Determination of the Systematics Related to Polarization Angle Uncertainty and Its Impact on r

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Overview

Polarization Angle Uncertainty

Polarization Angle Uncertainty

- One of the major systematics to be controlled is the uncertainty of the polarization sensitive angle of a polarimeter.
- Miscalibrated polarization angles induce a mixing of E- and B-modes which obscures the primordial B-mode signal.
- In-flight polarization angle calibration strategies only reach a $\sim 0.5^{\circ}$ uncertainty.[1]
- We introduce an iterative angular power-spectra maximum Likelihood-based method to calculate the polarization angles ($\bar{\alpha}$) from the multi-frequency signal by nulling the EB power-spectra.

[1] This uncertainty might be lower by a dedicated observational campaign for a better characterization of some astrophysical sources such as the Crab Nebula.

Formalism

Formalism I. Basis

The observed E and B spectra of the i-th channels are rotated as follows:

$$\begin{pmatrix} E_i \\ B_i \end{pmatrix} = \begin{pmatrix} \cos(2\alpha_i) & -\sin(2\alpha_i) \\ \sin(2\alpha_i) & \cos(2\alpha_i) \end{pmatrix} \begin{pmatrix} E_i^* \\ B_i^* \end{pmatrix} ,$$

The full log-likelihood (assumed to be Gaussian) of all possible EB-spectra is:

$$-2\log\mathcal{L} = \sum_{i=1}^{N_{ch}} \sum_{j=i}^{N_{ch}} \sum_{i'=1}^{N_{ch}} \sum_{j'=i'}^{N_{ch}} \sum_{\ell=\ell_{min}}^{\ell_{max}} \Delta_\ell^{ij} (\mathbf{C}^{-1})_{\ell,ij,i'j'} \Delta_\ell^{i'j'} ,$$

where $\Delta_\ell^{ij} = (\chi_\ell^{ij} - \xi_\ell^{ij} \tan(2\alpha_i + 2\alpha_j))$ and,

$$\chi_\ell^{ij} = \frac{C_\ell^{E_iB_j} + C_\ell^{B_iE_j}}{2} , \qquad \xi_\ell^{ij} = \frac{C_\ell^{E_iE_j} - C_\ell^{B_iB_j}}{2}$$

It is a modification of the Minami et al. 2019 methodology.

Formalism II. Assumptions

Assuming:

- Small angle approximation: $\alpha \ll 1 \rightarrow \tan(\alpha) \sim \alpha$.
- The covariance matrix $\mathbf{C}$ does not depend on α. To correct the mismatch induced by this approximation, we perform an iterative approach that updates the polarization angle in the covariance matrix with the one estimated in the previous step.

With these approximations, we achieve a linear system which enable us to obtain analytical equations to calculate the rotation angles. Moreover, the uncertainties can be evaluated from the Fisher matrix. This results in a very fast computational methodology.

Formalism III. Covariance Matrix Implementation

- The covariance matrix depends on $\widetilde{C}_\ell^{XY}$ which is the ensemble average XY power-spectrum ($X, Y \in \{E, B\}$)
- ⚠ Due to the lack of a reliable model of the foregrounds cross-spectra, we require a good estimator of it from the observed power-spectra.
- We have tried several estimators:
  - The observed power-spectra.
  - The observed power-spectra binned.
  - The observed power-spectra smooth with a square window ℓ-function convolution.
- Any of these previous approximations for the foregrounds cross-spectra model produce biases when a full covariance matrix including all the possible cross power-spectra are considered. Therefore in this work we use only the auto spectra.

Formalism IV. Rotation Angles Equations

The rotation angles are obtained solving the following system:

$$\mathbf{\Omega}\bar{\alpha} = \frac{1}{4}\bar{\eta}\,,$$

where $\mathbf{\Omega}$ and $\bar{\eta}$ elements are

$$\Omega_{ij} = \sum_{\ell=\ell_{min}}^{\ell_{max}} \xi_\ell^i \left(\mathbf{C}^{-1}\right)_{\ell,ij} \xi_\ell^j\,,$$

$$\eta_i = \sum_{j=1}^{N_{ch}} \sum_{\ell=\ell_{min}}^{\ell_{max}} \xi_\ell^i \left(\mathbf{C}^{-1}\right)_{\ell,ij} C_\ell^{EB_j}\,.$$

And the Fisher matrix is calculated with:

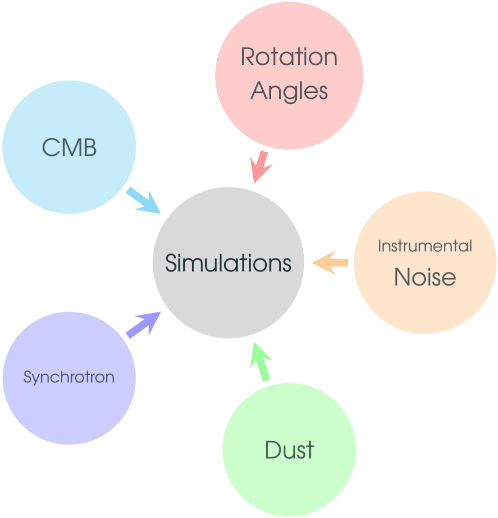
$$F_{ij} = \frac{1}{2}\frac{\partial^2(-2\log\mathcal{L})}{\partial\alpha_i\partial\alpha_j} = 16\sum_{\ell=\ell_{min}}^{\ell_{max}} \xi_\ell^i \left(\mathbf{C}^{-1}\right)_{\ell,ij} \xi_\ell^j\,.$$

Simulations

Simulations with Systematic

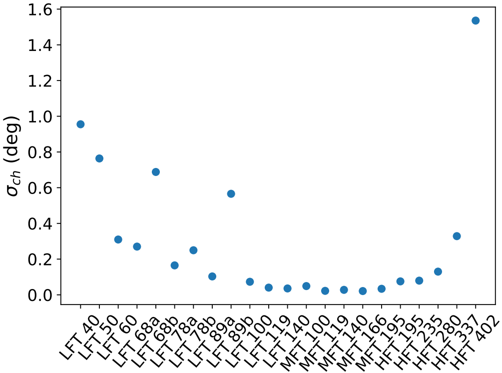
- Rotation Angles.
- Sky signal
  - CMB.
  - Polarized Foregrounds: synchrotron and dust.
- Instrumental Noise.

$$\begin{pmatrix} \bar{Q}^{rot} \\ \bar{U}^{rot} \end{pmatrix} = \begin{pmatrix} \cos(2\bar{\alpha}) & -\sin(2\bar{\alpha}) \\ \sin(2\bar{\alpha}) & \cos(2\bar{\alpha}) \end{pmatrix} \begin{pmatrix} \bar{Q} \\ \bar{U} \end{pmatrix} + \begin{pmatrix} \bar{n}^{Q} \\ \bar{n}^{U} \end{pmatrix}$$



Rotation Angles

Requirements with channels correlations.



The covariance among channels:

$$\mathbf{C}_\alpha = \begin{pmatrix} M_{LFT} & 0 & 0 \\ 0 & M_{MFT} & 0 \\ 0 & 0 & M_{HFT} \end{pmatrix},$$

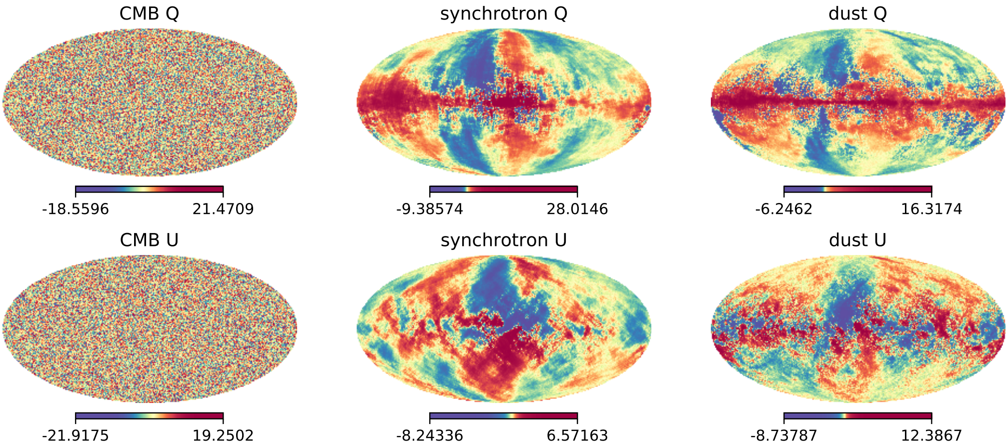
Using $\mathbf{L}$ the Cholesky decomposition of $\mathbf{C}_\alpha$, the rotation angles:

$$\bar{\alpha}_{ch} = \mathbf{L}\bar{x}\,,$$

where $\bar{x}$ random samples from the standard Gaussian distribution.

Based on the requirements shown by Enrique Martínez-González on Monday.

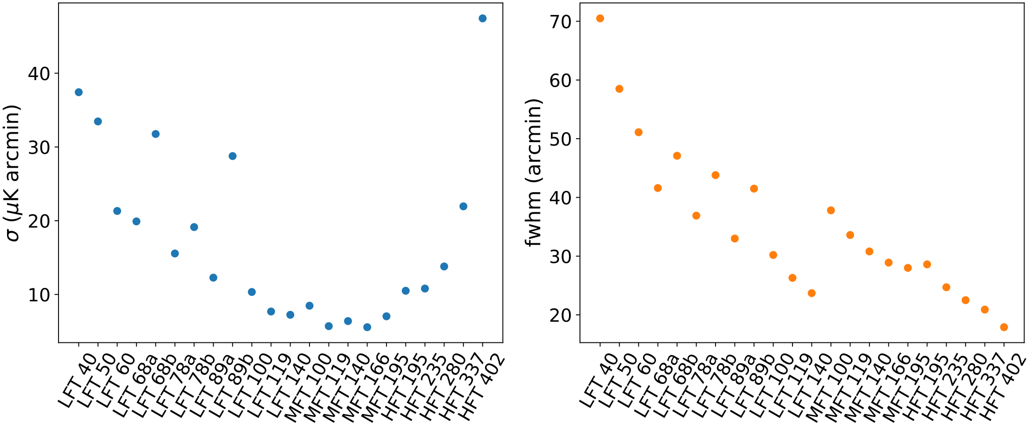
Sky Signal. CMB and foregrounds



CAMB. Lewis et al., astro-ph/9911177/ Planck 2018 results. VI. Cosmological parameters, 1807.06209

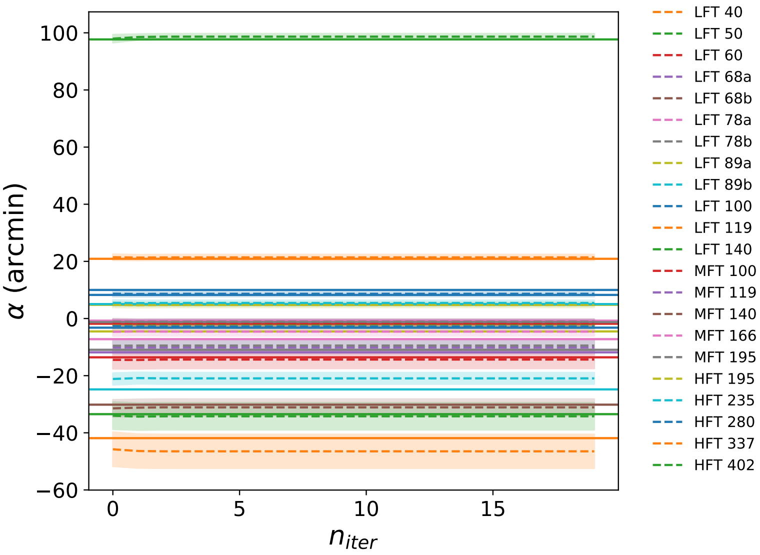
Thorne et al., 1608.02841 (PySM)

Instrumental Noise. LiteBIRD-like Experiment

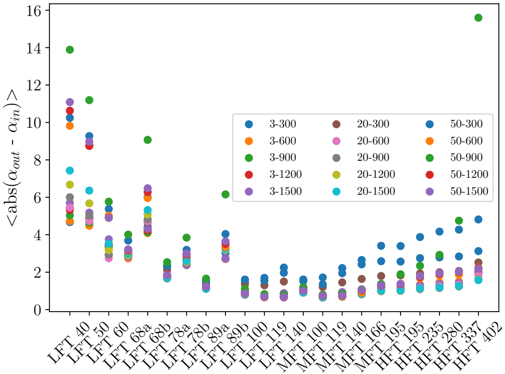


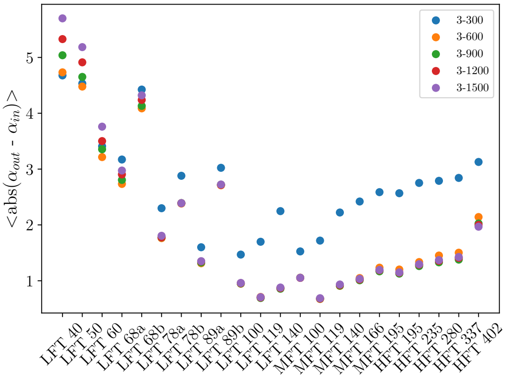
Validation with simulations

Convergence

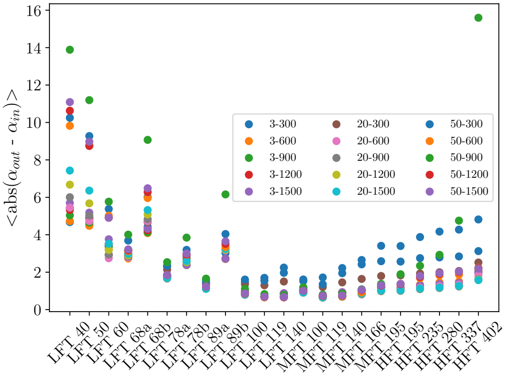


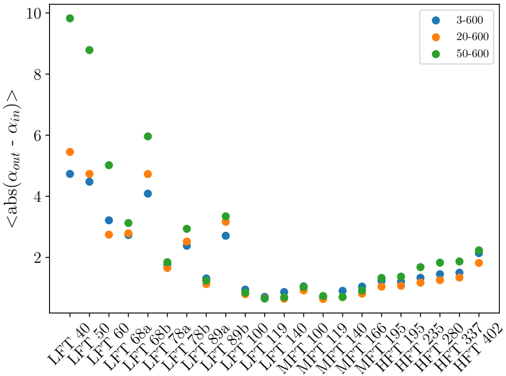
Multipole Range



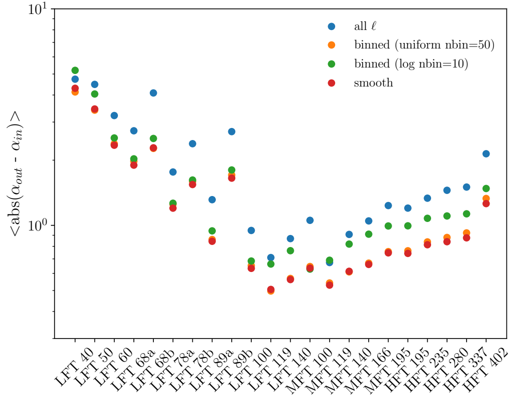


Multipole Range



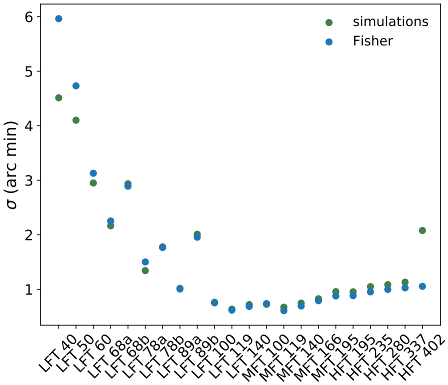


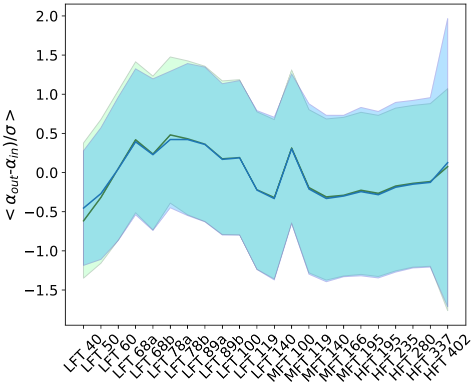
Covariance Model. Estimator



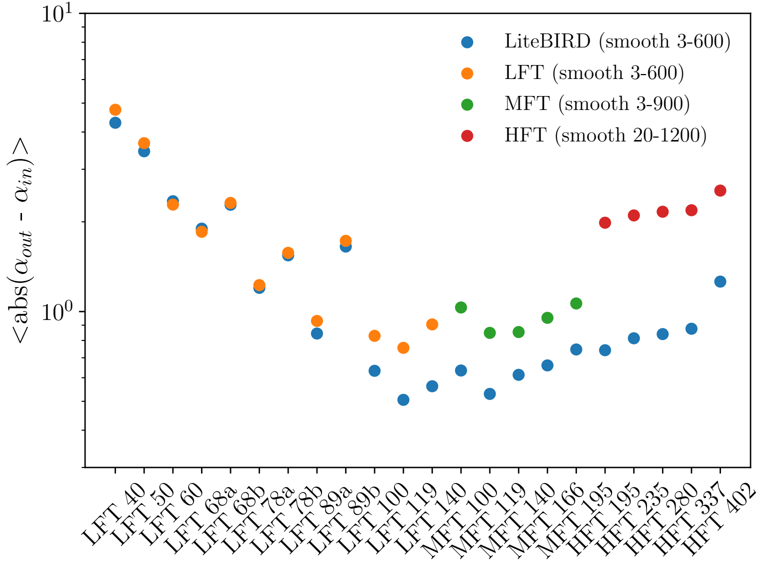
- **all** ℓ: original power-spectra. ($\ell_{min} = 3$-$\ell_{max} = 600$)
- **binned (uniform nbin=50)**: power-spectra binned in ℓ chunks of equal length ($n_{|ell} \sim 12$ per bin).
- **binned (log nbin=10)**: power-spectra binned in ℓ chunks with logarithmically increasing length.
- **smooth**: power-spectra smoothed by two subsequent convolutions with square window functions ($n_\ell = 5$, $n_\ell = 10$).

Uncertainty and Bias

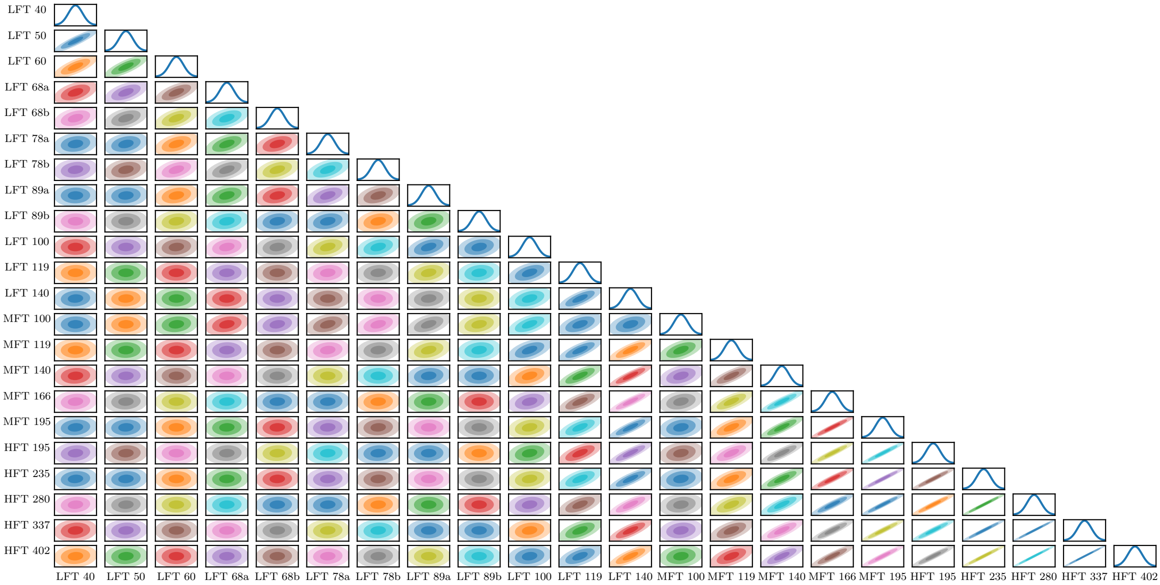




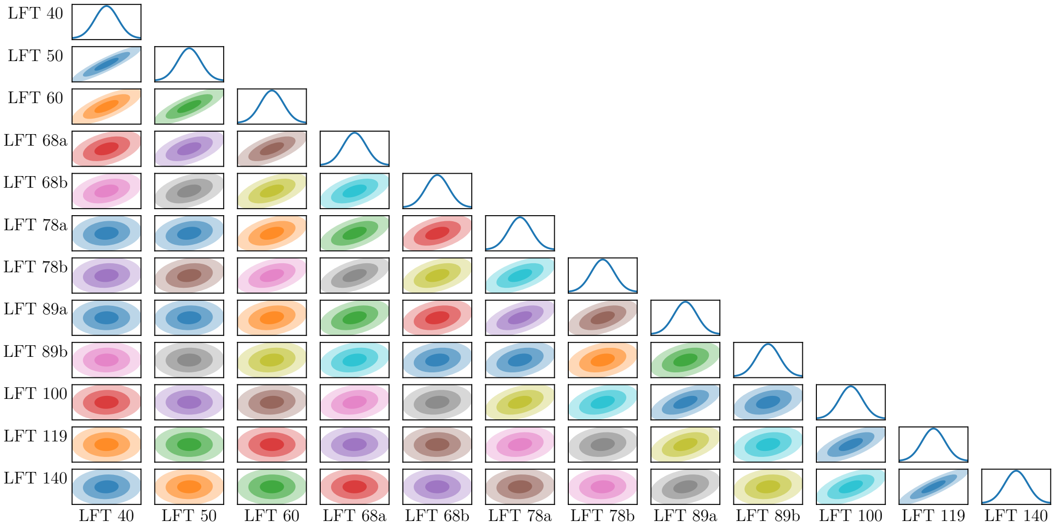
Independent Telescope Study



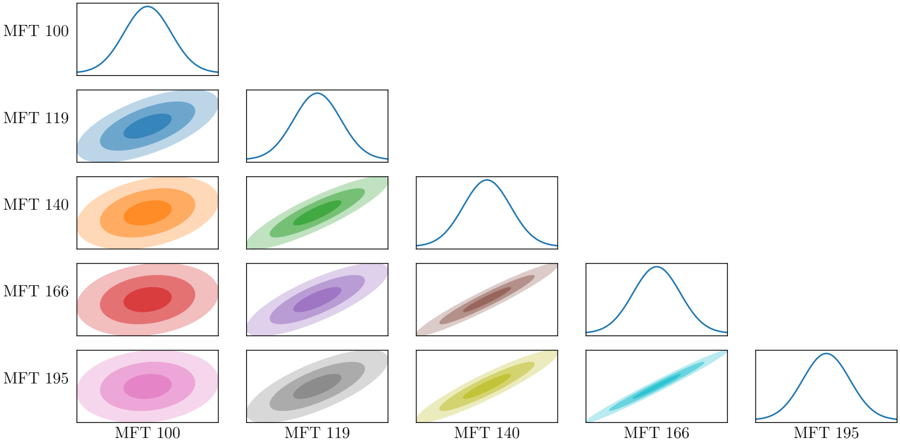
Correlations among Rotation Angles



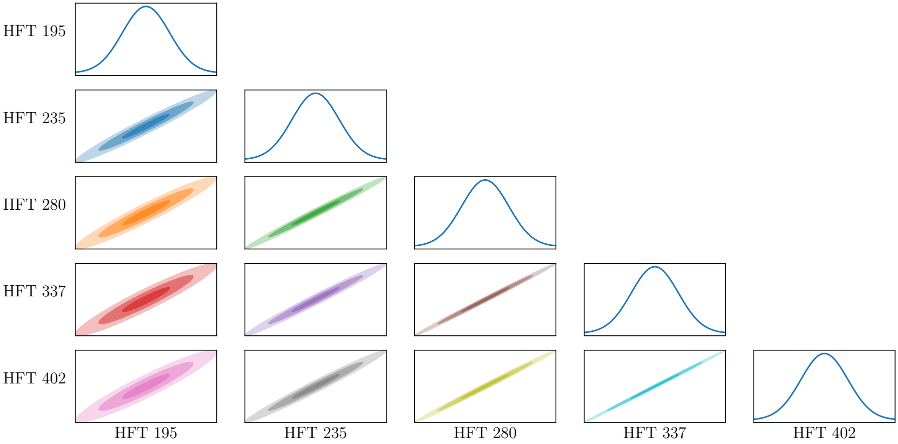
Correlations among Rotation Angles



Correlations among Rotation Angles



Correlations among Rotation Angles



Impact on r

Assesing the systematic

We perform component separation on the following set of maps:

- **Non-rotated maps.** The original signal maps without any rotation.
- **Rotated maps.** The previous maps with the corresponding rotation applied.
- **De-rotated maps.** The resulting maps after de-rotating the rotated maps with the solutions obtained with the described methodologies.

To study the impact introduced by the systematic.

Parametric Component Separation

The likelihood for a given pixel is:

$$\mathcal{L}(\theta_p|\bar{d}_p) = \frac{1}{\sqrt{(2\pi)^{2N_{ch}}\det(\mathbf{C})}} \exp\left(-\frac{1}{2}\left(\bar{d}_p - m(\bar{\nu};\theta_p)\right)^T \mathbf{C}^{-1}\left(\bar{d}_p - m(\bar{\nu};\theta_p)\right)\right),$$

The sky model is:

$$\begin{bmatrix} S^Q \\ S^U \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{c}^Q \\ \mathbf{c}^U \end{bmatrix}}_{\text{CMB}} + \underbrace{\begin{bmatrix} \mathbf{a_s}^Q \\ \mathbf{a_s}^U \end{bmatrix} \left(\frac{\nu}{\nu_s}\right)^{\boldsymbol{\beta_s}+\mathbf{c_s}(\nu/\nu_{cs})}}_{\text{Synchrotron}} + \underbrace{\begin{bmatrix} \mathbf{a_d}^Q \\ \mathbf{a_d}^U \end{bmatrix} \left(\frac{\nu}{\nu_d}\right)^{\boldsymbol{\beta_d}} \frac{B(\nu,\mathbf{T_d})}{B(\nu_d,\mathbf{T_d})}}_{\text{Dust}}$$

The amplitudes and spectral parameters are updated in different steps. Convergence is obtained quickly.

Component separation methodology used is an improved version of the one presented in de la Hoz, E., et al. JCAP 2020.06 (2020): 006

Map Processing before component separation

non-rotated/rotated/de-rotated maps
converted to spherical harmonics

↓

$a_{\ell m}^{i}$ deconvolved by corresponding LiteBIRD's i-th channel beam

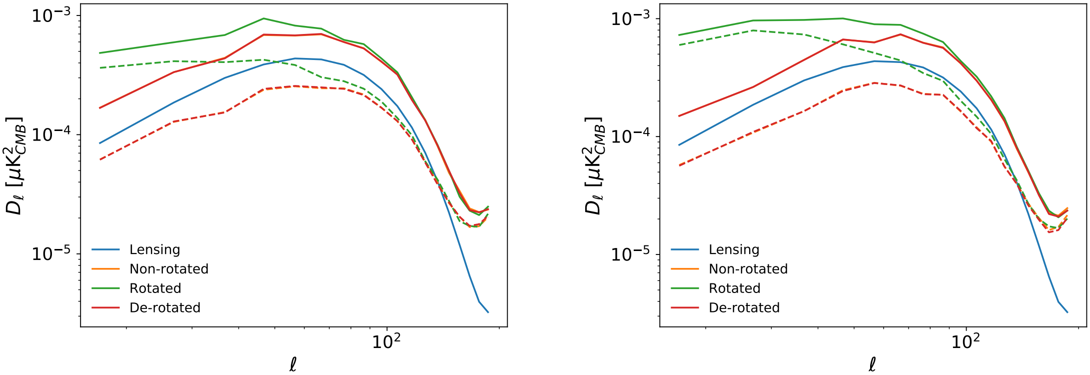
↓

Convolve $a_{\ell m}^{i}$ with a Gaussian beam of 132 arcmin $\forall i$

↓

Downgrade to nside 64 maps

Power-spectra residuals comparison



One minute summary

One Minute Summary

- Current polarization angles calibration strategies have uncertainties of $\sim 0.5^\circ$. Primordial B-mode search requires more stringent limits.
- We introduce an iterative angular power-spectra maximum likelihood-based method to calculate the polarization angles ($\bar{\alpha}$) from the multi-frequency signal by nulling the C_ℓ^{EB}.
- Two major assumptions are made: i) the rotation angles are small ($\lesssim$6deg), and, ii) the covariance matrix does not depend on $\bar{\alpha}$.
- We obtain an analytical linear system which leads to a very fast computational implementation.
- With this methodology we reach uncertainties on the order of a few arc minutes.
- We show that this accuracy is enough to remove the systematics by applying a parametric component separation technique to recover the CMB in three scenarios: i) rotated signal, ii) non-rotated signal, and iii) de-rotated signal.
- We find that the systematic introduced by leftover polarization angles in r is removed after the signal is corrected.