

On the arithmetic of Calabi-Yau manifolds: periods, zeta functions and attractor varieties

Friday, 4 June 2021 19:00 (1h 15m)

In this seminar I will discuss the arithmetic of Calabi-Yau 3-folds. The main goal is to explore whether there are questions of common interest in this context to physicists, number theorists and geometers. The main quantities of interest in the arithmetic context are the numbers of points of the manifold considered as a variety over a finite field. We are interested in the computation of these numbers and their dependence on the moduli of the variety. The surprise for a physicist is that the numbers of points over a finite field are also given by expression that involve the periods of a manifold. The number of points are encoded in the local zeta function, about which much is known in virtue of the Weil conjectures. I will discuss interesting topics related to the zeta function and the appearance of modularity for one parameter families of Calabi-Yau manifolds.

A topic I will stress is that for these families there are values of the parameter for which the manifold becomes singular and for these values the zeta function degenerates and exhibits modular behaviour. I will report (on joint work with Philip Candelas, Mohamed Elmi and Duco van Straten) on an example for which the quartic numerator of the zeta function factorises into two quadrics at special values of the parameter which satisfy an algebraic equation with coefficients in \mathbb{Q} (so independent of any particular prime), and for which the underlying manifold is smooth. We note that these factorisations are due to a splitting of the Hodge structure and that these special values of the parameter are rank two attractor points in the sense of type IIB supergravity. Modular groups and modular forms arise in relation to these attractor points.

To our knowledge, the rank two attractor points that were found by the application of these number theoretic techniques, provide the first explicit examples of such points for Calabi-Yau manifolds of full $SU(3)$ holonomy.

Presenter: DE LA OSSA, Xenia (Oxford U.)