pair annihilation and bound states in a thermal plasma[1]

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what are we talking about on the QCD side?

$\Rightarrow$ charm and bottom quarks at $T \sim 150...450$ MeV

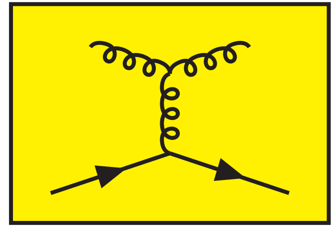
$\Rightarrow$ non-equilibrium: gluons and $N_{\rm f}$ light quarks are thermalized, charm and bottom quarks are "probes" in this background

$\Rightarrow$ bottom quark is non-relativistic ($m_b \sim 10...30\ T$), charm quark is a borderline case ($m_c \sim 2...6\ T$)

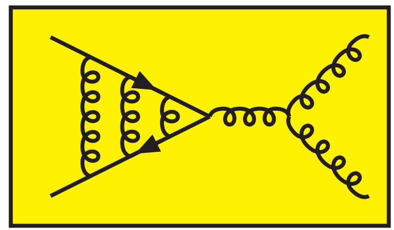
$\Rightarrow$ denote pole mass by M, with $M \gg \Lambda$, where $\Lambda =$ confinement scale; sometimes keep apart M_{rest} and M_{kin}

conceptual issues

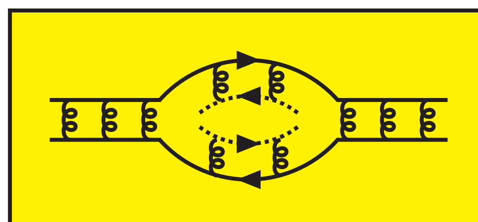
kinetic equilibration of quarks:
how fast does velocity adjust to overall plasma flow?



chemical equilibration [this talk]:
how fast does number density adjust to boltzmann weight?



$\sim$ kinetic equilibration of pairs:
how likely do $q\bar{q}$ states appear as scattering or bound states?
("dissociation / recombination")



brief history of chemical equilibration of heavy quarks

⇒ basic analysis with boltzmann equations[2]

⇒ non-perturbative formulation of rate coefficient[3]

⇒ inspired by cosmology, include sommerfeld effect[4]

⇒ digging deeper, rediscover bound state effects[5]

⇒ but after all this, interest remains academic within QCD...

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[2] e.g. T. Matsui *et al*, *...chemical kinetics...*, PRD 34 (1986) 783

[3] D. Bödeker and ML, *...chemical equilibration rate as a transport coefficient*, 1205.4987

[4] D. Bödeker and ML, *Sommerfeld effect in ... chemical equilibration*, 1210.6153

[5] S. Kim and ML, *Rapid ...co-annihilation through bound states in QCD*, 1602.08105

basic equations, assuming kinetic equilibrium

boltzmann equations ($n =$ number density):[6]

$$\dot{n} = -\langle \sigma v \rangle \, (n^2 - n^2_{\text{eq}})$$

$\Rightarrow$ non-perturbative formulation of rate [most of this talk]

$$\langle \sigma v \rangle \overset{[3]}{=} \frac{\Gamma_{\text{chem}}}{2 n_{\text{eq}}} \overset{[5]}{=} 4 \sum_i c_i \frac{\langle \mathcal{O}_i \rangle_T}{n^2_{\text{eq}}}$$

$\Rightarrow$ non-perturbative formulation of density [brief remark later on]

$$n^2 \overset{[7]}{\Rightarrow} \{ e^{\beta \mu(n)} n_{\text{eq}} \}^2 \, , \quad e^{\beta \mu(n)} n_{\text{eq}} \overset{[8]}{\approx} \frac{2n}{1 + \sqrt{1 + 8 \hat{p}_2 n}}$$

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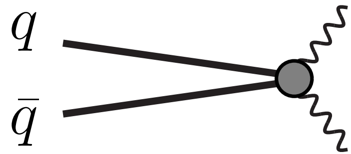
6 B.W. Lee and S. Weinberg, *Cosmological lower bound...,* PRL 39 (1977) 165

7 T. Binder, L. Covi and K. Mukaida, *...bound-state decay...,* 1808.06472

8 S. Biondini, S. Kim and ML, *Non-relativistic susceptibility...,* 1908.07541

physical picture of the decay



energy released in the inelastic reaction is $2M \gg T \Rightarrow$ the "hard" annihilation process is effectively local[9]

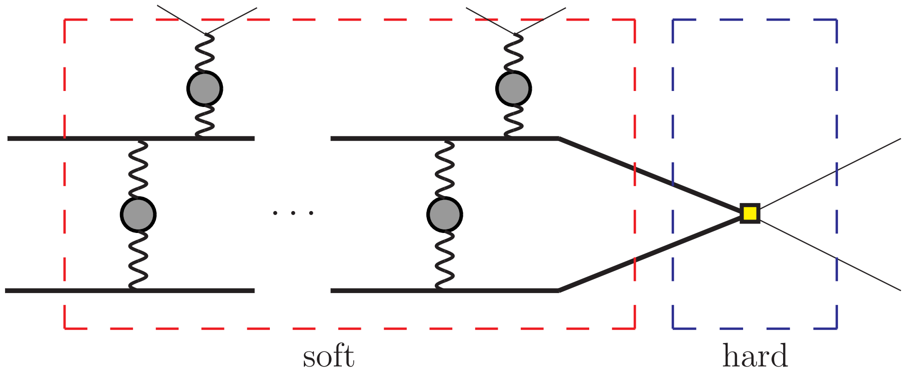
soft effects are encoded in the thermal expectation value of a 4-particle operator ("$\mathcal{M}^*\mathcal{M}$") describing the hard process[10]

[9] G.T. Bodwin *et al*, *Rigorous QCD analysis of ... annihilation ...*, hep-ph/9407339

[10] e.g. L.S. Brown and R.F. Sawyer, *Nuclear reaction rates in a plasma*, astro-ph/9610256

why soft effects are complicated

before annihilation there's time for plenty of initial-state effects:[11]



"debye screening", "landau damping", "salpeter correction", ...

in particular $2 \to 2$ scatterings, absent in vacuum computations of bound-state dissociation, do play an $\mathcal{O}(1)$ role

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[11] plot from S. Biondini and ML, *Re-derived overclosure bound...*, 1706.01894

definition of thermally averaged sommerfeld factor

if $\theta, \eta^\dagger$ annihilate q and $\bar{q}$, then the simplest operator structure is the "s-wave" or "singlet-channel" one:

$$\mathcal{O}_s = \mathcal{O}_1 = \frac{\theta^\dagger \eta \, \eta^\dagger \theta}{M^2}$$

$$\Rightarrow \quad \langle \mathcal{O}_s \rangle_T = \frac{1}{M^2} \frac{1}{\mathcal{Z}} \sum_{m \neq 0} e^{-E_m/T} \langle m | \theta^\dagger \eta \, \eta^\dagger \theta | m \rangle$$

denote by $\bar{S}_i$ enhancement factor over tree-level

$$\bar{S}_i \;\equiv\; \frac{\langle \mathcal{O}_i \rangle_T / \langle \mathcal{O}_i \rangle_{T,\,\text{tree}}}{n_{\text{eq}}^2 / (n_{\text{eq}}^2)_{\text{tree}}}$$

non-perturbative evaluation

express contractions in terms of full propagators

$$
\begin{aligned}
P_s &\equiv -\frac{\mathrm{Tr}\langle G^\theta(\frac{1}{T},\mathbf{0};0,\mathbf{0})G^\eta(0,\mathbf{0};\frac{1}{T},\mathbf{0})\rangle}{2N_\mathrm{c}}\,, \\
P_n &\equiv \frac{\mathrm{Tr}\langle G^\theta(\frac{1}{T},\mathbf{0};0,\mathbf{0})\rangle}{2N_\mathrm{c}}
\end{aligned}
$$

$$
\Rightarrow \quad \bar{S}_s = \frac{P_s}{P_n^2}
$$

this is straightforward to evaluate with lattice (NR)QCD[12]

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12 to lattice experts: real-time rate — yet no issues with analytic continuation!

perturbative evaluation

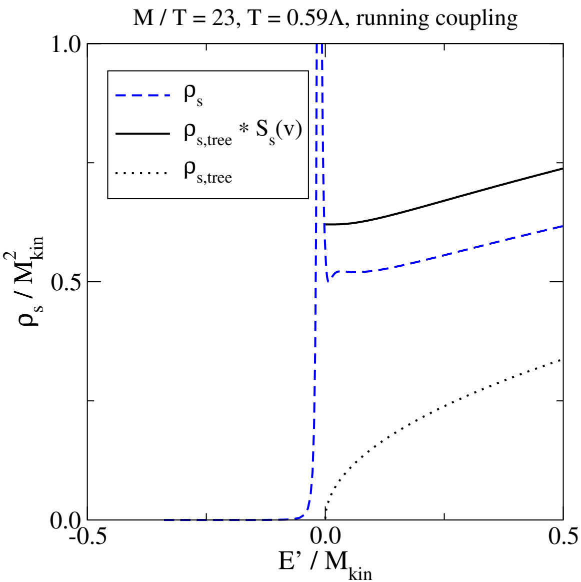
the 2-body problem can be reduced to a 1-body problem:

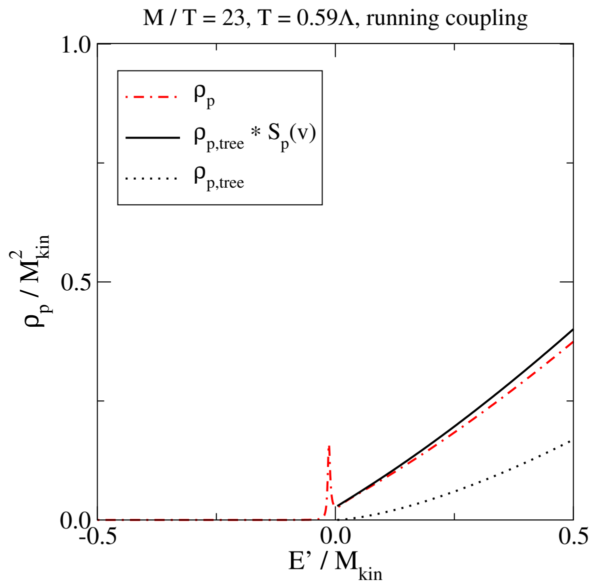
$$E_m \quad =: \quad E' + \underbrace{\left[2M_{\textsf{rest}} + \frac{k^2}{4M_{\textsf{kin}}} \right]}_{\textsf{center-of-mass energy}} \quad .$$

converting $\sum_m$ into integrals over E' and k and carrying out the integral over k we are left with

$$\langle \theta^\dagger \eta \, \eta^\dagger \theta \rangle_T = e^{-2M_{\textsf{rest}}/T} \left(\frac{M_{\textsf{kin}} T}{\pi} \right)^{3/2} \int_{-\Lambda}^{\infty} \frac{\mathrm{d}E'}{\pi} e^{-E'/T} \rho_s(E')$$

examples of perturbative evaluations of $\rho_{s,p}(E')$[13]

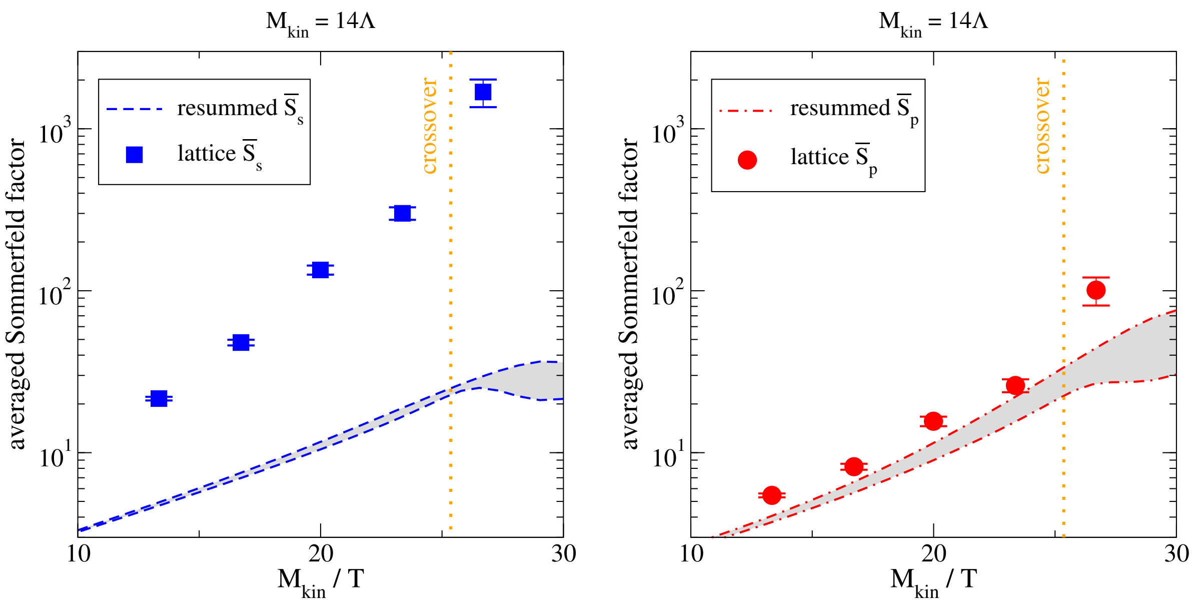




s-wave $\qquad$ p-wave

[13] S. Kim and ML, ... *thermally averaged p-wave Sommerfeld factor*, 1904.07882

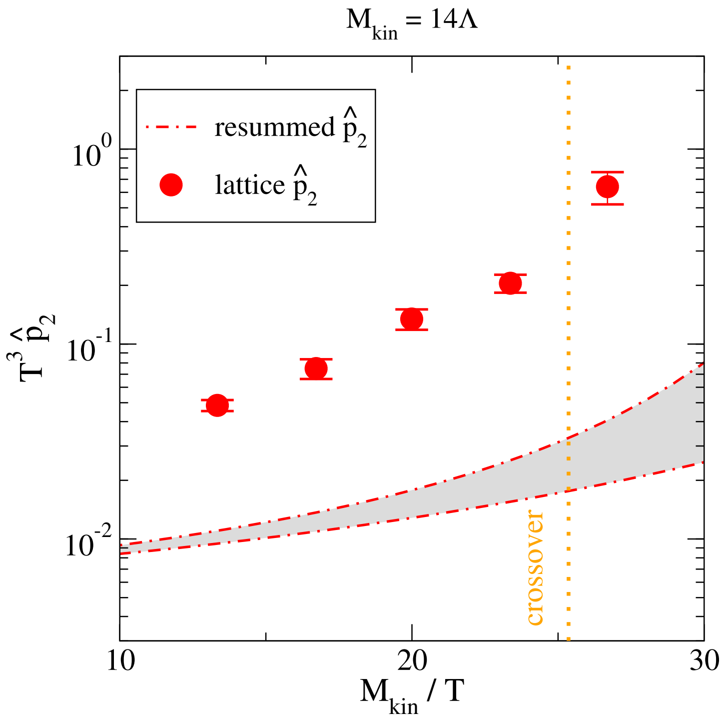
comparisons with non-perturbative data[13]



s-wave p-wave

time ⟶

"susceptibility" $\hat{p}_2$ can also be measured & compared[8]



time $\longrightarrow$

why this is of academic interest only within QCD itself

the process splits into "colour-singlet" and "colour-octet" parts

$$\Gamma_{\text{chem}} \approx \frac{g^4 C_{\rm F}}{8\pi M^2}\left(\frac{M_{\text{kin}}T}{2\pi}\right)^{3/2} e^{-M_{\text{rest}}/T} \times \left[\frac{1}{N_{\rm c}}\bar{S}_1 + \left(\frac{N_{\rm c}^2-4}{2N_{\rm c}} + N_{\rm f}\right)\bar{S}_8\right] .$$

for charm quarks: $\bar{S}_8 \simeq 0.8$ is weighted more than $\bar{S}_1 \simeq 15$

$\Rightarrow \Gamma^{-1}_{\text{chem}} \sim 150$ fm/c at $T \approx 400$ MeV,
$\Gamma^{-1}_{\text{chem}} \sim 40$ fm/c at $T \approx 600$ MeV

in practice the lifetime of the fireball is only $\sim 10 - 20$ fm/c

conclusions on heavy quark chemical equilibration

$\Rightarrow$ new ideas: cosmology to QCD

$\Rightarrow$ modern tools: QCD to cosmology

$\Rightarrow$ both lattice and perturbative methods available

$\Rightarrow$ channels with large IR exist, weighted by non-universal c_i

$\Rightarrow$ open: how fast are bound states formed ($\sim$ recombination)?