Dark matter bound states

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Quarkonia meet dark matter
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Frontiers in dark matter searches

- **Heavy DM**

  Particles with m $\gtrsim$ TeV coupled to SM via the Weak or other interactions not constrained by collider experiments

  → existing and upcoming telescopes observing multi-TeV sky with increasing sensitivity, e.g. HESS, IceCube, CTA, Antares

- **Light DM**

  Particles with m $\lesssim$ few GeV, possibly coupled to SM via a portal interaction, not constrained by older direct detection experiments

  → development of new generation of direct detection experiments

Frontiers in dark matter searches

- **Heavy DM**

  Particles with m ≳ TeV coupled to SM via the Weak or other interactions not constrained by collider experiments

  → existing and upcoming telescopes observing multi-TeV sky with increasing sensitivity, e.g. HESS, IceCube, CTA, Antares

- **Light DM**

  Partic

  ents

- Simple thermal-relic WIMP models live in the (multi-)TeV scale.
- Thermal-relic DM can be as heavy as few × 100 TeV.

How heavy can thermal-relic DM be, and what are the underlying dynamics of heavy (≳ TeV) thermal-relic DM?

Long-range interactions

If dark matter is very heavy, then:

$$\lambda_B \sim \frac{1}{\mu v_{\rm rel}}, \frac{1}{\mu \alpha} \lesssim \frac{1}{m_{\rm mediator}} \sim \text{interaction range}$$

μ: reduced mass $(m_{\rm DM}/2)$

Long-range interactions

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μ: reduced mass $(m_{\rm DM}/2)$

Relevant for various models

- Self-interacting DM
- DM explanations of astrophysical anomalies, e.g. galactic positrons, IceCube PeV neutrinos

- WIMP DM with m_{DM} > few TeV. [Hisano et al. 2002]
- WIMP DM with m_{DM} < TeV, in scenarios of DM co-annihilation with coloured partners.

Implications of long-range interactions

Sommerfeld effect

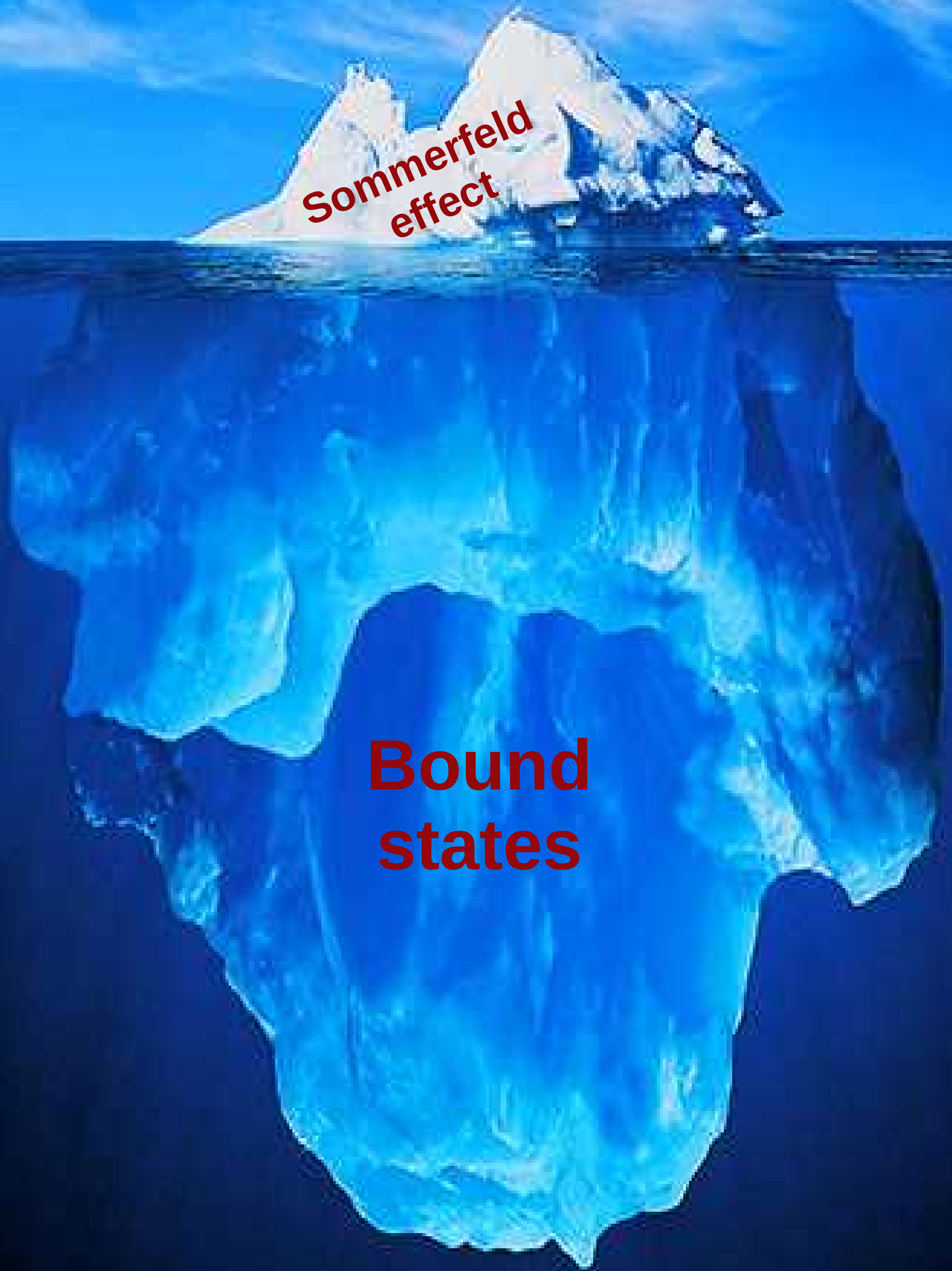
distortion of scattering-state wavefunctions
⇒ affects all cross-sections, incl annihilation

- Freeze-out ⇒ changes correlation of parameters (mass – couplings)
- Indirect detection signals
- Elastic scattering

Bound states

- **Unstable bound states**
  ⇒ extra annihilation channel
  - Freeze-out
  - Indirect detection
  - Novel low-energy indirect detection signals
- **Stable bound states (particularly important for asymmetric DM)**
  - Affect DM elastic scattering (screening)
  - Novel low-energy indirect detection signals
  - Inelastic scattering in direct detection experiments (?)

Outline



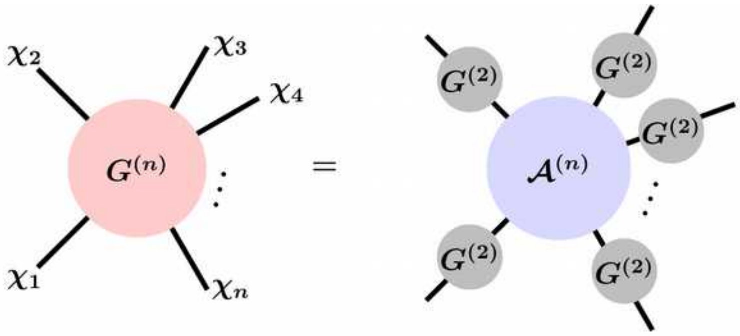
- Diagrammatic representation of long-range effects
- Dark U(1) sector

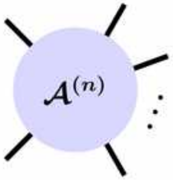
  Boltzmann equations for freeze-out
- Unitarity limit and long-range interactions
- Neutralino-squark coannihilation scenarios

Diagrammatic representation of long-range effects

Contact-type vs long-range interactions

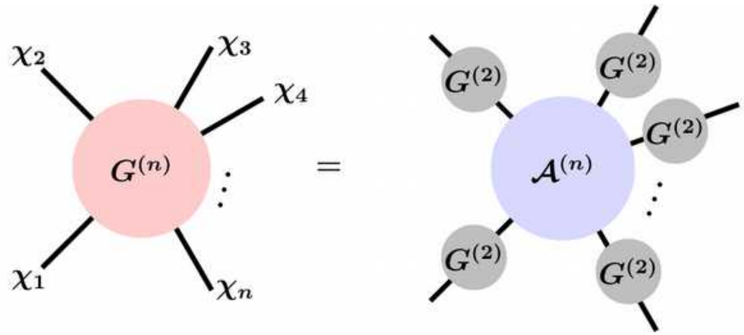
Scattering processes



where  includes all connected diagrams with the 1PI factors amputated.

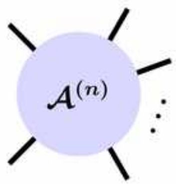
Contact-type vs long-range interactions

Scattering processes



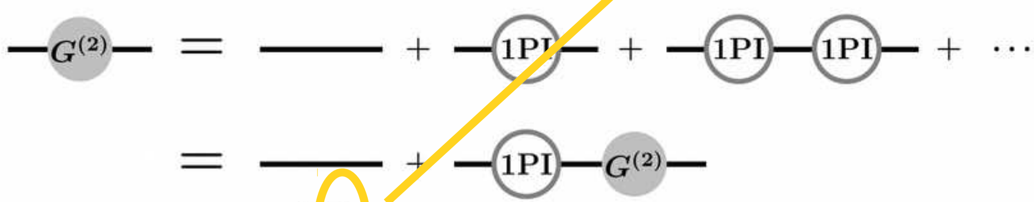
LSZ reduction $\Longleftrightarrow$

$$\mathcal{M} = \left(\prod_{j=1}^{n} \sqrt{Z_j} \right) \mathcal{A}$$

where 

includes all connected diagrams with the 1PI factors amputated.

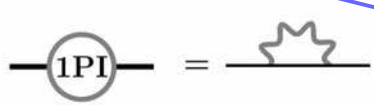
The properties of the asymptotic states are determined by resumming the self-interactions at infinity, via the Dyson-Schwinger equation



$$= \frac{iZ_j}{p_j^2 - m_j^2}$$

Field strength renormalization factor

Renormalized mass

where e.g. 

Contact-type vs **long-range** interactions

Scattering processes

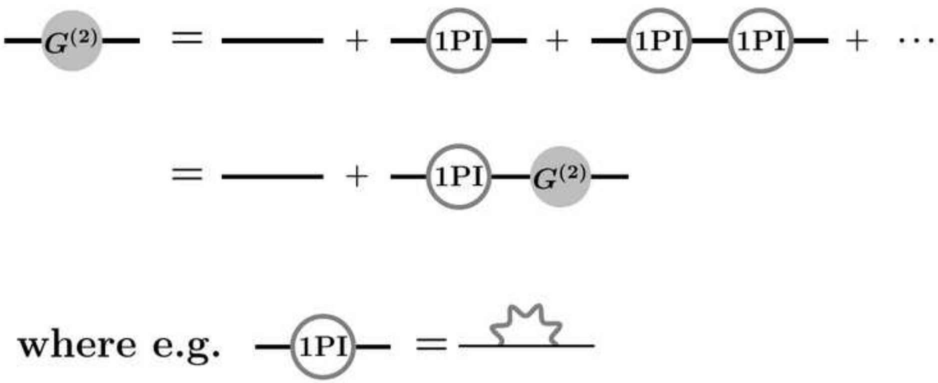
The particles interact at very large distance. We cannot define the asymptotic states by isolating the particles at infinity.

What do we do?

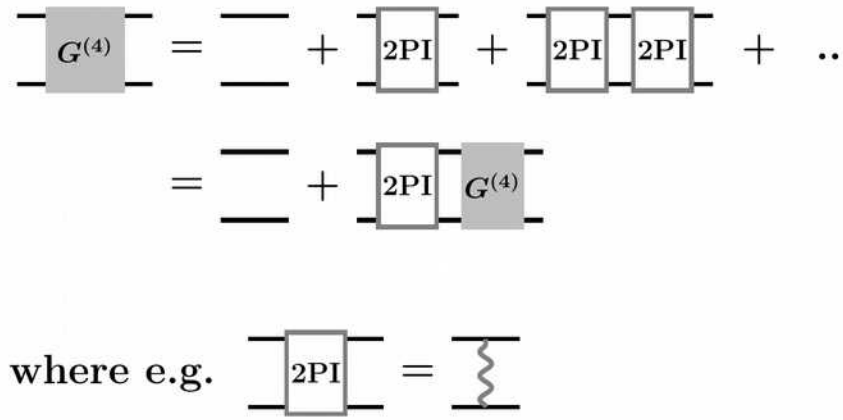
Resum 2-particle interactions at infinity!

Contact-type vs long-range interactions

Scattering processes



$$\mathcal{M} = \left(\prod_{j=1}^{n} \sqrt{Z_j} \right) \mathcal{A}$$



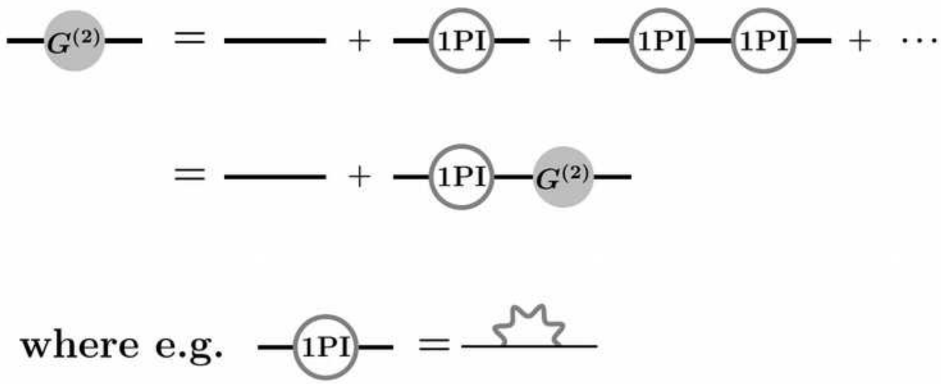
$$\mathcal{M} = \int \frac{d^3q}{(2\pi)^3} \phi_{\vec{k}}(\vec{q}) \mathcal{A}(\vec{q})$$

field strength renormalization factors / form factors / wavefunctions

$$G^{(2)} \sim Z/\text{singularity} \quad \leftrightarrow \quad G^{(4)} \sim [\phi_{\vec{k}}]^2/\text{singularity}$$

Contact-type vs long-range interactions

Scattering processes



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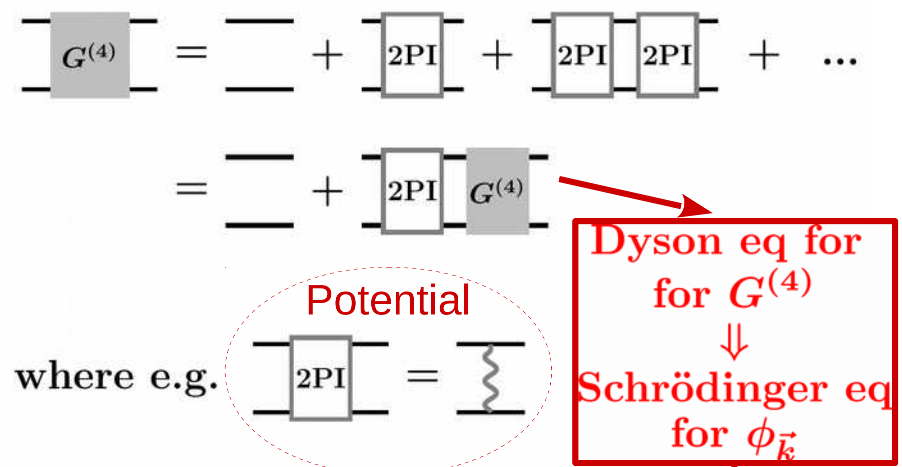

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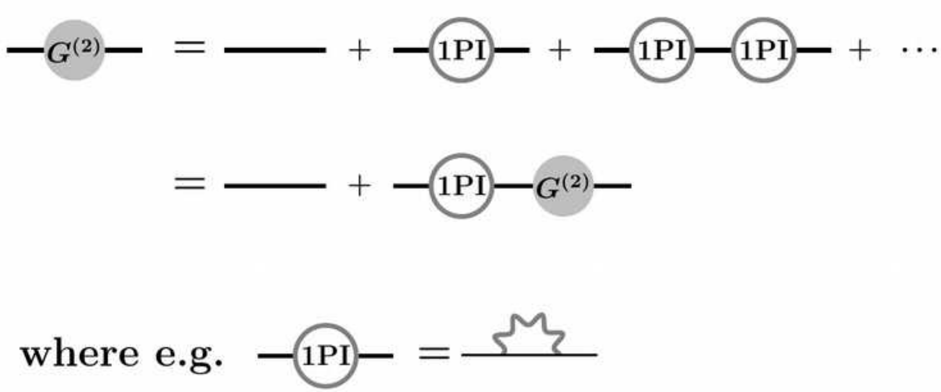
Contact-type vs long-range interactions

Scattering processes



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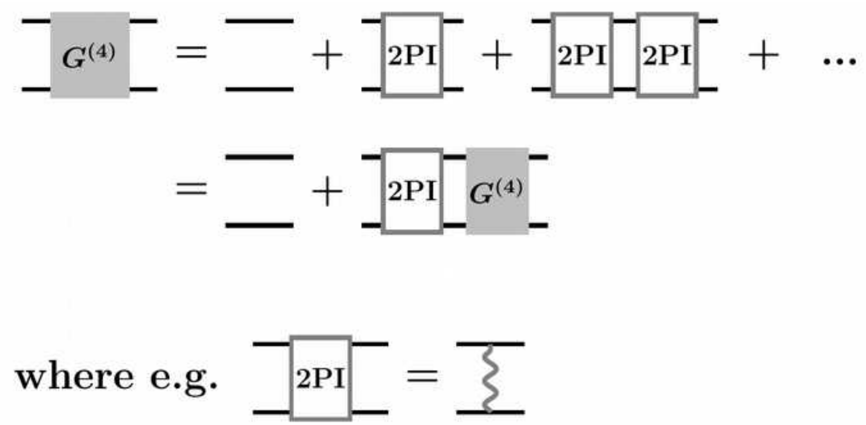


$$\mathcal{M} = \int \frac{d^3q}{(2\pi)^3}\, \phi_{\vec{k}}(\vec{q})\, \mathcal{A}(\vec{q})$$

Expectation value of relative momentum

Relative momentum of interacting particles

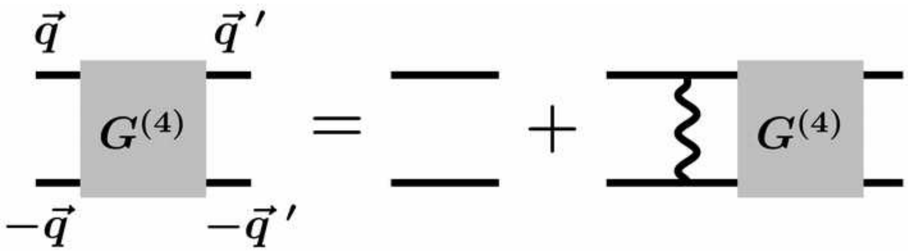
No long-range force: $\phi_{\vec{k}}(\vec{q}) = \delta^3(\vec{q} - \vec{k}) \quad \overset{\text{FT}}{\Leftrightarrow} \quad \tilde{\phi}_{\vec{k}}(\vec{r}) = e^{-i\vec{k}\cdot\vec{r}}$.

In the presence of a long-range interaction: $\tilde{\phi}_{\vec{k}}(\vec{r})$ is <u>not</u> a plane wave.

Long-range interactions

Scattering states and bound states

The Dyson-Schwinger equation with a Coulomb potential



where

$$G^{(4)} \sim [\phi_{\vec{k}}]^2/\text{singularity}$$

Solutions of the Schrödinger equation

continuous spectrum

$$\phi_{\vec{k}}(\vec{q}) \overset{\text{FT}}{\Leftrightarrow} \tilde{\phi}_{\vec{k}}(\vec{r})$$

$$\vec{k} = \mu \vec{v}_{\text{rel}}$$

$$E_{\vec{k}} = m_1 + m_2 + \vec{k}^2/(2\mu)$$

discrete spectrum

$$\psi_{n\ell m}(\vec{q}) \overset{\text{FT}}{\Leftrightarrow} \tilde{\psi}_{n\ell m}(\vec{r})$$

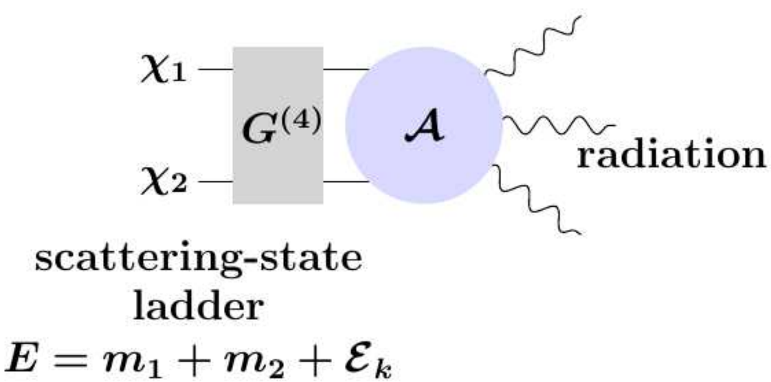
$$\kappa_n = \mu\alpha/n$$

$$E_n = m_1 + m_2 - \kappa_n^2/(2\mu)$$

where $\mu \equiv m_1 m_2/(m_1 + m_2)$ is the reduced mass

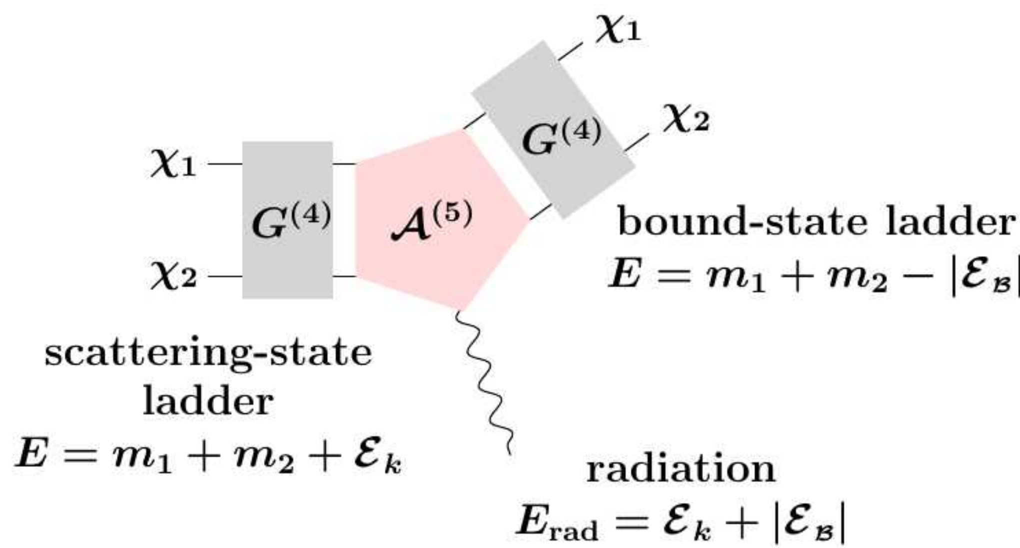
Computing cross-sections

Annihilation



$$\mathcal{M}_{\rm ann} \sim \int d^3k'\ \phi_k(k')\ \mathcal{A}(k')$$

Bound-state formation



$$\mathcal{M}_{\rm BSF} \sim \int d^3k'\, d^3p\ \phi_k(k')\ \mathcal{A}^{(5)}(k',p)\ \psi^*_{n\ell m}(p)$$

Many results (with analytical formulae in Coulomb limit):

KP, Postma, Wiechers: 1505.00109
KP, Postma, de Vries: 1611.01394
Harz, KP: 1711.03552, 1805.01200, 1901.10030
Oncala, KP: 1808.04854, 1911.02605, 2101.08666

More coming up: Filimonova et al.

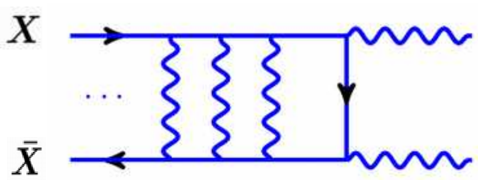
Dark U(1) sector

Thermal freeze-out with long-range interactions

Dark U(1) model: Dirac DM $X,\bar{X}$ coupled to γ_D

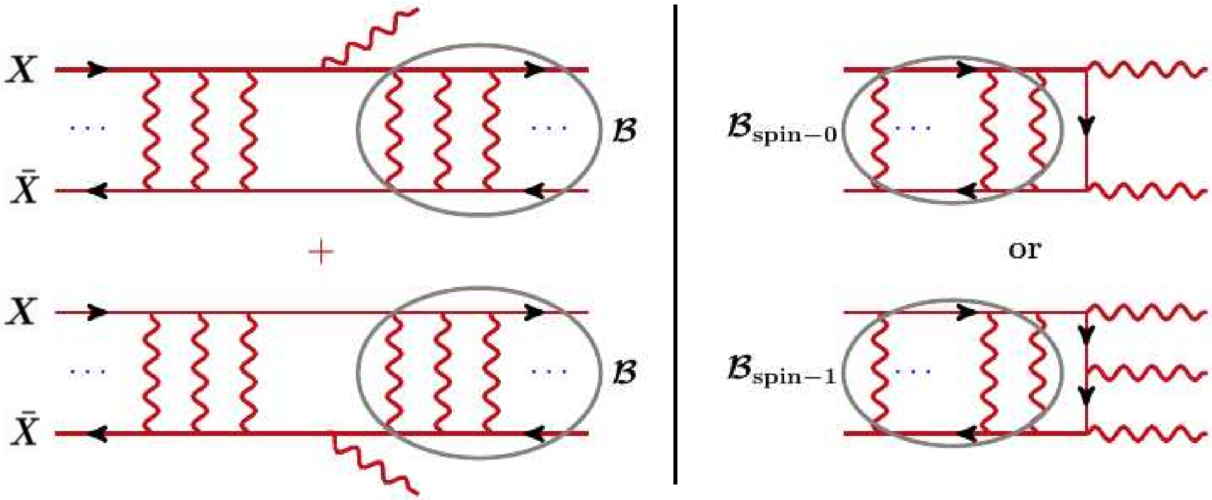
Direct annihilation

$X + \bar{X} \to 2\gamma_D$

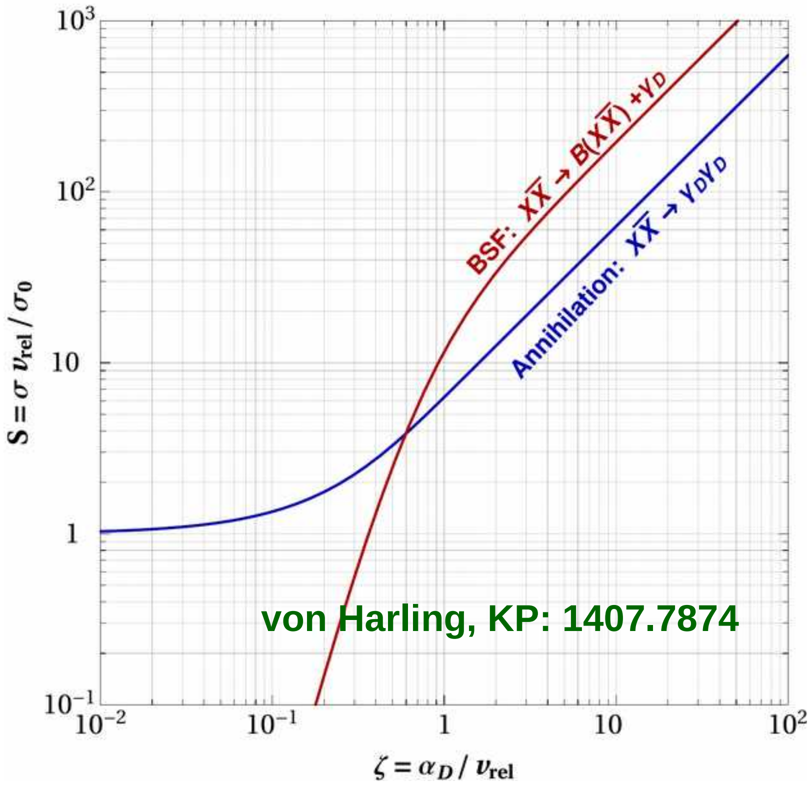


$$\sigma_{\rm ann} v_{\rm rel} = \frac{\pi \alpha_D^2}{m_X^2} \times S_{\rm ann}(\alpha_D / v_{\rm rel})$$

Bound-state formation and decay



$$\sigma_{\rm BSF} v_{\rm rel} = \frac{\pi \alpha_D^2}{m_X^2} \times S_{\rm BSF}(\alpha_D / v_{\rm rel})$$



$$S_{\rm ann} \simeq \left(\frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}\right) \xrightarrow{\zeta \gtrsim 1} 2\pi\zeta$$

$$S_{\rm BSF} \simeq \left(\frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}\right) \frac{2^9 \zeta^4 e^{-4\zeta \,{\rm arccot}\zeta}}{3(1+\zeta^2)^2} \xrightarrow{\zeta \gtrsim 1} 3.13 \times 2\pi\zeta$$

Thermal freeze-out with long-range interactions

Boltzmann equations

free particles: $$\frac{dn}{dt} + 3Hn = -\langle\sigma^{\text{ann}} v_{\text{rel}}\rangle \left(n^2 - n^{\text{eq}\,2}\right) - \sum_{\mathcal{B}} \left(\langle\sigma^{\text{BSF}}_{\mathcal{B}}\, v_{\text{rel}}\rangle\, n^2 - \Gamma^{\text{ion}}_{\mathcal{B}}\, n_{\mathcal{B}}\right)$$

bound states: $$\frac{dn_{\mathcal{B}}}{dt} + 3Hn_{\mathcal{B}} = + \left(\langle\sigma^{\text{BSF}}_{\mathcal{B}}\, v_{\text{rel}}\rangle\, n^2 - \Gamma^{\text{ion}}_{\mathcal{B}}\, n_{\mathcal{B}}\right) - \Gamma^{\text{dec}}_{\mathcal{B}} \left(n_{\mathcal{B}} - n^{\text{eq}}_{\mathcal{B}}\right) - \sum_{\mathcal{B}'\neq\mathcal{B}} \left(\Gamma^{\text{trans}}_{\mathcal{B}\to\mathcal{B}'} n_{\mathcal{B}} - \Gamma^{\text{trans}}_{\mathcal{B}'\to\mathcal{B}} n_{\mathcal{B}'}\right)$$

dilution due to expansion of the universe ($3Hn$, $3Hn_{\mathcal{B}}$)

Processes		Detailed balance
Bound state formation (BSF) Ionisation (ion)	$X + \bar{X} \to \mathcal{B}(X\bar{X}) + \gamma_D$ $\mathcal{B}(X\bar{X}) + \gamma_D \to X + \bar{X}$	$\langle\sigma^{\text{BSF}}_{\mathcal{B}} v_{\text{rel}}\rangle (n^{\text{eq}})^2 = \Gamma^{\text{ion}}_{\mathcal{B}}\, n^{\text{eq}}_{\mathcal{B}}$
Decay (dec)	$\mathcal{B}(X\bar{X}) \to 2\gamma_D$ or $3\gamma_D$	
Transitions (trans)	$\mathcal{B}(X\bar{X}) \to \mathcal{B}'(X\bar{X}) + \gamma_D$ $\mathcal{B}(X\bar{X}) + \gamma_D \to \mathcal{B}'(X\bar{X})$	$\Gamma^{\text{trans}}_{\mathcal{B}\to\mathcal{B}'}\, n^{\text{eq}}_{\mathcal{B}} = \Gamma^{\text{trans}}_{\mathcal{B}'\to\mathcal{B}}\, n^{\text{eq}}_{\mathcal{B}'}$

Thermal freeze-out with long-range interactions

Boltzmann equations

free particles: $$\frac{dn}{dt} + 3Hn = -\langle\sigma^{\rm ann} v_{\rm rel}\rangle \left(n^2 - n^{\rm eq\ 2}\right) - \sum_{\mathcal{B}} \left(\langle\sigma^{\rm BSF}_{\mathcal{B}}\, v_{\rm rel}\rangle\, n^2 - \Gamma^{\rm ion}_{\mathcal{B}}\, n_{\mathcal{B}}\right)$$

bound states: $$\frac{dn_{\mathcal{B}}}{dt} + 3Hn_{\mathcal{B}} = + \left(\langle\sigma^{\rm BSF}_{\mathcal{B}}\, v_{\rm rel}\rangle\, n^2 - \Gamma^{\rm ion}_{\mathcal{B}}\, n_{\mathcal{B}}\right) - \Gamma^{\rm dec}_{\mathcal{B}}\left(n_{\mathcal{B}} - n^{\rm eq}_{\mathcal{B}}\right) - \sum_{\mathcal{B}'\neq\mathcal{B}} \left(\Gamma^{\rm trans}_{\mathcal{B}\to\mathcal{B}'} n_{\mathcal{B}} - \Gamma^{\rm trans}_{\mathcal{B}'\to\mathcal{B}} n_{\mathcal{B}'}\right)$$

dilution due to expansion of the universe

Typically large enough, $\Gamma^{\rm dec}_{\mathcal{B}} \gg H$,
to keep bound states close to equilibrium

$\Rightarrow$ set $dn_{\mathcal{B}}/dt + 3Hn_{\mathcal{B}} \simeq 0$

$\Rightarrow$ get algebraic equations for $n_{\mathcal{B}}$ in terms of n, $n^{\rm eq}_{\mathcal{B}}$

$\Rightarrow$ re-employ it in Boltzmann equation for n

Ellis, Luo, Olive: 1503.07142

Thermal freeze-out with long-range interactions

Boltzmann equations

free particles: $$\frac{dn}{dt} + 3Hn = -\langle\sigma^{\rm ann} v_{\rm rel}\rangle\,(n^2 - n^{{\rm eq}\,2}) - \sum_{\mathcal{B}} \left(\langle\sigma_{\mathcal{B}}^{\rm BSF}\, v_{\rm rel}\rangle\, n^2 - \Gamma_{\mathcal{B}}^{\rm ion}\, n_{\mathcal{B}}\right)$$

bound states: $$\frac{dn_{\mathcal{B}}}{dt} + 3Hn_{\mathcal{B}} = + \left(\langle\sigma_{\mathcal{B}}^{\rm BSF}\, v_{\rm rel}\rangle\, n^2 - \Gamma_{\mathcal{B}}^{\rm ion}\, n_{\mathcal{B}}\right) - \Gamma_{\mathcal{B}}^{\rm dec}\,(n_{\mathcal{B}} - n_{\mathcal{B}}^{\rm eq}) - \sum_{\mathcal{B}' \neq \mathcal{B}} \left(\Gamma^{\rm trans}_{\mathcal{B}\to\mathcal{B}'} n_{\mathcal{B}} - \Gamma^{\rm trans}_{\mathcal{B}'\to\mathcal{B}} n_{\mathcal{B}'}\right)$$

$$\frac{dn}{dt} + 3Hn = -\langle\sigma^{\rm eff} v_{\rm rel}\rangle\,(n^2 - n^{{\rm eq}\,2})$$

where, neglecting bound-to-bound transitions,

$$\langle\sigma^{\rm eff} v_{\rm rel}\rangle \equiv \langle\sigma^{\rm ann} v_{\rm rel}\rangle + \sum_{\mathcal{B}} \langle\sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle \times \frac{\Gamma_{\mathcal{B}}^{\rm dec}}{\Gamma_{\mathcal{B}}^{\rm dec} + \Gamma_{\mathcal{B}}^{\rm ion}}$$

For transitions, see poster by Graham White

Thermal freeze-out with long-range interactions

Boltzmann equations

$$\frac{dn}{dt} + 3Hn = -\langle \sigma^{\rm eff} v_{\rm rel}\rangle \left(n^2 - n^{{\rm eq}\ 2}\right)$$

where, neglecting bound-to-bound transitions,

$$\langle \sigma^{\rm eff} v_{\rm rel}\rangle \equiv \langle \sigma^{\rm ann} v_{\rm rel}\rangle + \sum_{\mathcal{B}} \langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle \times \frac{\Gamma_{\mathcal{B}}^{\rm dec}}{\Gamma_{\mathcal{B}}^{\rm dec} + \Gamma_{\mathcal{B}}^{\rm ion}}$$

At $T \gg$ Binding Energy $\Rightarrow$ $\Gamma_{\mathcal{B}}^{\rm ion} \gg \Gamma_{\mathcal{B}}^{\rm dec}$,

$$\langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle \frac{\Gamma_{\mathcal{B}}^{\rm dec}}{\Gamma_{\mathcal{B}}^{\rm dec} + \Gamma_{\mathcal{B}}^{\rm ion}} \simeq \langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle \frac{\Gamma_{\mathcal{B}}^{\rm dec}}{\Gamma_{\mathcal{B}}^{\rm ion}} = \frac{n_{\mathcal{B}}^{\rm eq}}{(n^{\rm eq})^2}\, \Gamma_{\mathcal{B}}^{\rm dec}$$

$$\simeq \frac{g_{\mathcal{B}}}{g_X^2} \left(\frac{4\pi}{m_X T}\right)^{3/2} \times e^{|E_{\mathcal{B}}|/T}\, \Gamma_{\mathcal{B}}^{\rm dec}$$

$\Downarrow$

Independent of actual BSF cross-section!

$\Gamma_{\mathcal{B}}^{\rm dec} \propto (\sigma^{\rm ann} v_{\rm rel}) \to$ modest increase over the direct annihilation, but increases exponentially as T drops.

"Ionisation equilibrium"
Binder, Covi, Mukaida: 1808.06472

At $T \lesssim$ Binding Energy $\Rightarrow$ $\Gamma_{\mathcal{B}}^{\rm ion} \ll \Gamma_{\mathcal{B}}^{\rm dec}$,

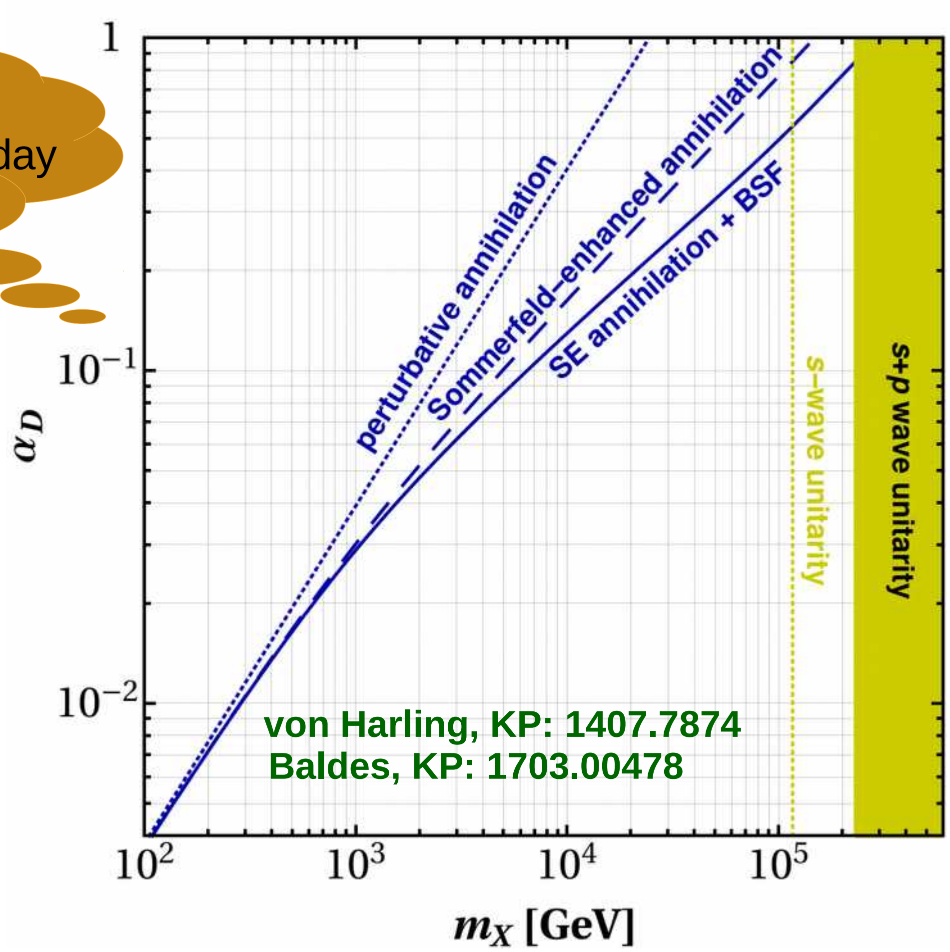
$$\langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle \frac{\Gamma_{\mathcal{B}}^{\rm dec}}{\Gamma_{\mathcal{B}}^{\rm dec} + \Gamma_{\mathcal{B}}^{\rm ion}} \simeq \langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle.$$

Typically, most of DM destruction due to BSF occurs in this regime.

Thermal freeze-out with long-range interactions

Dark U(1) model: Dirac DM $X,\bar{X}$ coupled to γ_D

Important because it determines DM interactions today (direct, indirect detection)



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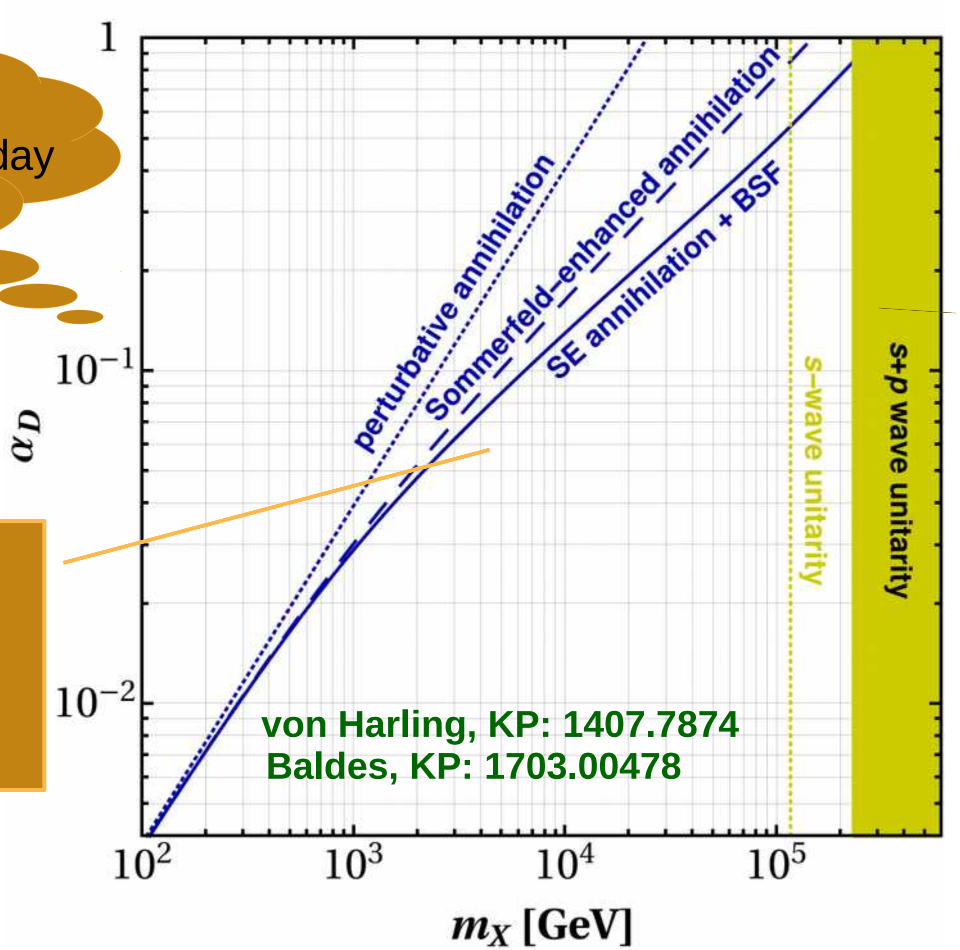


Long-range effects indeed become at $m_{DM} \gtrsim$ few TeV.

Verifies expectation from unitarity arguments!

Dominant annihilation mode: ***s*-wave**.

Dominant BSF mode: ***p*-wave**

Same order!

Higher partial waves Important / dominant in multi-TeV regime.

DM may be even heavier!

Unitarity limit and long-range interactions

Partial-wave unitarity limit

$$\sigma_{\rm inel}^{(\ell)} v_{\rm rel} \leqslant \sigma_{\rm uni}^{(\ell)} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M_{\rm DM}^2 v_{\rm rel}}$$

Implies upper bound on the mass of thermal-relic DM

Griest, Kamionkowski (1990)

$$\sigma_{\rm ann} v_{\rm rel} \simeq 2.2\times 10^{-26}\ {\rm cm^3/s} \leqslant \frac{4\pi}{M_{\rm DM}^2 v_{\rm rel}}$$

$$\langle v_{\rm rel}^2\rangle^{1/2} = (6T/M_{\rm DM})^{1/2} \xrightarrow[M_{\rm DM}/T\approx 25]{\rm freeze-out} 0.49$$

$$\Rightarrow M_{\rm uni} \simeq \begin{cases} 117\ {\rm TeV}, & \text{self-conjugate DM} \\ 83\ {\rm TeV}, & \text{non-self-conjugate DM} \end{cases}$$

Two assumptions to be questioned

1. "one does not expect $\sigma v_{\rm rel} \propto 1/v_{\rm rel}$ for annihilation channels in a non-relativistic expansion."
2. The s-wave yields the dominant contribution to the annihilation cross-section.

Partial-wave unitarity limit

$$\sigma^{(\ell)}_{\rm inel} v_{\rm rel} \leqslant \sigma^{(\ell)}_{\rm uni} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M^2_{\rm DM} v_{\rm rel}}$$

Implies upper bound on t... of thermal-relic DM

Griest, K...

What are the underlying dynamics
of heavy thermal-relic DM?

What interactions can approach / attain the unitarity limit?

$$\langle v^2_{\rm rel}\rangle^{1/2} = (6T/M_{\rm DM})^{1/2} \xrightarrow[M_{\rm DM}/T \approx 25]{\text{freeze-out}} 0.49$$

$$\Rightarrow M_{\rm uni} \simeq \begin{cases} 117\ \text{TeV}, & \text{self-conjuga...} \\ 83\ \text{TeV}, & \text{non-self-conjugate DM} \end{cases}$$

...ions
... questioned

1. "one does not expect $\sigma v_{\rm rel} \propto 1/v_{\rm rel}$ for annihilation channels in a ...
   ... yields the dominant contribution to the annihilation cross-section.

What are the implications for experiments?

Partial-wave unitarity limit

$$\sigma^{(\ell)}_{\rm inel} v_{\rm rel} \leqslant \sigma^{(\ell)}_{\rm uni} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M^2_{\rm DM} v_{\rm rel}}$$

1) Velocity dependence of σ_{uni}

Assuming σv_{rel} = constant, setting it to maximal (inevitably for a fixed v_{rel}) and thermal averaging is formally incorrect!

⇒ Unitarity violation at larger v_{rel}, non-maximal cross-section at smaller v_{rel}.

Sommerfeld-enhanced inelastic processes exhibit exactly this velocity dependence at large couplings / small velocities, e.g. in QED

$$\sigma^{\ell=0}_{\rm ann} v_{\rm rel} \simeq \frac{\pi\alpha_D^2}{M^2_{\rm DM}} \times \frac{2\pi\alpha_D/v_{\rm rel}}{1-\exp(-2\pi\alpha_D/v_{\rm rel})} \xrightarrow{\alpha_D \gg v_{\rm rel}} \frac{2\pi^2\alpha_D^3}{M^2_{\rm DM} v_{\rm rel}}$$

⇒ Velocity dependence of σ_{uni} definitely *not* unphysical!

Baldes, KP: 1703.00478

Partial-wave unitarity limit

$$\sigma_{\rm inel}^{(\ell)} v_{\rm rel} \leqslant \sigma_{\rm uni}^{(\ell)} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M_{\rm DM}^2 v_{\rm rel}}$$

Parametric

1) ~~Velocity~~ dependence of σ_{uni}

What can we learn?

For a contact-type interaction, mediated by heavy particle with $m_{\rm med} \gtrsim M_{\rm DM}$,

$$\sigma_{\rm ann} v_{\rm rel} \sim \frac{\alpha_D^2 M_{\rm DM}^2}{m_{\rm med}^4} \lesssim \frac{4\pi}{M_{\rm DM}^2 v_{\rm rel}}.$$

Approaching unitarity limit requires large coupling (no surprise)

$$\alpha_D \sim m_{\rm med}^4 / M_{\rm DM}^4 \gtrsim 1.$$

Calculation violates unitarity if

$$m_{\rm med} < \alpha_D^{1/2} M_{\rm DM} \lesssim \alpha_D M_{\rm DM}.$$

Comparison between physical scales
$\Rightarrow$ violation signals new effect at play!

Hisano, Matsumoto, Nojiri, hep-ph/0212022
Baldes, KP: 1703.00478

Partial-wave unitarity limit

$$\sigma^{(\ell)}_{\rm inel} v_{\rm rel} \leqslant \sigma^{(\ell)}_{\rm uni} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M^2_{\rm DM} v_{\rm rel}}$$

Parametric

1) ~~Velocity~~ dependence of σ_{uni}

For a contact-type interaction, mediated by heavy particle with $m_{\rm med} \gtrsim M_{\rm DM}$,

$$\sigma_{\rm ann} v_{\rm rel} \sim \frac{\alpha_D^2 M^2_{\rm DM}}{m^4_{\rm med}} \lesssim \frac{4\pi}{M^2_{\rm DM} v_{\rm rel}}.$$

Approaching unitarity limit requires large coupling (no surprise)

$$\alpha_D \sim m^4_{\rm med}/M^4_{\rm DM} \gtrsim 1.$$

Calculation violates unitarity if

$$m_{\rm med} < \alpha_D^{1/2} M_{\rm DM} \lesssim \alpha_D M_{\rm DM}.$$

Comparison between physical scales
⇒ violation signals new effect at play!

What can we learn?

Including the Sommerfeld enhancement, for a light mediator, e.g. dark QED

$$\sigma_{\rm ann} v_{\rm rel} \simeq \frac{2\pi^2 \alpha_D^3}{M^2_{\rm DM} v_{\rm rel}} \lesssim \frac{4\pi}{M^2_{\rm DM} v_{\rm rel}}.$$

Unitarity indicates range of validity

$$\alpha_D \lesssim 0.86$$

Only numerical bound on a dimensionless coupling
⇒ include (resummed) higher order corrections

Hisano, Matsumoto, Nojiri, hep-ph/0212022
Baldes, KP: 1703.00478

Partial-wave unitarity limit

$$\sigma_{\rm inel}^{(\ell)} v_{\rm rel} \leqslant \sigma_{\rm uni}^{(\ell)} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M_{\rm DM}^2 v_{\rm rel}}$$

1) Velocity dependence of $\sigma_{\rm uni}$

Proper thermal average and taking into account delayed chemical decoupling

$$M_{\rm uni} \simeq \begin{cases} 117\ \text{TeV}, & \text{self-conjugate DM} \\ 83\ \text{TeV}, & \text{non-self-conjugate DM} \end{cases}$$

$$M_{\rm uni} \simeq \begin{cases} 198\ \text{TeV}, & \text{self-conjugate DM} \\ 138\ \text{TeV}, & \text{non-self-conjugate DM} \end{cases}$$

s-wave annihilation

Baldes, KP: 1703.00478

Partial-wave unitarity limit

$$\sigma^{(\ell)}_{\rm inel} v_{\rm rel} \leqslant \sigma^{(\ell)}_{\rm uni} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M_{\rm DM}^2 v_{\rm rel}}$$

2) Higher partial waves

In direct annihilation processes, *s*-wave dominates.

- For contact-type interactions, higher ℓ are $v_{\rm rel}^{2\ell}$ suppressed:

$$\sigma_{\rm ann} v_{\rm rel} = \sum_{\ell} \sum_{r=0}^{\infty} c_{\ell r}\, v_{\rm rel}{}^{2\ell+2r}$$

- For long-range interactions:

$$\sigma^{(\ell=0)} v_{\rm rel} \sim \frac{\pi\alpha_D^2}{M_{\rm DM}^2} \times \left(\frac{2\pi\alpha_D/v_{\rm rel}}{1-e^{-2\pi\alpha_D/v_{\rm rel}}}\right) \xrightarrow{\alpha_D \gg v_{\rm rel}} \frac{2\pi^2\alpha_D^3}{M_{\rm DM}^2 v_{\rm rel}}$$

$$\sigma^{(\ell=1)} v_{\rm rel} \sim \frac{\pi\alpha_D^2}{M_{\rm DM}^2} v_{\rm rel}^2 \times \left(\frac{2\pi\alpha_D/v_{\rm rel}}{1-e^{-2\pi\alpha_D/v_{\rm rel}}}\right)\left(1+\frac{\alpha_D^2}{v_{\rm rel}^2}\right) \xrightarrow{\alpha_D \gg v_{\rm rel}} \frac{2\pi^2\alpha_D^5}{M_{\rm DM}^2 v_{\rm rel}}$$

Same $v_{\rm rel}$ scaling (as expected from unitarity!), albeit $v_{\rm rel}^2 \to \alpha_D^2$ suppression.

Baldes, KP: 1703.00478

Partial-wave unitarity limit

$$\sigma_{\text{inel}}^{(\ell)} v_{\text{rel}} \leqslant \sigma_{\text{uni}}^{(\ell)} v_{\text{rel}} = \frac{4\pi(2\ell+1)}{M_{\text{DM}}^2 v_{\text{rel}}}$$

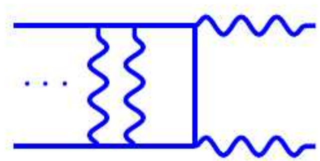
2) Higher partial waves

In direct annihilation processes, *s*-wave dominates.

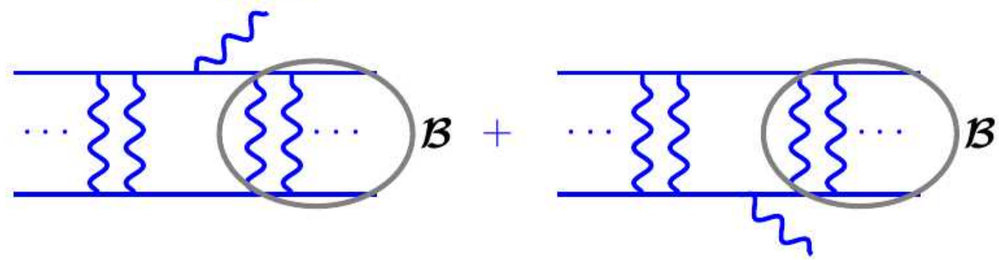
However, DM may annihilate via formation and decay of bound states.

dark QED

$$\sigma_{\text{ann}}^{(\ell=0)} v_{\text{rel}} \xrightarrow{\alpha_D \gg v_{\text{rel}}} \frac{2\pi^2\alpha_D^3}{M_{\text{DM}}^2 v_{\text{rel}}}$$



$$\sigma_{\text{BSF}}^{(\ell=1)} v_{\text{rel}} \xrightarrow{\alpha_D \gg v_{\text{rel}}} 3.13 \times \frac{2\pi^2\alpha_D^3}{M_{\text{DM}}^2 v_{\text{rel}}}$$



Baldes, KP: 1703.00478

Partial-wave unitarity limit

$$\sigma^{(\ell)}_{\rm inel} v_{\rm rel} \leqslant \sigma^{(\ell)}_{\rm uni} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M^2_{\rm DM} v_{\rm rel}}$$

2) Higher partial waves

In direct annihilation processes, *s*-wave dominates.

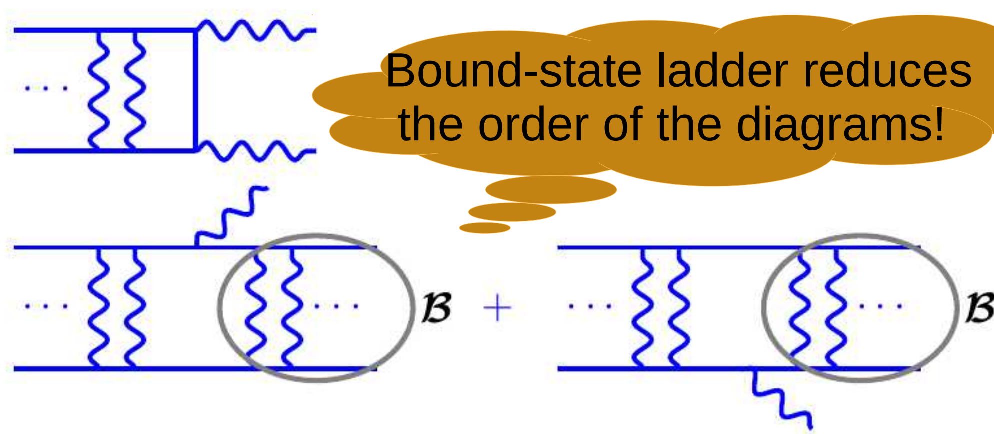
However, DM may annihilate via formation and decay of bound states.

dark QED

$$\sigma^{(\ell=0)}_{\rm ann} v_{\rm rel} \xrightarrow{\alpha_D \gg v_{\rm rel}} \frac{2\pi^2 \alpha_D^3}{M^2_{\rm DM} v_{\rm rel}}$$

same order!

$$\sigma^{(\ell=1)}_{\rm BSF} v_{\rm rel} \xrightarrow{\alpha_D \gg v_{\rm rel}} 3.13 \times \frac{2\pi^2 \alpha_D^3}{M^2_{\rm DM} v_{\rm rel}}$$



Bound-state ladder reduces the order of the diagrams!

Both *s*-wave and *p*-wave saturate their unitarity limit at $\alpha_D \simeq 0.86$.

$\Rightarrow$ Consider combined bound on the DM mass, $M^{s+p}_{\rm uni} \simeq 276$ TeV.

Higher partial waves important for DM destruction in early universe

$\Rightarrow$ higher $M_{\rm uni}$

Baldes, KP: 1703.00478

Partial-wave unitarity limit

$$\sigma_{\text{inel}}^{(\ell)} v_{\text{rel}} \leqslant \sigma_{\text{uni}}^{(\ell)} v_{\text{rel}} = \frac{4\pi(2\ell+1)}{M_{\text{DM}}^2 v_{\text{rel}}}$$

Can be approached or attained only by long-range interactions

Baldes, KP: 1703.00478

Generic conclusion:

In viable thermal-relic DM scenarios, expect long-range behaviour at $m_{DM} \gtrsim$ few TeV!

- **Freeze-out**
  Sommerfeld & BSF alter predicted mass – coupling relation.
  Important for all experimental probes.

- **Indirect detection**
  Sommerfeld & BSF must be considered in computing signals.
  Novel lower energy signals produced in BSF.

Poster by Iason Baldes

Neutralino-squark co-annihilation scenarios

Squark-neutralino co-annihilation scenarios

- Degenerate spectrum → soft jets → evade LHC constraints
- Large stop-Higgs coupling reproduces measured Higgs mass and brings the lightest stop close in mass with the LSP

⇒ DM density determined by "effective" Boltzmann equation

$$n_{\rm tot} = n_{\rm LSP} + n_{\rm NLSP}$$

$$\sigma^{\rm eff}_{\rm ann} = [\, n^2_{\rm LSP}\, \sigma^{\rm LSP}_{\rm ann} + n^2_{\rm NLSP}\, \sigma^{\rm NLSP}_{\rm ann} + n_{\rm LSP}\, n_{\rm NLSP}\, \sigma^{\rm LSP-NLSP}_{\rm ann} \,]/n^2_{\rm tot}$$

Scenario probed in colliders.
Important to compute DM density accurately!
→ **QCD corrections**

DM coannihilation with scalar colour triplet

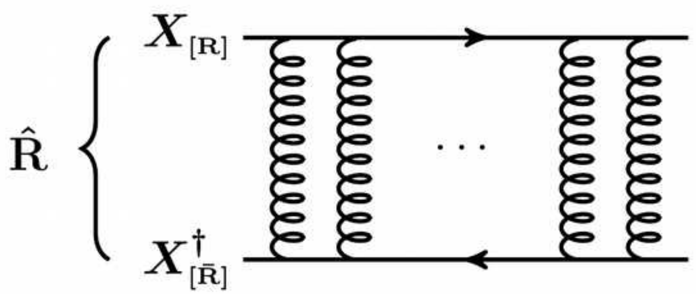
MSSM-inspired toy model

$$
\begin{aligned}
\mathcal{L} \quad \supset \quad & \frac{1}{2}\overline{\chi^c}\, i\partial\!\!\!/ \chi - \frac{1}{2} m_\chi \overline{\chi^c}\chi \\
+ \quad & \left[(\partial_\mu + i g_s G^a_\mu T^a) X\right]^\dagger \left[(\partial^\mu + i g_s G^{a,\mu} T^a) X\right] - m_X^2 |X|^2 \\
+ \quad & (\chi \leftrightarrow X, X^\dagger) \text{ interactions in chemical equilibrium during freeze-out}
\end{aligned}
$$

DM coannihilation with scalar colour triplet

MSSM-inspired toy model

Long-range interaction



$$R \otimes \bar{R} = \sum_{\hat{R}} \hat{R} = 1 \oplus \text{adj} + \cdots$$

$$V(r) = -\,\alpha_{g,[\hat{R}]} \,/\, r$$

$$\alpha_{g,[\hat{R}]} = \alpha_s \times \left[C_2(R) - C_2(\hat{R})/2 \right]$$

where $\alpha_s = g_s^2/(4\pi)$

for SU(3)

$$3 \otimes \bar{3} = 1 \oplus 8$$

$$\alpha_{g,[1]} = +\,(4/3)\alpha_s \quad \text{attractive}$$

$$\alpha_{g,[8]} = -\,(1/6)\alpha_s \quad \text{repulsive}$$

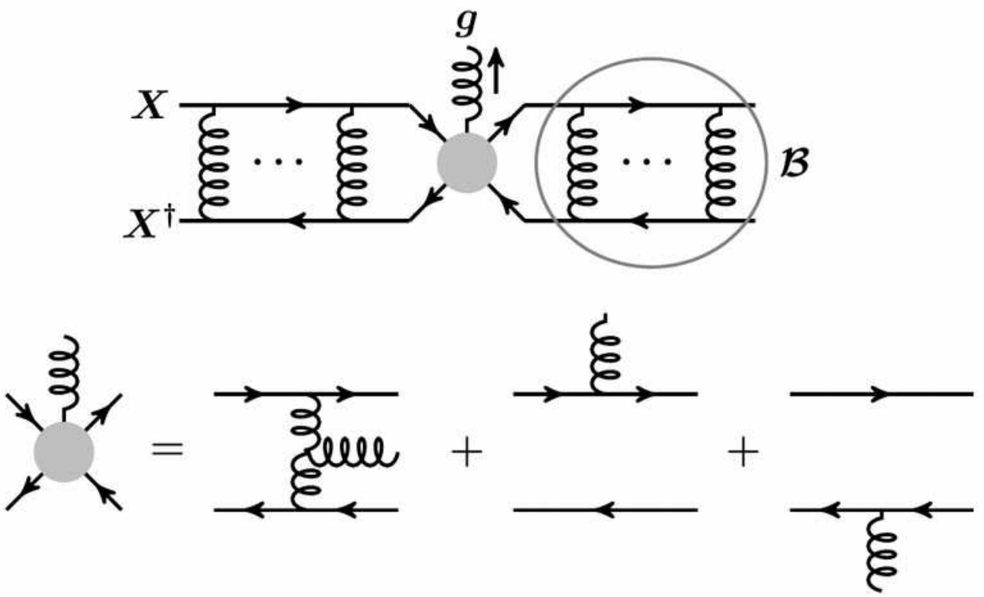
with $\alpha_s \sim 0.1$ at $m_X \sim \text{TeV}$

Kats, Schwartz 0912.0526

DM coannihilation with scalar colour triplet

MSSM-inspired toy model

Bound-state formation and decay

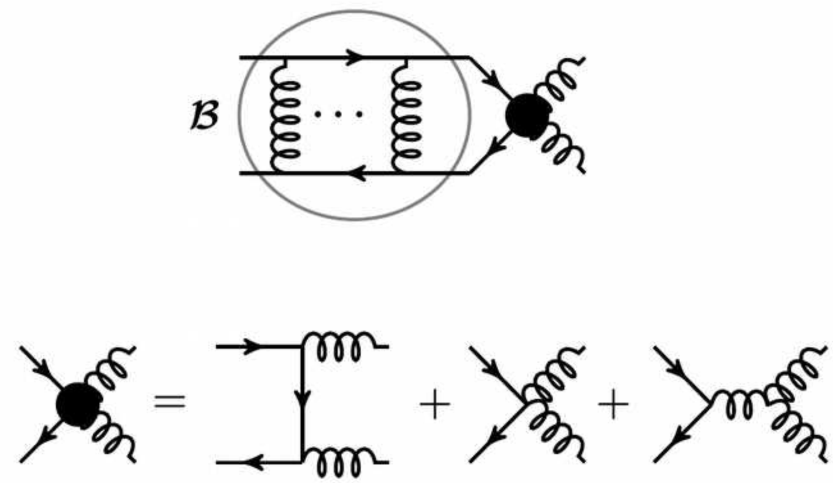


Allowed transitions

$$(X + X^\dagger)_{[8]} \rightarrow (X + X^\dagger)_{[1]} + g_{[8]}$$

$$(X + X^\dagger)_{[1]} \rightarrow (X + X^\dagger)_{[8]} + g_{[8]}$$

$$(X + X^\dagger)_{[8]} \rightarrow (X + X^\dagger)_{[8]} + g_{[8]}$$



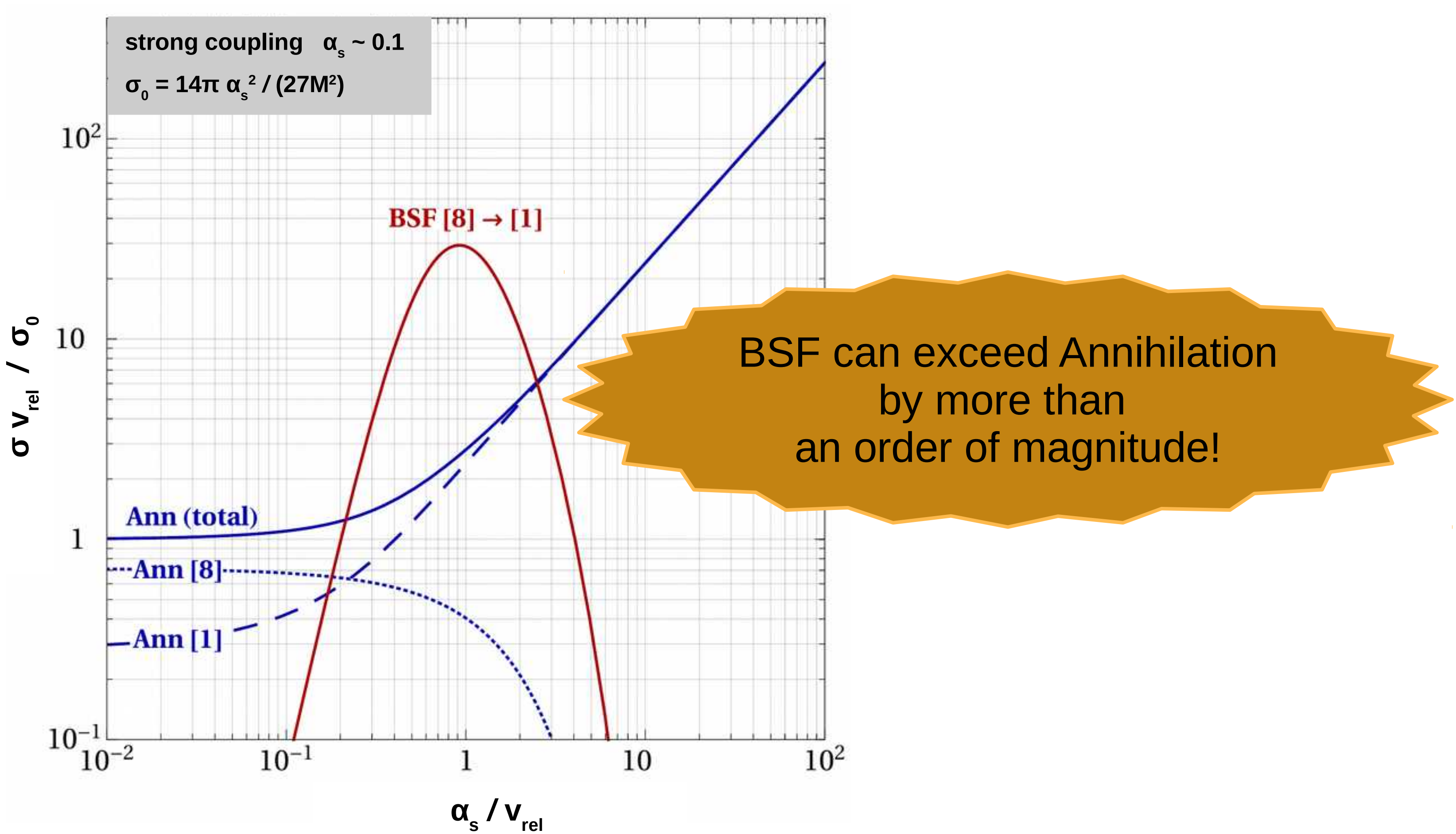
Harz, KP 1805.01200: Cross-sections for radiative BSF in non-Abelian theories

In agreement with Brambilla, Escobedo, Ghiglieri, Vairo 1109.5826:
Gluo-dissociation of quarkonium in pNRQCD

DM coannihilation with scalar colour triplet

MSSM-inspired toy model

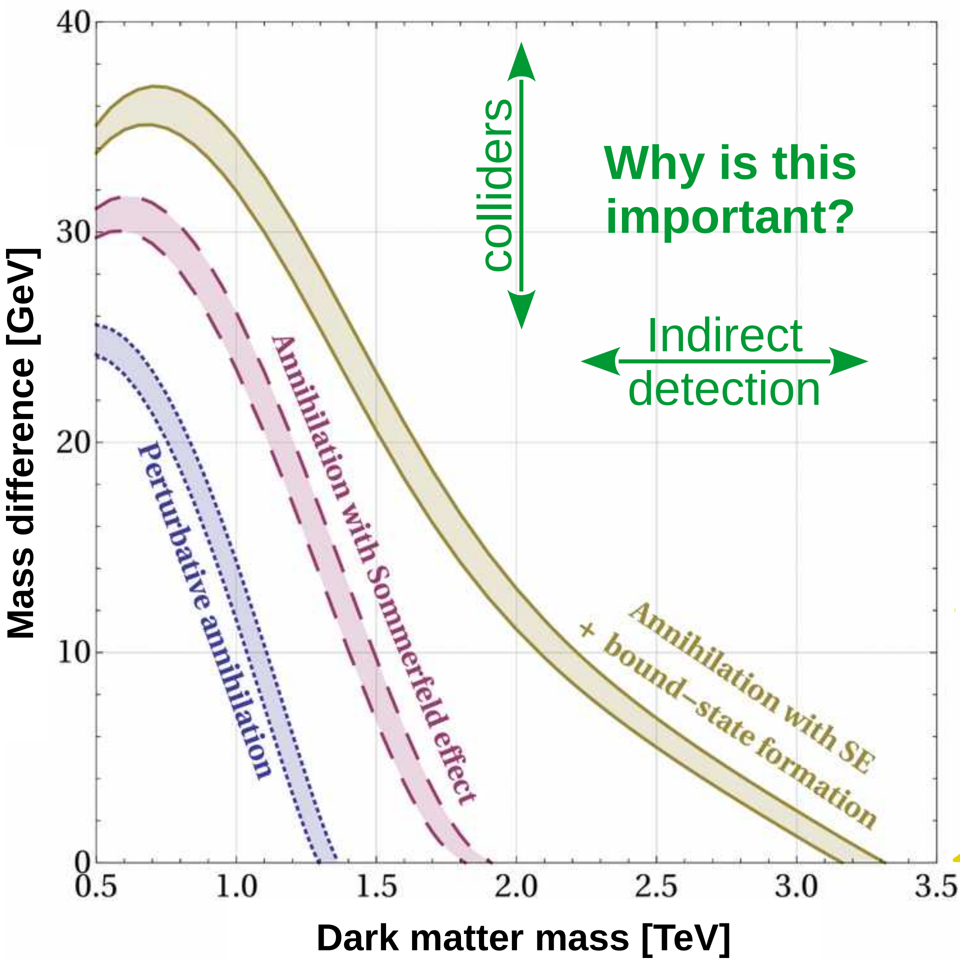
Bound-state formation *vs* Annihilation



Harz, KP: 1805.01200

DM coannihilation with scalar colour triplet

MSSM-inspired toy model



Effect on relic density: much much larger than obs uncertainty in Ω_{DM}

See also talks by Mikko Laine and Simone Biondini for a different approach

Not the final picture!

Harz, KP: 1805.01200

Squark-neutralino co-annihilation scenarios

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⇒ DM density determined by "effective" Boltzmann equation

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Scenario probed in colliders.
Important to compute DM density accurately!
→ **QCD corrections**

The Higgs as a light mediator

- Sommerfeld enhancement of direct annihilation — Harz, KP: 1711.03552
- Binding of bound states — Harz, KP: 1901.10030

Thursday talk by Julia Harz

- Formation of bound states via Higgs (*doublet*) emission ?

Emission of a charged scalar [or its Goldstone mode] results in very very rapid monopole transitions !

Ko,Matsui,Tang: 1910:04311
Oncala, KP: 1911.02605
Oncala, KP: 2101.08666
Oncala, KP: 2101.08667

My Friday talk

Conclusion

- **Bound states impel complete reconsideration of thermal decoupling at / above the TeV scale.**

  Unitarity limit can be approached / realised only by attractive long-range interactions ⇒ bound states play very important role! **Baldes, KP: 1703.00478**

- **Important experimental implications:**
  - DM heavier than anticipated: multi-TeV probes very important.
  - Indirect detection
    - Enhanced rates due to BSF
    - Novel signals: low-energy radiation emitted in BSF
    - Indirect detection of asymmetric DM
  - Colliders: improved detection prospects due increased mass gap in coannihilation scenarios

Poster by Iason Baldes