Effect of Half-Wave Plate systematics on the estimate of r

based on Giardiello et al., in preparation

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1-MINUTE SUMMARY

- we characterize HWP non-ideal parameters
  - differential efficiency,
  - phase-shift,
  - cross-polarization
- simulation pipeline with parameters varying in frequency, including foreground, no noise
- we estimate Δr caused by limited sensitivity on each non-ideal parameters
- requirements on sensitivity for each systematic parameter setting a threshold on Δr

Introduction

A rapidly rotating Half-Wave Plate (HWP) is introduced in the design of different future CMB experiments

Realistic HWP has non-idealities measured with a given precision.

We want to asses the accuracy required in measurements of HWP non-idealities in order to mitigate possible bias on r, in the case of LiteBIRD.

Sensitivity constraints

$$\Delta r(\Delta_{syst.}) < 10^{-5} \longrightarrow \text{limit on } \Delta_{syst.}$$

Simulation scheme

- We adopt a pair of orthogonal detectors at boresight.
- We consider light orthogonally incident on the HWP
- Systematics described in Jones formalism, results in Mueller formalism

Our system:

source $\rightarrow$ rotating HWP $\rightarrow$ polarizer $\rightarrow$ detector

$\swarrow$ $\searrow$

For an rotating HWP:

$$J_{rHWP} = J^T_{rot}(\theta)\, J_{HWP}\, J_{rot}(\theta)$$

For our two detectors:

$$J_{pol} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} / \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Modulation of observed light:

$$E_{out} = \begin{pmatrix} E_{x,out} \\ E_{y,out} \end{pmatrix} = J_{pol}\, J_{rHWP}\, J_{detector}\, E_{in} = J \begin{pmatrix} E_{x,in} \\ E_{y,in} \end{pmatrix}$$

Ideal case

In Jones formalism:

$$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \qquad \longleftarrow$$

ideal HWP, with a phase-shift $\psi = 180°$

TOD (time ordered data) for a single frequency, along x:

$$V_i = \frac{1}{2}[T(\hat{n}) + \cos(4\theta_i)Q(\hat{n}) + \sin(4\theta_i)U(\hat{n})] + n_i$$

Non-ideal case

$$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = \begin{pmatrix} 1 + h_1 & \zeta_1 e^{i\chi_1} \\ \zeta_2 e^{i\chi_2} & -(1 + h_2) e^{i\beta} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Meaning of non-ideal parameters:

- h_1 and h_2 is the transmissivity of light components E_x, E_y: i.e. $h_{1,2} < 0 \rightarrow$ light absorption
- $\beta = \psi - 180°$, describes a departure of the phase-shift ψ between x and y axes from its ideal value $\psi = 180$
- ζ_1 and ζ_2 describe the leakage from one axes to the other, giving cross-polarization; χ_1 and χ_2: their phases

The signal in the Stokes parameters is modulated by:

$$\begin{pmatrix} T+Q & U \\ U & T-Q \end{pmatrix}_{obs} = J \begin{pmatrix} T+Q & U \\ U & T-Q \end{pmatrix} J^{\dagger}$$

where $J = J_{pol}\, J^{T}_{rot}(\theta)\, J_{HWP}\, J_{rot}(\theta)\, J_{detector}$.

A detector with a polarizer along x, for a single frequency receives:

$$\begin{aligned} V_i &= \frac{1}{2}\left[(|J_{11}|^2 + |J_{12}|^2)_i T(\hat{n}) + (|J_{11}|^2 - |J_{12}|^2)_i Q(\hat{n}) + 2(J_{11}J^{*}_{12})_i U(\hat{n})\right] + n_i = \\ &= \frac{1}{2}\left[M^{TT}_i\, T(\hat{n}) + M^{TQ}_i\, Q(\hat{n}) + M^{TU}_i\, U(\hat{n})\right] + n_i \end{aligned}$$

Let us focus on $J_{pol} J_{rHWP}$. At linear order:

$$|J_{11}|^2 + |J_{12}|^2 = \underline{1} + h_1 + h_2 + (h_1 - h_2)\cos(2\theta) + $$
$$+ (\zeta_1 \cos\chi_1 \cos\beta - \zeta_2 \cos\chi_2) \sin(2\theta);$$
$$|J_{11}|^2 - |J_{12}|^2 \simeq \frac{1}{2}(1 + h_1 + h_2)(1 + \cos\beta)\underline{\cos(4\theta)} + (h_1 - h_2)\cos(2\theta)$$
$$+ \frac{1}{2}(1 + h_1 + h_2)(1 - \cos\beta) - (\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2 \cos\beta)\sin(2\theta)$$
$$- \frac{1}{2}(\zeta_1 \cos\chi_1 + \zeta_2 \cos\chi_2)(1 + \cos\beta)\sin(4\theta);$$
$$2(J_{11}J_{12}^*) \simeq \frac{1}{2}(1 + h_1 + h_2)(1 + \cos\beta)\underline{\sin(4\theta)} + (h_1 - h_2)\sin(2\theta) +$$
$$\frac{1}{2}(\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2)(1 - \cos\beta) + (\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2 \cos\beta)\cos(2\theta)$$
$$+ \frac{1}{2}(\zeta_1 \cos\chi_1 + \zeta_2 \cos\chi_2)(1 + \cos\beta)\cos(4\theta)$$

With only $h_1,\, h_2 \neq 0$:

$$|J_{11}|^2 + |J_{12}|^2 = 1 + h_1 + h_2 + (h_1 - h_2)\cos(2\theta)$$

$$+ (\zeta_1 \cos\chi_1 \cos(\beta) - \zeta_2 \cos\chi_2) \sin(2\theta);$$

$$|J_{11}|^2 - |J_{12}|^2 \simeq (1 + h_1 + h_2) \cos(4\theta) + (h_1 - h_2)\cos(2\theta)$$

$$\frac{1}{2}(1 - \cos\beta)(1 + h_1 + h_2) - (\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2 \cos\beta) \sin(2\theta)$$

$$- \frac{1}{2}(\zeta_1 \cos\chi_1 + \zeta_2 \cos\chi_2)(1 + \cos\beta) \sin(4\theta);$$

$$2(J_{11} J_{12}^*) \simeq (1 + h_1 + h_2) \sin(4\theta) + (h_1 - h_2)\sin(2\theta)$$

$$\frac{1}{2}(\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2)(1 - \cos\beta) + (\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2 \cos\beta) \cos(2\theta) +$$

$$+ \frac{1}{2}(\zeta_1 \cos\chi_1 + \zeta_2 \cos\chi_2)(1 + \cos\beta) \cos(4\theta)$$

With only $\beta \neq 0$:

$$|J_{11}|^2 + |J_{12}|^2 = 1 + h_1 + h_2 + (h_1 - h_2)\cos(2\theta)$$
$$+ (\zeta_1 \cos\chi_1 \cos(\beta) - \zeta_2 \cos\chi_2) \sin(2\theta);$$

$$|J_{11}|^2 - |J_{12}|^2 \simeq \frac{1}{2}(1 + \cos\beta)\cos(4\theta) + (1 + h_1 + h_2) + (h_1 - h_2)\cos(2\theta)$$
$$+ \frac{1}{2}(1 - \cos\beta)(1 + h_1 + h_2) - (\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2 \cos\beta)\sin(2\theta)$$
$$- \frac{1}{2}(\zeta_1 \cos\chi_1 + \zeta_2 \cos\chi_2)(1 + \cos\beta)\sin(4\theta);$$

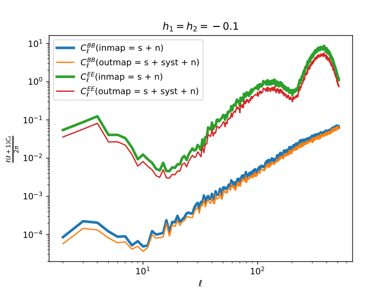
$$2(J_{11}J_{12}^*) \simeq \frac{1}{2}(1 + \cos\beta)\sin(4\theta)(1 + h_1 + h_2) + (h_1 - h_2)\sin(2\theta) +$$
$$\frac{1}{2}(\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2)(1 - \cos\beta) + (\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2 \cos\beta)\cos(2\theta)$$
$$+ \frac{1}{2}(\zeta_1 \cos\chi_1 + \zeta_2 \cos\chi_2)(1 + \cos\beta)\cos(4\theta)$$

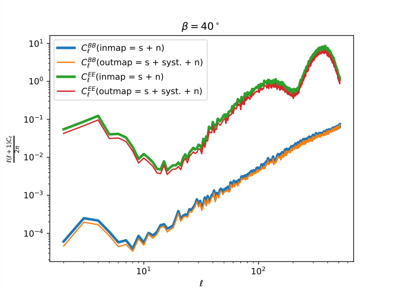
With only $\zeta_1,\ \zeta_2, \chi_1, \chi_2 \neq 0$:

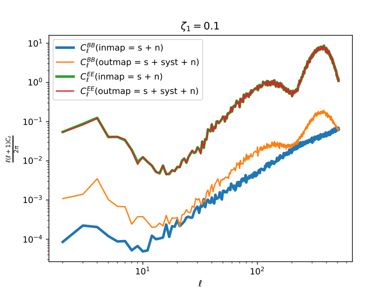
$$|J_{11}|^2 + |J_{12}|^2 = 1 + h_1 + h_2 + (h_1 - h_2)\cos(2\theta)$$
$$+ (\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2)\sin(2\theta);$$
$$|J_{11}|^2 - |J_{12}|^2 \simeq \cos(4\theta) + (1 + h_1 + h_2)\cos(4\theta) + (h_1 - h_2)\cos(2\theta)$$
$$- (\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2)\sin(2\theta)\frac{1}{2}(1 - \cos\beta)(1 + h_1 + h_2)$$
$$- (\zeta_1 \cos\chi_1 + \zeta_2 \cos\chi_2)\sin(4\theta);$$
$$2(J_{11}J_{12}^*) \simeq \sin(4\theta) + +(h_1 - h_2)\sin(2\theta)$$
$$+ (\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2)\cos(2\theta)\frac{1}{2}(\zeta_1 \cos\chi_1 - \zeta_2 \cos\chi_2)(1 - \cos\beta) +$$
$$+ (\zeta_1 \cos\chi_1 + \zeta_2 \cos\chi_2)\cos(4\theta)$$

$\longrightarrow$ we neglect the phases $\chi_{1,2}$

Effect of systematics on C_ℓ(CMB), monochromatic case







Map-making, single frequency

The TOD is given by: $V = Am + n$,

- A is the pointing matrix, $A(h_1, h_2, \beta, \zeta_1, \zeta_2)$
- n is the noise vector, with $N = \langle n^T n \rangle$

$$m = \begin{pmatrix} T_{p1} \\ Q_{p1} \\ U_{p1} \\ \dots \end{pmatrix}.$$

Map-making procedure, with matrix $B(h_{1,s}, h_{2,s}, \beta_s, \zeta_{1,s}, \zeta_{2,s})$:

$$\tilde{m} = \begin{pmatrix} \tilde{T}_{p1} \\ \tilde{Q}_{p1} \\ \tilde{U}_{p1} \\ \dots \end{pmatrix} = (B^T B)^{-1} B^T V = (B^T B)^{-1} B^T A m + (B^T B)^{-1} B^T n \; .$$

A includes all the systematic effects affecting the observation.
If $\mathrm{B} \neq \mathrm{A}$, which is the effect on $\tilde{m}$?

In a realistic experiment:

- non-ideal parameters depend on frequency
- modulated signal includes also chromatic foreground emissions.

We build the TOD in this more realistic setting (no noise):

$$V_i(\hat{n}) = \frac{\int d\nu \frac{\partial BB(\nu,T)}{\partial T_{CMB}} \tau(\nu) \left\{ M_i^{TT}(\nu) T_{CMB}(\hat{n}) + M_i^{TQ}(\nu) Q_{CMB}(\hat{n}) + M_i^{TU}(\nu) U_{CMB}(\hat{n}) \right\}}{\int d\nu \frac{\partial BB(\nu,T)}{\partial T_{CMB}} \tau(\nu)}$$

$$+ \frac{\int d\nu \frac{\partial BB(\nu,T)}{\partial T_{RJ}} \tau(\nu) \left\{ M_i^{TT}(\nu) T_{FG}(\nu,\hat{n}) + M_i^{TQ}(\nu) Q_{FG}(\nu,\hat{n}) + M_i^{TU}(\nu) U_{FG}(\nu,\hat{n}) \right\}}{\int d\nu \frac{\partial BB(\nu,T)}{\partial T_{CMB}} \tau(\nu)}$$

In matrix form: $V(\hat{n}) = A_{CMB}\, m_{CMB}(\hat{n}) + \int A_{FG}(\nu)\, m_{FG}(\nu,\hat{n})\, d\nu$.

Solver parameters $h_{1,2,s}(\nu)$, $\beta_s(\nu)$, $\zeta_{1,2,s}(\nu) \leftarrow$ from our model

used in

$$B_i = \left(\frac{\int d\nu \frac{\partial BB(\nu,T)}{\partial T_{CMB}} \tau(\nu) M^{TT}_{s,i}(\nu)}{\int d\nu \frac{\partial BB(\nu,T)}{\partial T_{CMB}} \tau(\nu)}, \frac{\int d\nu \frac{\partial BB(\nu,T)}{\partial T_{CMB}} \tau(\nu) M^{TQ}_{s,i}(\nu)}{\int d\nu \frac{\partial BB(\nu,T)}{\partial T_{CMB}} \tau(\nu)}, \frac{\int d\nu \frac{\partial BB(\nu,T)}{\partial T_{CMB}} \tau(\nu) M^{TU}_{s,i}(\nu)}{\int d\nu \frac{\partial BB(\nu,T)}{\partial T_{CMB}} \tau(\nu)} \right)$$

to compute: $m_{out} = (B^T B)^{-1} B^T A_{CMB}\, m_{CMB} + (B^T B)^{-1} B^T \int A_{FG}(\nu)\, m_{FG}(\nu, \hat{n})\, d\nu$

We can compute a template with $M^{XX} = M^{XX}_s \rightarrow A_{CMB} = B, A_{FG} = B_{FG}$:
$m_{templ} = m_{CMB} + (B^T B)^{-1} B^T \int B_{FG}(\nu)\, m_{FG}(\nu, \hat{n})\, d\nu$

which is used to get the residual map:
$m_{res} = m_{out} - m_{templ} = (B^T B)^{-1} B^T A_{CMB}\, m_{CMB} - m_{CMB} +$
$(B^T B)^{-1} B^T \int A_{FG}(\nu)\, m_{FG}(\nu, \hat{n})\, d\nu - (B^T B)^{-1} B^T \int B_{FG}(\nu)\, m_{FG}(\nu, \hat{n})\, d\nu$

Comparison residual maps

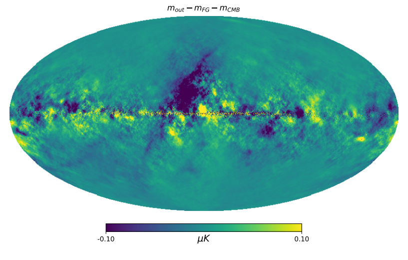


Figure 1: $m_{out} - m_{CMB} - m_{FG}$

Map with error in some systematic parameter when removing m_{CMB} and m_{FG}

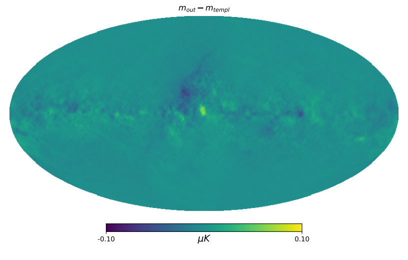


Figure 2: $m_{out} - m_{templ}$

Same map when removing m_{templ}

residual map as $m_{out} - m_{templ}$ depends just on the mismatch $h_{1,2,s}(\nu)$, $\beta_s(\nu)$, $\zeta_{1,2,s}(\nu) \neq h_{1,2}(\nu)$, $\beta(\nu)$, $\zeta_{1,2}(\nu)$

Choice of non-ideal parameters

For four MFT LiteBIRD bands:

simulated profiles for
$h_{1,2,s}(\nu)$, $\beta_s(\nu)$, $\zeta_s = 0.01$ $\longrightarrow$ matrix B

$h(\nu) = h_s(\nu) + \Delta h(\nu)$;
$\beta(\nu) = \beta_s(\nu) + \Delta\beta(\nu)$; $\longrightarrow$ matrix A_{CMB}, A_{FG}
$\zeta = \zeta_s + \Delta\zeta$

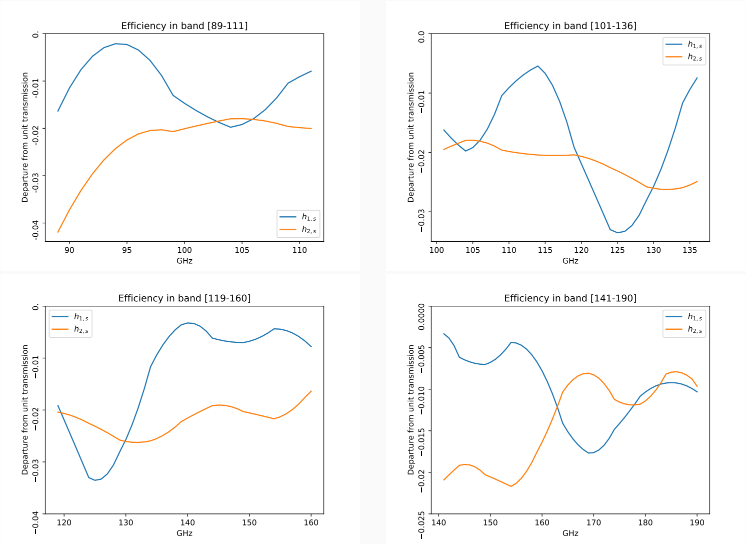
These errors are gaussian distributed and uncorrelated (both in frequency and between different systematics):

$$\overline{\Delta h}(\nu), \overline{\Delta\zeta}(\nu) = 0 \quad \sigma_{\Delta_h}, \sigma_{\Delta_\zeta} = 10^{-3}, 2\times10^{-3}, 5\times10^{-3}, \ldots \frac{1}{\sqrt{\mathrm{GHz}}}$$

$$\overline{\Delta\beta}(\nu) = 0, \quad \sigma_{\Delta\beta} = 0.5^\circ, 1^\circ, 2^\circ, \ldots \frac{1}{\sqrt{\mathrm{GHz}}}$$

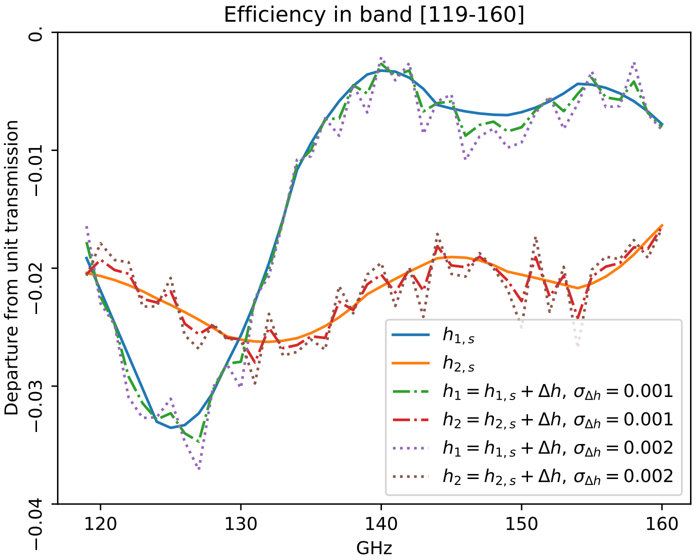
We do 10 end-to-end simulations for each value of $\sigma_{\Delta syst}$, keeping CMB and FG fixed.

h_1, h_2 FOR EACH FREQUENCY BAND[1]

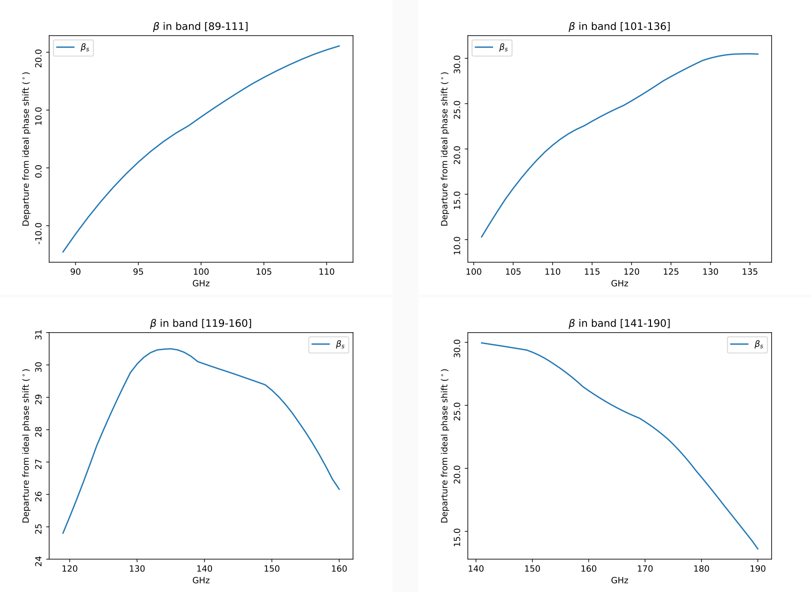


[1]we thank Giampaolo Pisano for providing the simulated profiles

h with Δh, channel 140 GHz



β FOR EACH FREQUENCY BAND[2]



[2]we thank Giampaolo Pisano for providing the simulated profiles

Estimate of r

Given the fiducial C_ℓ^{BB} with $r_{\rm fid} = 0$, we compute the likelihoods:

- $\mathcal{L}(\hat{C}_\ell^{BB}|C_\ell^{BB})$, where $\hat{C}_\ell^{BB} = C_\ell^{BB}(m_{CMB})$;
- $\mathcal{L}(\tilde{C}_\ell^{BB}|C_\ell^{BB})$, where $\tilde{C}_\ell^{BB} = C_\ell^{BB}(m_{CMB}) + \frac{C_\ell^{BB}(m_{res} \times \mathrm{mask})}{fsky}$.

using the exact log-likelihood for both $\hat{C}_\ell^{BB}/\tilde{C}_\ell^{BB}$:

$$-2\ln\mathcal{L}(\hat{C}_\ell^{BB}|C_\ell^{BB}(r)) =$$

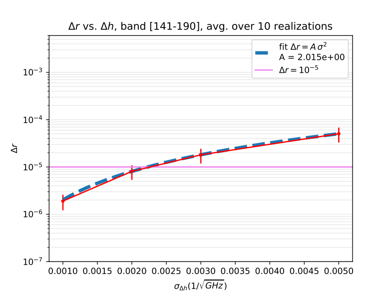
$$= fsky \sum_\ell (2\ell+1)\left\{\hat{C}_\ell^{BB}(C_\ell^{BB}(r))^{-1} - ln[\hat{C}_\ell^{BB}(C_\ell^{BB}(r))^{-1}] - 1\right\}.$$

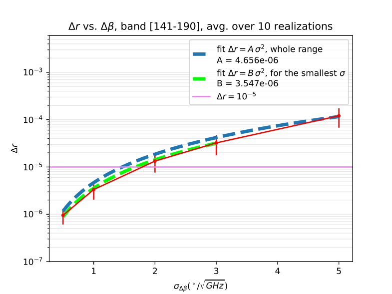
- $\mathcal{L}(\hat{C}_\ell^{BB}|C_\ell^{BB}(r))$ peaks around $r_{fid} = 0$,
- $\mathcal{L}(\tilde{C}_\ell^{BB}|C_\ell^{BB}(r))$ peaks at $r = r_{fid} + \Delta r$

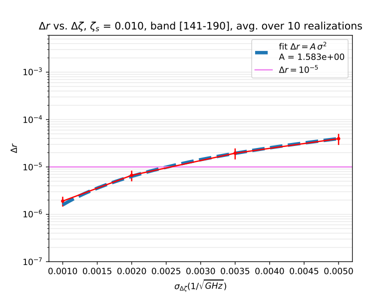
$\Delta r = r_{peak(\tilde{p})} - r_{peak(\hat{p})}$, for $\mathcal{L}$ computed between $30 < \ell < 600$

To extend to lowest ℓ, still looking for better C_ℓ estimator

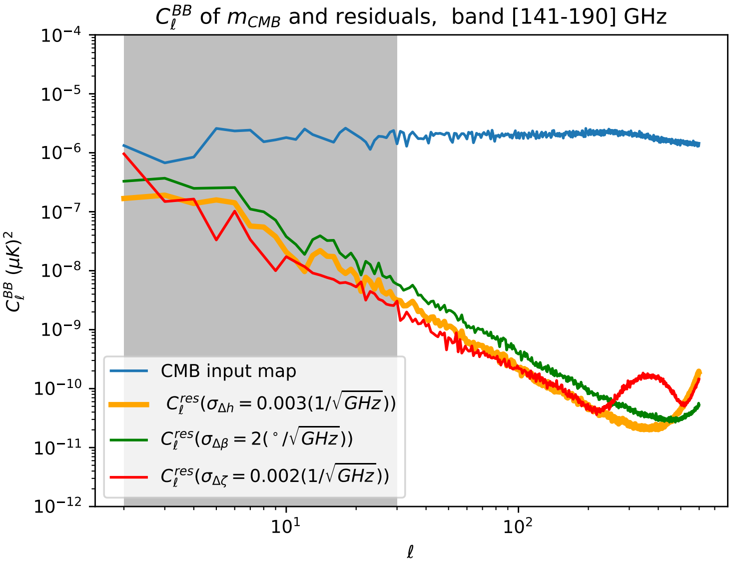
Δr VS. Δ SYST., FOR BAND [141-190] MFT







C_ℓ^{BB} OF (MASKED) RESIDUAL MAPS WITH ERRORS IN ONE SYSTEMATIC



Constraints on sensitivity

- $\Delta r \propto \sigma^2_{\Delta\, syst.}$ for small errors;
- Taking the most conservative constraint for the sensitivity in each systematic, among the four bands:

	$\sigma_{\Delta h}(\Delta r \simeq 10^{-5})$	$\sigma_{\Delta \beta}(\Delta r \simeq 10^{-5})$	$\sigma_{\Delta \zeta}(\Delta r \simeq 10^{-5})$
MFT	$\leq 0.0023\ \frac{1}{\sqrt{\mathrm{GHz}}}$	$\leq 1.34\ \frac{^\circ}{\sqrt{\mathrm{GHz}}}$	$\leq 0.0026\ \frac{1}{\sqrt{\mathrm{GHz}}}$

- we are working on the coaddition of the constraints from each band using weights from component separation

Other considerations

- for our analysis, not really dependent on the shape of $h(\nu), \beta(\nu), \zeta(\nu)$
- when combining more systematics, $\Delta r(\sigma_x, \sigma_y) \simeq \Delta r(\sigma_x) + \Delta r(\sigma_y)$, where $x, y = \Delta h, \Delta\beta, \Delta\zeta$
- we expect to set more stringent requirements when also including $\ell \leq 30$ in the analysis
- we are working on a more sophisticated model for systematics, where we would also take into account correlated errors

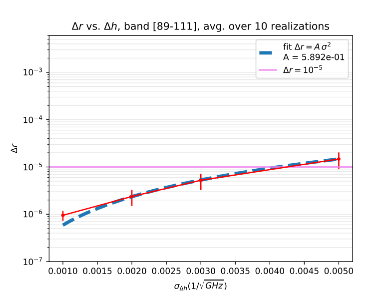
Thank you

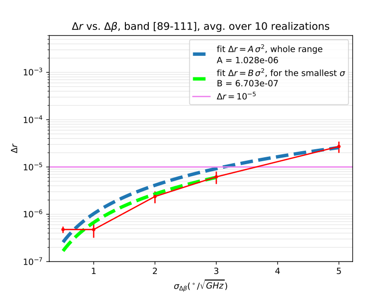
Backup slides

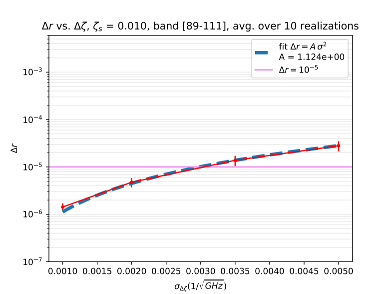
Setup

- Foreground model: d1 (dust modelled as a single-component modified black body), s1 (power law scaling for the synchrotron emission, with a spatially varying spectral index), a1 (AME modeled as a sum of two spinning dust populations based on the Commander code), f1 (free-free emission modeled using the analytic model assumed in the Commander fit to the Planck 2015 data)
- CMB fiducial parameters: $r = 0$, $\tau = 0.0544$
- nside = 512, smoothing with gaussian beam with fwhm = [37.8',33.6',30.8',28.9'] for [100,119,140,166] GHz centered bands
- HWP revolution rate = 39 rpm for MFT, sampling rate = 19 Hz

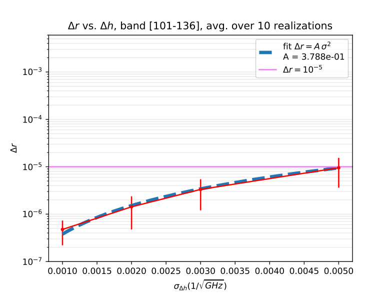
Δr vs. Δ syst., for band [89-111] MFT

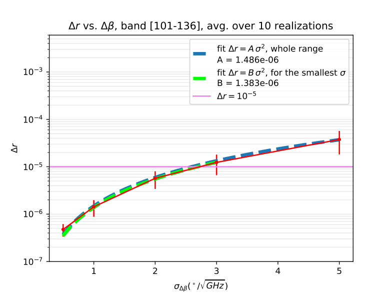


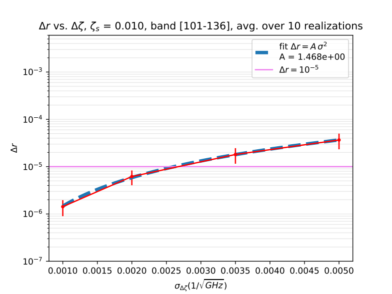




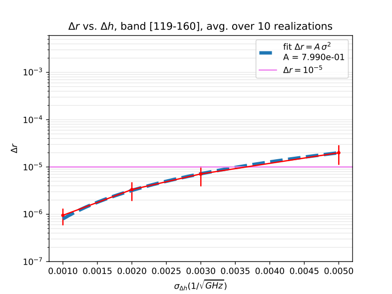
Δr vs. Δ syst., for band [101-136] MFT

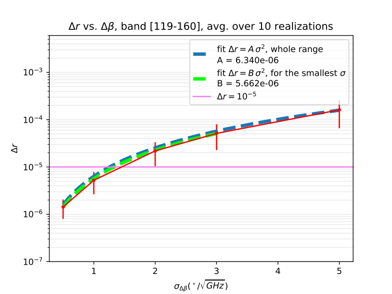


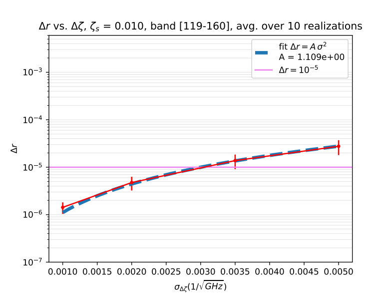




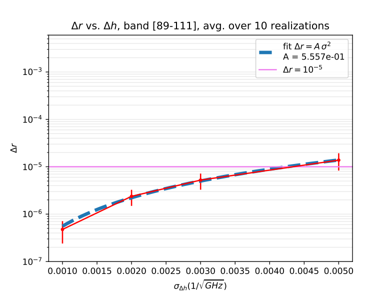
Δr VS. Δ SYST., FOR BAND [119-160] MFT

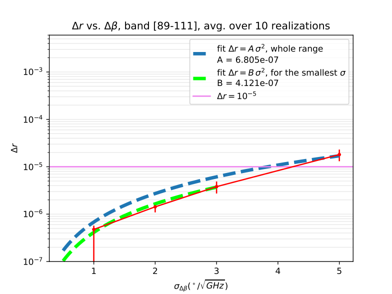


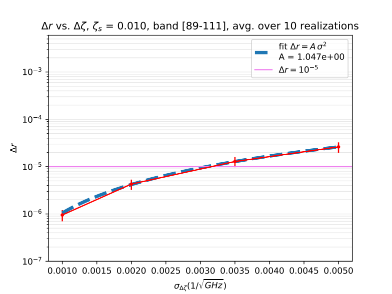




Δr vs. Δ syst., for band [89-111] LFT, flat profiles







Δr vs. Δ syst., for band [166-224] HFT flat profiles