A minimal power-spectrum-based moment expansion for CMB B-mode searches

CMB systematics and calibration focus workshop, Virtually @ Kavli IPMU 30/11-3/12/2020

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In collaboration with Max Abitbol, David Alonso, Alex Gough, Tomotake Matsumura, Nobuhiko Katayama

Based on arXiv:2011.11575

< 1 minute summary

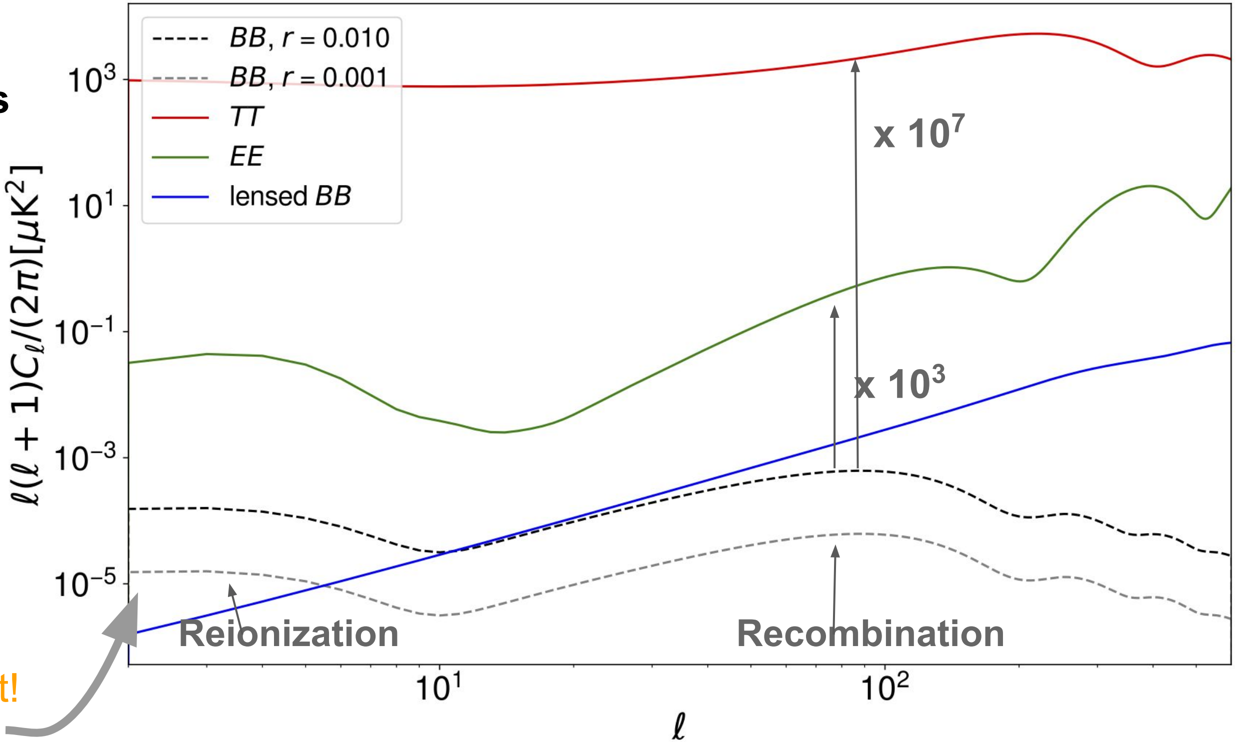
- We propose a simple extension to the usual power-spectrum-based likelihood which accounts for spatially varying spectral indices
  - We look at the minimal moments-space expansion at the power-spectrum level for B-modes in ground based telescopes
- We test the method on simulations of varying complexities and on real data and we find that it is able to correct for the kind of biases that we expect for future CMB B-modes experiments

Detecting B-modes

Small B-mode amplitude:

- Need **sensitive instruments**
- Need **good calibration**
- Removal of **foregrounds**

to reduce bias on **r.**



r ~ O(10^{-2}-10^{-3})

CMB-S4, LiteBIRD target!

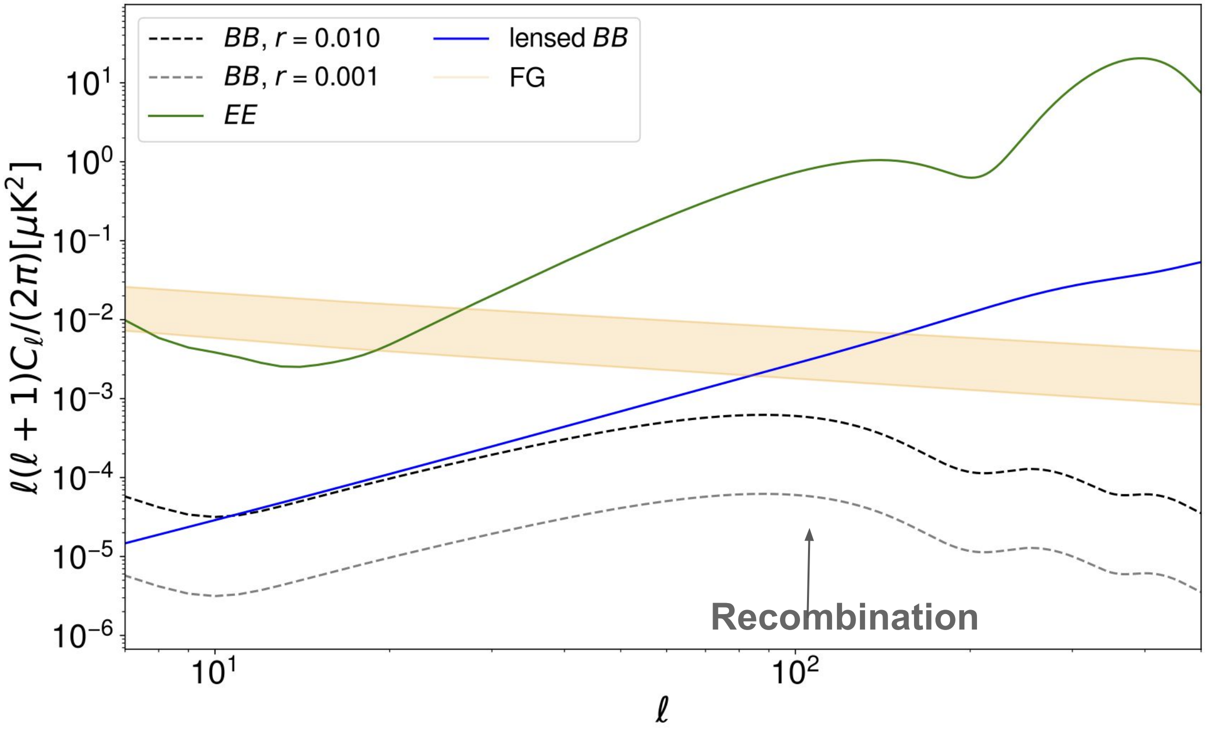
Focus of this work

Small B-mode amplitude:

- Need **sensitive instruments**
- Need **good calibration**
- Removal of **foregrounds**

❖ **Component separation** to isolate the CMB signal

❖ **Characterize varying foreground emission** to properly model the sky



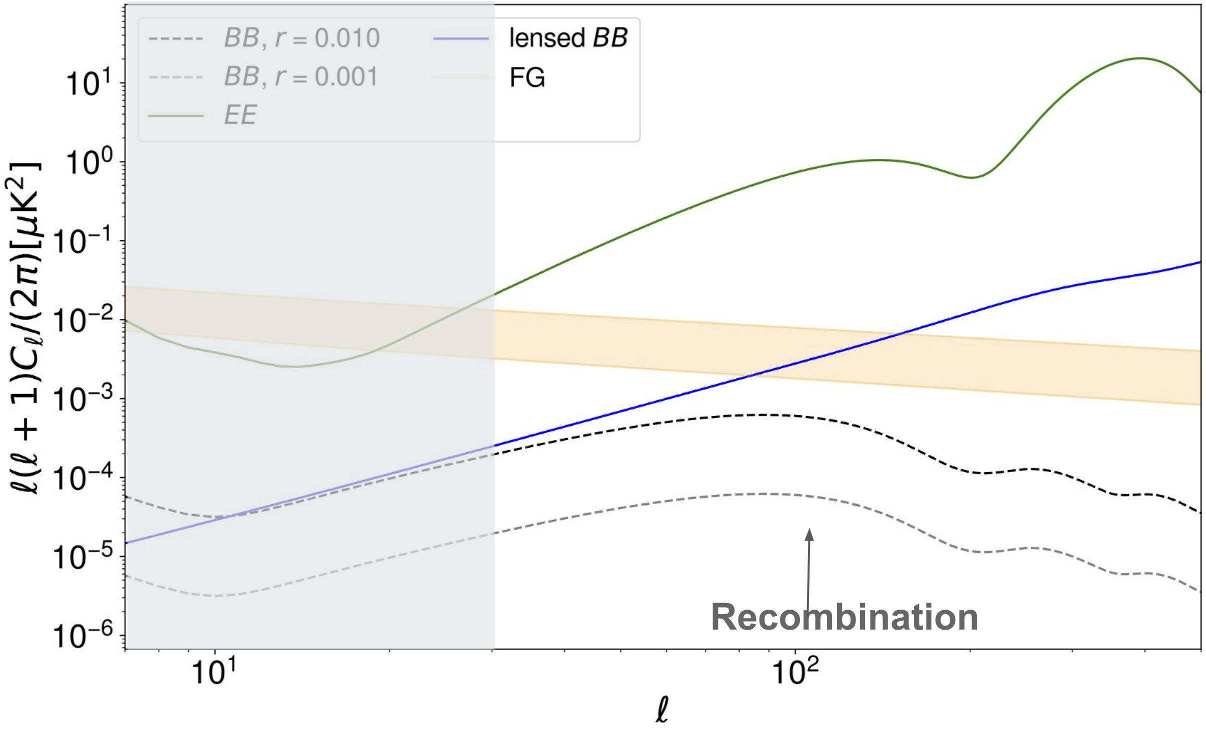
Focus of this work

Small B-mode amplitude:

- Need **sensitive instruments**
- Need **good calibration**
- Removal of **foregrounds**

Additional challenges for **ground-based experiments**:

- Atmosphere
- Ground pickup



Foregrounds

Observed sky = (Polarized dust + synchrotron) + Polarized CMB

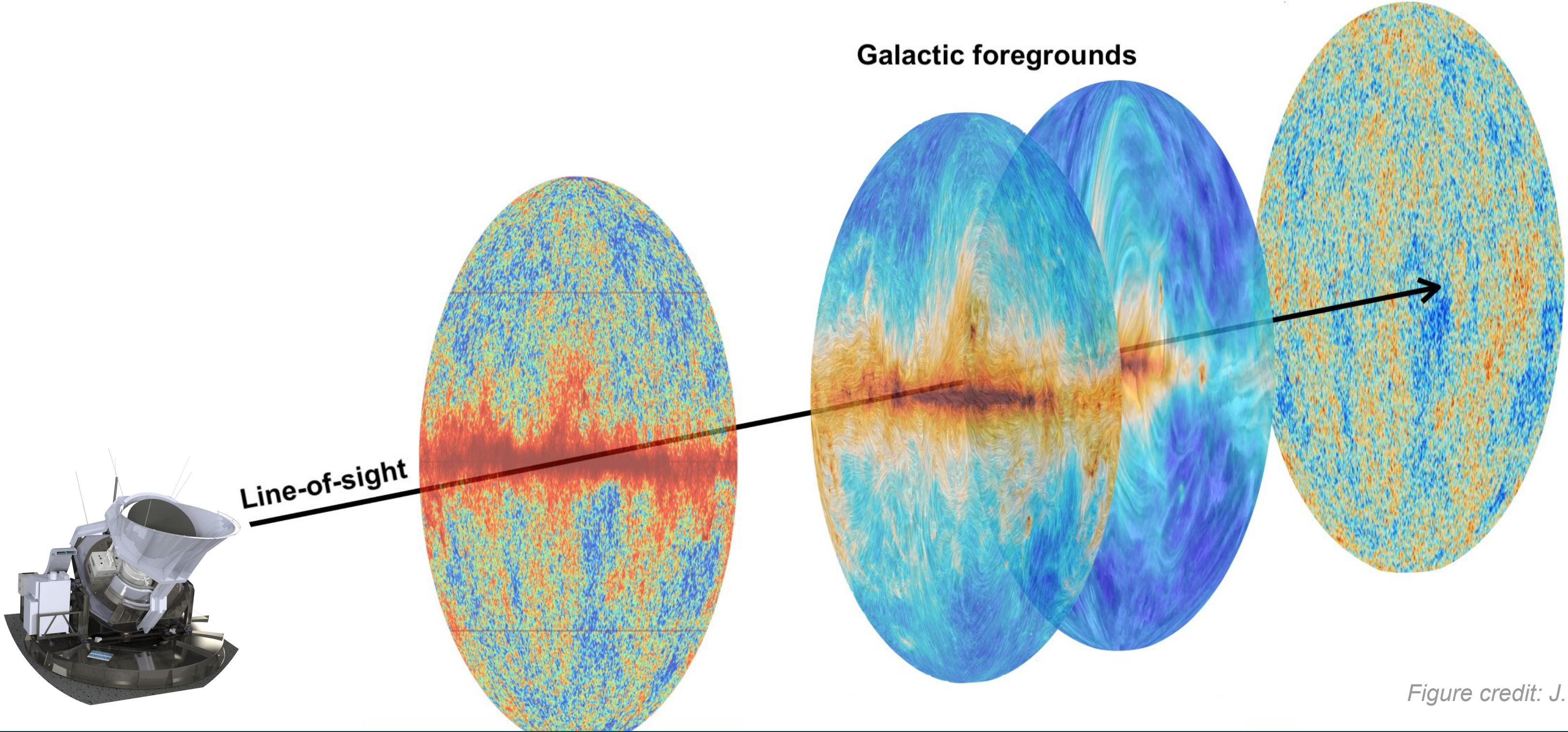


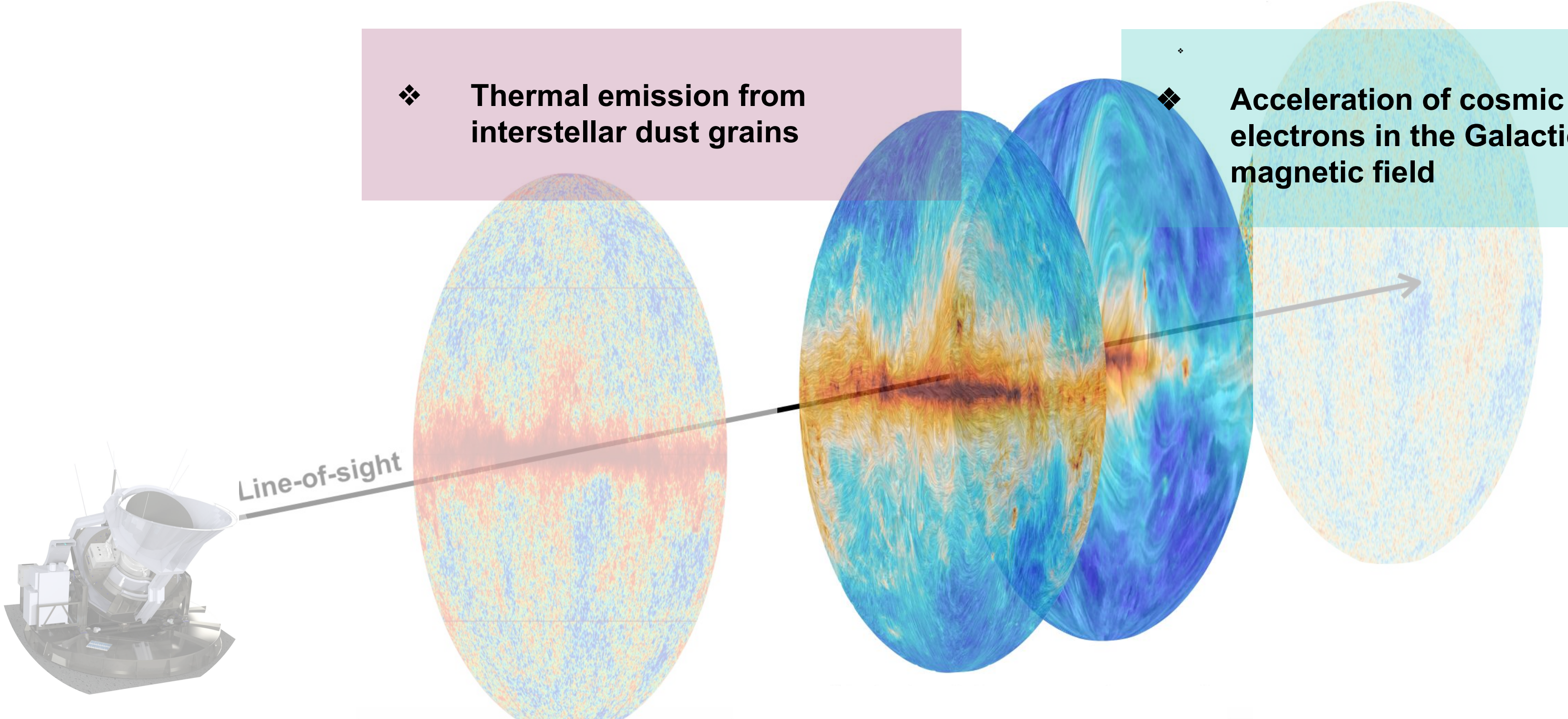
Figure credit: J. Errard, adapted

Foregrounds

Observed sky = (Polarized **dust** + **synchrotron**) + Polarized CMB

- ❖ **Thermal emission from interstellar dust grains**

- ❖ **Acceleration of cosmic rays electrons in the Galactic magnetic field**

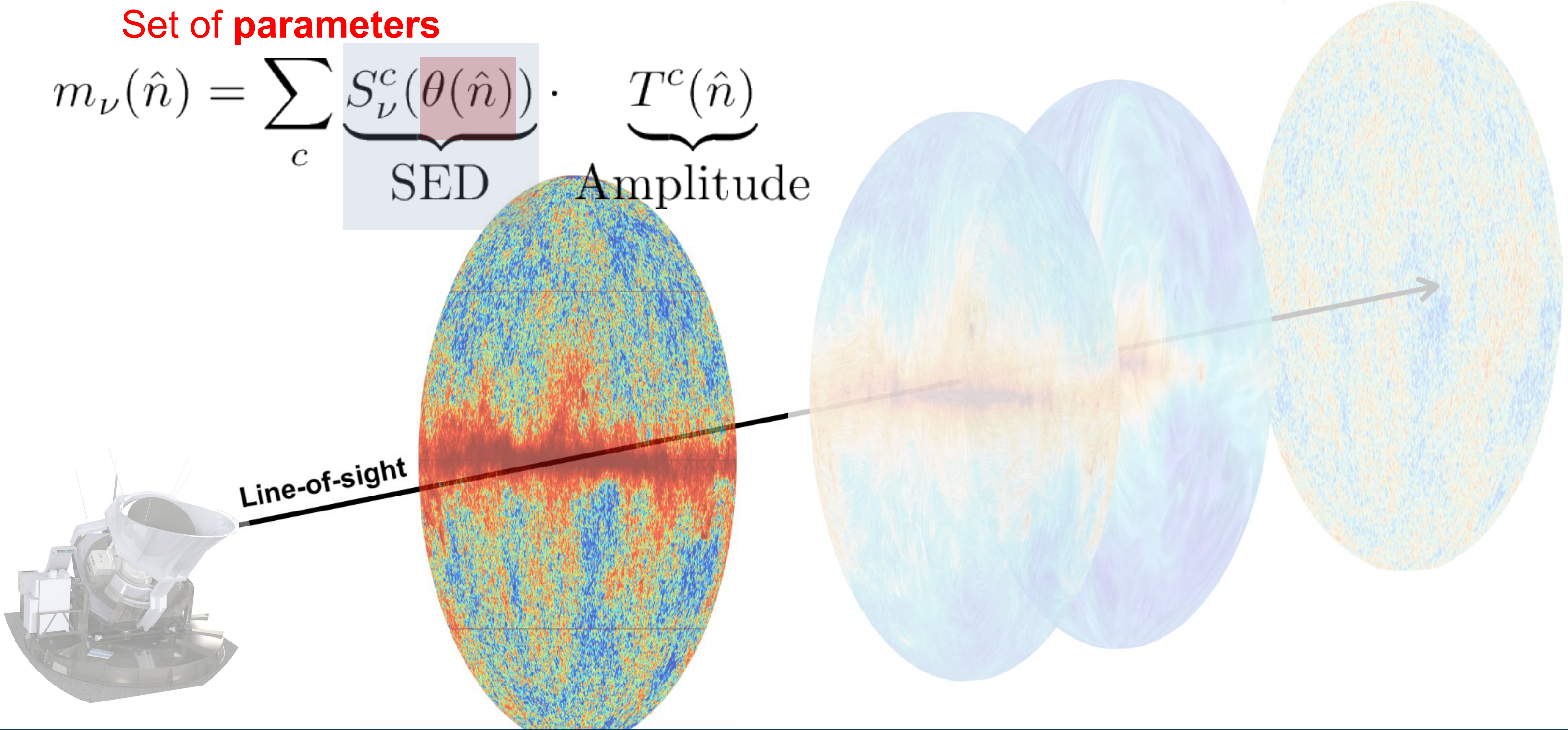


Foregrounds

Observed sky = (Polarized dust + synchrotron)+ Polarized CMB

Set of **parameters**

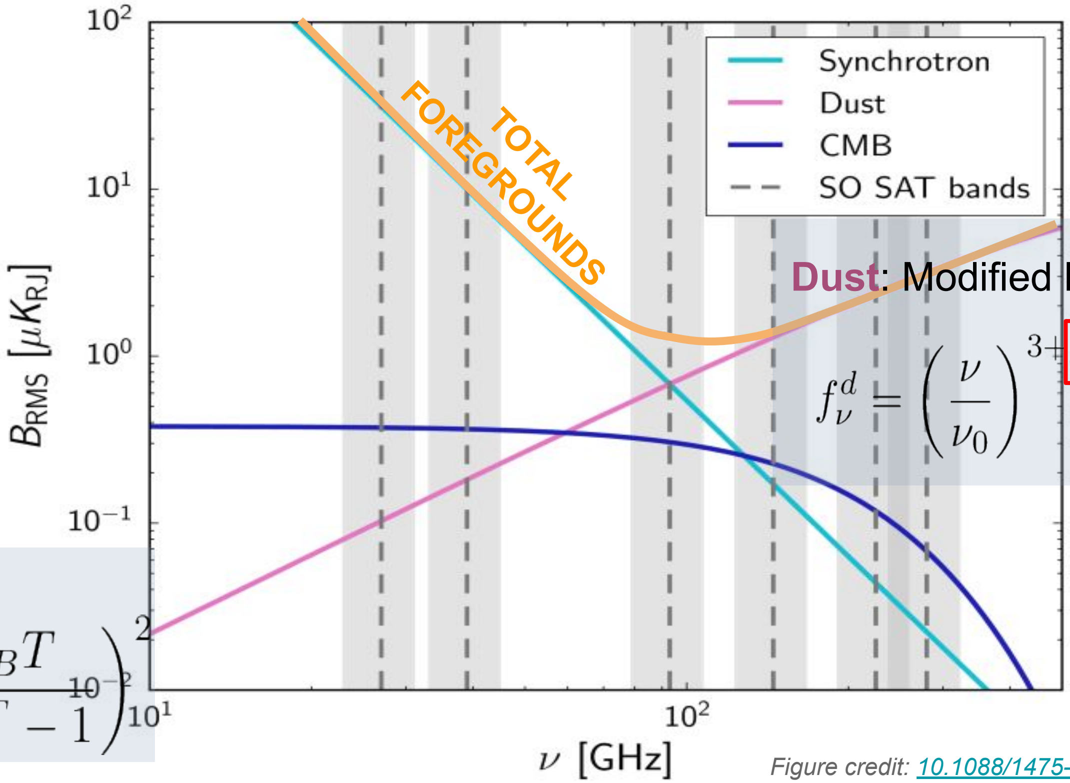
$$m_\nu(\hat{n}) = \sum_c \underbrace{S_\nu^c(\theta(\hat{n}))}_{\text{SED}} \cdot \underbrace{T^c(\hat{n})}_{\text{Amplitude}}$$



Foregrounds SEDs

Synchrotron: Power law

$$f_\nu^s = \left(\frac{\nu}{\nu_0}\right)^{2+\beta_s}$$



Dust: Modified Blackbody

$$f_\nu^d = \left(\frac{\nu}{\nu_0}\right)^{3+\beta_d} \frac{e^{h\nu_0/k_B T_d} - 1}{e^{h\nu/k_B T_d} - 1}$$

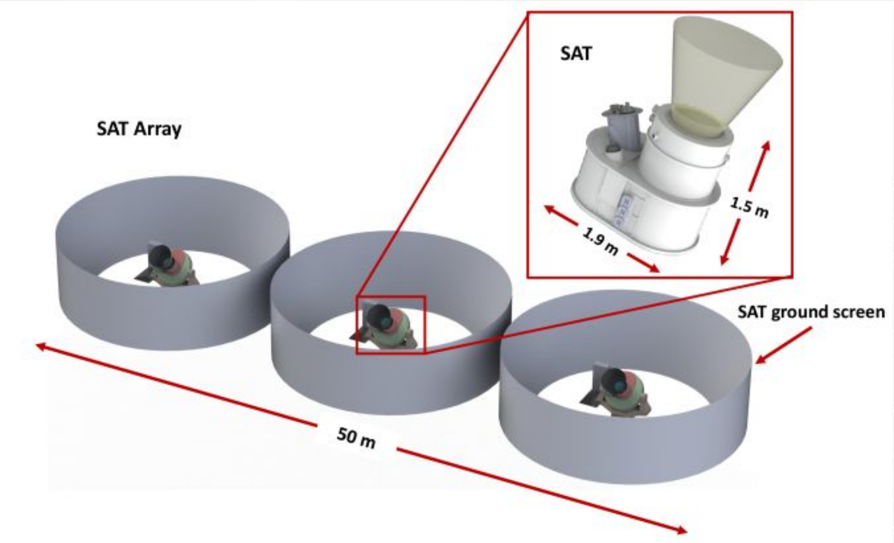
CMB: Blackbody

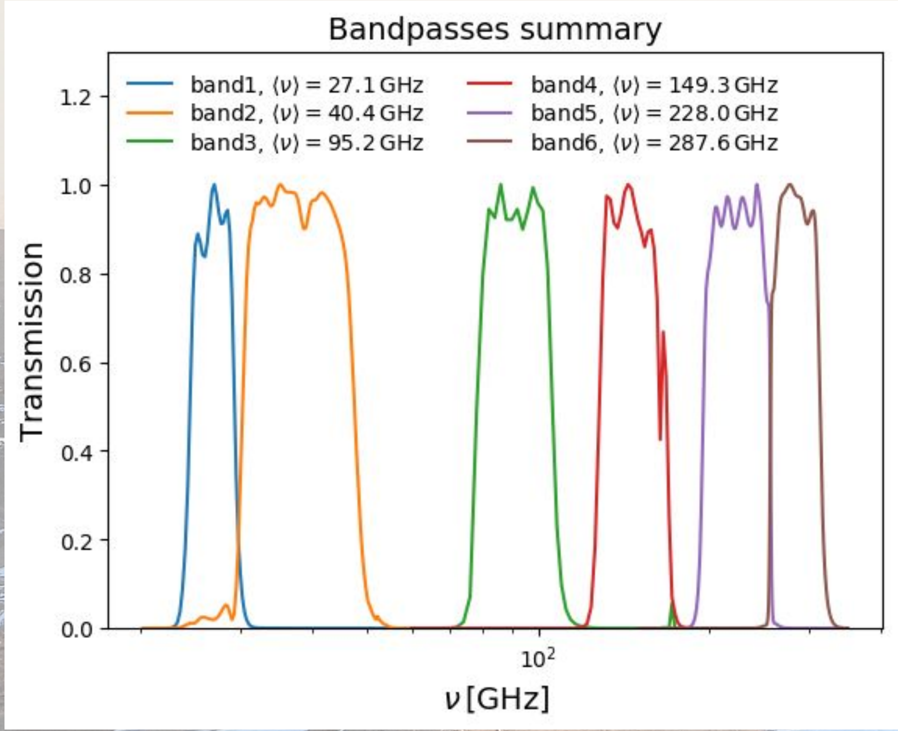
$$f_\nu^{\rm CMB} = e^{\frac{h\nu}{k_B T}} \left(\frac{h\nu/k_B T}{e^{h\nu/k_B T} - 1}\right)^2$$

Figure credit: 10.1088/1475-7516/2019/02/056, adapted

Simons Observatory

Simons Observatory

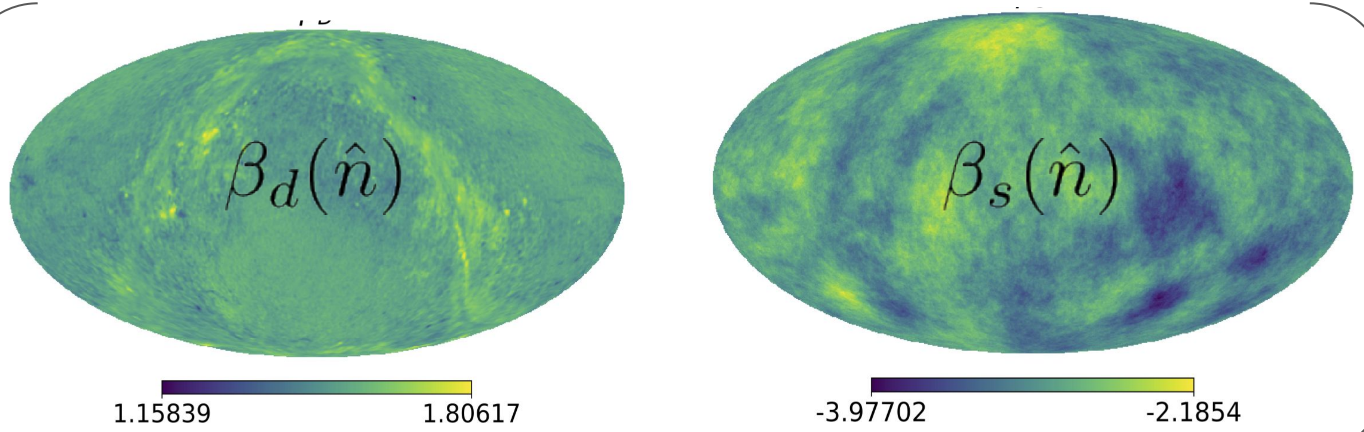




(Spatially-varying) Foregrounds

Set of parameters → space dependent

$\theta(\hat{\eta}) =$



SEDs spatially varying!

$$m_\nu(\hat{n}) = \sum_c \underbrace{S_\nu^c(\vec{\beta}(\hat{n}))}_{\text{SED}} \cdot \underbrace{T^c(\hat{n})}_{\text{Amplitude}}$$

Q: How do we remove spatially varying foregrounds?

Foregrounds removal methods

Map-based: model the contribution of each component at each pixel and at each frequency (*real space*)

- Exact likelihood function in real space
- *BUT* Expensive computational cost for ℓ_{max} > few hundreds

C_ℓ -based: compute all spectra between different frequencies (*harmonic space)*

- Easier to account for systematics effects in harmonic space
- *BUT* Harder to account for spatial variations

[*H.K Eriksen et al., (2008) [0709.1058]*](https://arxiv.org/abs/0709.1058)
[*J. Dunkley et al., (2009) [0811.4280]*](https://arxiv.org/abs/0811.4280)
[*Planck Collab., (2020) [1807.06208]*](https://arxiv.org/abs/1807.06208)
[*M. Remazeilles et al., (2020) [2006.08628]*](https://arxiv.org/abs/2006.08628)
[*R.Stompor et al., (2016) [1609.03807]*](https://arxiv.org/abs/1609.03807)
[*R. D. P. Grumitt et al., (2020) [1910.14170]*](https://arxiv.org/abs/1910.14170)

[*BICEP2 Collab., (2018) [1810.05216]*](https://arxiv.org/abs/1810.05216)
[*BICEP2/Keck, Planck, (2015), [1502.00612]*](https://arxiv.org/abs/1502.00612)
[*J. Dunkley et al., (2013), [1301.0776]*](https://arxiv.org/abs/1301.0776)
[*S.K.Choi et al., (2020) [2007.07289]*](https://arxiv.org/abs/2007.07289)

Foregrounds removal methods

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C_ℓ -based: compute all spectra between different frequencies (*harmonic space)*

- Easier to account for systematics effects in harmonic space
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Especially **Ideal for ground based experiments (e.g. SO, CMB-S4)**
→ **Additional systematics** e.g. filtering, ground pickup, atmospheric noise, ...

Required to analyse data with higher sensitivity over wider patches of the sky for the forthcoming B-modes experiments

Need to address this!

Method

(*I*,*Q*,*U*) signal maps

$$m_\nu(\hat{n}) = \sum_c T_c(\hat{n}) S^c_\nu(\vec{\beta}_c(\hat{n}))$$

Expand map in spherical harmonics:

$$m^\nu_{\ell m} = \sum_c \left[T^c S^c_\nu(\vec{\beta}_c) \right]_{\ell m}$$

Power Spectra

$$C^{\nu\nu'}_\ell = \langle m^\nu_{\ell m} m^{\nu'}_{\ell' m'} \rangle$$

Compute Likelihood
Using:

- Covariance matrix
- Fiducial Cl
- Noise Cl

→ Measure *r*

Method

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$$m_\nu(\hat{n}) = \sum_c T_c(\hat{n}) S_\nu^c(\vec{\beta}_c(\hat{n}))$$

Expand map in spherical harmonics:

$$m_{\ell m}^\nu = \sum_c \left[T^c S_\nu^c(\vec{\beta}_c)\right]_{\ell m}$$

Power Spectra

$$C_\ell^{\nu\nu'} = \langle m_{\ell m}^\nu m_{\ell' m'}^{\nu'} \rangle$$

Compute Likelihood
Using:

- Covariance matrix
- **Fiducial Cl**
- Noise Cl

→ Measure *r*

Q: How do we model spatially varying components?

1. Specify templates for **spectral indices** and **amplitudes**
2. **Propagate model from map to C_ℓ**

Formalism

1) **Spectral indices**

Assume **small spatial variation**

$$\beta(\hat{\eta}) = \beta_0 + \delta\beta(\hat{\eta})$$

Based on existing ``**moment expansion**'' *Chluba, Hill & Abitbol (2017)*
Taylor expand SEDs, additional parameters

$$S_\nu^c(\beta(\hat{\eta})) = S_\nu^c(\beta_0) + \delta\beta(\hat{\eta}) \frac{\partial S_\nu^c}{\partial \beta}\Big|_{\beta_0} + \frac{1}{2!}[\delta\beta(\hat{\eta})]^2 \frac{\partial^2 S_\nu^c}{\partial \beta^2}\Big|_{\beta_0} + \dots$$

Amplitudes

for dust, synchrotron, spectral indices: **power-laws**

$$C_\ell^{cc} = \langle T^c T^c \rangle_\ell = A_c \left(\frac{\ell}{80}\right)^{\alpha_c} , \quad C_\ell^{\beta_c \beta_c} = \langle \beta_c \beta_c \rangle_\ell = A_{\beta_c} \left(\frac{\ell}{80}\right)^{\gamma_c}$$

2) **Propagate moments into the power spectrum**

- Parameterize the C_ℓ of the moment parameters
- Model SED spectral index as power law:

$$C_\ell^{\nu\nu'} = C_\ell^{\nu\nu'}|_{0\times0} + C_\ell^{\nu\nu'}|_{0\times1} + C_\ell^{\nu\nu'}|_{1\times1} + C_\ell^{\nu\nu'}|_{0\times2},$$

$$C_\ell^{\nu\nu'}|_{0\times0} \equiv \sum_{cc'} \bar{S}_\nu^c\, \bar{S}_{\nu'}^{c'}\, C_\ell(T_c, T_{c'}),$$

$$C_\ell^{\nu\nu'}|_{0\times1} \equiv \sum_{cc'} \partial_i \bar{S}_\nu^c\, \bar{S}_{\nu'}^{c'}\, C_\ell(T_c \delta\beta_c^i, T_{c'}) + (\nu \leftrightarrow \nu'),$$

$$C_\ell^{\nu\nu'}|_{1\times1} \equiv \sum_{cc'} \partial_i \bar{S}_\nu^c\, \partial_j \bar{S}_{\nu'}^{c'}\, C_\ell(T_c \delta\beta_c^i, T_{c'} \delta\beta_{c'}^j),$$

$$C_\ell^{\nu\nu'}|_{0\times2} \equiv \frac{1}{2} \sum_{cc'} \partial_i \partial_j \bar{S}_\nu^c\, \bar{S}_{\nu'}^{c'}\, C_\ell(T_c \delta\beta_c^i \delta\beta_c^j, T_{c'}) + (\nu \leftrightarrow \nu'),$$

2) Propagate moments into the power spectrum

- Parameterize the C_ℓ of the moment parameters
- Model SED spectral index as power law:

$$C_\ell^{\beta_c} = \boxed{B_c}\left(\frac{\ell}{\ell_0}\right)^{\boxed{\gamma_c}}$$

Additional parameters

$$C_\ell^{\nu\nu'} = C_\ell^{\nu\nu'}|_{0\times0} + C_\ell^{\nu\nu'}|_{0\times1} + C_\ell^{\nu\nu'}|_{1\times1} + C_\ell^{\nu\nu'}|_{0\times2},$$

$$C_\ell^{\nu\nu'}|_{0\times0} \equiv \sum_{cc'} \bar{S}_\nu^c\, \bar{S}_{\nu'}^{c'}\, C_\ell(T_c, T_{c'}),$$

$$C_\ell^{\nu\nu'}|_{0\times1} \equiv \sum_{cc'} \partial_i \bar{S}_\nu^c\, \bar{S}_{\nu'}^{c'}\, C_\ell(T_c\delta\beta_c^i, T_{c'}) + (\nu \leftrightarrow \nu'),$$

$$C_\ell^{\nu\nu'}|_{1\times1} \equiv \sum_{cc'} \partial_i \bar{S}_\nu^c\, \partial_j \bar{S}_{\nu'}^{c'}\, C_\ell(T_c\delta\beta_c^i, T_{c'}\delta\beta_{c'}^j),$$

$$C_\ell^{\nu\nu'}|_{0\times2} \equiv \frac{1}{2}\sum_{cc'} \partial_i\partial_j \bar{S}_\nu^c\, \bar{S}_{\nu'}^{c'}\, C_\ell(T_c\delta\beta_c^i\delta\beta_c^j, T_{c'}) + (\nu \leftrightarrow \nu'),$$

Simplifying assumptions:

1. spectral index variations are Gaussianly distributed
2. foreground amplitudes and spectral index variations are uncorrelated
3. spectral index variations of different foreground sources are uncorrelated

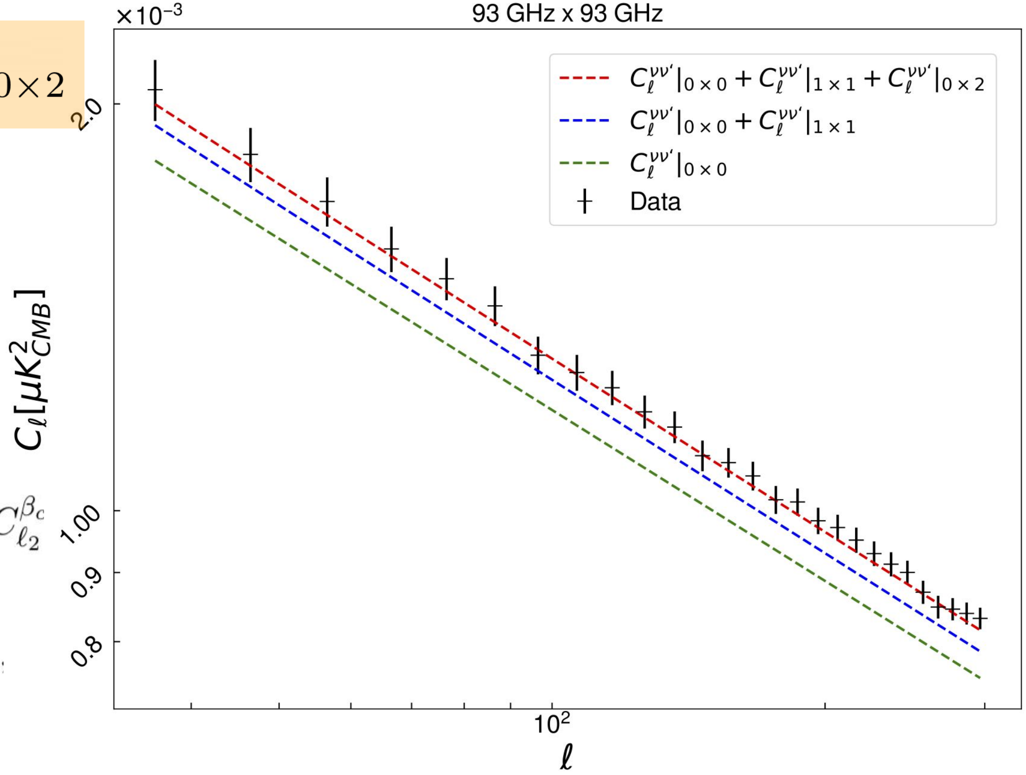
2) Propagate moments into the power spectrum

$$C_\ell^{\nu\nu'} = C_\ell^{\nu\nu'}|_{0\times0} + C_\ell^{\nu\nu'}|_{1\times1} + C_\ell^{\nu\nu'}|_{0\times2}$$

$$C_\ell^{\nu\nu'}|_{0\times0} = \bar{S}_\nu^{\rm D}\bar{S}_{\nu'}^{\rm D}\,C_\ell^{\rm DD} + \bar{S}_\nu^{\rm S}\bar{S}_{\nu'}^{\rm S}\,C_\ell^{\rm SS} + \left(\bar{S}_\nu^{\rm D}\bar{S}_{\nu'}^{\rm S} + \bar{S}_\nu^{\rm S}\bar{S}_{\nu'}^{\rm D}\right)C_\ell^{\rm SD},$$

$$C_\ell^{\nu\nu'}|_{1\times1} = \sum_{c\in\{\rm D,S\}} \partial_\beta\bar{S}_\nu^c\,\partial_\beta\bar{S}_{\nu'}^c \sum_{\ell_1\ell_2}\frac{(2\ell_1+1)(2\ell_2+1)}{4\pi}\begin{pmatrix}\ell & \ell_1 & \ell_2\\ 0 & 0 & 0\end{pmatrix}^2 C_{\ell_1}^{cc}C_{\ell_2}^{\beta_c}$$

$$C_\ell^{\nu\nu'}|_{0\times2} = \sum_{c\in\{\rm D,S\}}\frac{1}{2}\left[\bar{S}_\nu^c\,\partial_\beta^2\bar{S}_{\nu'}^c + \bar{S}_{\nu'}^c\,\partial_\beta^2\bar{S}_\nu^c\right]C_\ell^{cc}\sigma_{\beta_c}^2$$



Simulations

(*I*,*Q*,*U*) signal maps

$$m_\nu(\hat{n}) = \sum_c T_c(\hat{n}) S_\nu^c(\vec{\beta}_c(\hat{n}))$$

Include instrument
(e.g. bandpass, beams)

Add Noise + splits

Apply mask

Expand map in spherical harmonics

$$m^\nu_{\ell m} = \sum_c \left[T^c S^c_\nu(\vec{\beta}_c)\right]_{\ell m}$$

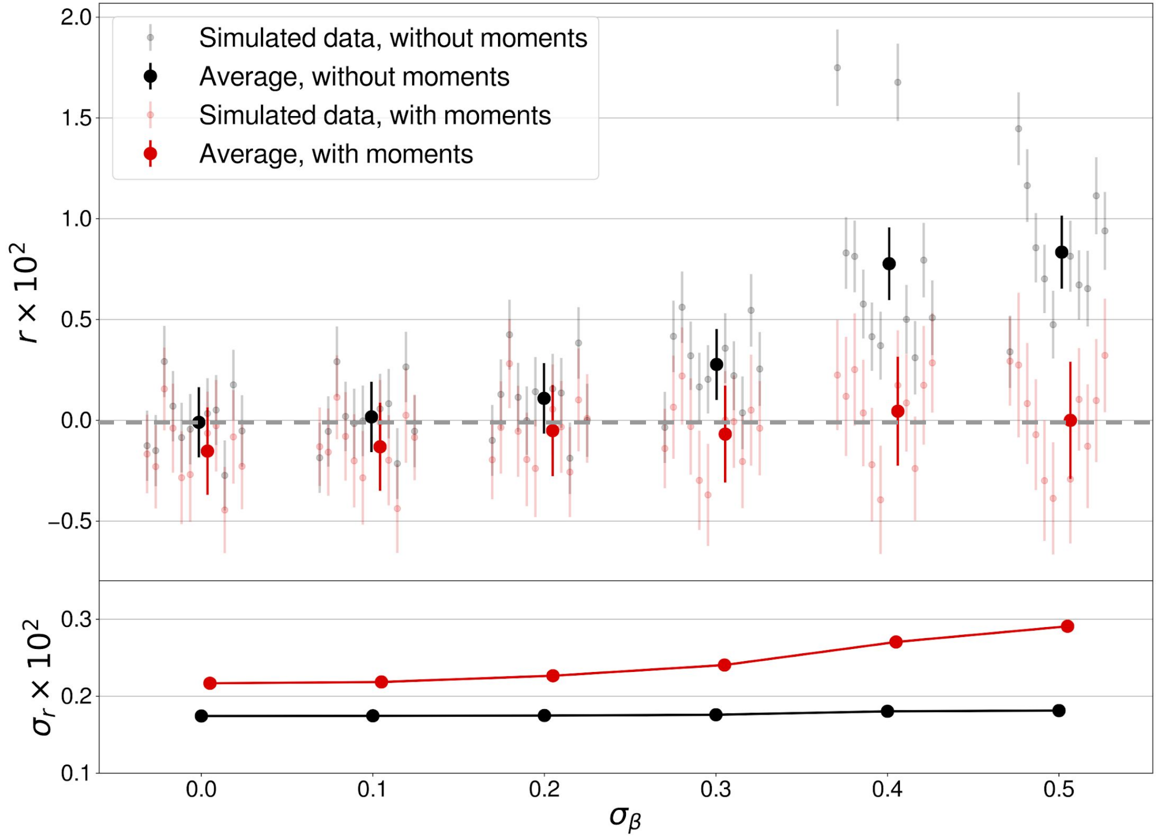
Power Spectra

$$C_\ell^{\nu\nu'} = \langle m^\nu_{\ell m} m^{\nu'}_{\ell' m'} \rangle$$

→ Compute binned PS
→ Covariance matrix

1) **Gaussian simulations**
2) **Realistic simulations (PySM)**
3) **Simulations challenge**

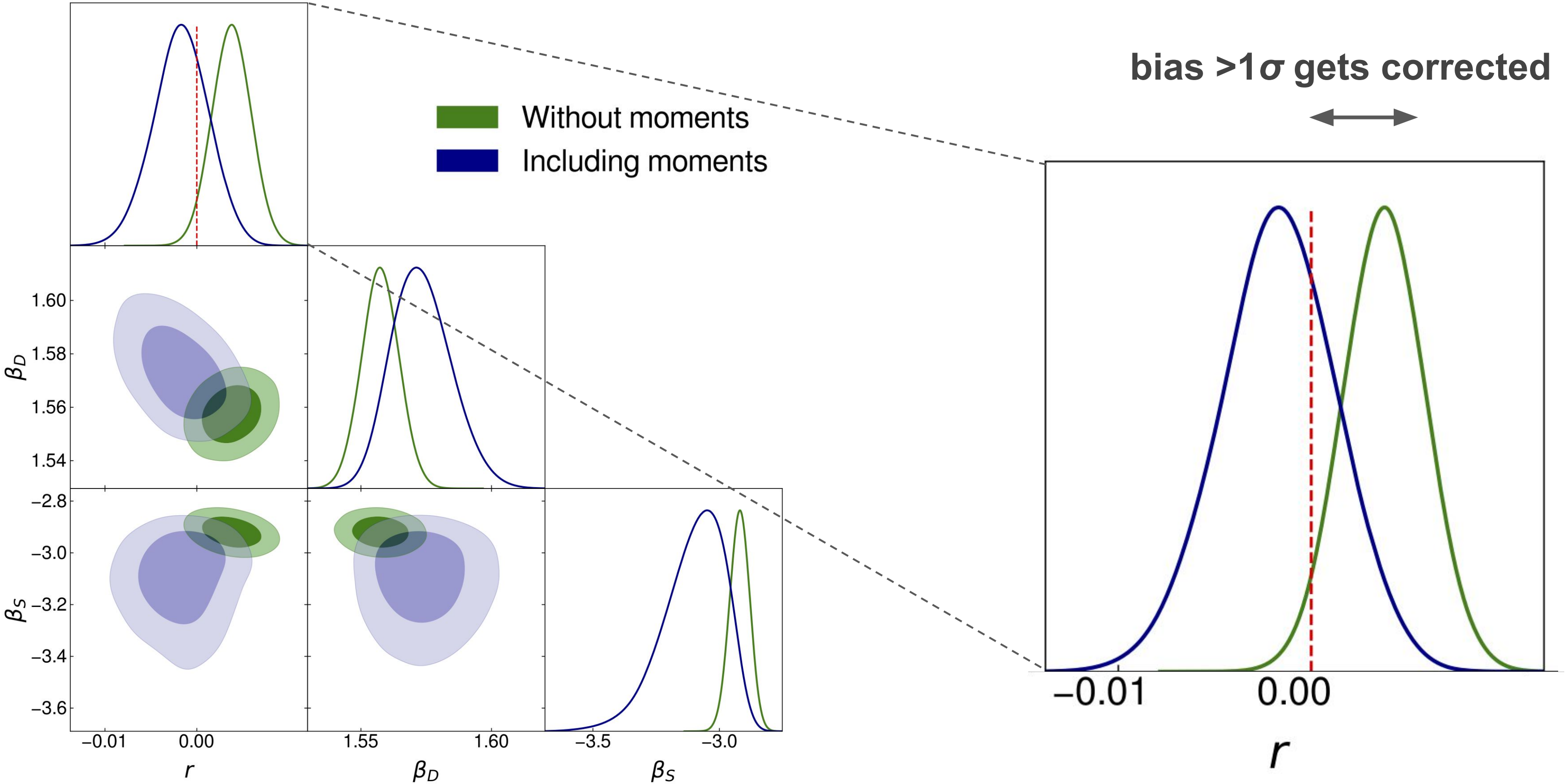
Application to **Gaussian simulations** with increasing spectral indices variation



biases on *r* (1-2 *σ*) are corrected

small penalty in uncertainty on *r* (~30%)

Application to **PySM simulations**



$$m_\nu(\hat{n}) = \sum_c T_c(\hat{n}) S^c_\nu(\vec{\beta}_c(\hat{n}))$$

Simulation challenge

Simulation		No moments		With moments	
Description; $(\sigma_{\beta_D}, \sigma_{\beta_S})$	$r_{\rm true}$	$r_{\rm fit} \pm \sigma_r$	χ^2/d.o.f.	$r_{\rm fit} \pm \sigma_r$	χ^2/d.o.f.
G, MBB; $\sigma_\beta = (0, 0)$	0	-0.0013 ± 0.0021	0.8	-0.0024 ± 0.0024	0.8
G, MBB; $\sigma_\beta = (0, 0)$	0.01	0.0116 ± 0.0022	0.8	0.0099 ± 0.0025	0.8
G, MBB; $\sigma_\beta = (0.2, 0.3)$	0	0.0088 ± 0.0023	0.9	0.0038 ± 0.0035	0.8
G, MBB; $\sigma_\beta = (0.2, 0.3)$	0.01	0.0158 ± 0.0025	0.9	0.0098 ± 0.0035	0.9
P, MBB; σ_β = PySM	0	0.0051 ± 0.0022	0.9	0.0036 ± 0.0026	0.9
P, MBB; σ_β = PySM	0.01	0.0130 ± 0.0023	0.9	0.0104 ± 0.0027	0.9
G, H&D; $\sigma_\beta = (0, 0)$	0	0.0058 ± 0.0026	1.1	0.0003 ± 0.0037	1.1
G, H&D, $\sigma_\beta = (0, 0)$	0.01	0.0122 ± 0.0024	1.1	0.0055 ± 0.0038	1.1
P, H&D; σ_β = PySM	0	0.0052 ± 0.0025	1.1	0.0001 ± 0.0033	1.1
P, H&D; σ_β = PySM	0.01	0.0120 ± 0.0024	1.1	0.0069 ± 0.0034	1.1
P, VS; $\sigma_\beta = (0.13, N.A.)$	0	0.0114± 0.0024	1.0	-0.0036 ± 0.0036	1.0
P, VS; $\sigma_\beta = (0.13, N.A.)$	0.01	0.0184 ± 0.0025	1.0	0.0029 ± 0.0034	1.0

$$m_\nu(\hat{n}) = \sum_c T_c(\hat{n}) S^c_\nu(\vec{\beta}_c(\hat{n}))$$

Simulation challenge

Amplitudes:

Gaussian

PySM templates

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Simulation challenge

Foreground spectral indices:

Gaussian fields w/ std σ_β

PySM templates

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Thermal dust spectrum:

Modified Black Body

Hensley & Draine *[1611.08607]*

Van Syngel et al *[1611.02577]*

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Simulation challenge

Amplitudes:

Gaussian

PySM templates

Foreground spectral indices:

Gaussian fields w/ std σ_β

PySM templates

Thermal dust spectrum:

Modified Black Body

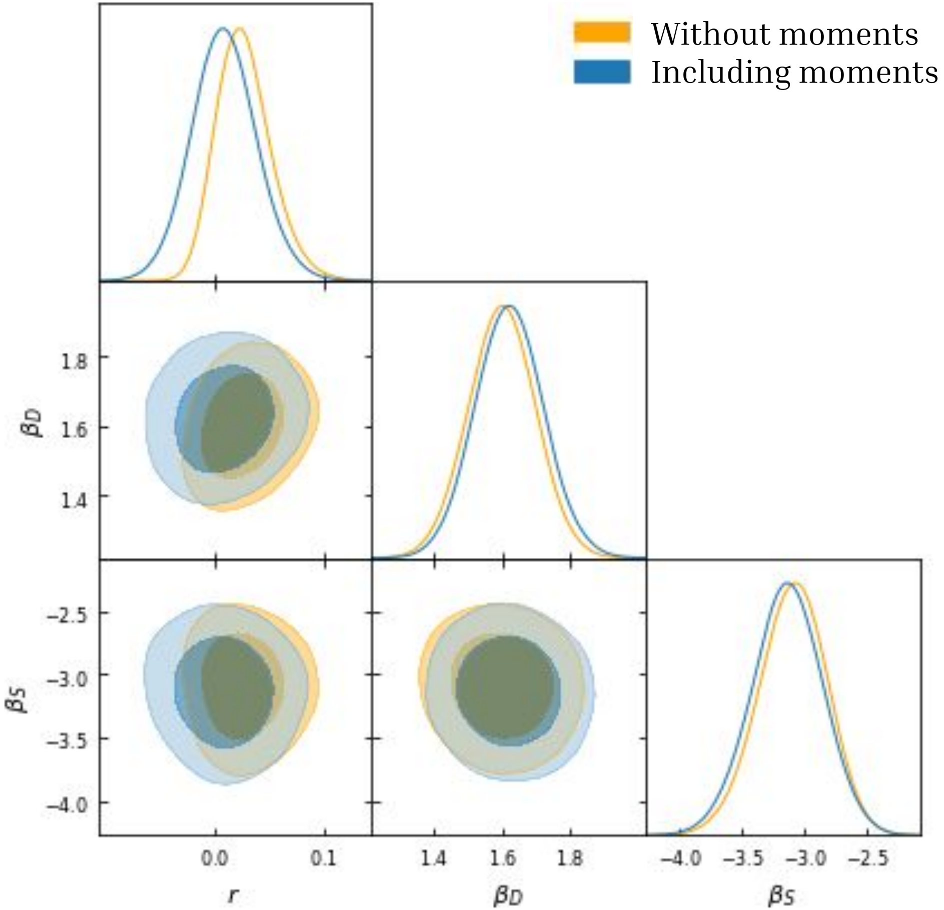
Hensley & Draine *[1611.08607]*

Van Syngel et al *[1611.02577]*

Simulation		No moments		With moments	
Description; $(\sigma_{\beta_D}, \sigma_{\beta_S})$	r_{true}	$r_{fit} \pm \sigma_r$	χ^2/d.o.f.	$r_{fit} \pm \sigma_r$	χ^2/d.o.f.
G, MBB; $\sigma_\beta = (0,0)$	0	-0.0013 ± 0.0021	0.8	-0.0024 ± 0.0024	0.8
G, MBB; $\sigma_\beta = (0,0)$	0.01	0.0116 ± 0.0022	0.8	0.0099 ± 0.0025	0.8
G, MBB; $\sigma_\beta = (0.2,0.3)$	0	0.0088 ± 0.0023	0.9	0.0038 ± 0.0035	0.8
G, MBB; $\sigma_\beta = (0.2,0.3)$	0.01	0.0158 ± 0.0025	0.9	0.0098 ± 0.0035	0.9
P, MBB; σ_β = PySM	0	0.0051 ± 0.0022	0.9	0.0036 ± 0.0026	0.9
P, MBB; σ_β = PySM	0.01	0.0130 ± 0.0023	0.9	0.0104 ± 0.0027	0.9
G, H&D; $\sigma_\beta = (0,0)$	0	0.0058 ± 0.0026	1.1	0.0003 ± 0.0037	1.1
G, H&D, $\sigma_\beta = (0,0)$	0.01	0.0122 ± 0.0024	1.1	0.0055 ± 0.0038	1.1
P, H&D; σ_β = PySM	0	0.0052 ± 0.0025	1.1	0.0001 ± 0.0033	1.1
P, H&D; σ_β = PySM	0.01	0.0120 ± 0.0024	1.1	0.0069 ± 0.0034	1.1
P, VS; $\sigma_\beta = (0.13, N.A.)$	0	0.0114± 0.0024	1.0	-0.0036 ± 0.0036	1.0
P, VS; $\sigma_\beta = (0.13, N.A.)$	0.01	0.0184 ± 0.0025	1.0	0.0029 ± 0.0034	1.0

Red: results with a bias $|r_{fit} - r_{true}| \geq 2\sigma_r$

Application to **BK15 data**



- Consistent with BK15X results [BICEP2 & Keck Array Collaboration (2018)]
- Adding spatial variability shifts the posterior down
- Small penalty in uncertainty on *r* (~20%)
- Comparable to the effects of decorrelation

Take-away message

- **Implemented component separation method accounting for moment expansion of the dust/synchrotron moments in power-spectrum space**
  - Moment expansion of a foreground SED is a general parametrization of additional features of the underlying distribution of physical parameters
  - Very few a priori assumptions, captures the spectral and spatial variations of the SED

- **Study limited to the analysis of primordial B-modes from ground-based facilities**
  - targeting the recombination bump on scales $30 < \ell < 300$
  - its applicability to space missions may be limited, the use of pixel-based methods is likely more appropriate.

- **It is a promising tool to model the foreground components at a level of precision that will be useful in the analysis of future observatories to characterize spatially-varying foregrounds and marginalize over them in order to achieve reliable constraints on r**

Thank you!