# Towards ending partial sky *E-B* ambiguity in CMB observations

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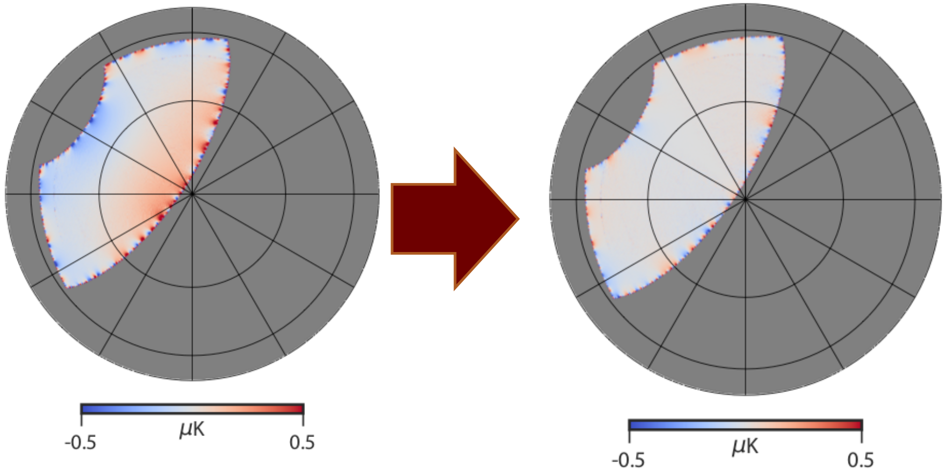
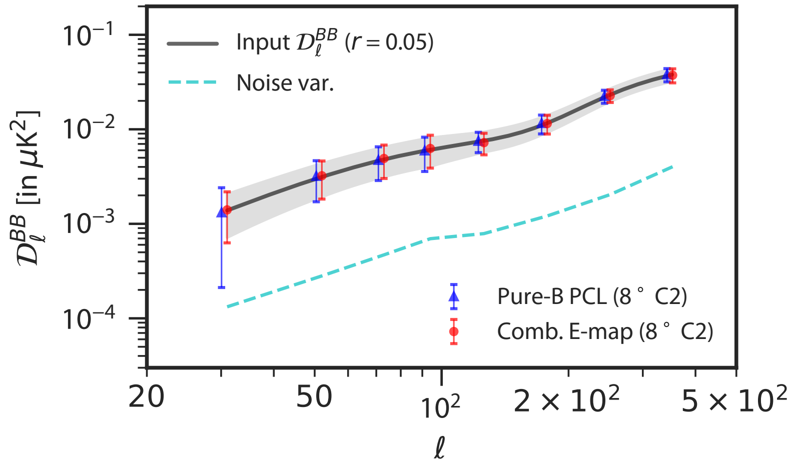
*CMB Systematics and Calibration Focus Workshop*

*Kavli IPMU, 3rd December 2020*

# 1 minute Summary

- Leakage of power from *E*-to-*B* modes on an incomplete sky patch is a critical problem for search for primordial *B* modes.
- We demonstrate that it is possible to remove this leakage contribution by completing the *E*-mode information in suborbital experiments with ancillary *E*-mode information from satellite experiments.
- For Planck *E*-modes which have considerably higher noise than current and upcoming ground-based mission, we can Wiener filter the E-modes.
- We combine the filtered *E*-mode only QU maps from Planck outside the observed sky patch and show that we effective remove the impact of leakage.
- We show that for AliCPT-like or LSPE-like cases we outperform pure-*B* PCL estimator giving unbiased, with optimal PCL error bars.
- For CMB-S4, with signal dominated LiteBIRD *E*-mode data, the *E*-to-*B* leakage can be nearly eliminated.




**This talk is based on arXiv: *[2007.09928]***

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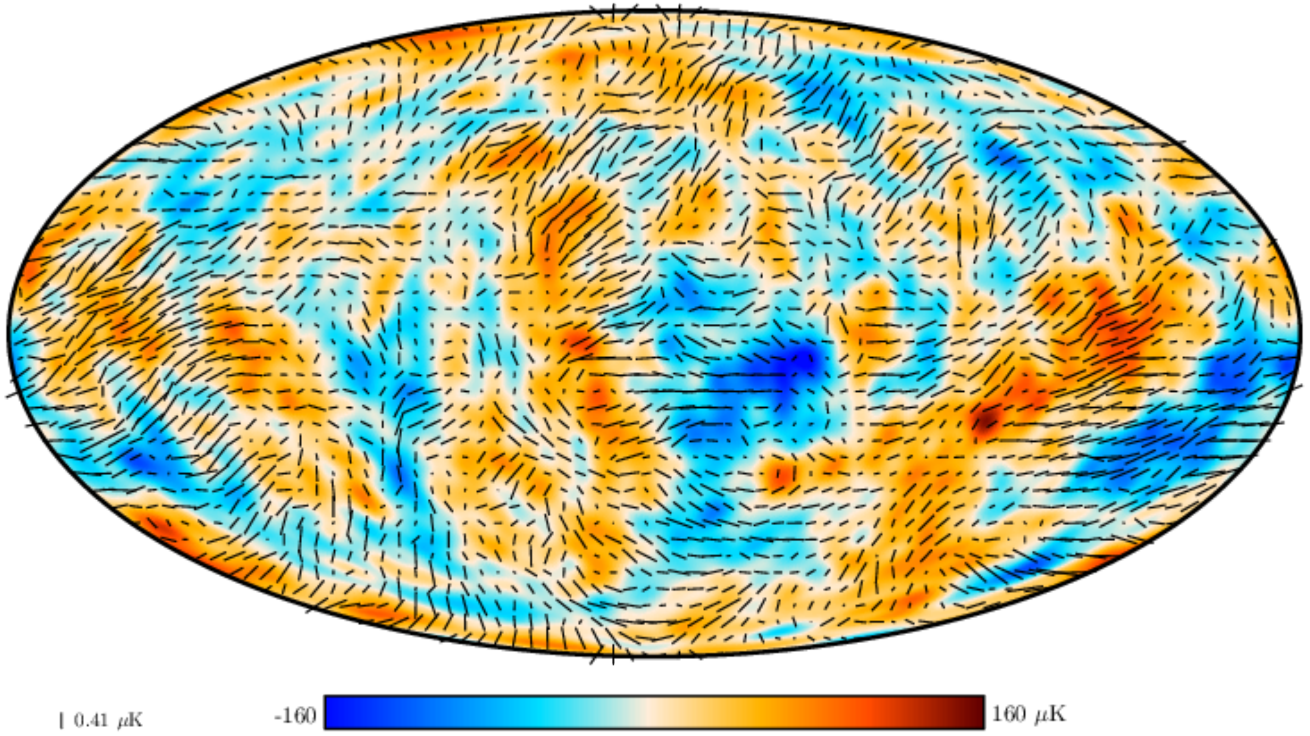
*Work done in collaboration with:*

Jacques Delabrouille, APC

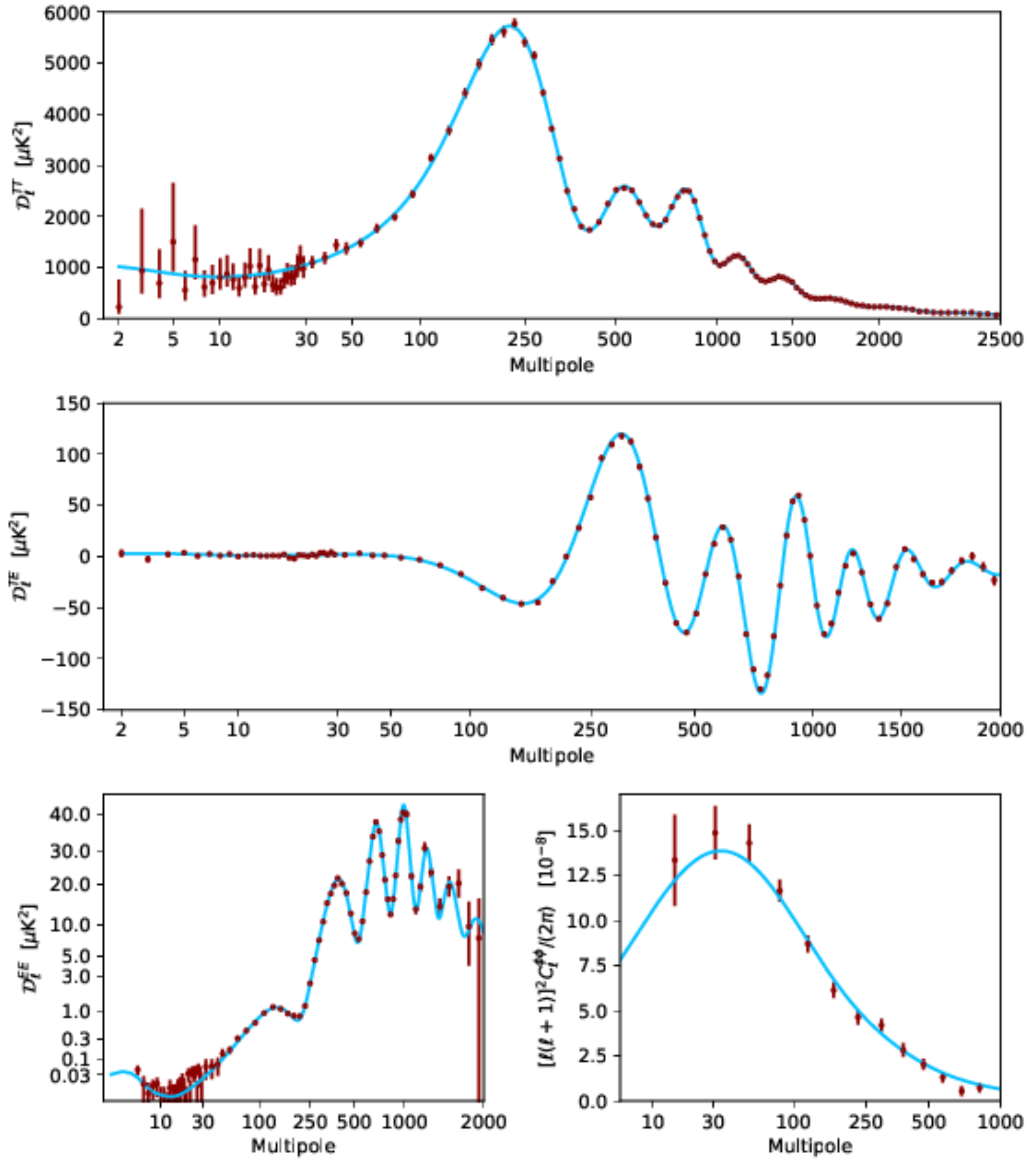
Wen Zhao, USTC

Larissa Santos, Yangzhou University

# CMB Post-Planck




Planck polarization anisotropies shown with rods, overlayed on the Planck temperature anisotropies smoothed with 5 deg beam. [Ref: Planck Picture Gallery]




[Ref: Planck Picture Gallery]

# Introduction

- Planck mission is complete and LiteBIRD is still little far in the future

- In between there are large number of sub-orbital missions both in northern and southern hemisphere

- Goal is to observe the primordial *B* modes in a small sky patch with low noise and relatively free from foregrounds

- The Ali CMB Polarization Telescope will begin observing the northern sky sometime next year at 95 and 150 GHz

Ali CMB Polarization Telescope site

# *Q-U* to *E-B*

We observe Stokes *Q* and *U* fields in experiments

$$P_{\pm}(\hat{n}) = Q(\hat{n}) \pm iU(\hat{n})$$

The Stokes *Q* and *U* fields are not scalars. $P_{\pm}$ represents a spin-$(\pm 2)$ field.

$$P_{\pm}(\hat{n}) = \sum_{\ell m} a_{\pm 2,\ell m} \, {}_{\pm 2}Y_{\ell m}(\hat{n})$$

The *E-B* decomposition is unique for FULL SKY FIELDS ONLY!

Complication for incomplete sky.

The spherical harmonic coefficients for *E*- and *B*-fields are defined as:

$$a_{E,\ell m} = -\frac{1}{2}\left(a_{2,\ell m} + a_{-2,\ell m}\right) = -\frac{1}{2}\left[\int P_{+}(\hat{n})\,{}_{2}Y^{*}_{\ell m}(\hat{n})d\Omega + \int P_{-}(\hat{n})\,{}_{-2}Y^{*}_{\ell m}(\hat{n})d\Omega\right]$$

$$a_{B,\ell m} = \frac{i}{2}\left(a_{2,\ell m} - a_{-2,\ell m}\right) = \frac{i}{2}\left[\int P_{+}(\hat{n})\,{}_{2}Y^{*}_{\ell m}(\hat{n})d\Omega - \int P_{-}(\hat{n})\,{}_{-2}Y^{*}_{\ell m}(\hat{n})d\Omega\right].$$

# The *E-B* ambiguity

For an incomplete sky observation the E-B mode definition becomes:

$$\tilde{a}_{E,\ell m} = -\frac{1}{2}\left[\int P_+(\hat{n})W(\hat{n})_2Y^*_{\ell m}(\hat{n})d\Omega + \int P_-(\hat{n})W(\hat{n})_{-2}Y^*_{\ell m}(\hat{n})d\Omega\right],$$

$$\tilde{a}_{B,\ell m} = \frac{i}{2}\left[\int P_+(\hat{n})W(\hat{n})_2Y^*_{\ell m}(\hat{n})d\Omega - \int P_-(\hat{n})\underbrace{W(\hat{n})}_{}{}_{-2}Y^*_{\ell m}(\hat{n})d\Omega\right].$$

Window defining the sky patch

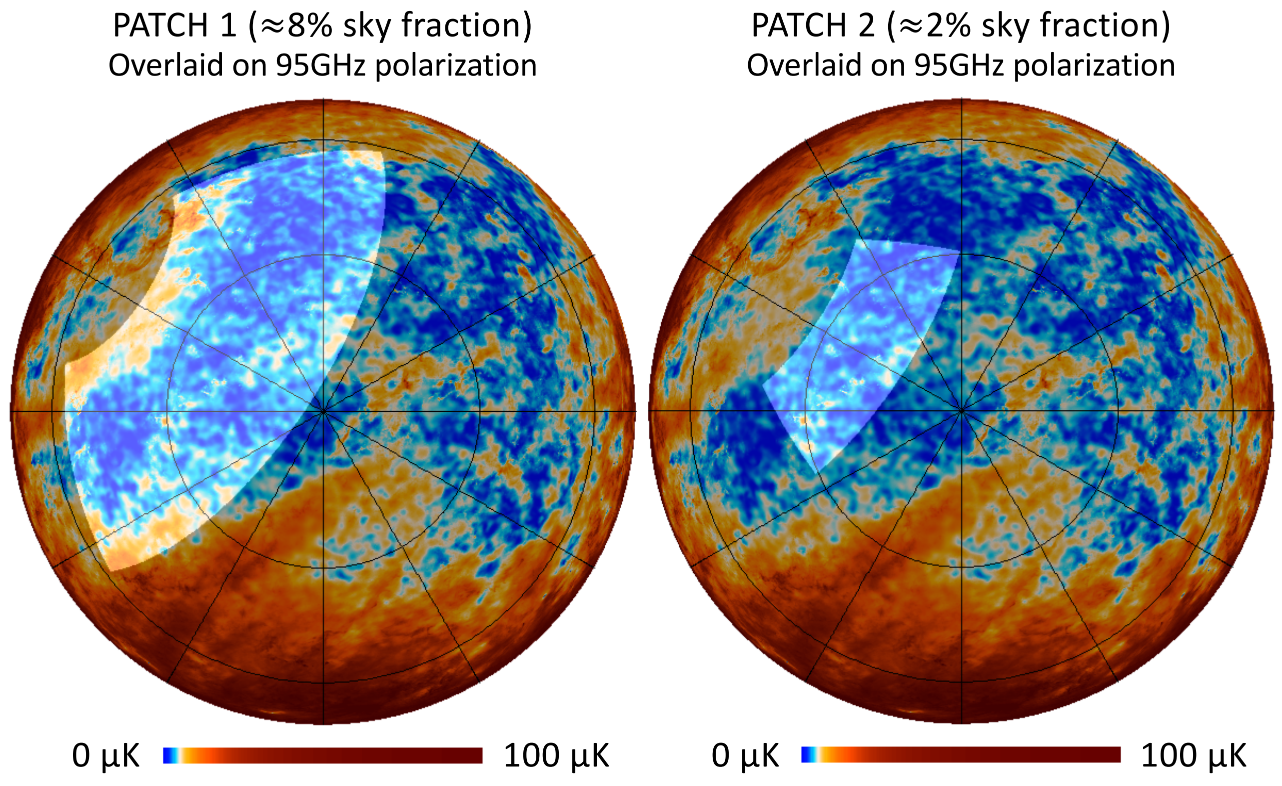
Rewriting partial sky *E-B* harmonics to full sky *E-B* harmonics:

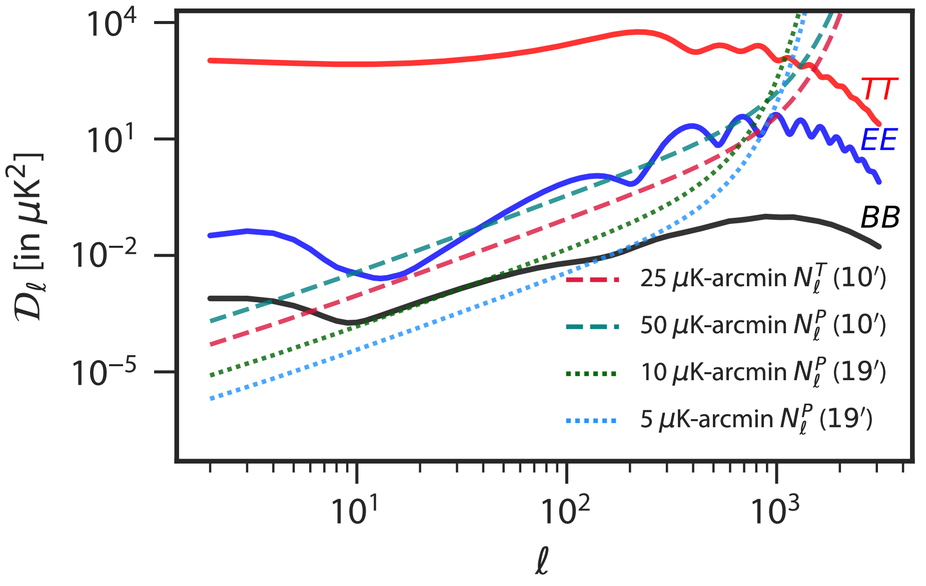
$$\tilde{a}_{E,\ell m} = \sum_{\ell' m'}\left[K^{EE}_{\ell m \ell' m'}a_{E,\ell' m'} + iK^{EB}_{\ell m \ell' m'}a_{B,\ell' m'}\right],$$

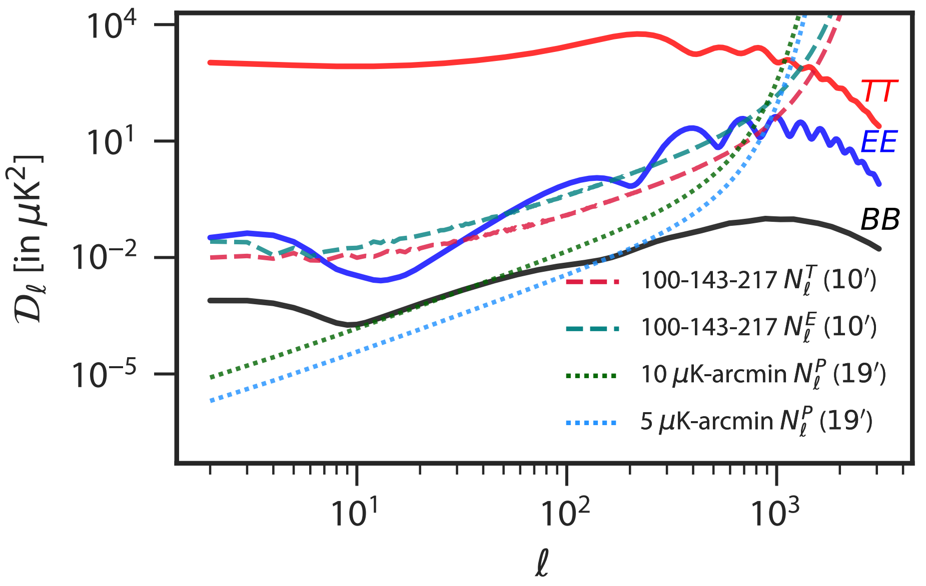
$$\tilde{a}_{B,\ell m} = \sum_{\ell' m'}\left[-iK^{BE}_{\ell m \ell' m'}a_{E,\ell' m'} + K^{BB}_{\ell m \ell' m'}a_{B,\ell' m'}\right].$$

$K^{XY}_{\ell m \ell' m'}$ ⟹ Kernel matrix

# Simulation setup

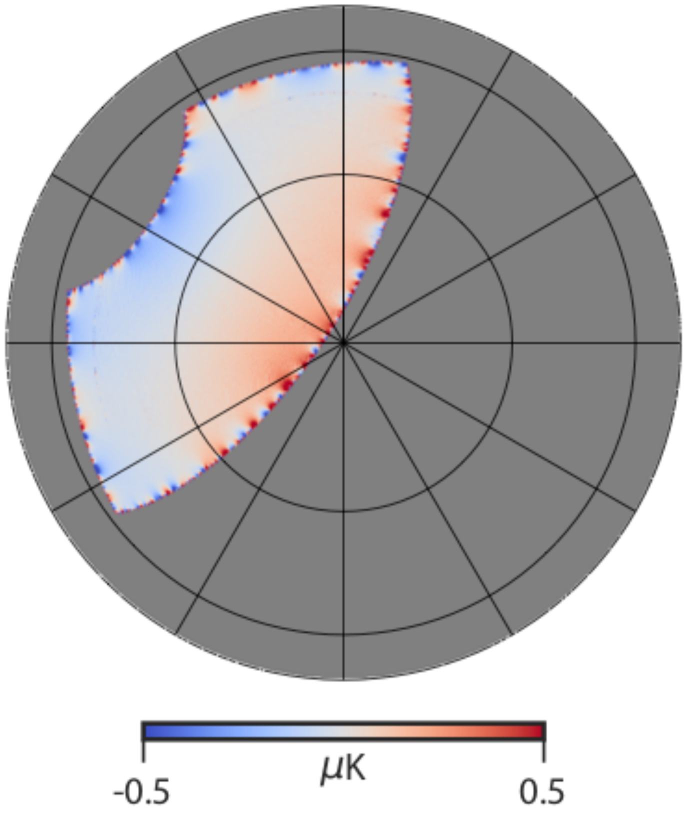


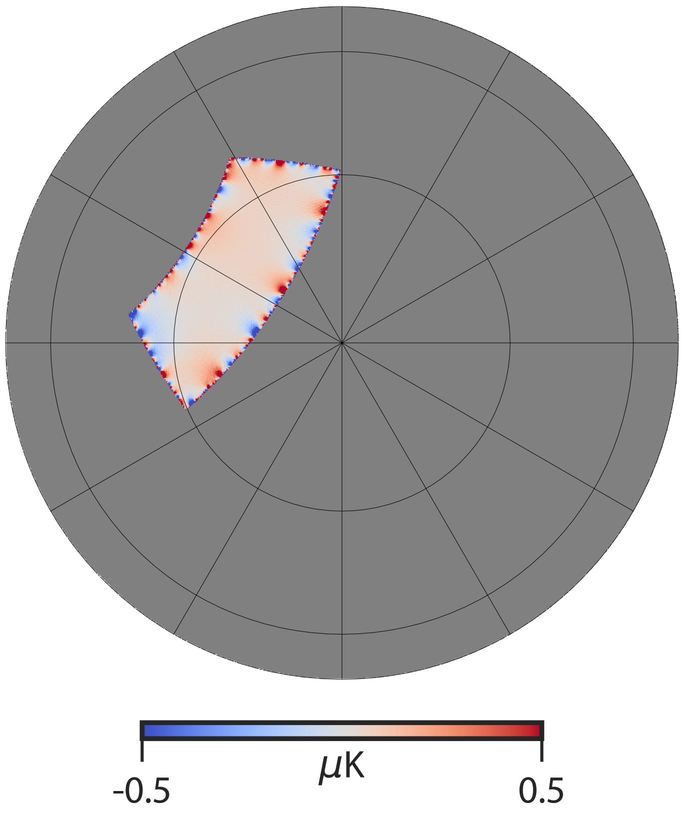



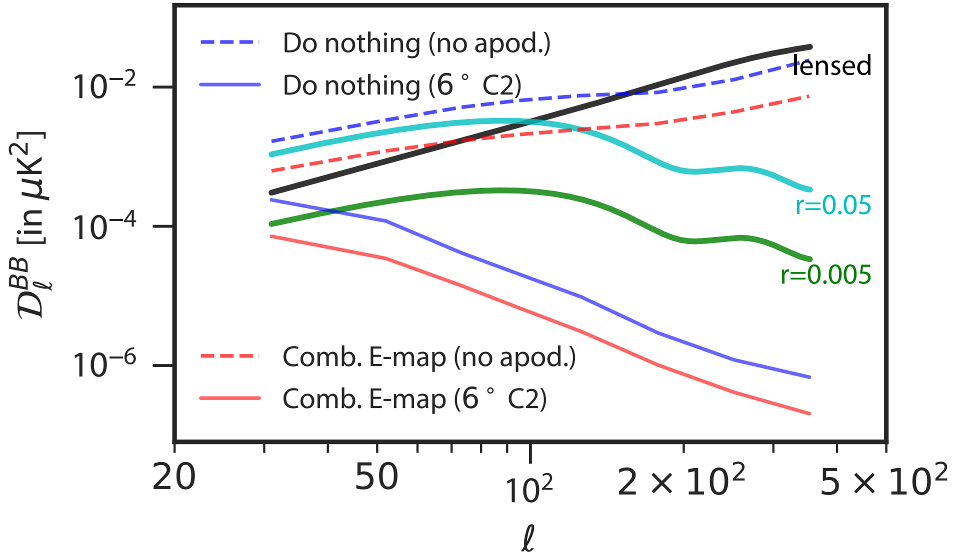



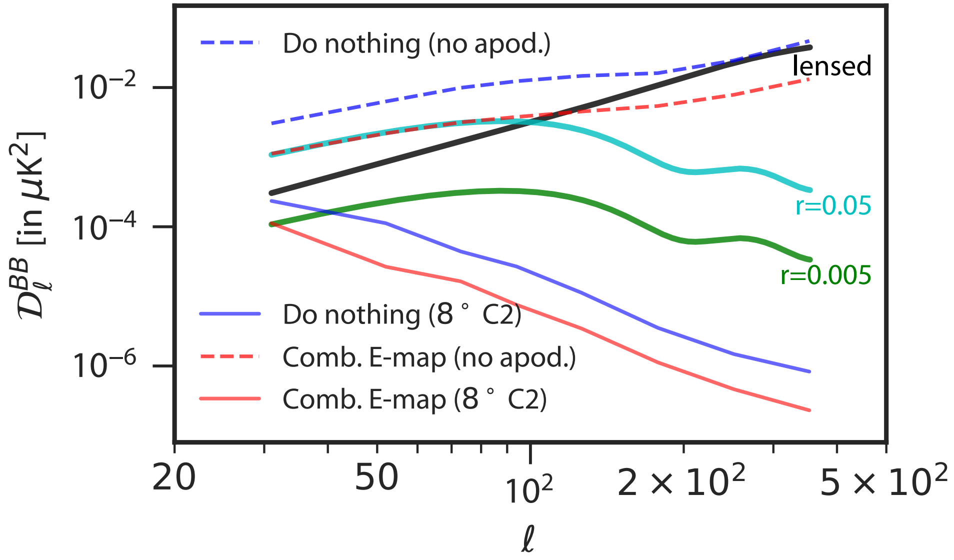

We simulate CMB+noise only maps. This assumes that there are no foreground residuals or time stream filtering.

# The *E*-to-*B* Leakage



# *E*-to-*B* Leakage Control

Standard PCL method:

$$\langle \tilde{C}_\ell^{EE} \rangle = \sum_{\ell'} \left[ M_{\ell\ell'}^{EE} \langle C_{\ell'}^{EE} \rangle + M_{\ell\ell'}^{EB} \langle C_{\ell'}^{BB} \rangle \right]$$

$$\langle \tilde{C}_\ell^{BB} \rangle = \sum_{\ell'} \left[ M_{\ell\ell'}^{BE} \langle C_{\ell'}^{EE} \rangle + M_{\ell\ell'}^{BB} \langle C_{\ell'}^{BB} \rangle \right]$$

'pure' fields:

$$\mathcal{E}(\hat{n}) = -\frac{1}{2} \left[ \bar{\eth}\bar{\eth} P_+(\hat{n}) + \eth\eth P_-(\hat{n}) \right]$$

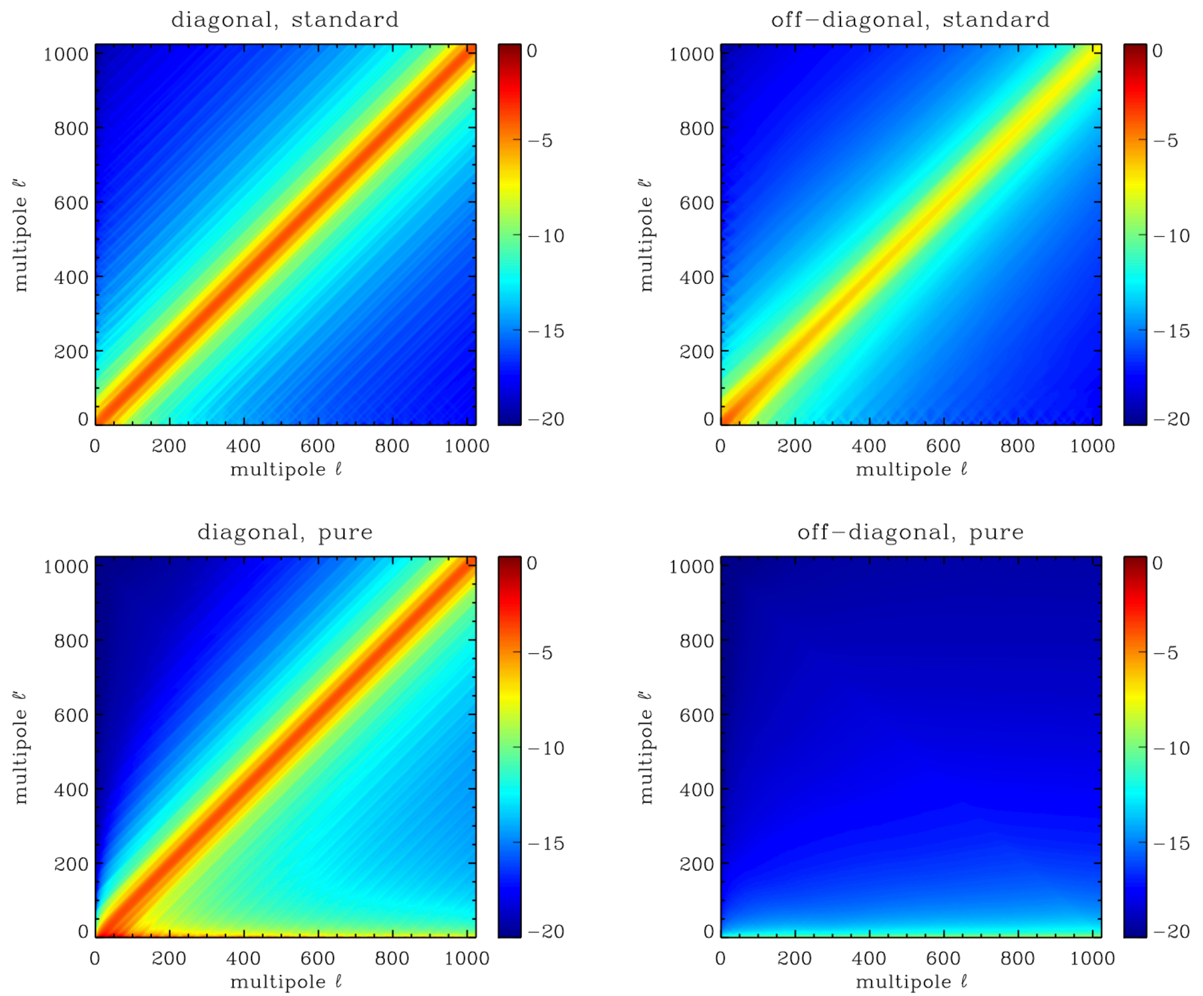
$$\mathcal{B}(\hat{n}) = \frac{i}{2} \left[ \bar{\eth}\bar{\eth} P_+(\hat{n}) - \eth\eth P_-(\hat{n}) \right]$$

Pure-*B* method:

$$\tilde{a}_{B,\ell m} = \sqrt{\frac{(\ell-2)!}{(\ell+2)!}} \int W(\hat{n}) \mathcal{B}(\hat{n}) Y_{\ell m}^* d\Omega$$

$$M_{\ell\ell'}^{XY} = \sum_{mm'} \frac{1}{2\ell+1} \left| K_{\ell m \ell' m'}^{XY} \right|^2$$

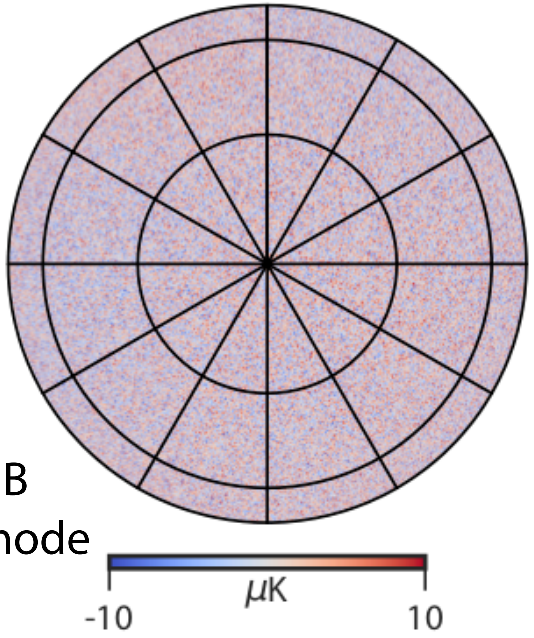
Mixing matrix figure from Grain et al. 2009



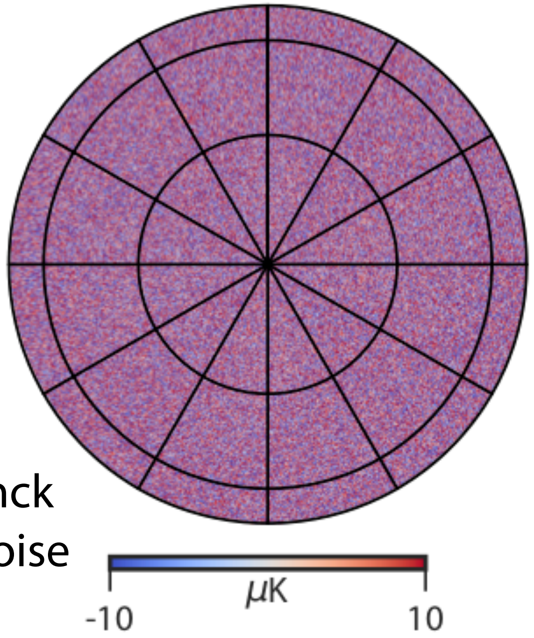
# Combined *E*-map method

The *E*-to-*B* leakage originates from completeness of the *E*-mode information.

So if we use full sky *E*-mode maps from Planck to fill-in the *E*-mode information,
it can reduce and possibly remove leakage




CMB
*E*-mode




Planck
*E*-noise

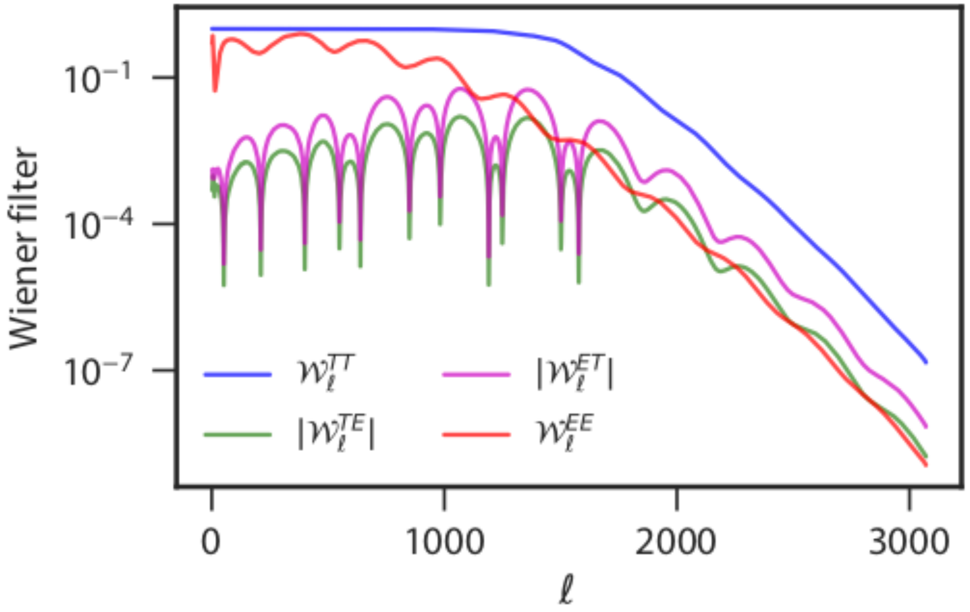
However, Planck is too noisy for direct combination.

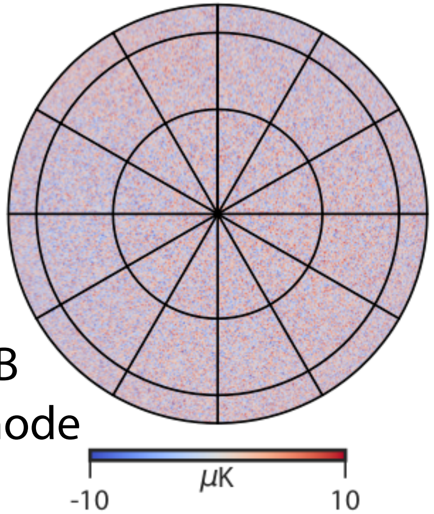
# Wiener filtered *E*-maps

$$\mathcal{W} = CI^\dagger \left[ ICI^\dagger + N \right]^{-1}$$

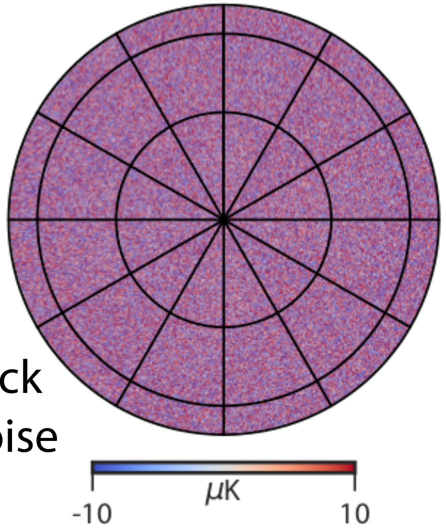
$$\mathcal{W} = \begin{bmatrix} \mathcal{W}_\ell^{TT} & \mathcal{W}_\ell^{TE} \\ \mathcal{W}_\ell^{ET} & \mathcal{W}_\ell^{EE} \end{bmatrix} \quad N = \begin{bmatrix} N_\ell^{TT} & 0 \\ 0 & N_\ell^{EE} \end{bmatrix}.$$

$$C = \begin{bmatrix} C_\ell^{TT} B_{T,\ell}^2 & C_\ell^{TE} B_{T,\ell} B_{E,\ell} \\ C_\ell^{TE} B_{T,\ell} B_{E,\ell} & C_\ell^{EE} B_{E,\ell}^2 \end{bmatrix}$$

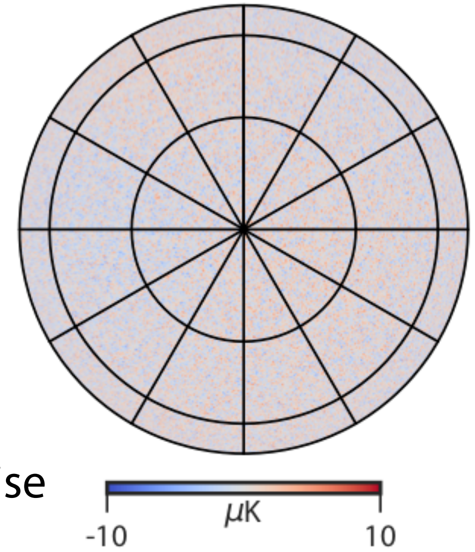



CMB
*E*-mode

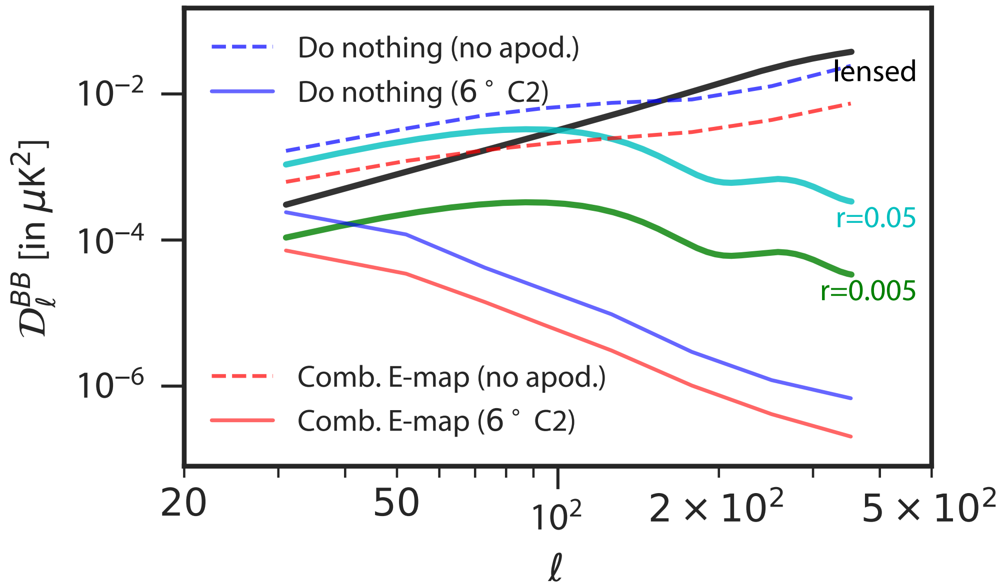



Planck
*E*-noise




Wiener
filtered
*E*-mode
CMB+noise

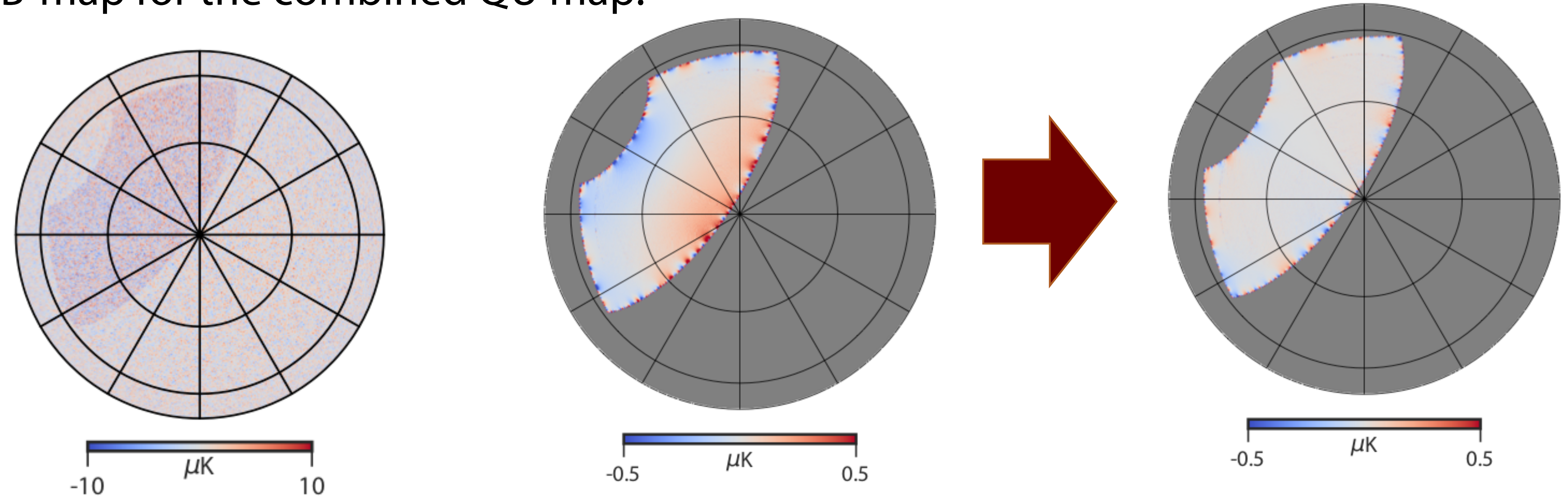
# Combined *E*-map




1. Use Wiener filtered E-modes to generate QU maps at ground-based exp. resolution.
2. Combine the QU maps:

$$d_{\rm com} = \widehat{d}_{\rm sat}\left(1 - W_{\rm grd}\right) + d_{\rm grd} W_{\rm grd}.$$

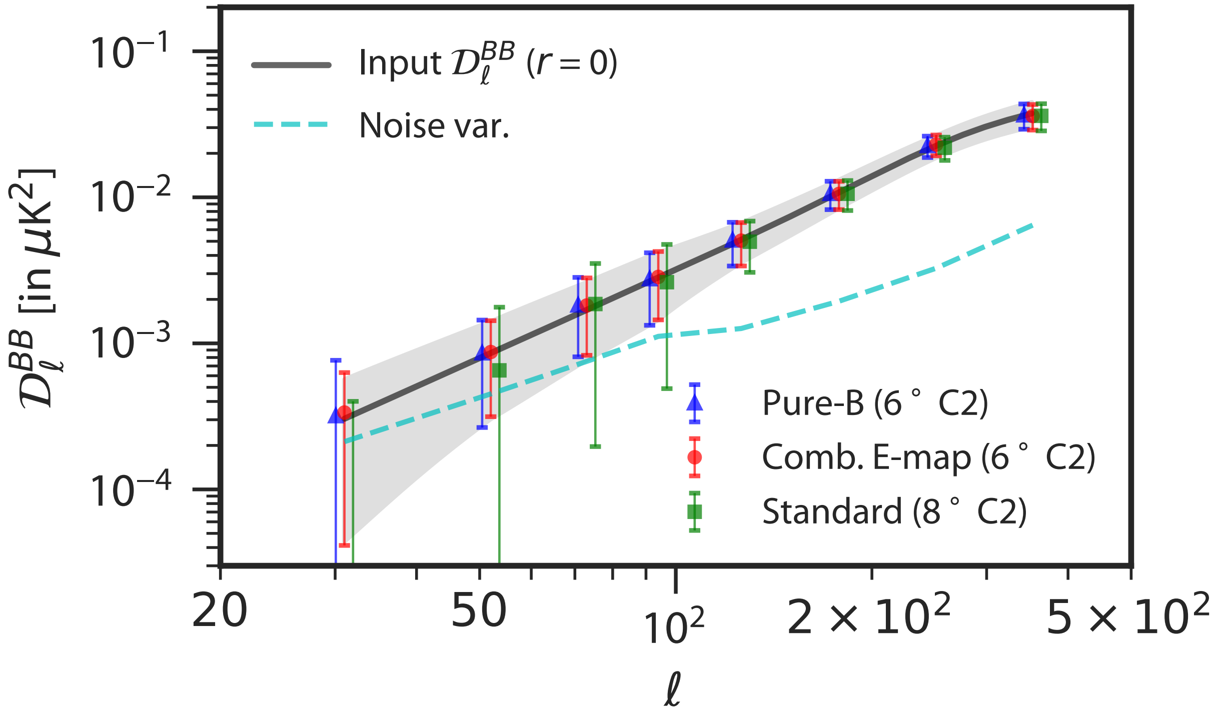
3. Get B-map for the combined QU map.



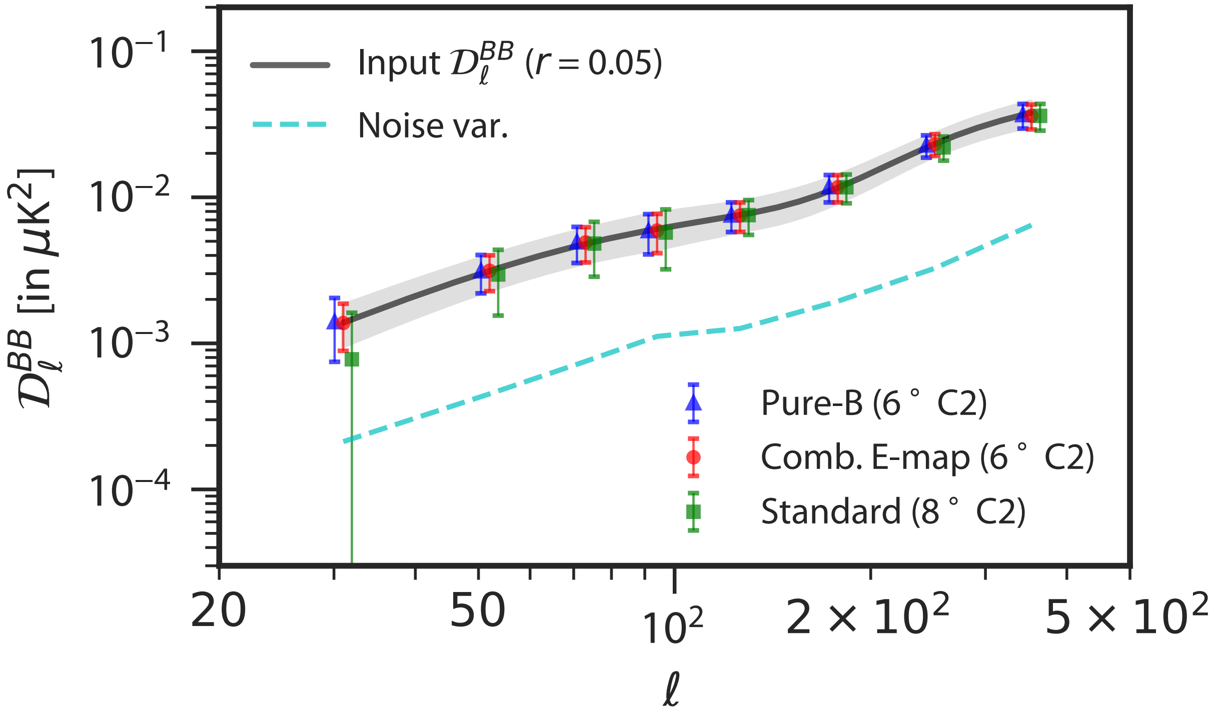
# Results

fsky = 8% patch, 10 uK arcmin noise;
Full sky: 50 uK-arcmin Planck noise

$r = 0$




$r = 0.05$




Our method results shown in red
Pure-B method shown in blue
Standard method in green

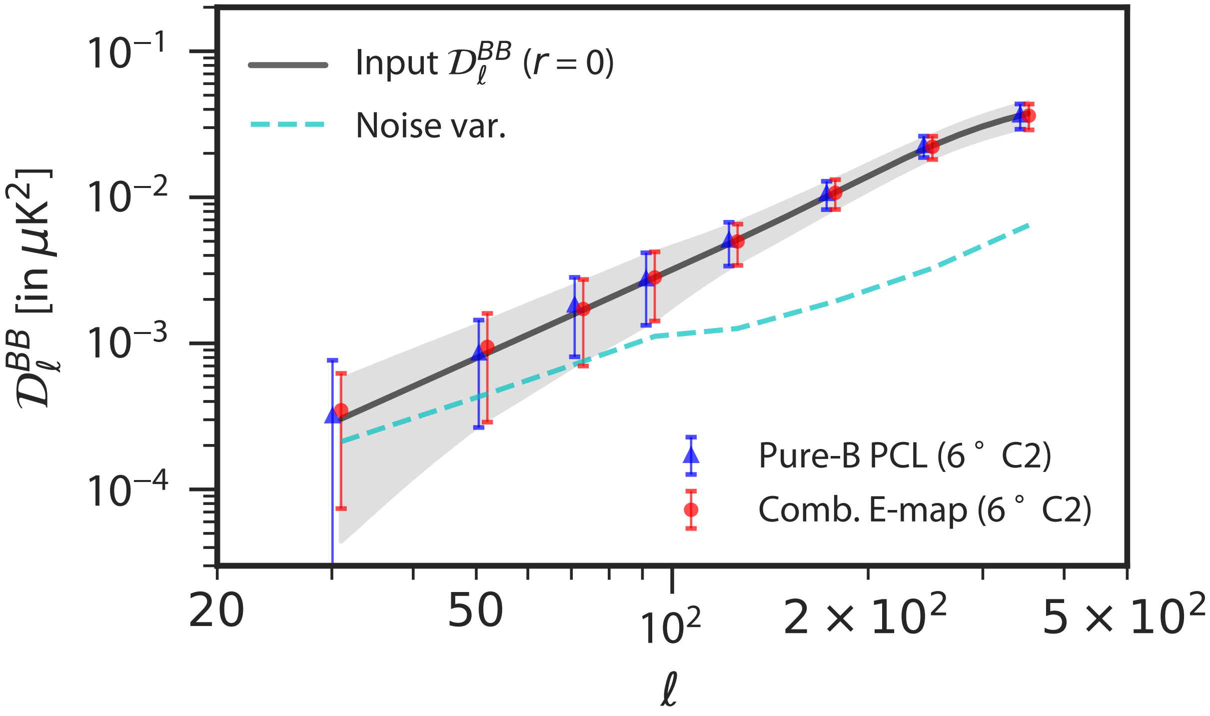
Optimal error band:

$$\Delta\mathcal{D}_{\bar\ell;\text{optimal}} = \sqrt{\frac{2}{(2\bar\ell+1)f_{\text{sky}}\Delta\ell}}\left[\mathcal{D}_{\bar\ell}+\frac{\mathcal{N}_{\bar\ell}}{B^2_{\bar\ell}}\right]\left[\frac{w_{(4)}}{w^2_{(2)}}\right]^{1/2}$$

# Results

fsky = 8% patch, 10 uK arcmin noise;
Full sky: isotropized 100-143-217 Planck noise

$r = 0$

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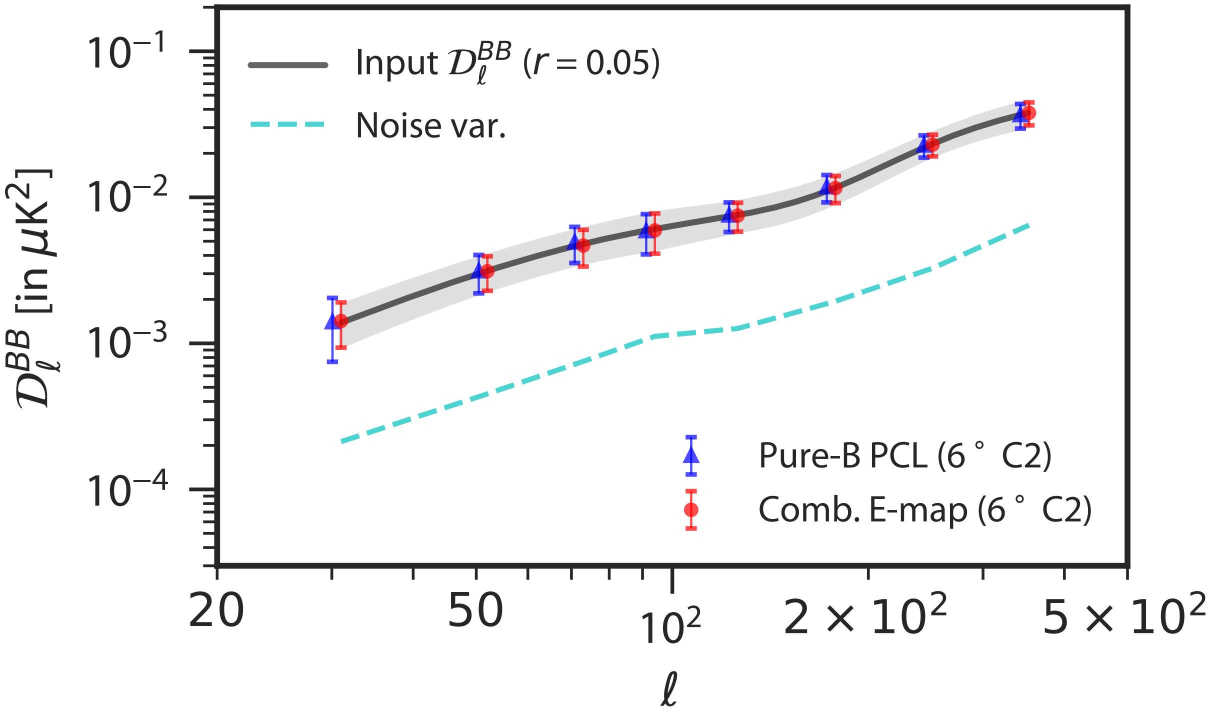


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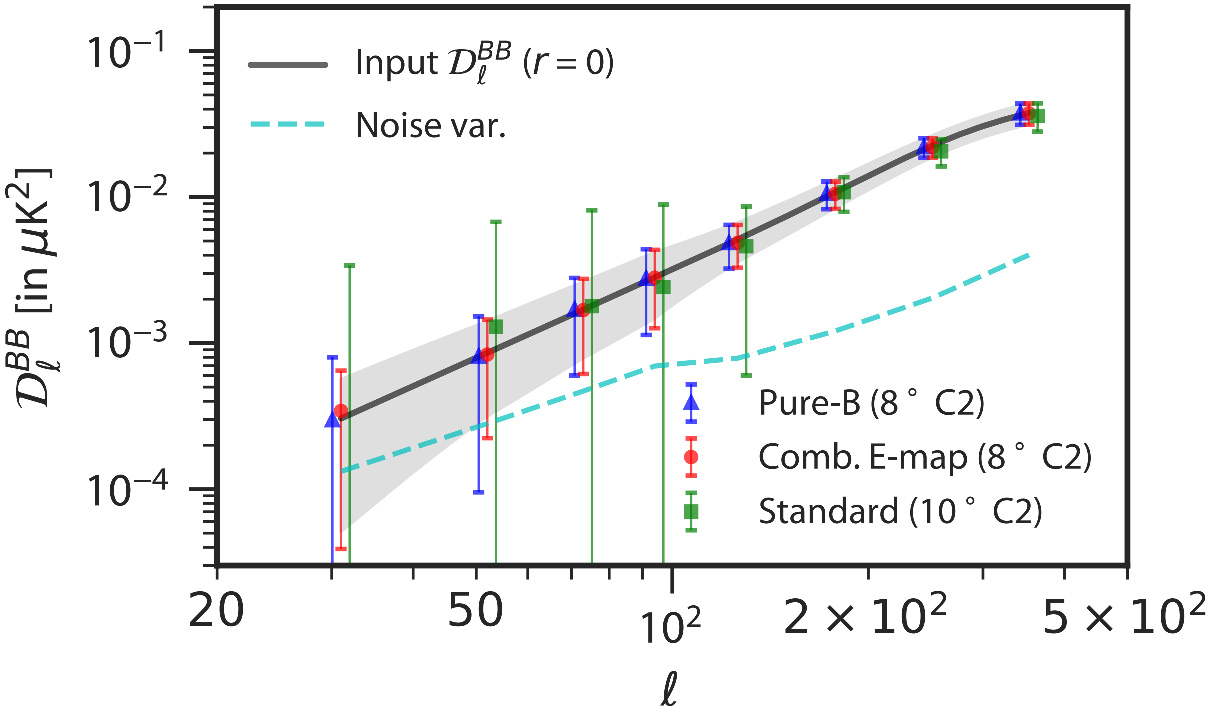
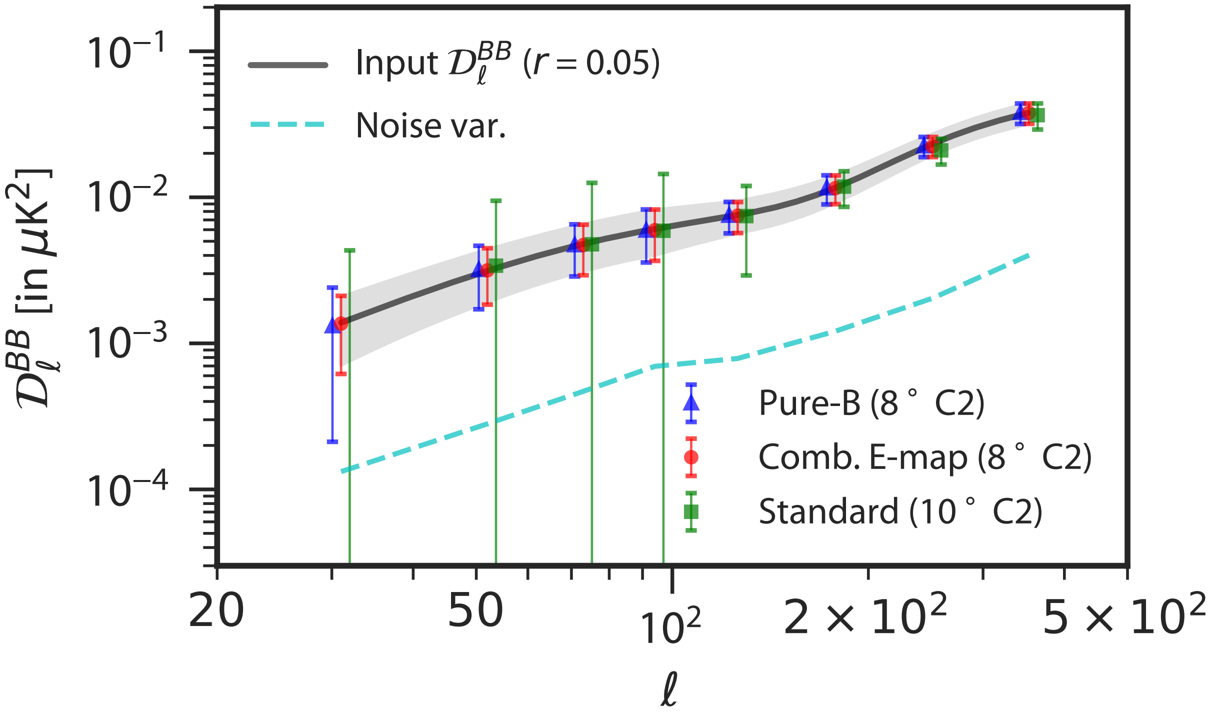
# Results

fsky = 2% patch, 5 uK arcmin noise;
Full sky: 50 uK-arcmin Planck noise

$r = 0$




$r = 0.05$

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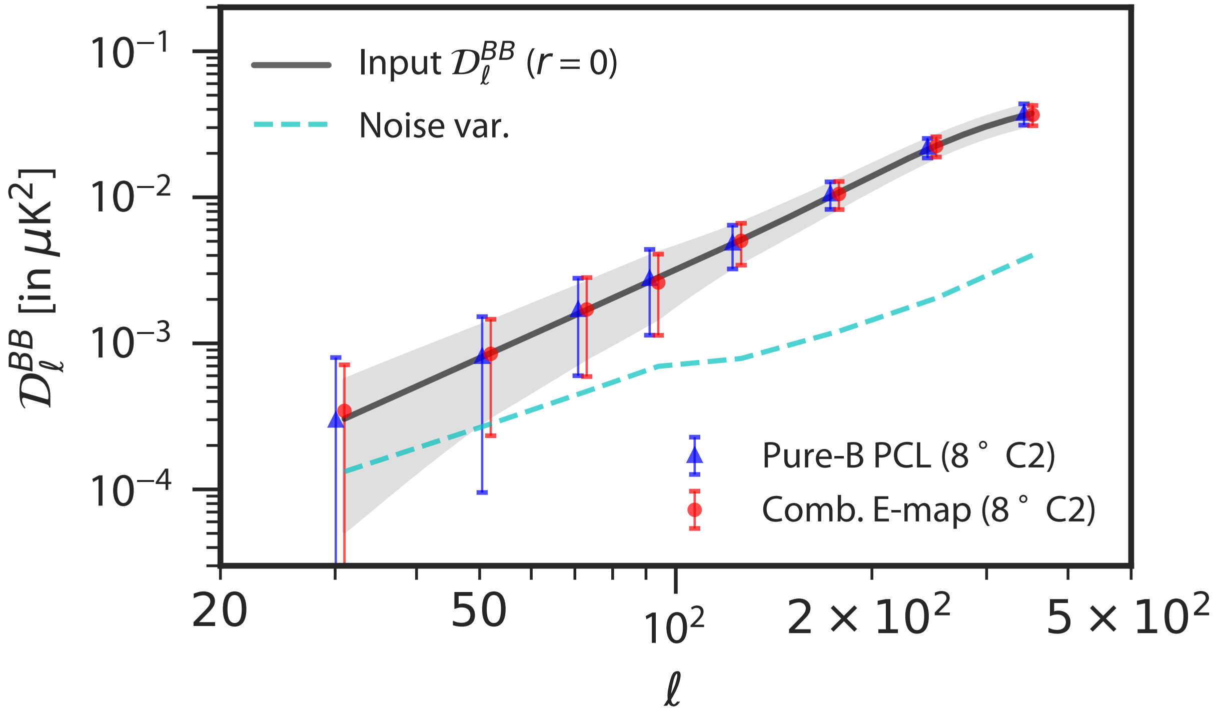
# Results

fsky = 2% patch, 5 uK arcmin noise;
Full sky: isotropized 100-143-217 Planck noise

$r = 0$

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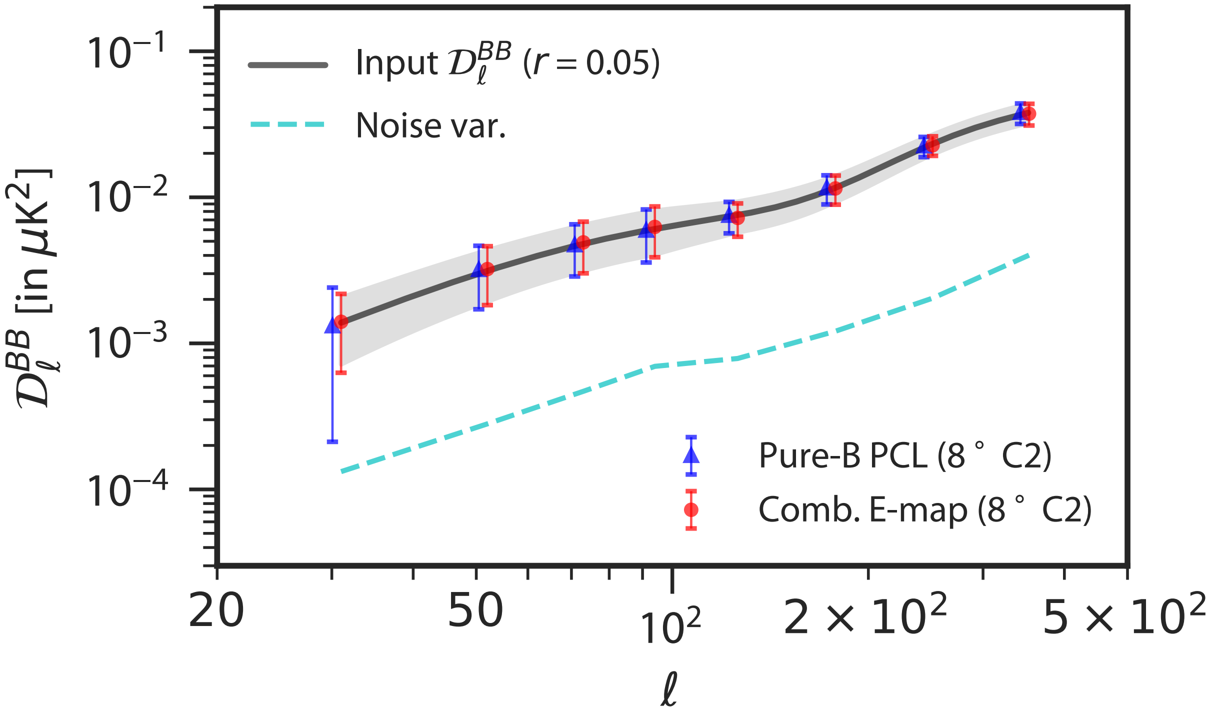


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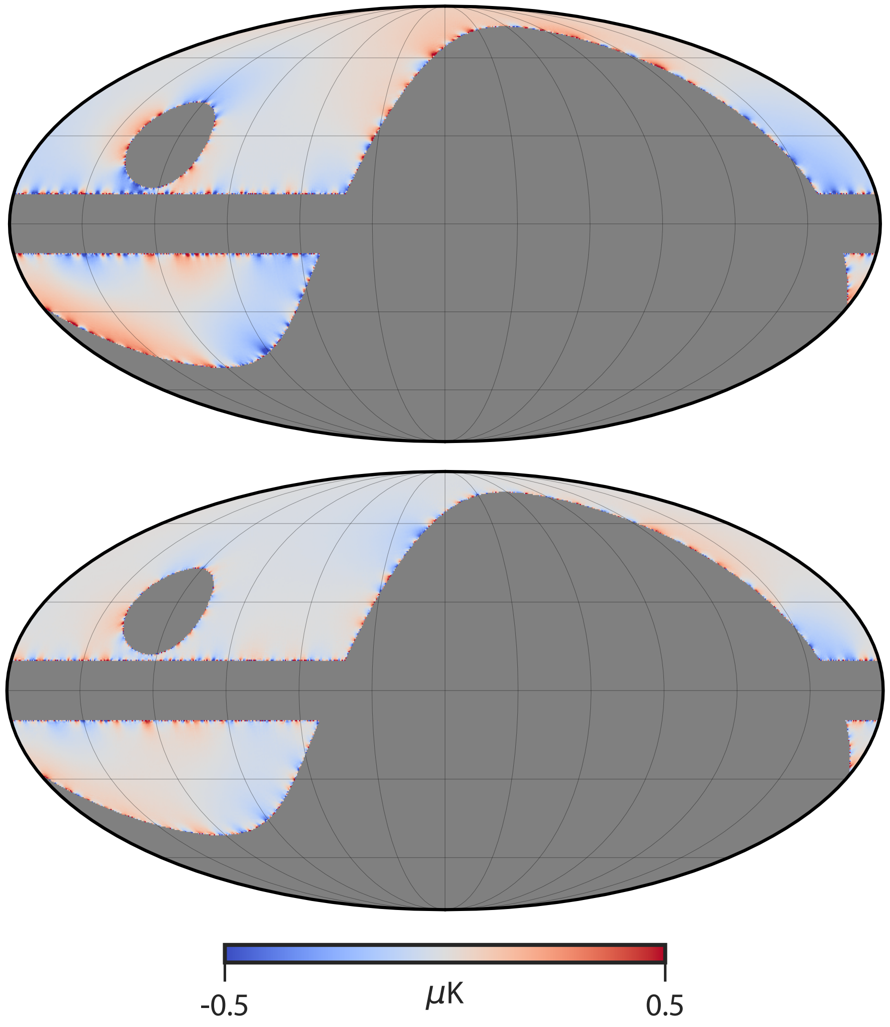
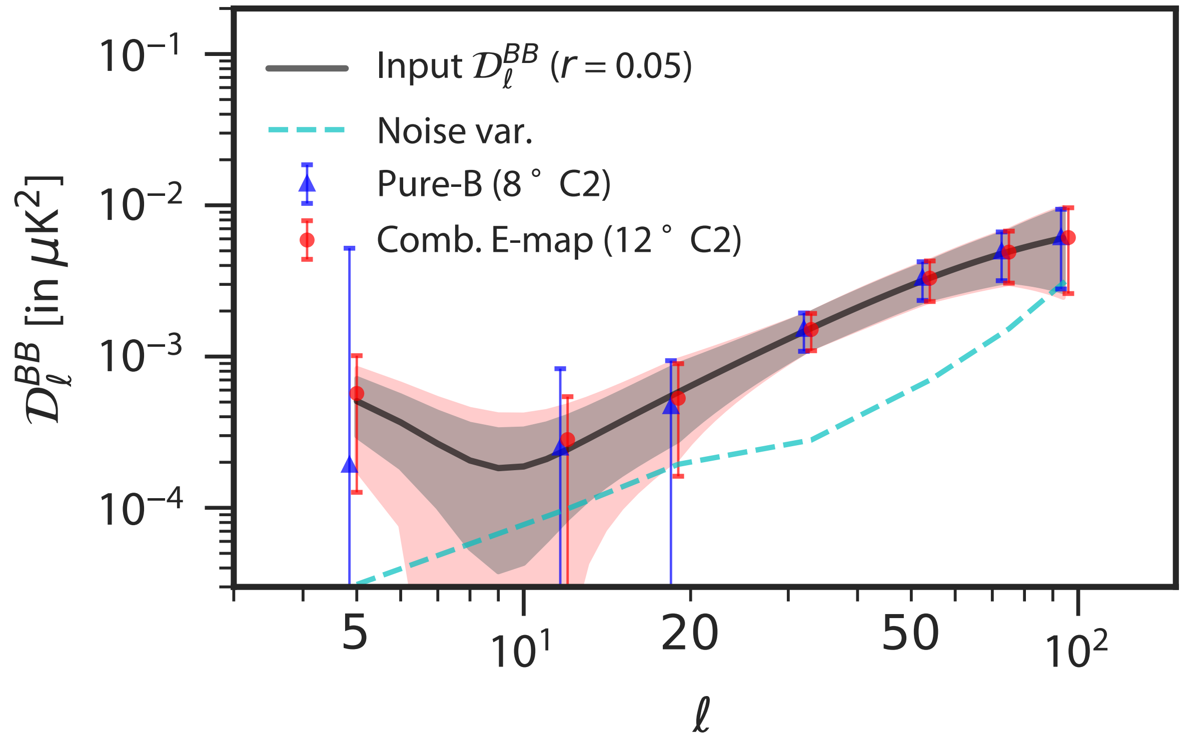
# LSPE case

fsky ~ 30% patch, combined SWIPE noise, 85′ beam;
Full sky: isotropized 100-143-217 Planck noise

$r = 0.05$




Optimal error band:

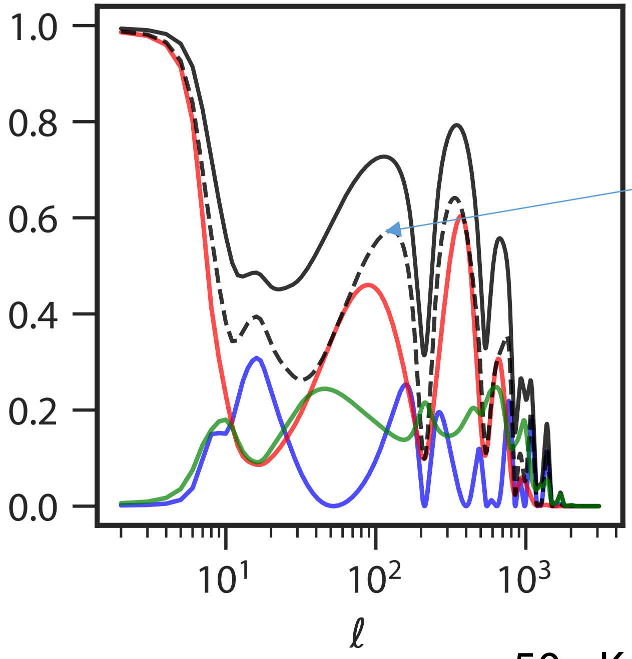
$$\Delta\mathcal{D}_{\bar{\ell};\text{optimal}} = \sqrt{\frac{2}{(2\bar{\ell}+1)f_{\text{sky}}\Delta\ell}\left[\mathcal{D}_{\bar{\ell}} + \frac{\mathcal{N}_{\bar{\ell}}}{B_{\bar{\ell}}^2}\right]\left[\frac{w_{(4)}}{w_{(2)}^2}\right]}^{\,1/2}$$

# How important is SNR?

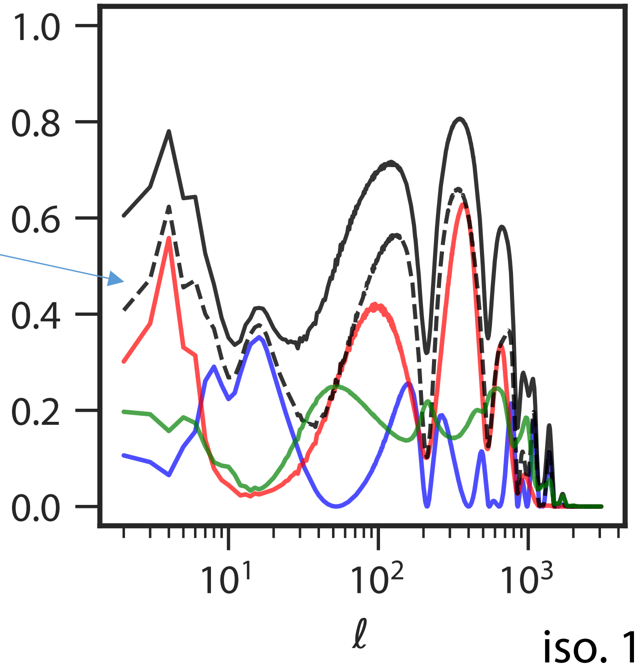
The reconstruction ratio:

$$r_{\rm rec} = \frac{1}{C_\ell^{EE} B^2_{E,\ell}} \left[ \langle \widehat{d}_{E,\ell m} \mathcal{W}^{EE}_{\ell'} s^*_{E,\ell' m'} \rangle + \langle \widehat{d}_{E,\ell m} \mathcal{W}^{ET}_{\ell'} s^*_{T,\ell' m'} \rangle \right]$$

Quantifies how much of the E-mode signal is present after Wiener filtering.



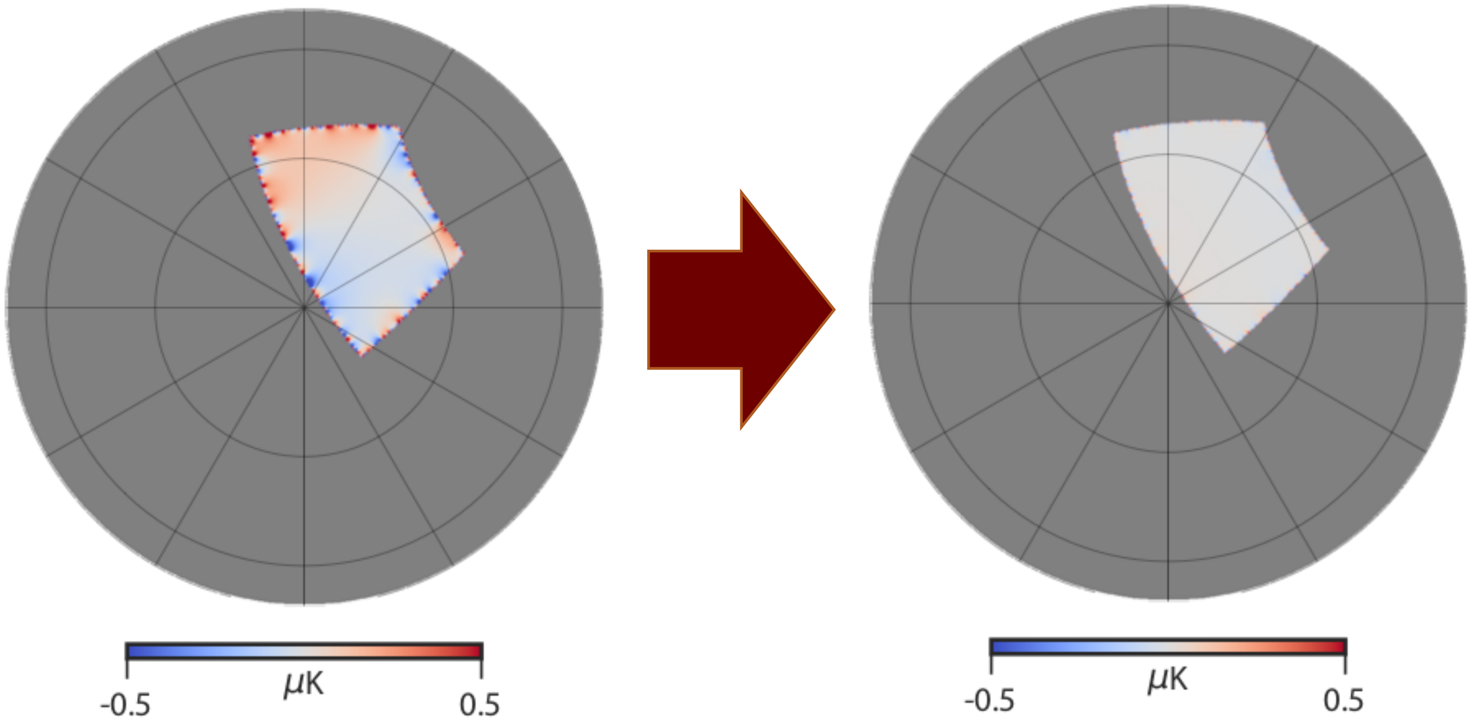

Follow the dashed black line!

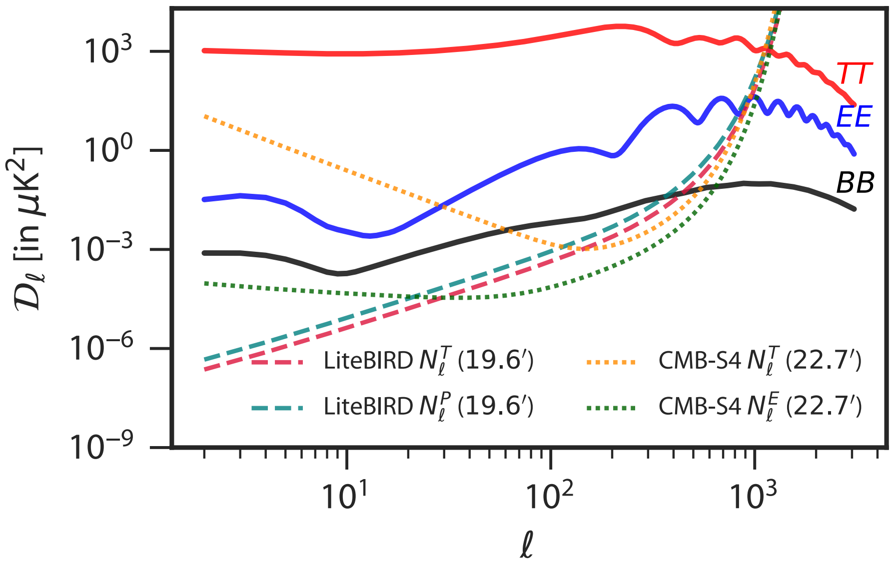


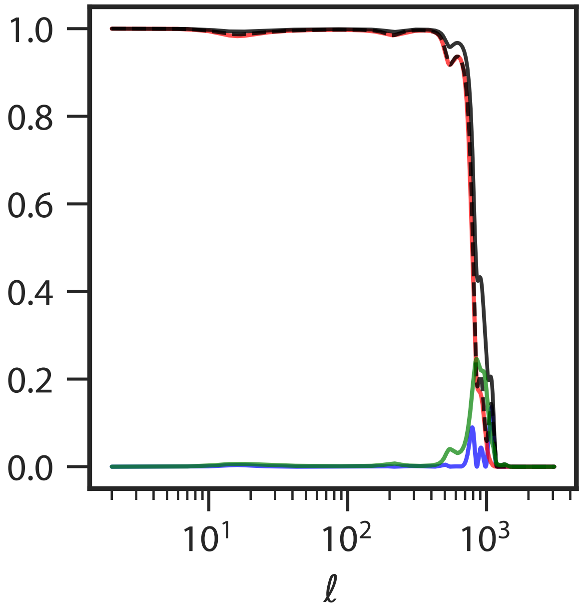

50 uK-arcmin

iso. 100-143-217

# Potential: LiteBIRD - CMB-S4



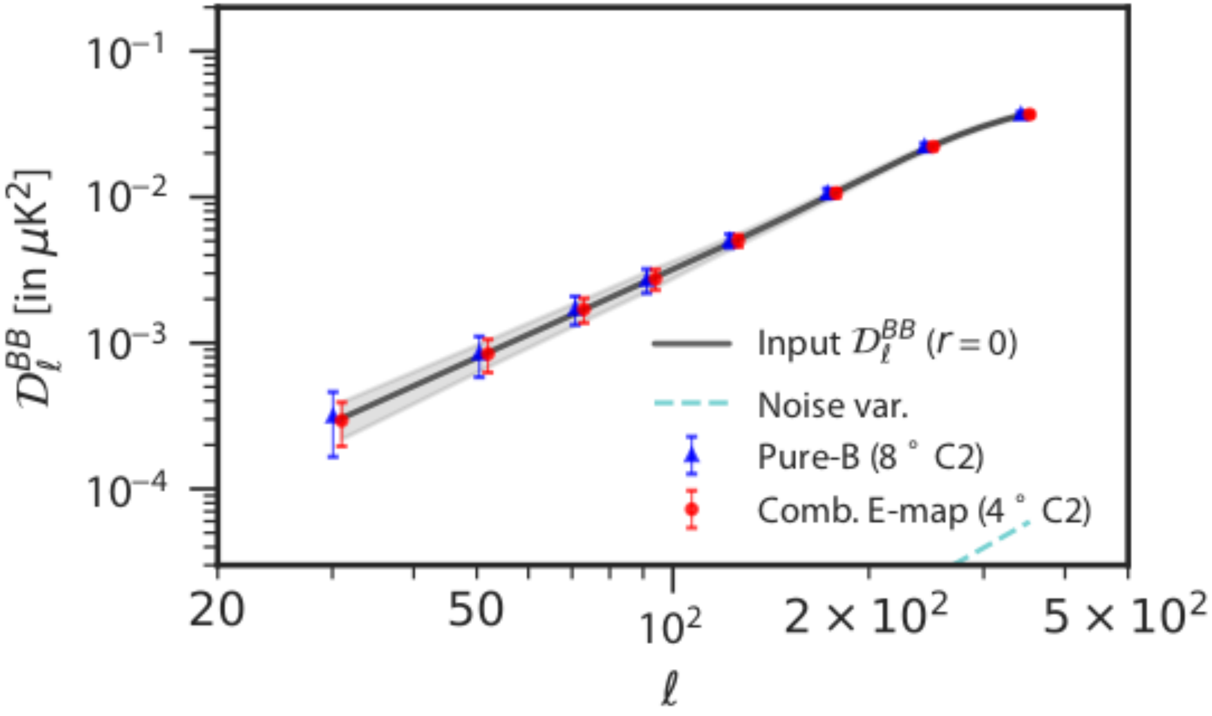




fsky ~ 3% patch, CMB-S4 Lo-Res Ultra-Deep patch
patch 95-145-155 GHz channel noise, 22.7′ resolution

# LiteBIRD - CMB-S4 Results

$r = 0$

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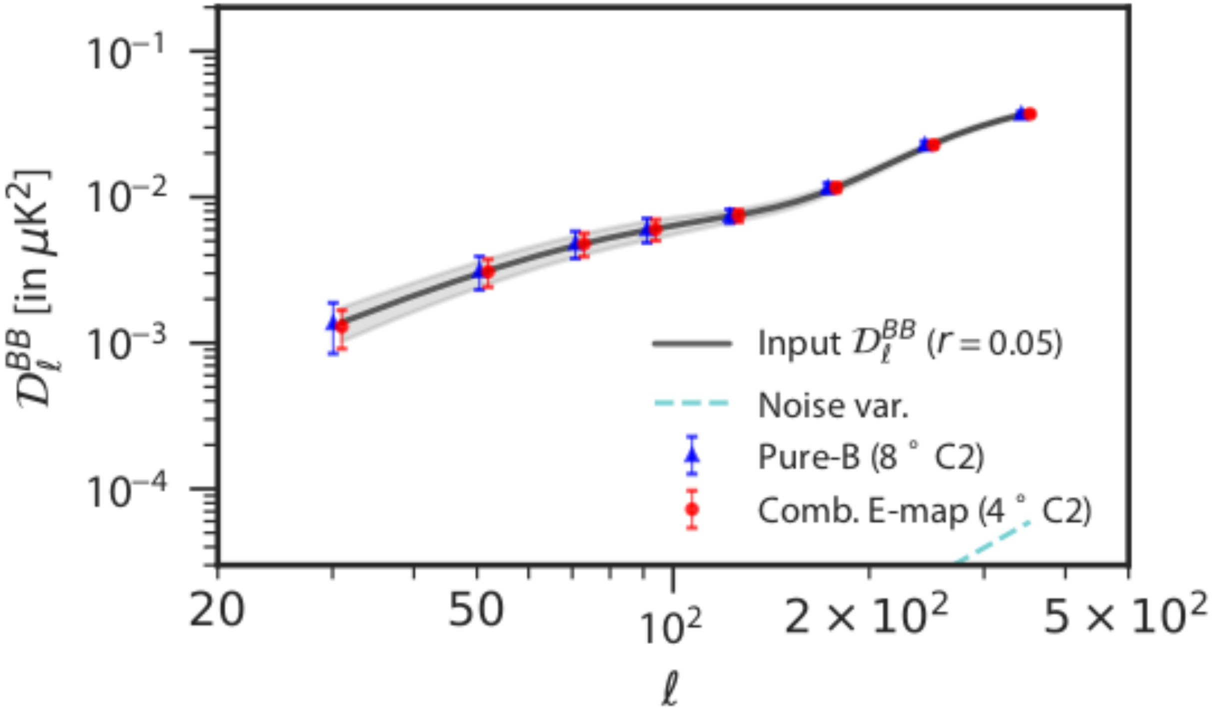


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# Summary and conclusions

- We demonstrate that it is possible to remove this leakage contribution by completing the *E*-mode information in suborbital experiments with ancillary *E*-mode information from satellite experiments.

- For Planck *E*-modes which have considerably higher noise than current and upcoming ground-based mission, we can Wiener filter the E-modes.

- We combine the filtered *E*-mode only QU maps from Planck outside the observed sky patch and show that we effective remove the impact of leakage.

- We show that for AliCPT-like or LSPE-like cases we outperform pure-*B* PCL estimator giving unbiased, with optimal PCL error bars.

- For CMB-S4, with signal dominated LiteBIRD *E*-mode data, the *E*-to-*B* leakage can be nearly eliminated.

- Next we need to update the method for inhomogeneous and correlated noise, with foreground residuals etc.