Quarkonium in medium

a NR EFT approach to dissociation and recombination

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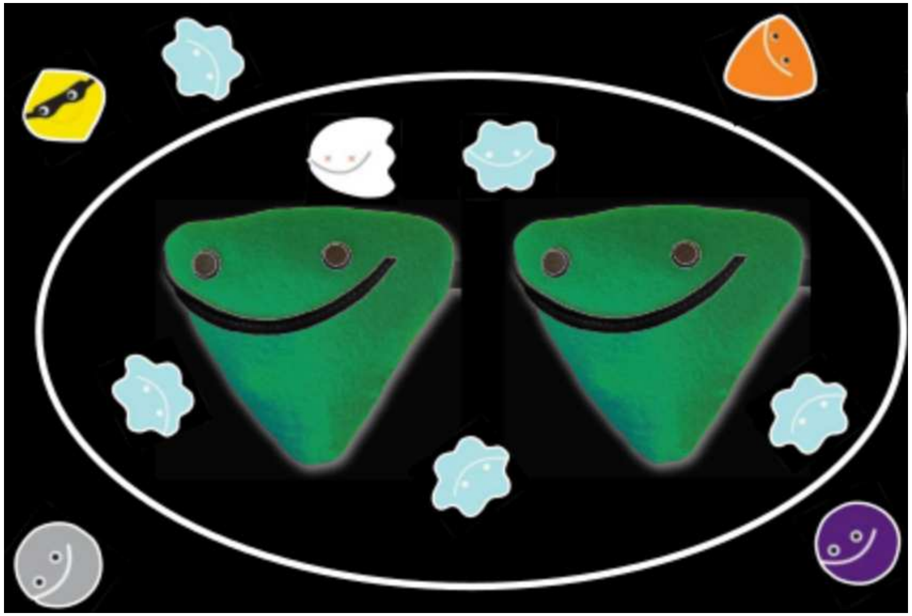
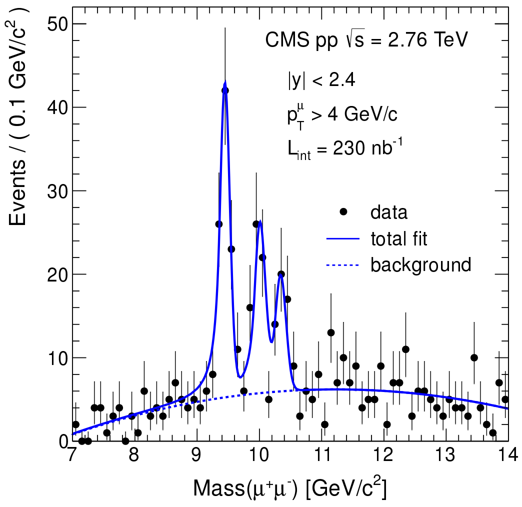
Outline



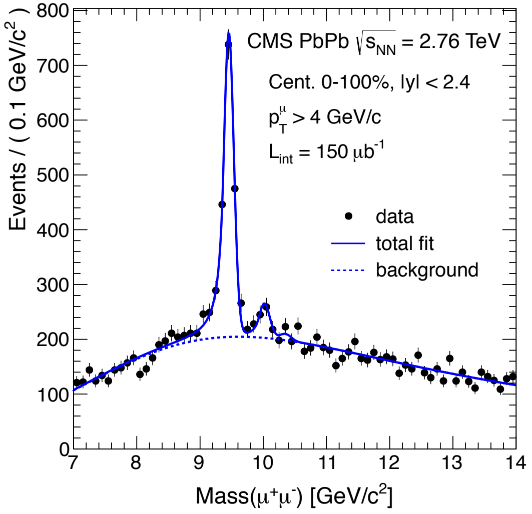
Motivation

One of the most striking signatures of quark-gluon plasma formation in heavy-ion collisions is quarkonium suppression. We will study it using EFTs of QCD to disentangle the different energy scales characterizing the system to arrive at a simple but rigorous evolution equation encompassing dissociation and recombination mechanisms.



$\Upsilon(nS)$ spectrum

@CMS



- CMS PRL 109 (2012) 222301

Non-relativistic EFTs

Quarkonium as a multiscale system

Quarkonium being a composite system is characterized by several energy scales:

- the scales of a non-relativistic bound state
  (v is the relative heavy-quark velocity; $v \sim \alpha_{\rm s}$ for a Coulombic bound state):
  M (mass),
  Mv (momentum transfer, inverse distance),
  Mv^2 (kinetic energy, binding energy, potential V), ...

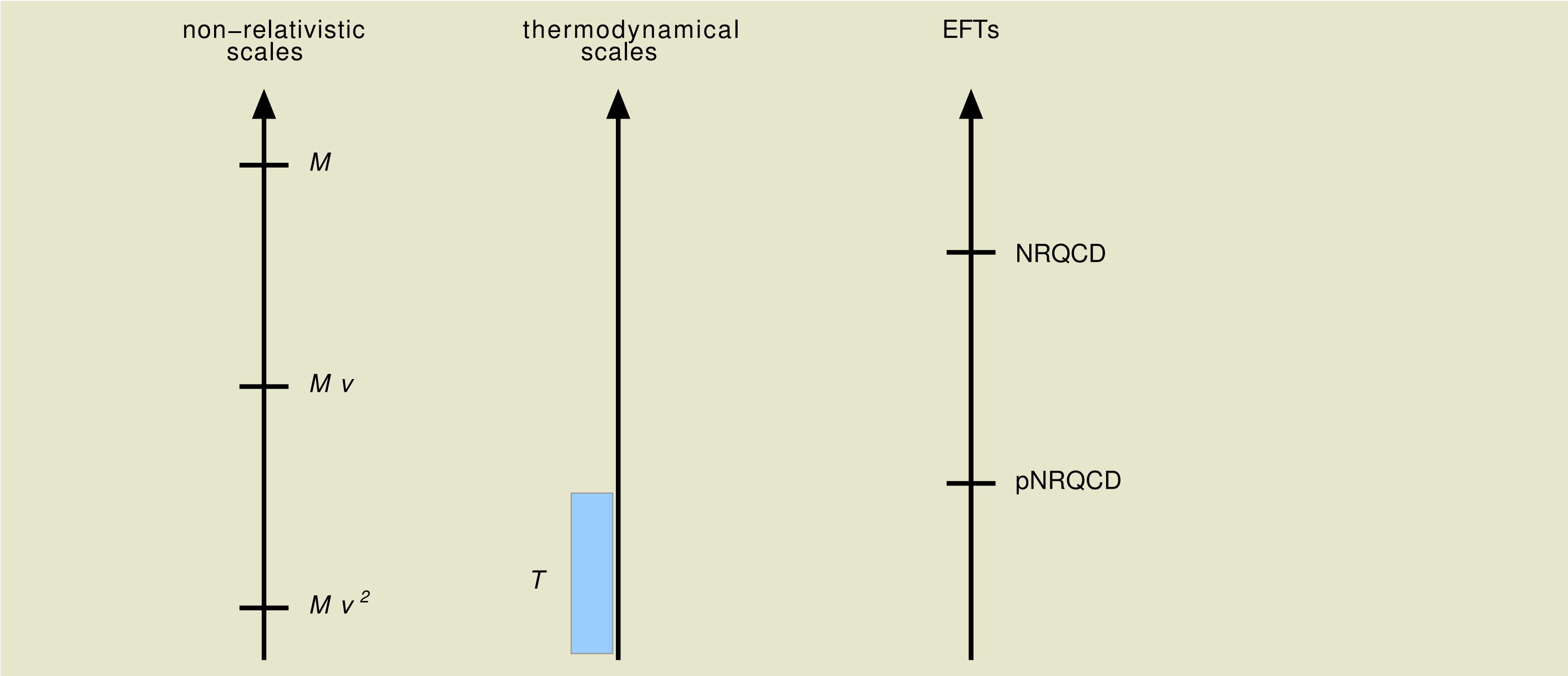
- the thermodynamical scales:
  T (temperature), ...

  T stands for a generic inverse correlation length characterizing the medium.
  For definiteness we will assume that the system is locally in thermal equilibrium so that a slowly varying time-dependent temperature can be defined.

The non-relativistic scales are hierarchically ordered: $M \gg Mv \gg Mv^2$

Non-relativistic EFTs of QCD

The existence of a hierarchy of energy scales calls for a description of the system in terms of a hierarchy of EFTs. We assume T $(\sim 400$ MeV$)$ $< Mv$ $(\sim 1.5$ GeV, for $\Upsilon)$.



- Brambilla Pineda Soto Vairo RMP 77 (2005) 1423
  Brambilla Ghiglieri Petreczky Vairo PRD 78 (2008) 014017

pNRQCD

$$\mathcal{L} = -\frac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i + \int d^3r \mathrm{Tr}\left\{\mathrm{S}^\dagger\left(i\partial_0 - h_s\right)\mathrm{S} + \mathrm{O}^\dagger\left(iD_0 - h_o\right)\mathrm{O}\right\}$$

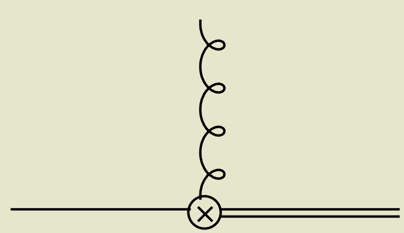
- LO in r

$\theta(T)\, e^{-iTh_s}$

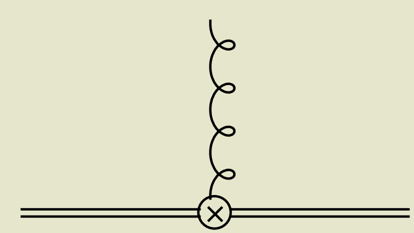
$\theta(T)\, e^{-iTh_o}\left(e^{-i\int dt\, A^{\mathrm{adj}}}\right)$

$$+V_A \mathrm{Tr}\left\{\mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{S} + \mathrm{S}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{O}\right\} + \frac{V_B}{2}\mathrm{Tr}\left\{\mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{O} + \mathrm{O}^\dagger\mathrm{O}\mathbf{r}\cdot g\mathbf{E}\right\}$$

- NLO in r



$O^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{S}$



$O^\dagger\{\mathbf{r}\cdot g\mathbf{E},\, \mathrm{O}\}$

$$+\cdots$$

pNRQCD: fields and potentials

Fields:

- $S^\dagger$ creates a quark-antiquark pair in a color singlet configuration.
- $O^\dagger$ creates a (unbound) quark-antiquark pair in a color octet configuration.

Potentials:

- quark-antiquark color singlet Hamiltonian $= h_s = \dfrac{\mathbf{p}^2}{M} - \dfrac{4}{3}\dfrac{\alpha_{\rm s}}{r} + \dots$
- quark-antiquark color octet Hamiltonian $= h_o = \dfrac{\mathbf{p}^2}{M} + \dfrac{\alpha_{\rm s}}{6r} + \dots$
- $V_A = V_B = 1$ up to higher corrections in $\alpha_{\rm s}$.

Annihilation

The pNRQCD Hamiltonian contains at order $1/M^2$ contact terms with imaginary coefficients (Im f):

$$h_s \supset \frac{\text{Im}\, f}{M^2}\delta^{(3)}(\boldsymbol{r})$$

They describe the heavy particle pair annihilation width in the effective field theory:

$$2\,\frac{\text{Im}\, f}{M^2}|\phi(\mathbf{0})|^2$$

where ϕ is the heavy particle pair wave function.

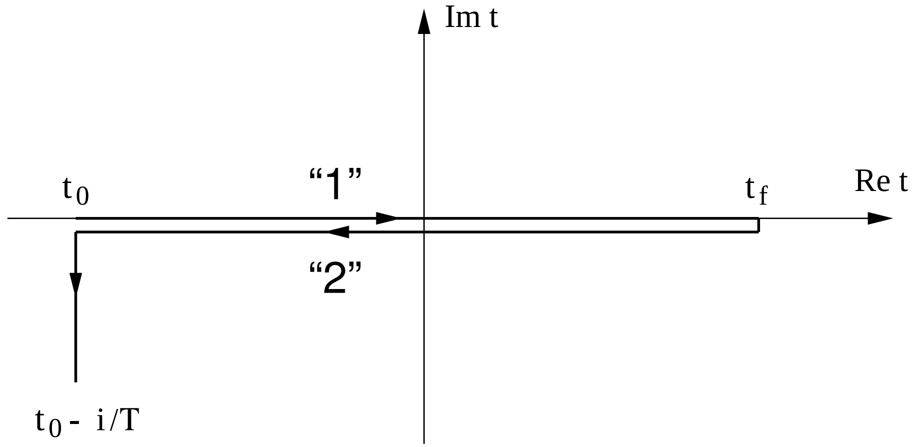
- If ϕ is an S-wave bound state then $|\phi(\mathbf{0})|^2 = 1/(\pi a_0^3)$; $a_0 =$ Bohr radius.
- If ϕ is an S-wave scattering state then $|\phi(\mathbf{0})|^2 = 2\pi\zeta/(1 - e^{-2\pi\zeta})$; $\zeta =$ Coulomb coupling$/v_{\text{rel}}$. $|\phi(\mathbf{0})|^2$ is called Sommerfeld factor.

In heavy ion collisions, quarkonium annihilation (whose width is tiny) happens after the disappearance of the quark gluon plasma, hence annihilation is commonly not accounted for in the evolution equations.

Quarkonium dissociation in a medium

Real-time formalism

Temperature enters the partition function.
Dynamical properties are better described in the real-time formalism.



In real time, the degrees of freedom double (“1” and “2”), however, the advantages are

- the framework becomes very close to the one for $T = 0$ EFTs;
- in the heavy-particle sector, the second degrees of freedom, labeled “2”, decouple from the physical degrees of freedom, labeled “1”.

This usually leads to a simpler treatment with respect to alternative calculations in imaginary time formalism.

Real-time heavy-particle propagator

- The free heavy-particle propagator is proportional to

$$\mathbf{S}^{(0)}(p) = \begin{pmatrix} \dfrac{i}{p^0 + i\epsilon} & 0 \\ 2\pi\delta(p^0) & \dfrac{-i}{p^0 - i\epsilon} \end{pmatrix}$$

Since $[\mathbf{S}^{(0)}(p)]_{12} = 0$, the static quark fields labeled “2” never enter in any physical amplitude, i.e. any amplitude that has the physical fields, labeled “1”, as initial and final states.

These properties hold also for interacting heavy particle(s): interactions do not change the nature (“1” or “2”) of the interacting fields.

◦ Brambilla Ghiglieri Vairo Petreczky PRD 78 (2008) 014017

Dissociation mechanisms at LO

A key quantity for describing the observed quarkonium dilepton signal suppression is the quarkonium thermal dissociation width.

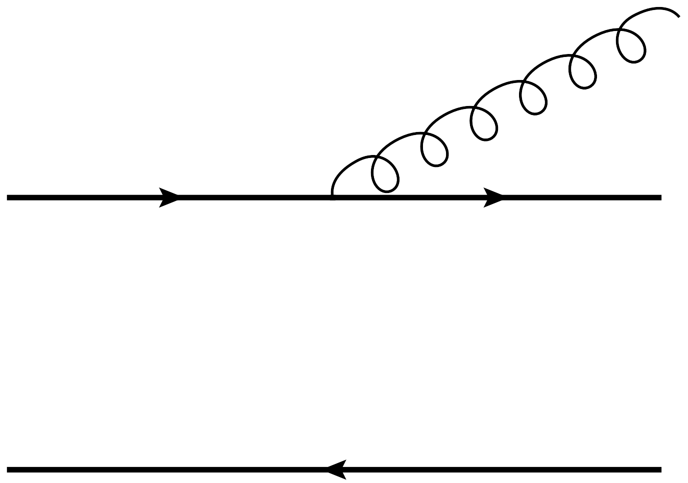
Two distinct dissociation mechanisms may be identified at leading order:

- gluodissociation
  (this is the dominant mechanism for $Mv^2 \gg m_D \sim gT$)
- dissociation by inelastic parton scattering
  (this is the dominant mechanism for $Mv^2 \ll m_D \sim gT$)

Beyond leading order the two mechanisms are intertwined and distinguishing between them becomes unphysical, whereas the physical quantity is the total decay width.

Gluodissociation

Gluodissociation is the dissociation of quarkonium by absorption of a gluon from the medium.

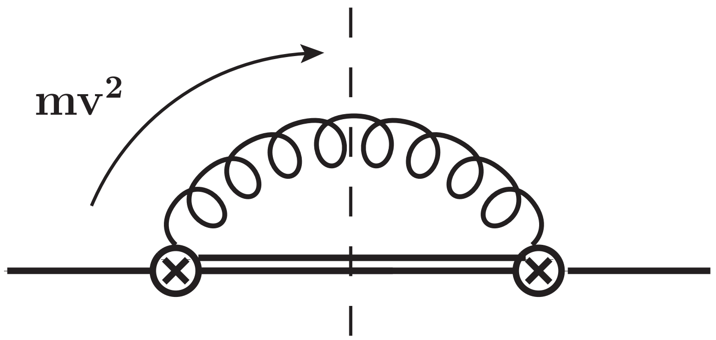


- The exchanged gluon is lightlike or timelike.
- The process happens when the gluon has an energy of order Mv^2.

○ `Kharzeev Satz PLB 334 (1994) 155`
`Xu Kharzeev Satz Wang PRC 53 (1996) 3051`

Gluodissociation

From the optical theorem, the gluodissociation width follows from cutting the gluon propagator in the following pNRQCD diagram



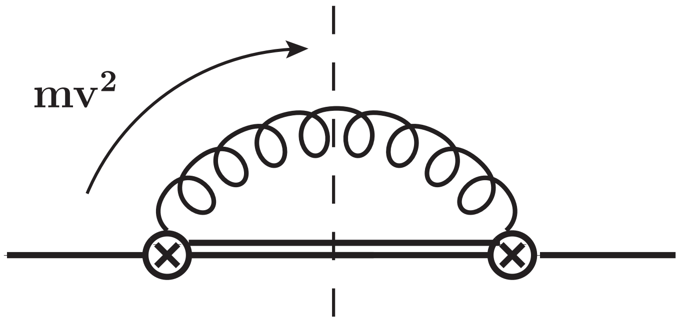
For a quarkonium at rest with respect to the medium, the width has the form

$$\Gamma_{nl} = \int_{q_{\min}} \frac{d^3q}{(2\pi)^3}\, n_{\rm B}(q)\, \sigma^{nl}_{\rm gluo}(q)\,.$$

- $\sigma^{nl}_{\rm gluo}$ is the in-vacuum cross section $(Q\overline{Q})_{nl} + g \to Q + \overline{Q}$.
- Gluodissociation is also known as singlet-to-octet break up.

○ Brambilla Ghiglieri Vairo Petreczky PRD 78 (2008) 014017

$1S$ gluodissociation at LO



The LO gluodissociation cross section for $1S$ Coulombic states is

$$\sigma^{1S}_{\rm gluo\,LO}(q) = \frac{\alpha_{\rm s} C_F}{3} 2^{10}\pi^2 \rho(\rho+2)^2 \frac{E_1^4}{M q^5} \left(t(q)^2+\rho^2\right) \frac{\exp\left(\frac{4\rho}{t(q)} \arctan\left(t(q)\right)\right)}{e^{\frac{2\pi\rho}{t(q)}}-1}$$

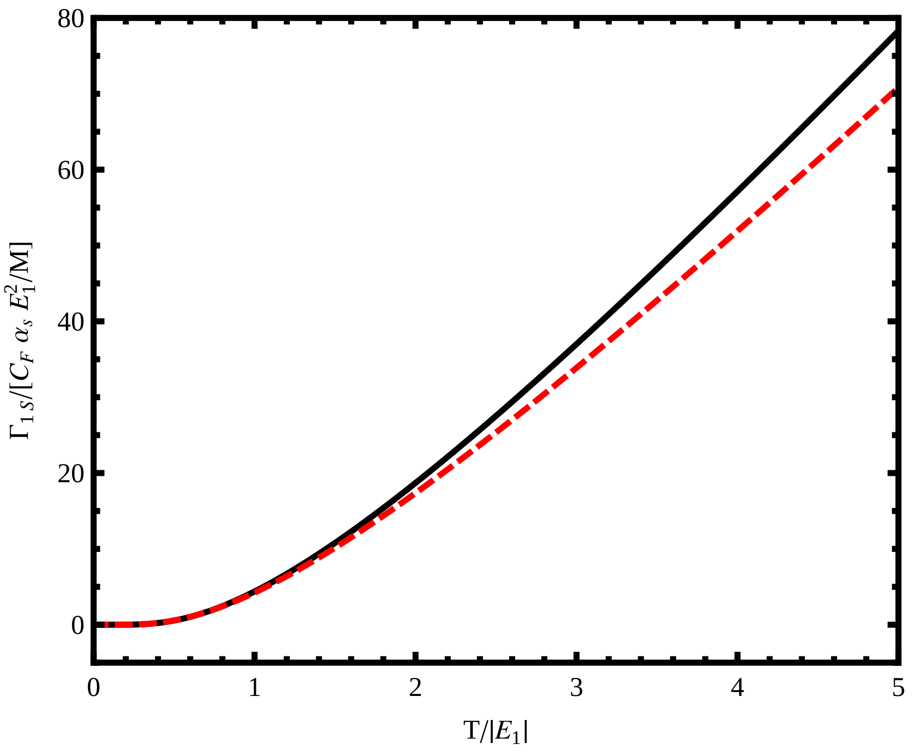
where $\rho \equiv 1/(N_c^2-1)$, $t(q) \equiv \sqrt{q/|E_1|-1}$ and $E_1 = -MC_F^2\alpha_{\rm s}^2/4$.

- Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116
  Brezinski Wolschin PLB 707 (2012) 534

The Bhanot–Peskin approximation corresponds to the large N_c limit, i.e. to neglecting final state interactions (the rescattering of a $Q\overline{Q}$ pair in a color octet configuration).

- Peskin NPB 156 (1979) 365, Bhanot Peskin NPB 156 (1979) 391

Gluodissociation width vs Bhanot–Peskin width



- Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116

Ionization and bound state formation cross sections

The gluodissociation cross section is analogous to the ionization cross section entering in many models of dark matter evolution. Ionization, i.e. the break up of a dark matter bound state into an unbound pair, happens only in the medium provided by the evolving universe.

The cross section for the inverse process, bound state formation, is related to the ionization cross section through the Milne relation

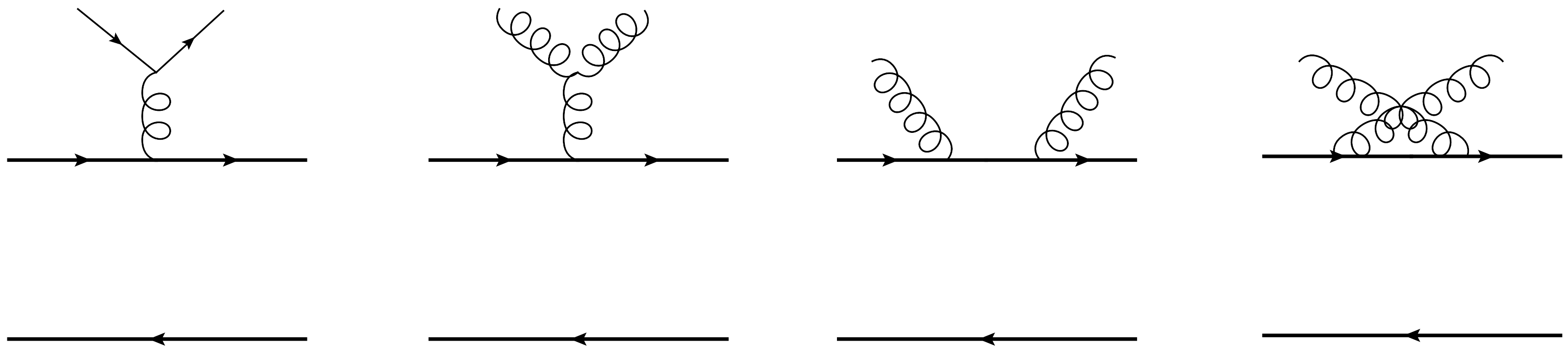
$$\frac{\sigma_{\text{ion}}}{\sigma_{\text{BSF}}} = \frac{M^2 v_{\text{rel}}^2}{8q^2}$$

where v_{rel} is the relative velocity of the unbound pair.

- E.g. in Harling Petraki JCAP 12 (2014) 033
  Biondini Brambilla Gramos Vairo, in preparation.

Dissociation by inelastic parton scattering

Dissociation by inelastic parton scattering is the dissociation of quarkonium by scattering with gluons and light-quarks in the medium.



○ `Grandchamp Rapp PLB 523 (2001) 60, NPA 709 (2002) 415`

- The exchanged gluon is spacelike.
- External thermal gluons are transverse.
- In the NRQCD power counting, each external transverse gluon is suppressed by T/M.

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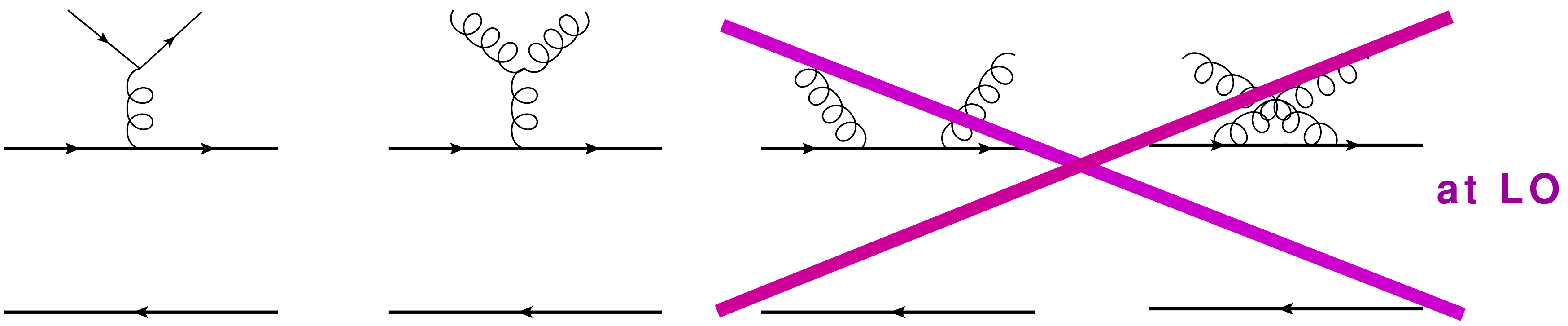
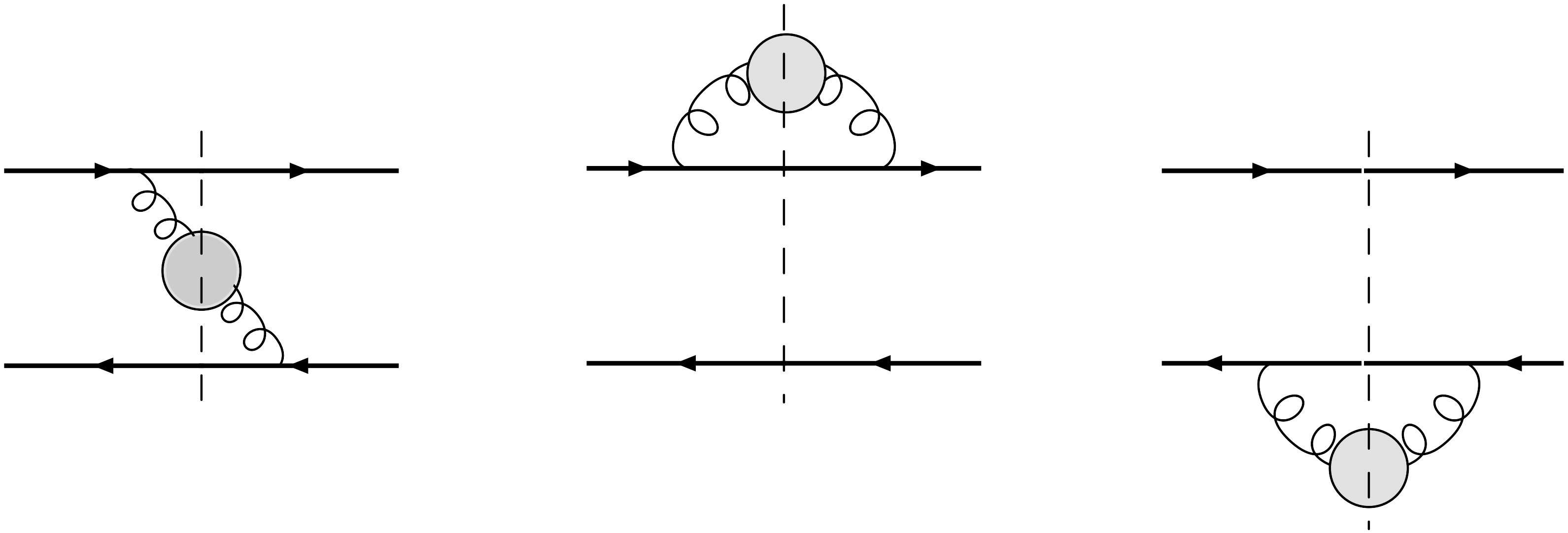


○ Grandchamp Rapp PLB 523 (2001) 60, NPA 709 (2002) 415

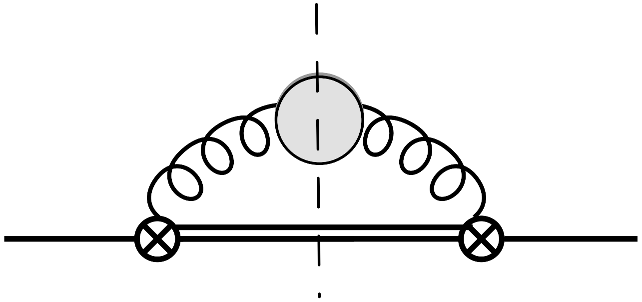
- The exchanged gluon is spacelike.
- External thermal gluons are transverse.
- In the NRQCD power counting, each external transverse gluon is suppressed by T/M.

Dissociation by inelastic parton scattering

From the optical theorem, the thermal width follows from cutting the gluon self-energy in the following NRQCD diagrams (momentum of the gluon $\gtrsim Mv$)



and/or pNRQCD diagram (momentum of the gluon $\ll Mv$)



- Dissociation by inelastic parton scattering is also known as Landau damping.

Dissociation by inelastic parton scattering

For a quarkonium at rest with respect to the medium, the thermal width has the form

$$\Gamma_{nl} = \sum_p \int_{q_{\min}} \frac{d^3q}{(2\pi)^3}\, f_p(q)\, [1 \pm f_p(q)]\, \sigma_p^{nl}(q)$$

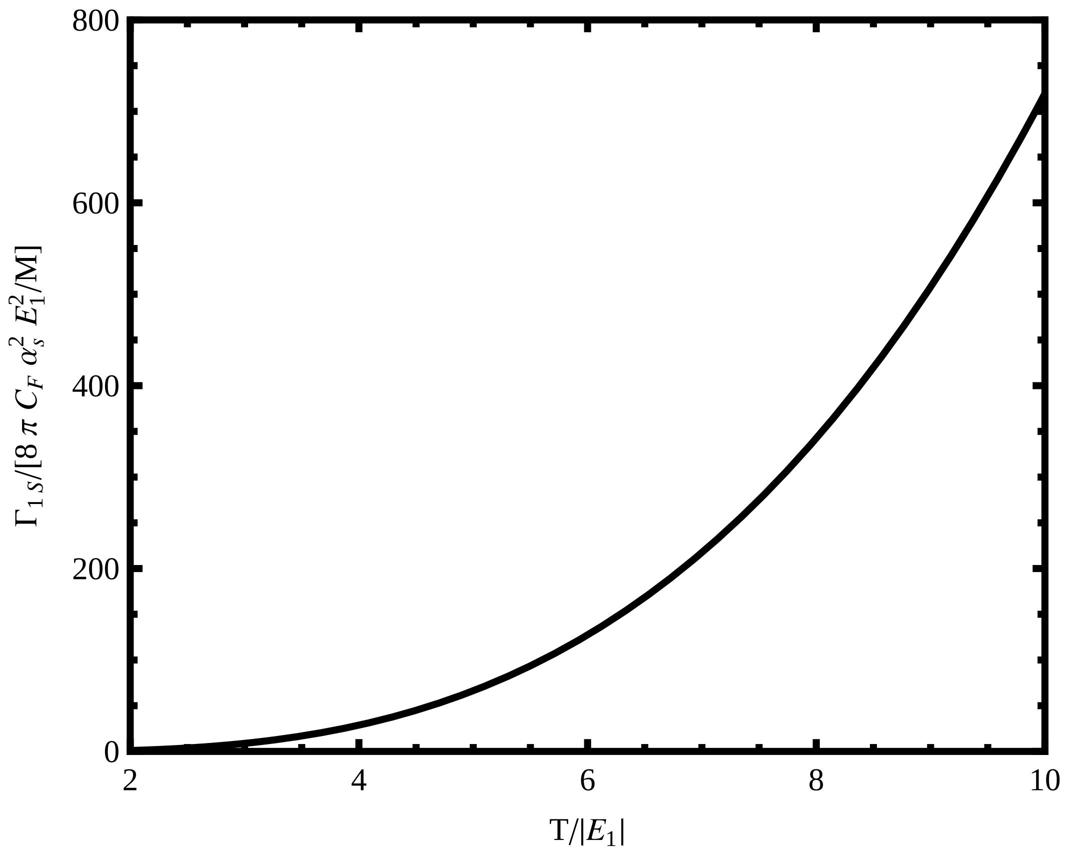
where the sum runs over the different incoming light partons and $f_g = n_{\mathrm{B}}$ or $f_q = n_{\mathrm{F}}$.

◦ Brambilla Escobedo Ghiglieri Vairo JHEP 05 (2013) 130

- σ_p^{nl} is the in-medium cross section $(Q\overline{Q})_{nl} + p \to Q + \overline{Q} + p$.
- The convolution formula correctly accounts for Pauli blocking in the fermionic case (minus sign).
- The formula differs from the gluodissociation formula.
- The formula differs from the one used for long in the literature, which has been inspired by the gluodissociation formula.

  ◦ Grandchamp Rapp PLB 523 (2001)
  Park Kim Song Lee Wong PRC 76 (2007) 044907, ...

Dissociation width



$m_D a_0 = 0.5$

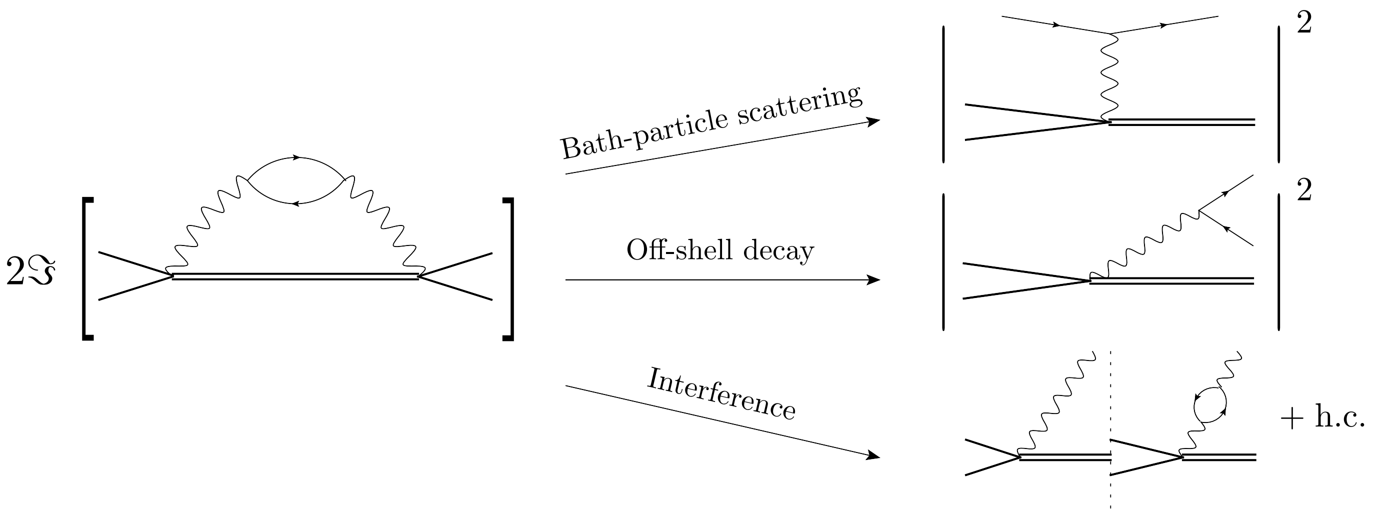
$|E_1|/m_D = 0.5$

$n_f = 3$

- Brambilla Escobedo Ghiglieri Vairo JHEP 05 (2013) 130
  Vairo EPJ Web Conf 71 (2014) 00135

Bath-particle scattering

The dissociation by inelastic parton scattering cross section is analogous to the bath-particle scattering entering dark matter scenarios with bound state formation.



- Biondini Laine JHEP 04 (2018) 072
  Binder Covi Mukaida PRD 98 (2018) 115023
  Biondini Vogl JHEP 02 (2019) 016, 11 (2019) 147
  Binder Mukaida Petraki PRL 124 (2020) 161102
  Binder Blobel Harz Mukaida JHEP 09 (2020) 086

Quarkonium out of equilibrium

Density matrix

An arbitrary statistical ensemble of quantum states can be represented by a density matrix ρ, which is

- Hermitian: $\rho^\dagger = \rho$;
- positive: $\langle\psi|\rho|\psi\rangle \geq 0$ for all nonzero states $|\psi\rangle$;
- and can be normalized to have unit trace: $\mathrm{Tr}\{\rho\} = 1$.

The time evolution of the density matrix is described by the von Neumann equation:

$$i\frac{d\rho}{dt} = [H, \rho]$$

which follows from the Schrödinger equation for $|\psi\rangle$. The evolution equation

- is linear in ρ;
- preserves the trace of ρ;
- is Markovian.

Open quantum system

Of physical relevance is the case when the full system separates into a subsystem of interest and an environment. A density matrix ρ of the subsystem can be obtained from the density matrix ρ_{full} of the full system by the partial trace over the environment states:

$$\rho = \mathrm{Tr}_{\mathrm{environment}} \left\{ \rho_{\mathrm{full}} \right\}$$

In general the evolution of ρ is non-Markovian.

The evolution is Markovian if the time during which the subsystem is observed is much larger than the time scale for correlations between the subsystem and the environment.

Lindblad equation

The density matrix ρ for the subsystem necessarily satisfies the three basic properties: it is Hermitian, positive, and it can be normalized.

If further the time evolution is linear in ρ, preserves the trace of ρ, is Markovian and the linear operator that determines the time evolution of ρ is completely positive
$\Rightarrow$ then this requires the time evolution equation to have the Lindblad form

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\})$$

where H is a Hermitian operator and the C_n's are an additional set of operators called collapse operators.

- Lindblad CMP 48 (1976) 119
  Gorini Kossakowski Sudarshan JMP 17 (1976) 821

Quarkonium in a fireball

- After the heavy-ion collisions, heavy quark-antiquarks propagate freely up to 0.6 fm.
- From 0.6 fm to the freeze-out time t_F they propagate in the medium.
- We assume the medium infinite, homogeneous and isotropic.
- We assume the heavy quarks comoving with the medium.
- We assume the medium to be locally in thermal equilibrium.
- The hydrodynamics can be realistically simulated.
  - Alqahtani Nopoush Strickland PPNP 101 (2018) 204

Density matrices

- Subsystem: heavy quarks/quarkonium
- Environment: quark gluon plasma

We may define a density matrix in pNRQCD for the heavy quark-antiquark pair in a singlet and octet configuration:

$$\begin{aligned}
\langle \mathbf{r}', \mathbf{R}' | \rho_s(t';t) | \mathbf{r}, \mathbf{R} \rangle &\equiv \mathrm{Tr}\{\rho_{\rm full}(t_0)\, S^\dagger(t,\mathbf{r},\mathbf{R}) S(t',\mathbf{r}',\mathbf{R}')\} \\
\langle \mathbf{r}', \mathbf{R}' | \rho_o(t';t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{8} &\equiv \mathrm{Tr}\{\rho_{\rm full}(t_0)\, O^{a\,\dagger}(t,\mathbf{r},\mathbf{R}) O^b(t',\mathbf{r}',\mathbf{R}')\}
\end{aligned}$$

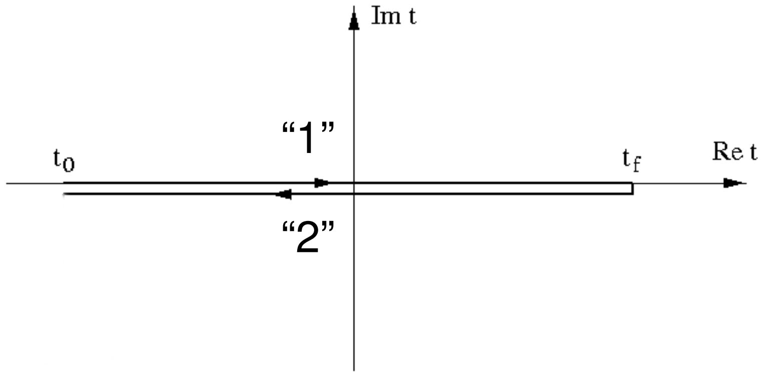
$t_0 \approx 0.6$ fm is the time formation of the plasma.

The system is in non-equilibrium because through interaction with the environment (quark gluon plasma) singlet and octet quark-antiquark states continuously transform in each other although the number of heavy quarks is conserved: $\mathrm{Tr}\{\rho_s\} + \mathrm{Tr}\{\rho_o\} = 1$.

Closed-time path formalism

In the closed-time path formalism we can represent the density matrices as 12 propagators on a closed time path:

$$\begin{aligned}
\langle \mathbf{r}', \mathbf{R}' | \rho_s(t'; t) | \mathbf{r}, \mathbf{R} \rangle &= \langle S_1(t', \mathbf{r}', \mathbf{R}') S_2^\dagger(t, \mathbf{r}, \mathbf{R}) \rangle \\
\langle \mathbf{r}', \mathbf{R}' | \rho_o(t'; t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{8} &= \langle O_1^b(t', \mathbf{r}', \mathbf{R}') O_2^{a\dagger}(t, \mathbf{r}, \mathbf{R}) \rangle
\end{aligned}$$



Differently from the thermal equilibrium case 12 propagators are relevant
(in thermal equilibrium they are exponentially suppressed).

12 propagators are not time ordered, while 11 and 22 operators select the forward time direction $\propto \theta(t - t')$, $\theta(t' - t)$.

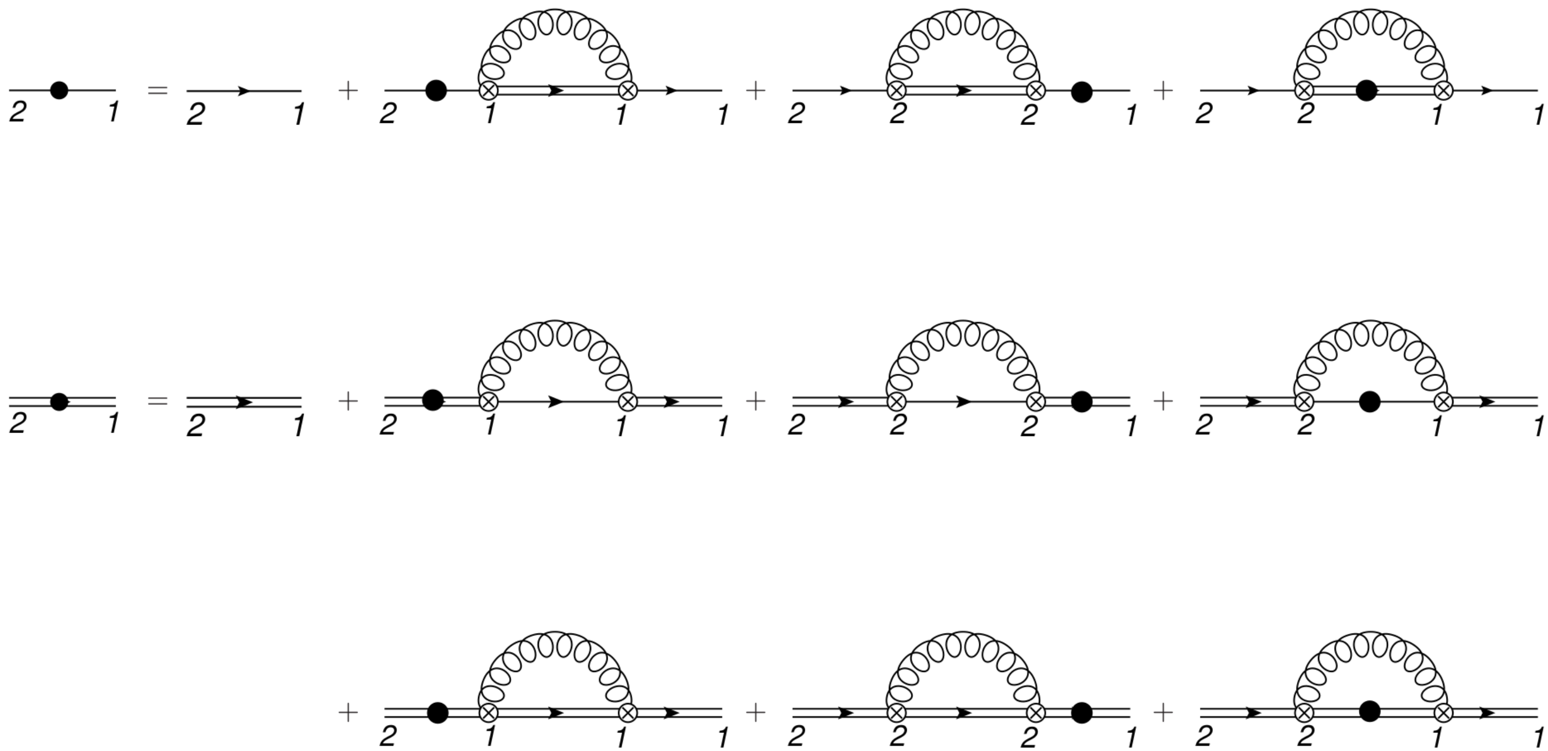
Expansions

- The density of heavy quarks is much smaller than the one of light quarks: we expand at first order in the heavy quark-antiquark density.
- We consider T much smaller than the inverse Bohr radius of the quarkonium: we expand up to order r^2 in the multipole expansion.

The evolution depends on the density at initial time: non Markovian evolution.

Resummation

Resumming $(t - t_0)\times$ self-energy contributions ...



(Resummation is accurate at order r^2 and consistent with unitary evolution at LO.)

Evolution equations I

... and differentiating over time we obtain the coupled evolution equations:

$$\frac{d\rho_s(t;t)}{dt} = -i[h_s, \rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) - \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_{so}(\rho_o(t;t), t)$$

$$\frac{d\rho_o(t;t)}{dt} = -i[h_o, \rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{os}(\rho_s(t;t), t) + \Xi_{oo}(\rho_o(t;t), t)$$

- The evolution equations are now valid for large time.
- The evolution equations are Markovian.

○ Brambilla Escobedo Soto Vairo PRD 96 (2017) 034021
Brambilla Escobedo Soto Vairo PRD 97 (2018) 074009

Interpretation

- The self energies Σ_s and Σ_o provide the in-medium induced mass shifts, $\delta m_{s,o}$, and widths, $\Gamma_{s,o}$, for the color-singlet and color-octet heavy quark-antiquark systems respectively:

$$
\begin{aligned}
-i\Sigma_{s,o}(t) + i\Sigma^{\dagger}_{s,o}(t) &= 2\,\mathrm{Re}\,(-i\Sigma_{s,o}(t)) = 2\delta m_{s,o}(t) \\
\Sigma_{s,o}(t) + \Sigma^{\dagger}_{s,o}(t) &= -2\,\mathrm{Im}\,(-i\Sigma_{s,o}(t)) = \Gamma_{s,o}(t)
\end{aligned}
$$

- Ξ_{so} accounts for the production of singlets through the decay of octets, and Ξ_{os} and Ξ_{oo} account for the production of octets through the decays of singlets and octets respectively. There are two octet production mechanisms/octet chromoelectric dipole vertices in the pNRQCD Lagrangian.
- The conservation of the trace of the sum of the densities, i.e., the conservation of the number of heavy quarks, follows from

$$
\begin{aligned}
\mathrm{Tr}\left\{\rho_s(t;t)\left(\Sigma_s(t) + \Sigma^{\dagger}_s(t)\right)\right\} &= \mathrm{Tr}\left\{\Xi_{os}(\rho_s(t;t),t)\right\} \\
\mathrm{Tr}\left\{\rho_o(t;t)\left(\Sigma_o(t) + \Sigma^{\dagger}_o(t)\right)\right\} &= \mathrm{Tr}\left\{\Xi_{so}(\rho_o(t;t),t) + \Xi_{oo}(\rho_o(t;t),t)\right\}
\end{aligned}
$$

Evolution equations II

An alternative way of writing the evolution equations is

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_{nm} h_{nm} \left(L_i^n \rho L_i^{m\,\dagger} - \frac{1}{2}\{L_i^{m\,\dagger} L_i^n, \rho\} \right)$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix} \qquad H = \begin{pmatrix} h_s + \frac{\Sigma_s - \Sigma_s^\dagger}{2i} & 0 \\ 0 & h_o + \frac{\Sigma_o - \Sigma_o^\dagger}{2i} \end{pmatrix}$$

$$\Sigma_s(t) = r^i\, A_i^{so\,\dagger}(t) \qquad \Sigma_o(t) = \frac{r^i\, A_i^{os\,\dagger}(t)}{8} + \frac{5}{16} r^i\, A_i^{oo\,\dagger}(t)$$

$$L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r^i \qquad L_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{5}{16} A_i^{oo\,\dagger} \end{pmatrix}$$

$$L_i^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} r^i \qquad L_i^3 = \begin{pmatrix} 0 & \frac{1}{8} A_i^{os\,\dagger} \\ A_i^{so\,\dagger} & 0 \end{pmatrix}$$

with $\quad A_i^{so}(t) = \frac{g^2}{6} \int_{t_0}^{t} dt_2\, e^{ih_s(t_2 - t)}\, r^j\, e^{ih_o(t-t_2)}\, \langle E^{a,j}(t_2, \mathbf{0}) E^{a,i}(t, \mathbf{0}) \rangle$

Positivity

The matrix h_{nm} is

$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} .$$

If h were a positive definite matrix then it would always be possible to redefine the operators L_i^n in such a way that the evolution equation would be of the Lindblad form.

Since, however, h is not a positive definite matrix, the Lindblad theorem does not guarantee that the equations may be brought into a Lindblad form.

Time scales

Environment correlation time: $\tau_E \sim \dfrac{1}{T}$

System intrinsic time scale: $\tau_S \sim \dfrac{1}{E}$

System relaxation time: $\tau_R \sim \dfrac{1}{\text{self-energy}} \sim \dfrac{1}{\alpha_{\rm s} a_0^2 \Lambda^3}$ $\quad a_0 =$ Bohr radius, $\Lambda = T, E$

- Because we have assumed $1/a_0 \gg \Lambda$, it follows $\tau_R \gg \tau_S, \tau_E$ which, after resummation, qualifies the system as Markovian.

- If $T \gg E$ then $\tau_S \gg \tau_E$ which qualifies the motion of the system as quantum Brownian.

○ Akamatsu PRD 91 (2015) 056002

From the evolution equations to the Lindblad equation

Under the Markovian

$$\tau_R \gg \tau_S, \tau_E \quad \text{or} \quad \frac{1}{a_0} \gg E, T$$

and quantum Brownian motion condition

$$\tau_S \gg \tau_E \quad \text{or} \quad T \gg E$$

at least at LO in E/T the evolution equations can be written in the Lindblad form.

Heavy quark-antiquarks in a strongly coupled medium: $T \gg E$

If $E \ll T$ the Lindblad equation for a strongly coupled plasma reads

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\})$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa(t)}{8}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{8} & 0 \end{pmatrix}, \qquad C_i^1 = \sqrt{\frac{5\kappa(t)}{16}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

- Brambilla Escobedo Soto Vairo PRD 96 (2017) 034021
  Brambilla Escobedo Soto Vairo PRD 97 (2018) 074009

Thermal width and mass shift

The quantity κ is related to the thermal decay width of the heavy quarkonium.
In particular for $1S$ states, we have

$$\Gamma(1S) = -2\langle \mathrm{Im}\,(-i\Sigma_s)\rangle = 3a_0^2\,\kappa$$

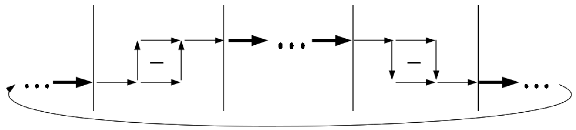
The quantity γ is related to the thermal mass shift of the heavy quarkonium.
In particular for $1S$ states, we have

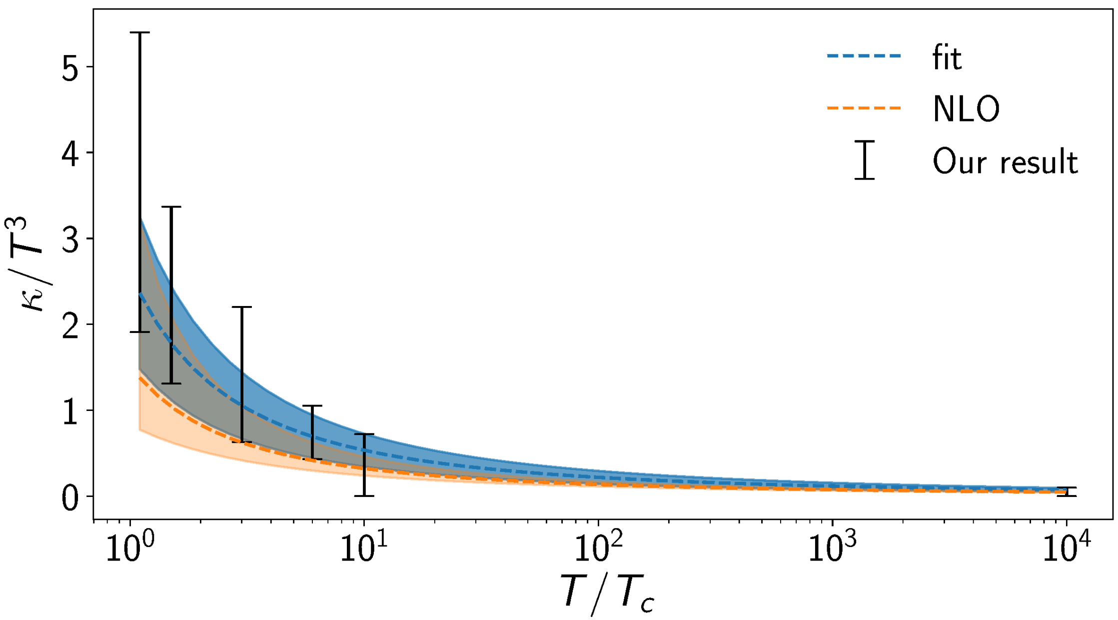
$$\delta M(1S) = \langle \mathrm{Re}\,(-i\Sigma_s)\rangle = \frac{3}{2}a_0^2\,\gamma$$

κ

Low energy parameters may be determined by numerical calculations in lattice QCD.
κ is the heavy-quark momentum diffusion coefficient:

$$\kappa = \frac{g^2}{18}\,\mathrm{Re}\int_{-\infty}^{+\infty} ds\,\langle\,\mathrm{T}\,E^{a,i}(s,\mathbf{0})\,\phi^{ab}(s,0)\,E^{b,i}(0,\mathbf{0})\rangle =$$

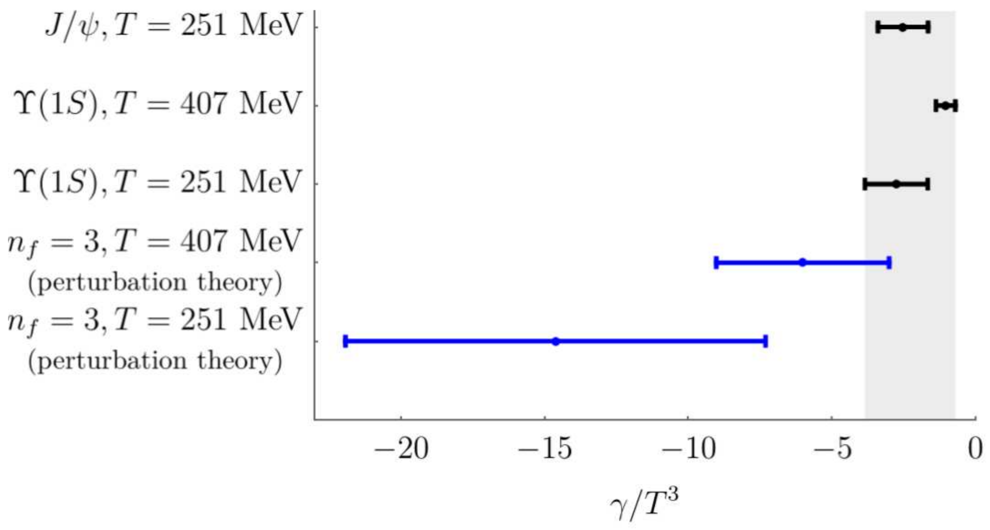




○ Brambilla Leino Petreczky Vairo PRD 102 (2020) 074503

γ

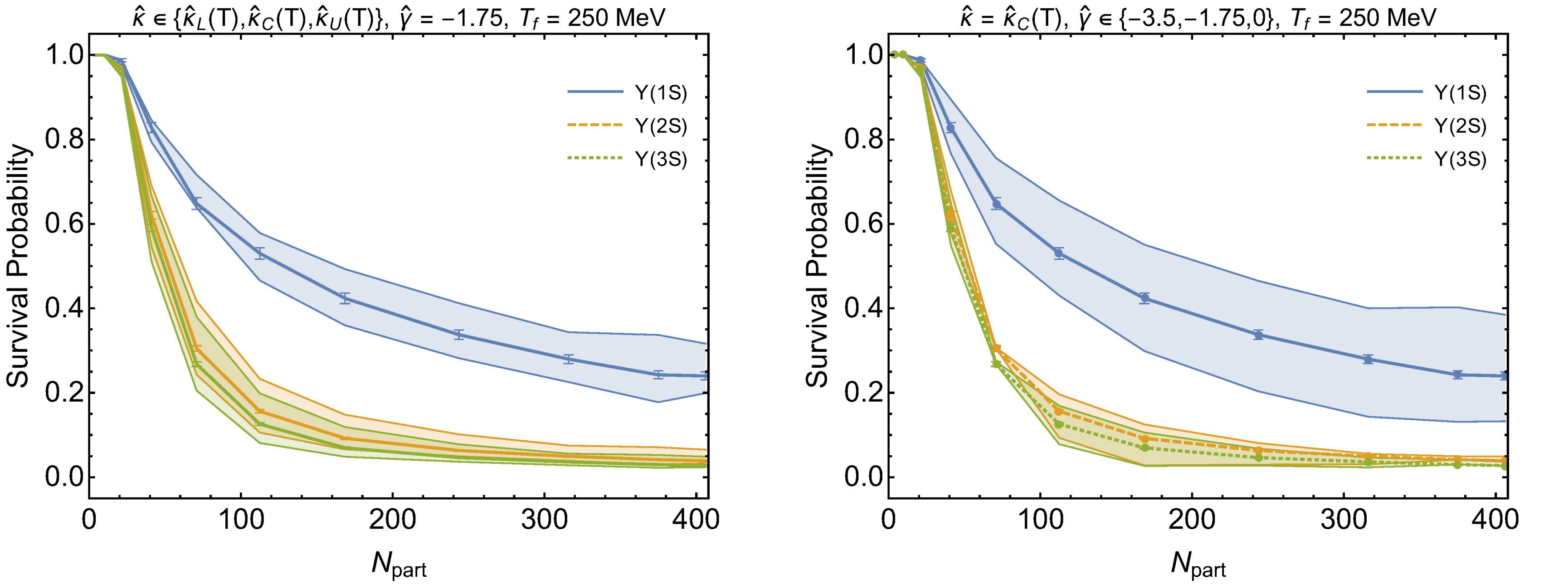
$$\gamma = \frac{g^2}{18}\,\mathrm{Im}\int_{-\infty}^{+\infty} ds\,\langle\,\mathrm{T}\,E^{a,i}(s,\mathbf{0})\,\phi^{ab}(s,0)\,E^{b,i}(0,\mathbf{0})\rangle$$



- Brambilla Escobedo Vairo Vander Griend PRD 100 (2019) 054025
  from the lattice data of Kim Petreczky Rothkopf JHEP 11 (2018) 088

  for an Euclidean version Eller Ghiglieri Moore PRD 99 (2019) 094042

Survival probability of $\Upsilon(nS)$



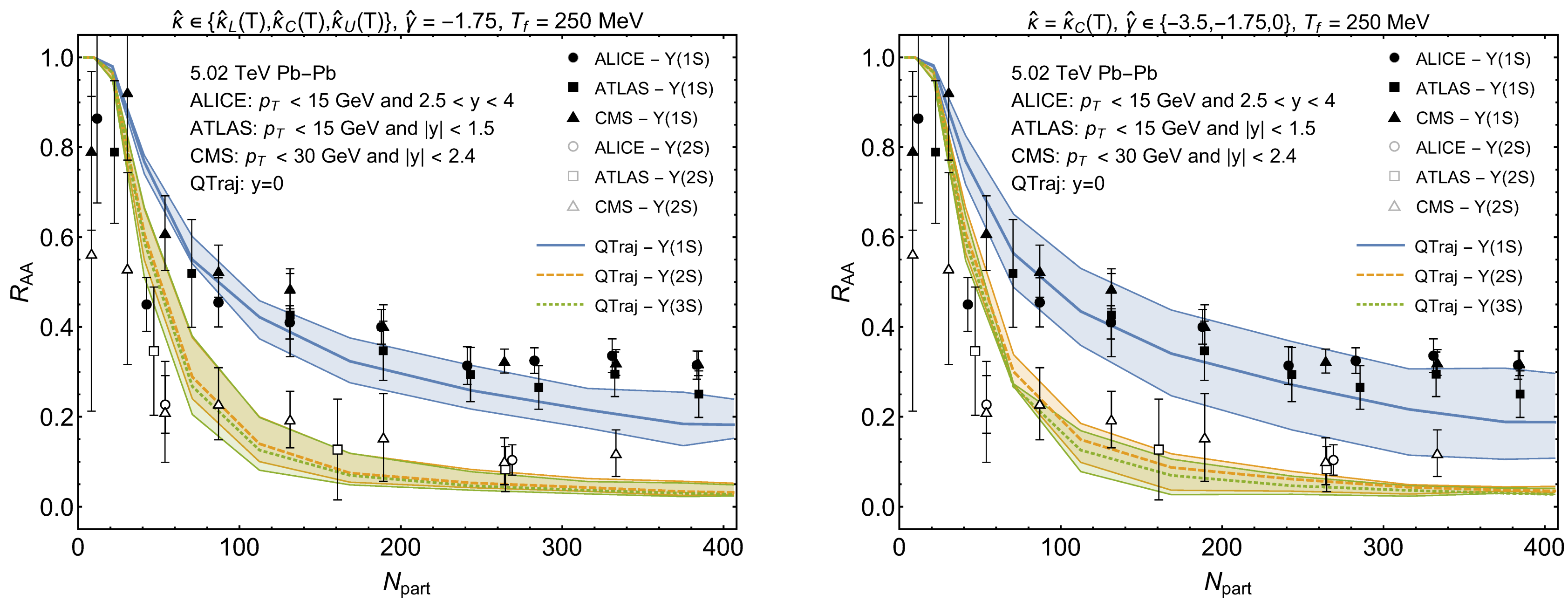
- Brambilla Escobedo Strickland Vairo Vander Griend JHEP 05 (2021) 136

Nuclear modification factor

We compute the nuclear modification factor R_{AA} from

$$R_{AA}(nS) = \frac{\langle n, \mathbf{q}|\rho_s(t_F; t_F)|n, \mathbf{q}\rangle}{\langle n, \mathbf{q}|\rho_s(0; 0)|n, \mathbf{q}\rangle}$$

Bottomonium nuclear modification factor vs CMS data



- data from Alice coll arXiv:2011.05758

  data from CMS coll PLB 790 (2019) 270

  Brambilla Escobedo Strickland Vairo Vander Griend JHEP 05 (2021) 136

Further reading/listening

- A. Rothkopf Phys. Rept. 858 (2020) 1
- Y. Akamatsu arXiv:2009.10559

- poster by P. Vander Griend, Tuesday 16:20 (CEST)
- talk by M. Escobedo, Wednesday 13:40 (CEST)
- talk by M. Strikland, Friday 14:20 (CEST)

Conclusions

We have shown how the heavy quark-antiquark pair out-of-equilibrium evolution can be treated in the framework of pNRQCD. With respect to previous determinations:

- the medium may be a strongly-coupled plasma (not necessarily a quark-gluon plasma) whose characteristics are determined by lattice calculations;
- the total number of heavy quarks, i.e., $\mathrm{Tr}\{\rho_s\} + \mathrm{Tr}\{\rho_o\}$, is preserved by the evolution equations;
- the non-abelian nature of QCD is fully accounted for;
- the treatment does not rely on classical approximations.

The evolution equations follow from assuming the inverse size of the quark-antiquark system to be larger than any other scale of the medium and from being accurate at first non-trivial order in the multipole expansion and at first order in the heavy-quark density.

Under some conditions (large time, quasistatic evolution, temperature much larger than the inverse evolution time of the quarkonium) the evolution equations are of the Lindblad form. Their numerical solution provides $R_{AA}(nS)$ close to experimental data.

Outlook

Quarkonium suppression and recombination in the quark-gluon plasma and dark matter bound state formation and dissociation, with dark matter candidates that are massive and weakly interacting, in the medium provided by the early universe describe very similar phenomena (bound state formation, ionization, Sommerfeld enhancement, bath-particle scattering ...) that may be treated by very similar methods (non relativistic effective field theories, real time formalism, out of equilibrium evolution equations ...). This workshop is a further step in exploiting this similarity.