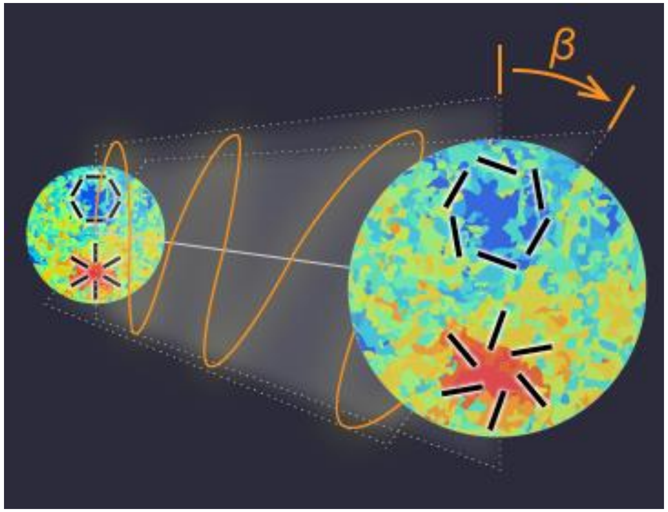
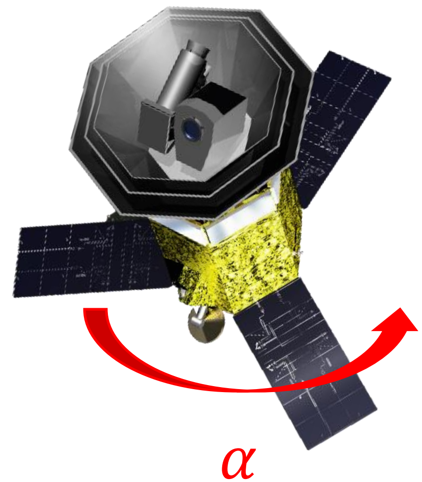
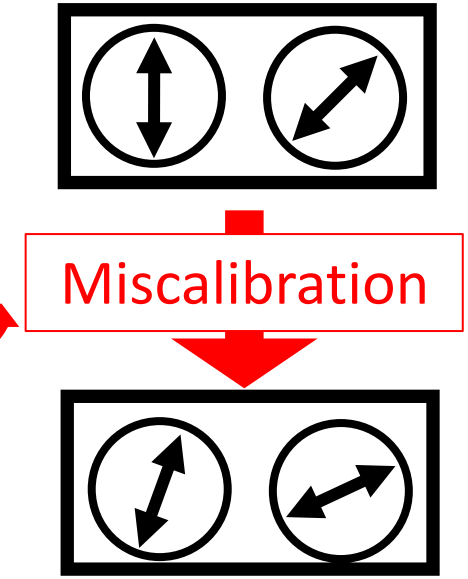
Simultaneous determination of the cosmic birefringence and miscalibrated polarisation angles (1 min slide)



Cosmic birefringence

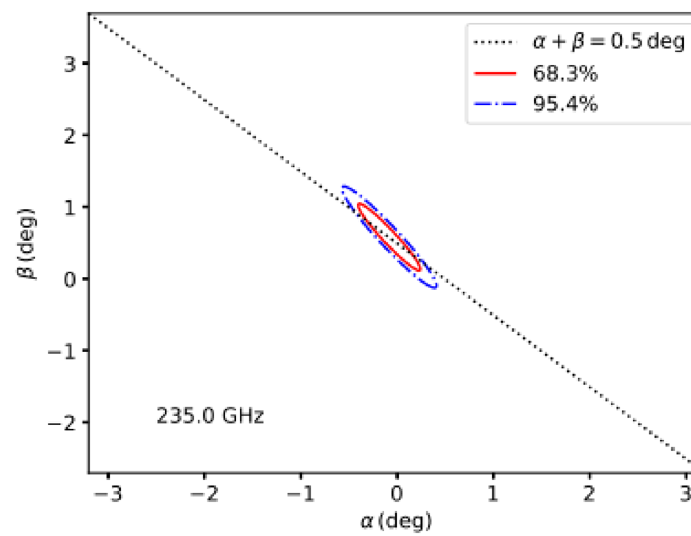
OR



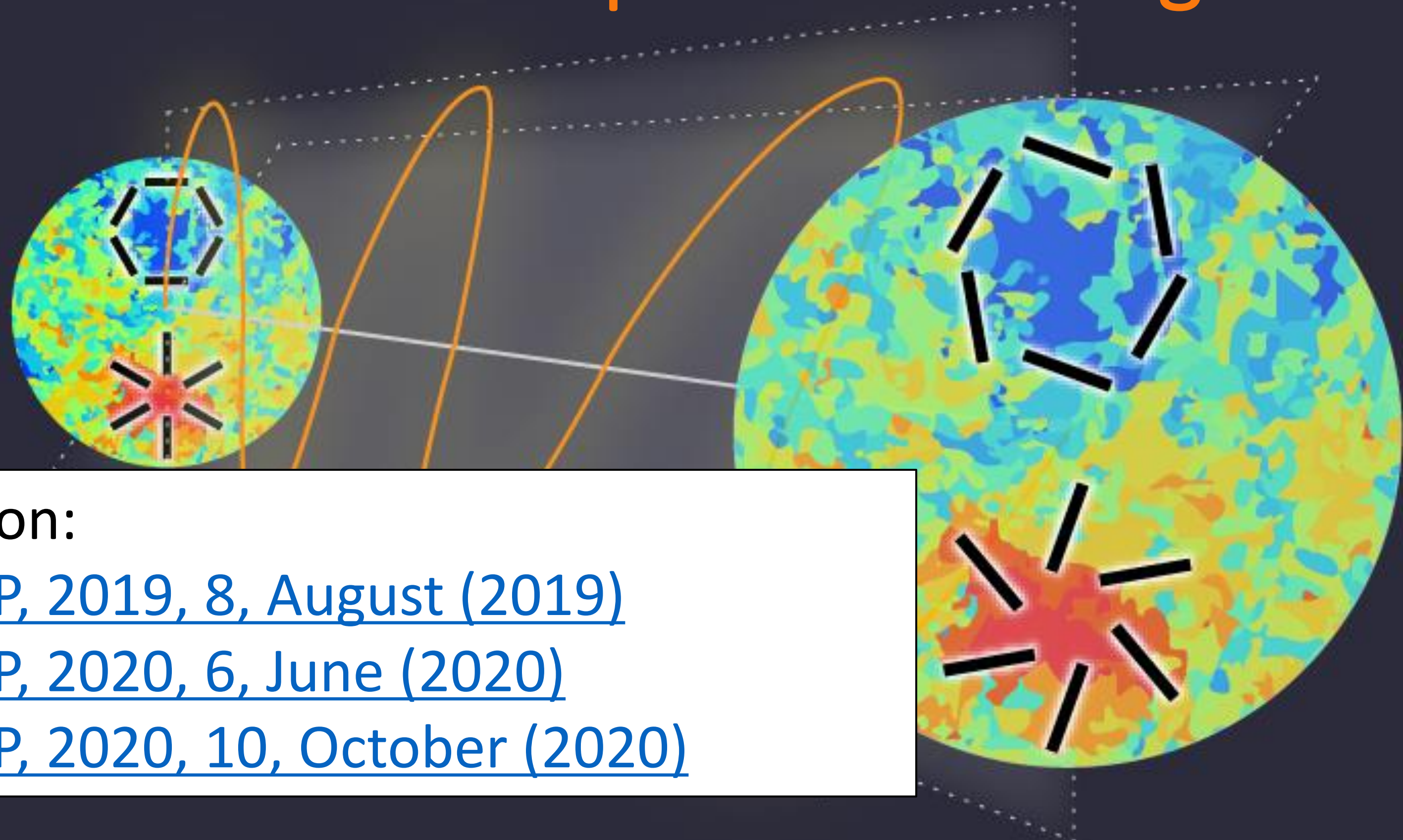


- With miscalibration, we can only measure the sum, $\alpha + \beta$
- We develop a method to determine α with the Galactic foreground and break the degeneracy!

Let's apply this method to other future and existing experiments!



Simultaneous determination of the cosmic birefringence and miscalibrated polarisation angles



Based on:
- PTEP, 2019, 8, August (2019)
- PTEP, 2020, 6, June (2020)
- PTEP, 2020, 10, October (2020)

Yuto Minami (RCNP, Osaka Univ.)

Pre-introduction

A new paper related to this talk was published

- “New Extraction of the Cosmic Birefringence from the Planck 2018 polarisation Data”, Yuto Minami and Eiichiro Komatsu, Phys. Rev. Lett. 125, 221301, 2020
  - https://doi.org/10.1103/PhysRevLett.125.221301

However, today, I will only talk about the methodology used in the analysis, since this workshop is for “systematics and calibration”

- I hope you can apply this method to your observed data

Overview

1. "Simultaneous determination of miscalibration angles and cosmic birefringence", PTEP, 2019, 8, August (2019)
    - Determine miscalibration angle, α, only with observed power spectra
    - Use prior knowledge of CMB power spectra to determine additional angle, β
2. "Simultaneous determination of the cosmic birefringence and miscalibrated polarisation angles II: Including cross-frequency spectra", PTEP, 2020, 10, October (2020)
    - Application to multi channel (detector) observations
    - Include cross correlation among different frequency channel
        - Improvement of uncertainty
3. "Determination of miscalibrated polarisation angles from observed CMB and foreground EB power spectra: Application to partial-sky observation", PTEP, 2020, 6, June (2020)
    - Include ℓ-to-ℓ covariance
    - Use prior knowledge of CMB power spectra to determine foreground EB correlation

Overview

1. “Simultaneous determination of miscalibration angles and cosmic birefringence”, PTEP, 2019, 8, August (2019)

- ➢ Determine miscalibration angle, α, only with observed power spectra
- ➢ Use prior knowledge of CMB power spectra to determine additional angle, β

2. “Simultaneous determination of the cosmic birefringence and miscalibrated polarisation angles II: Including cross-frequency spectra”, PTEP, 2020, 10, October (2020)

- ➢ Application to multi channel (detector) observations
- ➢ Include cross correlation among different frequency channel
  - ➢ Improvement of uncertainty

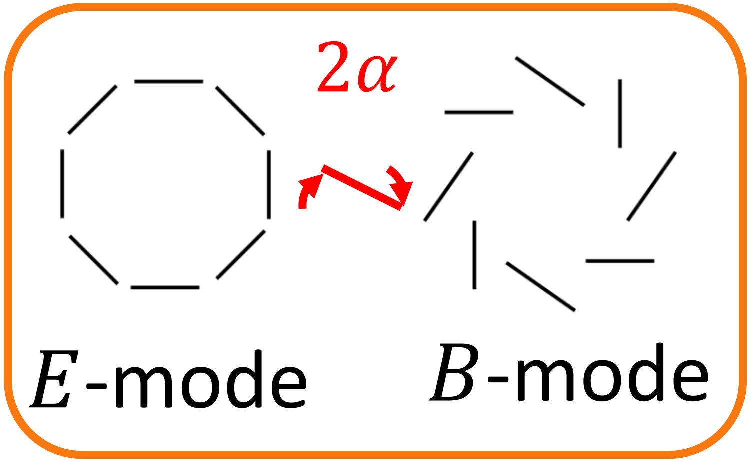
3. “Determination of miscalibrated polarisation angles from observed CMB and foreground EB power spectra: Application to partial-sky observation”, PTEP, 2020, 6, June

This talk only includes above 2 topics

- ➢ Use prior knowledge of CMB power spectra to determine foreground EB correlation

"Simultaneous determination of miscalibration angles and cosmic birefringence", PTEP, 2019, 8, August (2019)

Miscalibration



- Miscalibration of detector rotation angle (α) creates spurious B−mode from E−mode

$$C_\ell^{BB,o} = C_\ell^{EE}\sin^2(2\alpha) + C_\ell^{BB}\cos^2(2\alpha) \cdots (1)$$

observed　　before detectors

We need to determine α to calibrate rotation angle

- In past experiments, this α was calculated assuming that EB correlation of CMB is zero:

$$C_\ell^{EB,o} = \frac{1}{2}\left(C_\ell^{EE,CMB} - C_\ell^{BB,CMB}\right)\sin(4\alpha) \quad \cdots (2)$$

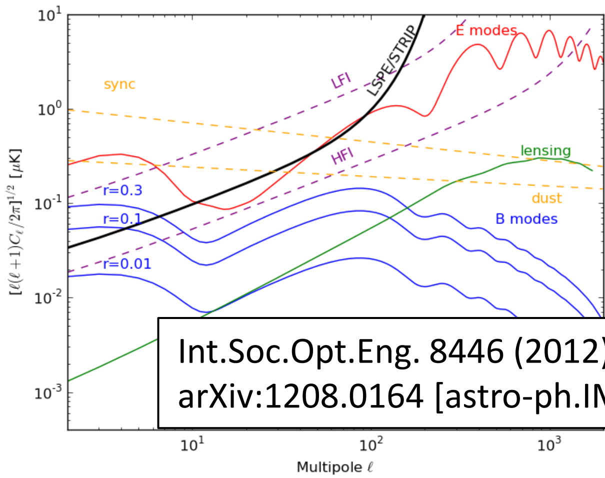
From theory

"Self-calibration" by Keating, Shimon, & Yadav (2013).

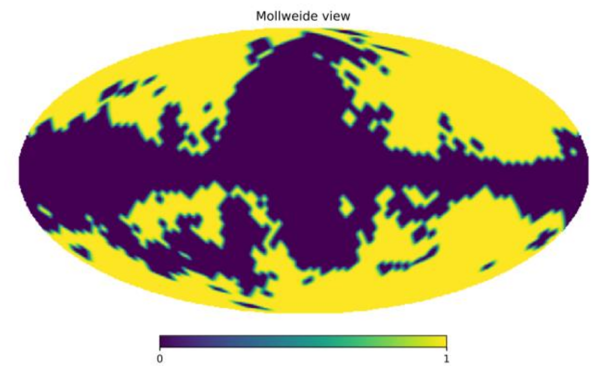
Restriction of “self-calibration” using theory

Foreground emissions should be small

- Because foreground signals also have α,
  - We need to know the foreground model
  - We need to mask the Galactic plane



Int.Soc.Opt.Eng. 8446 (2012) 84467C
arXiv:1208.0164 [astro-ph.IM]



Cosmological EB correlation should be zero

- We lose sensitivity to cosmic birefringence

To solve these issues

We relate observed E- and B- modes to the intrinsic ones as

$$E^{o}_{\ell,m} = E_{\ell,m}\cos(2\alpha) - B_{\ell,m}\sin(2\alpha)$$
$$B^{o}_{\ell,m} = E_{\ell,m}\sin(2\alpha) + B_{\ell,m}\cos(2\alpha) \quad \cdots(3)$$

From these equations, we find

$$C_\ell^{EB,o} = \frac{1}{2}\left(C_\ell^{EE,o} - C_\ell^{BB,o}\right)\tan(4\alpha) + \frac{C_\ell^{EB}}{\cos(4\alpha)} \quad \cdots(4)$$

G. B. Zhao et al.（2015）

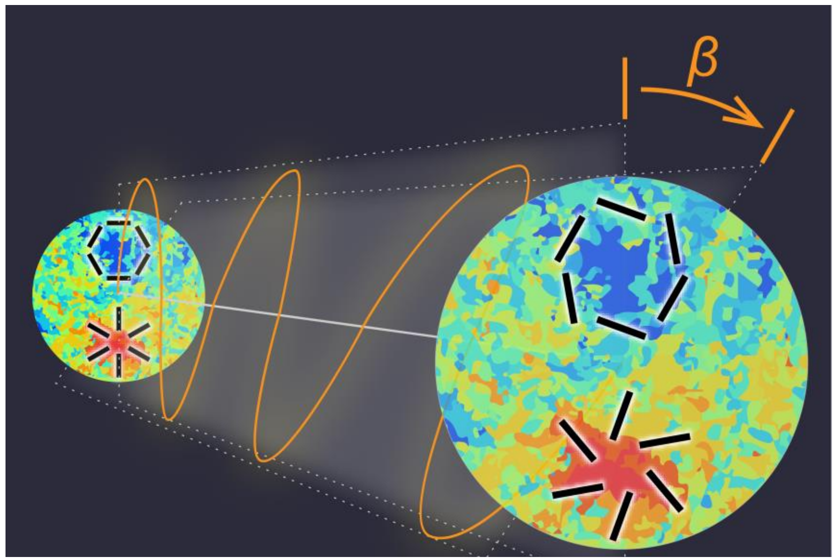
Our work [Y.Minami et al. (2019)]

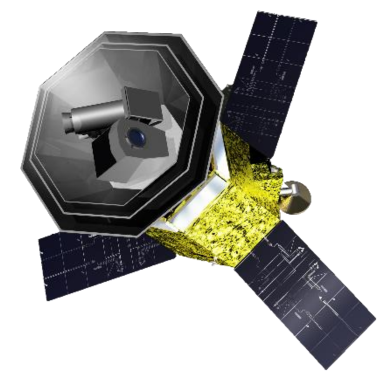
- We can estimate α with only observed data
- If we assume theory CMB power spectra, we can estimate an additional angle!

→ Cosmic birefringence

Cosmic birefringence

During the long travel from the last scattering surface to observer, parity-violating physics, e.g., axion-like particles (ALPs), rotate CMB linear polarisation by





$$\beta = \frac{1}{2} g_{\phi\gamma}(\bar{\phi}_{obs} - \bar{\phi}_{LSS} + \delta\phi_{obs})$$

Carroll, Field & Jackiw (1990); Harari & Sikivie (1992); Carroll (1998); [Fujita, Minami, et al. (2020)]

- Foreground: rotated only by α
- CMB: rotated by $\alpha + \beta$

Equations including birefringence rotation:

The coefficients become

$$E^{\mathrm{o}}_{\ell,m} = E^{\mathrm{fg}}_{\ell,m}\cos(2\alpha) - B^{\mathrm{fg}}_{\ell,m}\sin(2\alpha) + E^{\mathrm{CMB}}_{\ell,m}\cos(2\alpha+2\beta) - B^{\mathrm{CMB}}_{\ell,m}\sin(2\alpha+2\beta) + E^{\mathrm{N}}_{\ell,m}$$

$$B^{\mathrm{o}}_{\ell,m} = E^{\mathrm{fg}}_{\ell,m}\sin(2\alpha) + B^{\mathrm{fg}}_{\ell,m}\cos(2\alpha) + E^{\mathrm{CMB}}_{\ell,m}\sin(2\alpha+2\beta) + B^{\mathrm{CMB}}_{\ell,m}\cos(2\alpha+2\beta) + B^{\mathrm{N}}_{\ell,m}$$

noise

$\cdots (5)$

From them, we derived

$$\langle C_\ell^{EB,\mathrm{o}}\rangle = \frac{\tan(4\alpha)}{2}\left(\langle C_\ell^{EE,\mathrm{o}}\rangle - \langle C_\ell^{BB,\mathrm{o}}\rangle\right) + \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(\langle C_\ell^{EE,\mathrm{CMB}}\rangle - \langle C_\ell^{BB,\mathrm{CMB}}\rangle\right) \quad \cdots (6)$$

$$+ \frac{1}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{fg}}\rangle + \frac{\cos(4\beta)}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{CMB}}\rangle .$$

Assume these to be zero

Therefore, we can determine both miscalibration and birefringence-rotation angles simultaneously!

Construct a likelihood for determination of α and β

$$\langle C_\ell^{EB,\mathrm{o}} \rangle = \frac{\tan(4\alpha)}{2}\left(\langle C_\ell^{EE,\mathrm{o}}\rangle - \langle C_\ell^{BB,\mathrm{o}}\rangle\right) + \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(\langle C_\ell^{EE,\mathrm{CMB}}\rangle - \langle C_\ell^{BB,\mathrm{CMB}}\rangle\right) \quad \cdots (6)$$

$$+ \frac{1}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{fg}}\rangle + \frac{\cos(4\beta)}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{CMB}}\rangle \,.$$

Assume these to be zero

$$-2\ln\mathcal{L} = \sum_{\ell=2}^{\ell_{\max}} \frac{\left[C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right) - \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(C_\ell^{EE,\mathrm{CMB}} - C_\ell^{BB,\mathrm{CMB}}\right)\right]^2}{\mathrm{Var}\left(C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right)\right)} \quad \cdots (7)$$

Minimise $-2\ln\mathcal{L}$ to determine α and β

Variance

- With full-sky power spectra (not cut-sky pseudo power spectra), we can calculate variance exactly as

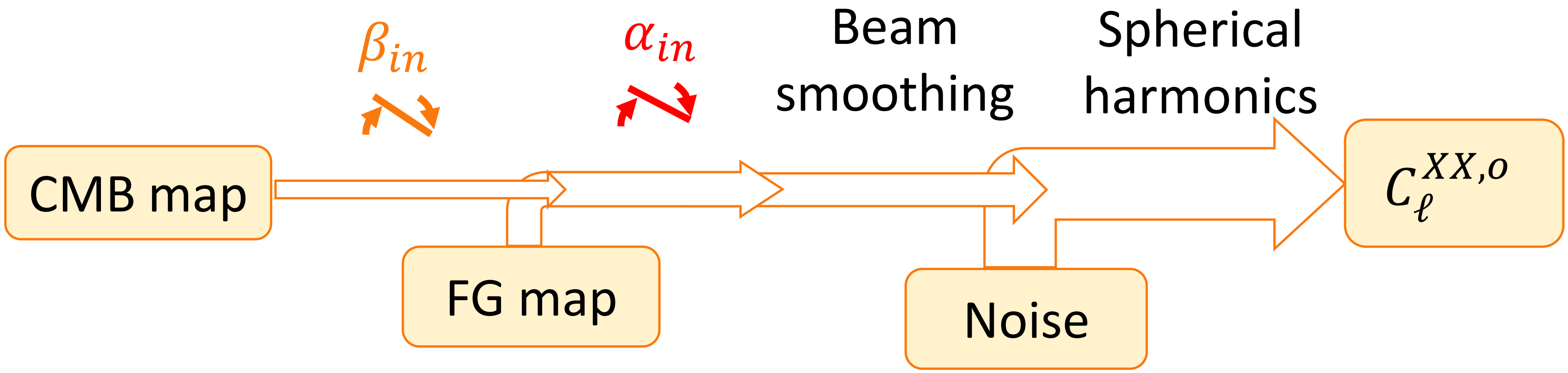
$$
\begin{aligned}
&\mathrm{Var}\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right] \\
&= \left\langle \left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right]^2 \right\rangle - \langle C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\rangle^2 \\
&= \frac{1}{2\ell+1}\langle C_\ell^{EE}\rangle\langle C_\ell^{BB}\rangle + \frac{\tan^2(4\alpha)}{4}\frac{2}{2\ell+1}\left(\langle C_\ell^{EE}\rangle^2 + \langle C_\ell^{BB}\rangle^2\right) \\
&\quad - \tan(4\alpha)\frac{2}{2\ell+1}\langle C_\ell^{EB}\rangle\left(\langle C_\ell^{EE}\rangle - \langle C_\ell^{BB}\rangle\right) + \boxed{\frac{1}{2\ell+1}\left(1-\tan^2(4\alpha)\right)\langle C_\ell^{EB}\rangle^2.}
\end{aligned}
\quad \cdots (9)
$$

$$= 0$$

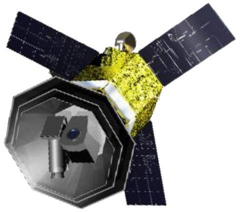
- We approximate $\langle C_\ell^{XY}\rangle \approx C_\ell^{XY,o}$
- We ignore $\langle C_\ell^{EB}\rangle^2$ term because it's small and yields bias
  - Even if $\langle C_\ell^{EB}\rangle \approx 0$, $C_\ell^{EB,o}$ has a small non-zero value with fluctuation, and ${C_\ell^{EB,o}}^2$ yields bias

Sky simulation setup for the validation

- Components
  - Thermal dust: modified black body
  - Synchrotron: simple power law
  - CMB: tensor-to-scalar ratio r = 0
  - Noise: white noise with LiteBIRD polarisation sensitivity
- N_{side} is 512 and l_{max} is 1024
- Compute EB power spectra from full-sky maps



Sky simulation setup for the validation: LiteBIRD



We extract representative 3 frequencies to show how the method works

	Frequency	polarisation sensitivity (uK’)	Beam size in FWHM (arcmin)
CMB	119	7.6	25
Dust + CMB	195	5.8	20
Dust	337	19.5	20

LiteBIRD parameters extracted from M. Hazumi et al., J. Low Temp. Phys. 194, 443 (2019).

Before the simultaneous determination: α only case

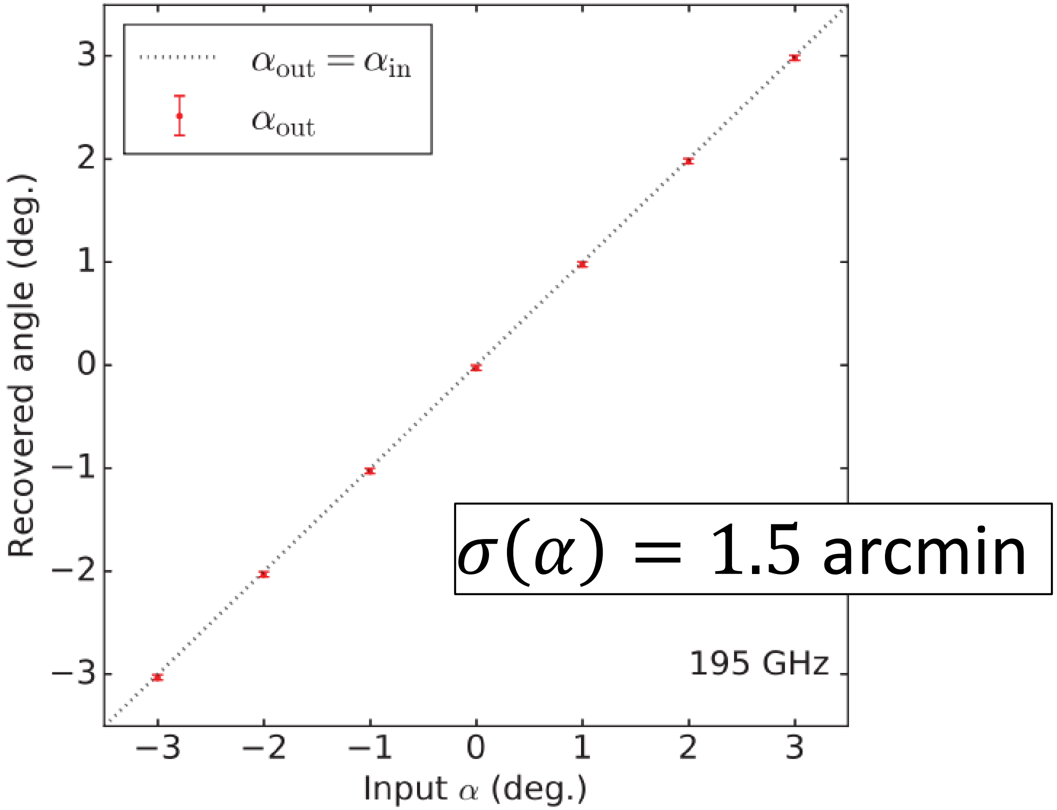
If we assume $\beta_{in}=0$, we can set $\beta=0$ in the Likelihood as,

$$-2\ln\mathcal{L} = \frac{\left[C_\ell^{EB,o} - \frac{1}{2}\left(C_\ell^{EE,o} - C_\ell^{BB,o}\right)\tan(4\alpha)\right]^2}{\mathrm{Var}\left(C_\ell^{EB,o} - \frac{1}{2}\left(C_\ell^{EE,o} - C_\ell^{BB,o}\right)\tan(4\alpha)\right)}. \qquad \cdots(10)$$

With this likelihood, we can determine α.

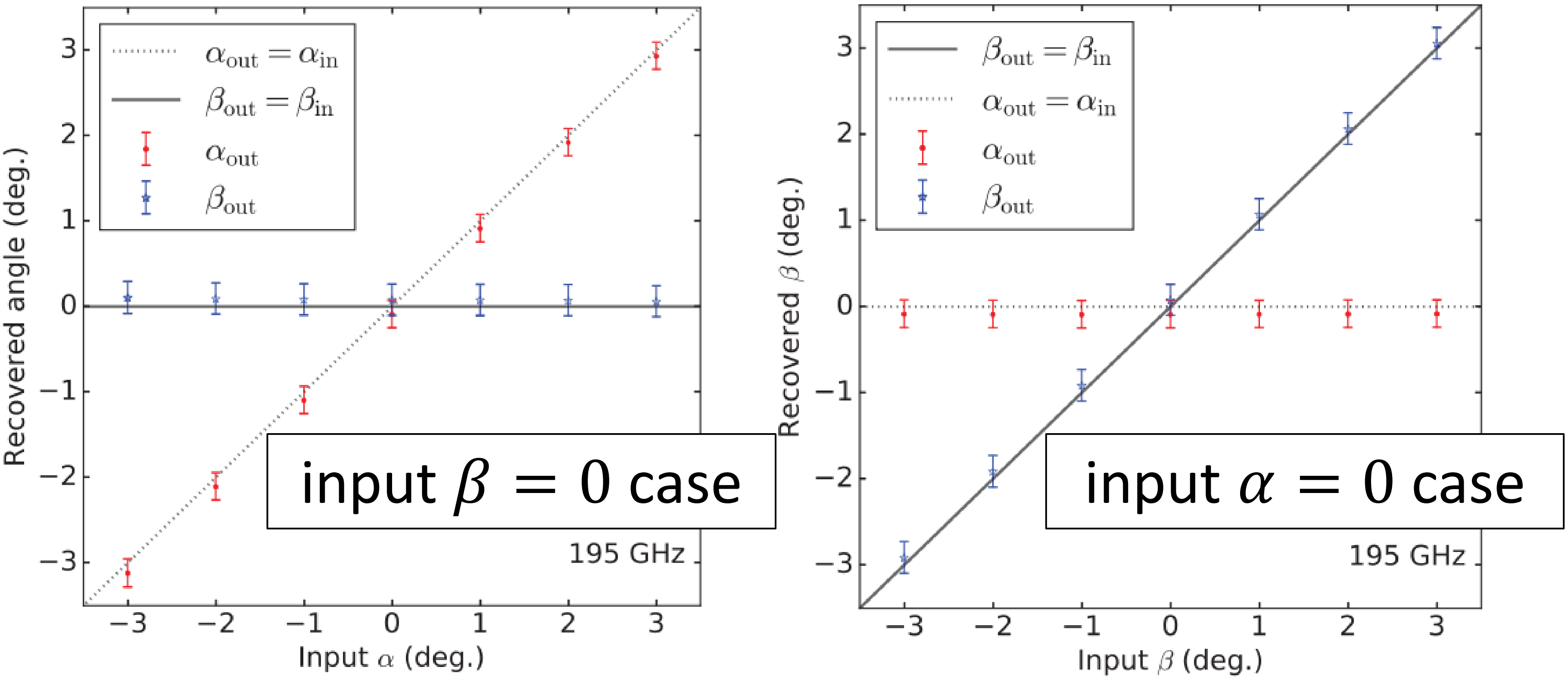
α only estimation at 195 GHz

We set β_{in}=0 and try whether we can determine α_{in}



We can recover the correct α without theoretical power spectra

Simultaneous determination with simulations: at LiteBIRD 195 GHz



$\sigma(\alpha) = 9.6$ arcmin and $\sigma(\beta) = 11$ arcmin

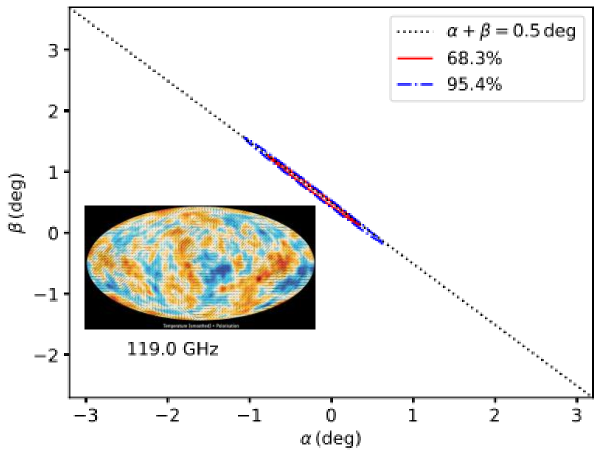
(Previous 1σ upper bound for β is 38 arcmin by Planck)

We can recover the correct α and β simultaneously

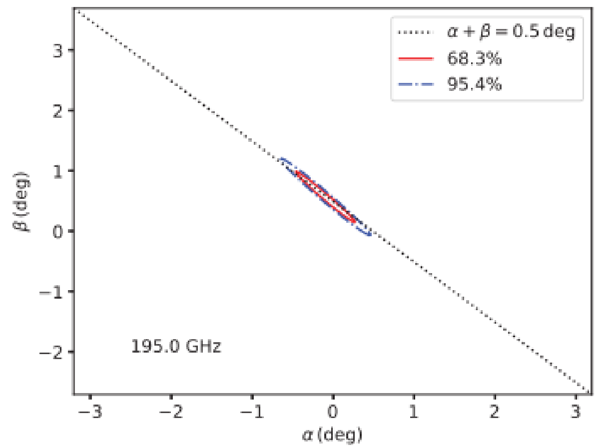
How does it work?

Minami et al. (2019)

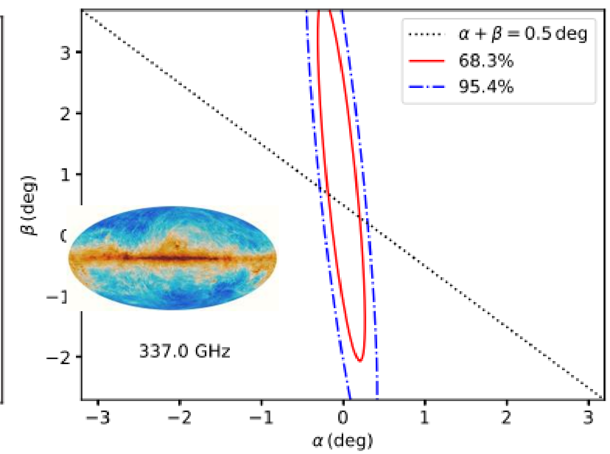
Simulation with future CMB data (LiteBIRD)



CMB channel (119 GHz)



Mid freq. (195 GHz)



Dust FG channel (337 GHz)

- The CMB signal determines the sum of two angles, $\alpha + \beta$
  - Diagonal line
- The FG determines only α
- Mid freq. : breaking the degeneracy with FG signal!
  - $\sigma(\beta) \sim \sigma(\alpha)$, since $\sigma(\alpha + \beta) \ll \sigma(\alpha)$

“Simultaneous determination of the cosmic birefringence and miscalibrated polarisation angles II: Including cross-frequency spectra”,
PTEP, 2020, 10, October (2020)

Application to multi channel (detector) observations

- Modern CMB experiments N_ν frequency channels
  - Auto correlation: N_ν
  - Cross correlation: $\frac{N_\nu(N_\nu-1)}{2}$

Including cross correlation, we can gain significantly more information

Extension with cross correlation:

- Consider two channels with different miscalibration angles, α_i and α_j
- When we take ensemble average of cross correlation between the two channels, we find

$$\begin{pmatrix} \langle C_\ell^{E_iE_j,\mathrm{o}} \rangle \\ \langle C_\ell^{B_iB_j,\mathrm{o}} \rangle \end{pmatrix} = \mathbf{R}(\alpha_i, \alpha_j) \begin{pmatrix} \langle C_\ell^{E_iE_j,\mathrm{fg}} \rangle \\ \langle C_\ell^{B_iB_j,\mathrm{fg}} \rangle \end{pmatrix} + \mathbf{R}(\alpha_i + \beta, \alpha_j + \beta) \begin{pmatrix} \langle C_\ell^{E_iE_j,\mathrm{CMB}} \rangle \\ \langle C_\ell^{B_iB_j,\mathrm{CMB}} \rangle \end{pmatrix} + \delta_{i,j} \begin{pmatrix} \langle C_\ell^{E_iE_i,\mathrm{N}} \rangle \\ \langle C_\ell^{B_iB_i,\mathrm{N}} \rangle \end{pmatrix} ,$$

$$\langle C_\ell^{E_iB_j,\mathrm{o}} \rangle = \vec{R}^T(\alpha_i, \alpha_j) \begin{pmatrix} \langle C_\ell^{E_iE_j,\mathrm{fg}} \rangle \\ \langle C_\ell^{B_iB_j,\mathrm{fg}} \rangle \end{pmatrix} + \vec{R}^T(\alpha_i + \beta, \alpha_j + \beta) \begin{pmatrix} \langle C_\ell^{E_iE_j,\mathrm{CMB}} \rangle \\ \langle C_\ell^{B_iB_j,\mathrm{CMB}} \rangle \end{pmatrix} ,$$

where

$$\mathbf{R}(\theta_i, \theta_j) = \begin{pmatrix} \cos(2\theta_i)\cos(2\theta_j) & \sin(2\theta_i)\sin(2\theta_j) \\ \sin(2\theta_i)\sin(2\theta_j) & \cos(2\theta_i)\cos(2\theta_j) \end{pmatrix} ,$$

$$\vec{R}(\theta_i, \theta_j) = \begin{pmatrix} \cos(2\theta_i)\sin(2\theta_j) \\ -\sin(2\theta_i)\cos(2\theta_j) \end{pmatrix} .$$

Log-likelihood:

- From the equations, we find

$$\left(-\vec{R}^{\mathrm{T}}(\alpha_i,\alpha_j)\mathbf{R}^{-1}(\alpha_i,\alpha_j) \quad 1\right)\begin{pmatrix}\langle C_\ell^{E_iE_j,\mathrm{o}}\rangle \\ \langle C_\ell^{B_iB_j,\mathrm{o}}\rangle \\ \langle C_\ell^{E_iB_j,\mathrm{o}}\rangle\end{pmatrix}$$

$A_{ij}\vec{C}_\ell^{o,ij}$

$$-\left[\vec{R}^{\mathrm{T}}(\alpha_i+\beta,\alpha_j+\beta)-\vec{R}^{\mathrm{T}}(\alpha_i,\alpha_j)\mathbf{R}^{-1}(\alpha_i,\alpha_j)\mathbf{R}(\alpha_i+\beta,\alpha_j+\beta)\right]\begin{pmatrix}\langle C_\ell^{E_iE_j,\mathrm{CMB}}\rangle \\ \langle C_\ell^{B_iB_j,\mathrm{CMB}}\rangle\end{pmatrix} = 0.$$

$B_{ij}\vec{C}_\ell^{CMB,ij}$

- From this equation, we construct a log-likelihood,

$$\ln\mathcal{L} = -\frac{1}{2}\sum_{\ell=2}^{\ell_{\max}}\left(\mathbf{A}\vec{C}_\ell^{\mathrm{o}} - \mathbf{B}\vec{C}_\ell^{\mathrm{CMB,th}}\right)^T \mathbf{C}^{-1}\left(\mathbf{A}\vec{C}_\ell^{\mathrm{o}} - \mathbf{B}\vec{C}_\ell^{\mathrm{CMB,th}}\right), \qquad \cdots (8)$$

where

$$\mathbf{C} = \mathbf{A}\mathrm{Cov}\left(\vec{C}_\ell^{o},\vec{C}_\ell^{o}\right)\mathbf{A}^{\mathrm{T}}$$

Approximated covariance

- For each element, we use power spectra from one realization instead of ensemble averaged ones

$$
\begin{aligned}
\mathrm{Cov}(C_\ell^{X,Y}, C_\ell^{Z,W}) &= \frac{1}{(2\ell+1)}(\langle C_\ell^{X,Z}\rangle\langle C_\ell^{Y,W}\rangle + \langle C_\ell^{X,W}\rangle\langle C_\ell^{Y,Z}\rangle) \\
&\approx \frac{1}{(2\ell+1)}(C_\ell^{X,Z} C_\ell^{Y,W} + C_\ell^{X,W} C_\ell^{Y,Z}).
\end{aligned}
$$

- In addition, we neglect off-diagonal elements because their fluctuation makes the approximation worse

$$
\mathrm{Cov}(C_\ell^{\vec{\mathrm{o}},ij}, {C_\ell^{\vec{\mathrm{o}},pq}}^T)
= \begin{pmatrix}
\mathrm{Cov}(C_\ell^{E_iE_j,\mathrm{o}}, C_\ell^{E_pE_q,\mathrm{o}}) & 0 & 0 \\
0 & \mathrm{Cov}(C_\ell^{B_iB_j,\mathrm{o}}, C_\ell^{B_pB_q,\mathrm{o}}) & 0 \\
0 & 0 & \mathrm{Cov}(C_\ell^{E_iB_j,\mathrm{o}}, C_\ell^{E_pB_q,\mathrm{o}})
\end{pmatrix}
$$

Binning

- Because statistical fluctuation of power spectra for each ℓ makes the approximation worse, we reduce the fluctuation by binning the power spectra,

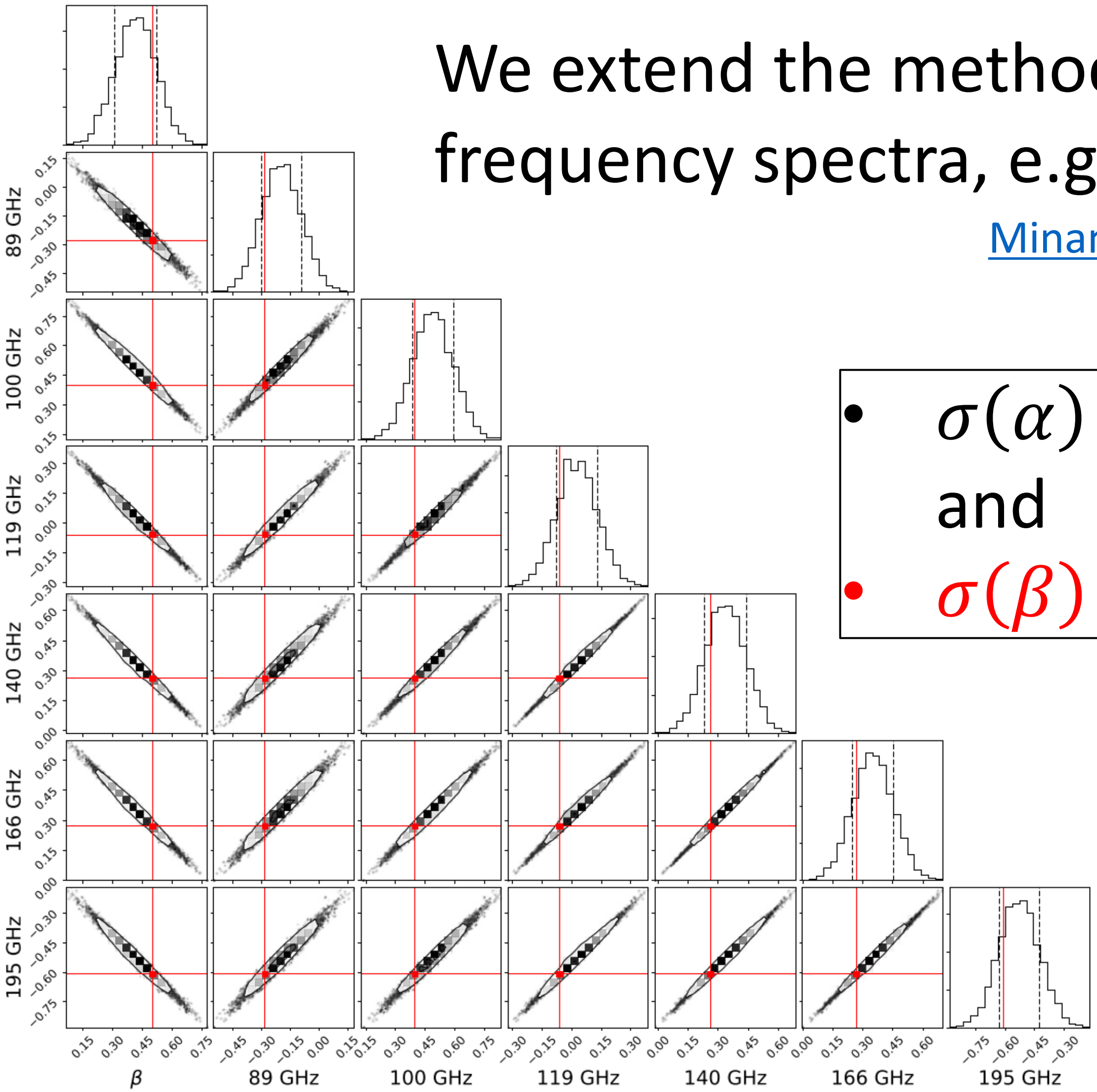
$$C_b^{X,Y} = \frac{1}{\Delta\ell} \sum_{\ell \in b} C_\ell^{X,Y}.$$

- We find corresponding covariance,

$$\mathrm{Cov}(C_b^{X,Y}, C_b^{Z,W}) = \frac{1}{\Delta\ell^2} \sum_{\ell \in b} \mathrm{Cov}(C_\ell^{X,Y}, C_\ell^{Z,W}).$$

- We simply use these binned variables to calculate the log–likelihood

Extend to cross frequency spectra: LiteBIRD 6 freqs. From 15 freqs



We extend the method by including cross frequency spectra, e.g., $E^{89}B^{195}$

Minami&Komatsu (2020b)

- $\sigma(\alpha) \approx 6.0$ arcmin and
- $\sigma(\beta) = 6.1$ arcmin

Summary

- Cosmic birefringence and miscalibration angle are degenerate in the CMB signal
- We lift the degeneracy using foreground power spectra
- We can constrain the birefringence angle with the precision of 6 arcmin with LiteBIRD

Why don't you apply this method to other future and existing experiments?