}\cos 4\varphi \rangle \\ \Delta U_{CO} \langle \frac{1}{2} - \frac{1}{2}\cos 4\varphi \rangle^{-1} \langle \frac{1}{2} - \frac{1}{2}\cos 4\varphi \rangle \end{pmatrix} \approx \begin{pmatrix} \Delta Q_{CO} \\ \Delta U_{CO} \end{pmatrix}$$

Intrinsic polarization maps from Giuseppe Puglisi: (assumption $P_{CO} < 1\% \ I_{CO}$).



Notch Filter Requirement

To inject the leakage I'm using only LFT (22 Hz sampling rate) cross link maps for simplicity, considering 3 years of observation.

- **Small effect up to 100% leakage!**
- **Is the assumption of 1% polarization correct?**

Thanks for listening!!!

Back up slides

Non ideal HWP

with Kunimoto Komatsu

Retarder

$$\Gamma = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & -\sin\delta \\ 0 & 0 & \sin\delta & \cos\delta \end{pmatrix}$$

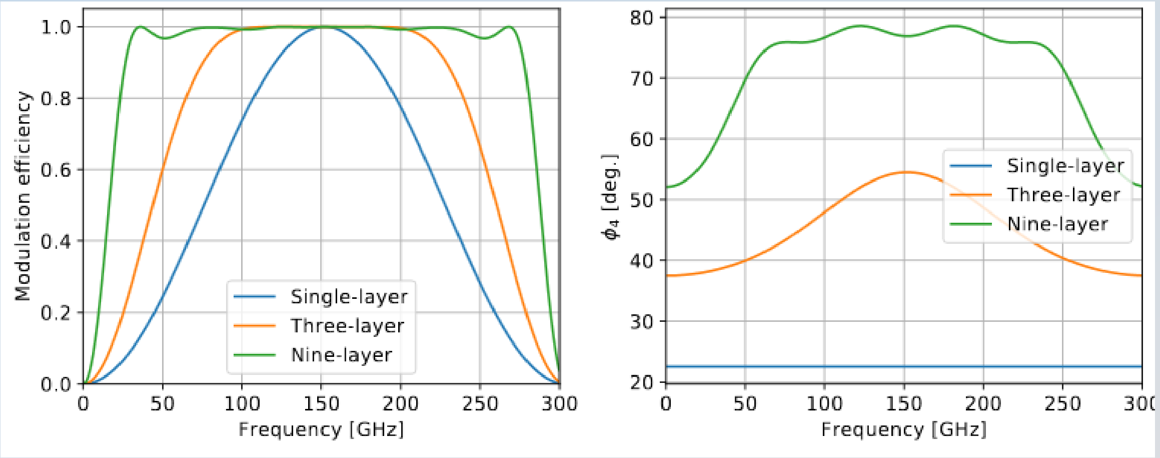
Rotator

$$R(\rho) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\rho & -\sin 2\rho & 0 \\ 0 & \sin 2\rho & \cos 2\rho & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Stack of n birefringent plates: $\Gamma_{stack} = \prod_{i=1}^{n} R(-\chi_i)\,\Gamma(\delta_i) R(\chi_i)$

$$\phi(\nu) = \frac{1}{4}\tan^{-1}\left(\frac{\Gamma_{QU} + \Gamma_{UQ}}{\Gamma_{QQ} - \Gamma_{UU}}\right)$$

- Retardance: $\delta = 2\pi \frac{\Delta n\, d\, \nu}{c}$
- For sapphire $n_o \sim 3.047$ and $n_e \sim 3.361$



Komatsu et al. 2019

Non ideal HWP

with Kunimoto Komatsu

$$d = \int d\nu\, G(\nu)\{I(\nu) + \boldsymbol{\varepsilon}(\boldsymbol{\nu})[Q(\nu)\cos(4\chi - 2\varphi + 4\boldsymbol{\phi}(\boldsymbol{\nu})) + U(\nu)\sin(4\chi - 2\varphi + 4\boldsymbol{\phi}(\boldsymbol{\nu}))]\}$$

For total intensity γ-factor does not change. However for Q and U becomes more complicated:

$$\gamma_i^{cos} = \frac{I_{cmb}(\nu_0)}{I_d(\nu_0)} \frac{\int d\nu\, G(\nu)\varepsilon(\nu)S_i(\nu)\cos 4\phi(\nu)}{\int d\nu\, G(\nu)I_{cmb}(\nu)}$$

$$\gamma_i^{sin} = \frac{I_{cmb}(\nu_0)}{I_d(\nu_0)} \frac{\int d\nu\, G(\nu)\varepsilon(\nu)S_i(\nu)\sin 4\phi(\nu)}{\int d\nu\, G(\nu)I_{cmb}(\nu)}$$

In the end we can rewrite in the usual form:

$$d(\nu_0) = I'(\nu_0) + Q'(\nu_0)\langle\cos(4\chi - 2\varphi)\rangle + U'(\nu_0)\langle\sin(4\chi - 2\varphi)\rangle$$