Potential non-relativistic QCD and open quantum system description of the non-equilibrium evolution of quarkonium inside the medium

Miguel A. Escobedo

Instituto Galego de Física de Altas Enerxías
Universidade de Santiago de Compostela

June 16, 2021

UNIÓN EUROPEA
FONDO EUROPEO DE DESENVOLVEMENTO REXIONAL
"Unha maneira de facer Europa"

Talk based on the following works

1. N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, Phys. Rev. D **96** (2017) no.3, 034021 [arXiv:1612.07248 [hep-ph]].
2. N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, Phys. Rev. D **97** (2018) no.7, 074009 [arXiv:1711.04515 [hep-ph]].
3. N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, Phys. Rev. D **100** (2019) no.5, 054025 [arXiv:1903.08063 [hep-ph]].
4. N. Brambilla, M. A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. H. Weber, [arXiv:2012.01240 [hep-ph]].

Outline

Heavy quarkonium in heavy-ion collisions

- Heavy quarkonium is a bound state of heavy quarks, whose mass is larger than Λ_{QCD}.

Heavy quarkonium in heavy-ion collisions

- Heavy quarkonium is a bound state of heavy quarks, whose mass is larger than Λ_{QCD}.
- Heavy quarks can only be created at the beginning of the collision. It is a hard process.

Heavy quarkonium in heavy-ion collisions

- Heavy quarkonium is a bound state of heavy quarks, whose mass is larger than Λ_{QCD}.
- Heavy quarks can only be created at the beginning of the collision. It is a hard process.
- However, the existence of a medium changes the probability that a bound state is formed and its lifetime.

Heavy quarkonium in heavy-ion collisions

- Heavy quarkonium is a bound state of heavy quarks, whose mass is larger than Λ_{QCD}.
- Heavy quarks can only be created at the beginning of the collision. It is a hard process.
- However, the existence of a medium changes the probability that a bound state is formed and its lifetime.
- Measuring R_{AA}, the ratio of quarkonium states measured in heavy-ion collisions divided by the naive extrapolation of pp data, we can extract information about the medium.

The mechanisms of dissociation

Screening

- Chromoelectric fields are screened at large distances due to the presence of a medium.

The mechanisms of dissociation

Screening

- Chromoelectric fields are screened at large distances due to the presence of a medium.
- The original idea of Matsui and Satz (1986). Dissociation of heavy quarkonium in heavy-ion collisions due to color screening signals the creation of a quark-gluon plasma.

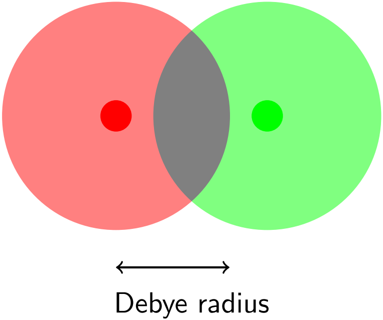
The mechanisms of dissociation

Screening

- Chromoelectric fields are screened at large distances due to the presence of a medium.
- The original idea of Matsui and Satz (1986). Dissociation of heavy quarkonium in heavy-ion collisions due to color screening signals the creation of a quark-gluon plasma.

$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

At finite temperature



The mechanisms of dissociation

Inelastic scattering with partons in the medium

- A singlet can decay into an octet. Interaction with the medium changes the color state.

The mechanisms of dissociation

Inelastic scattering with partons in the medium

- A singlet can decay into an octet. Interaction with the medium changes the color state.
- Dissociation without screening.

The mechanisms of dissociation

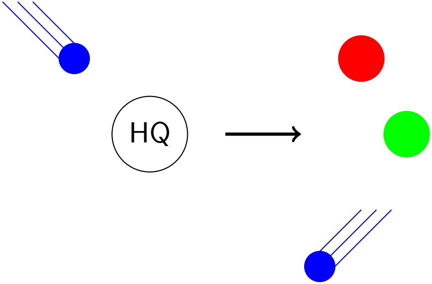
Inelastic scattering with partons in the medium

- A singlet can decay into an octet. Interaction with the medium changes the color state.
- Dissociation without screening.
- This is the mechanism behind the imaginary part of the potential (Laine et al. (2007)). Related to singlet to octet transitions (Brambilla et al. (2008)).

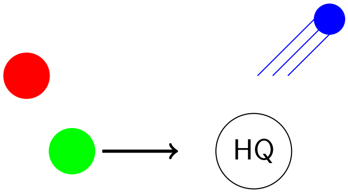
The mechanisms of dissociation

Inelastic scattering with partons in the medium

- A singlet can decay into an octet. Interaction with the medium changes the color state.
- Dissociation without screening.
- This is the mechanism behind the imaginary part of the potential (Laine et al. (2007)). Related to singlet to octet transitions (Brambilla et al. (2008)).



Recombination



Two heavy quarks coming from different origin may recombine to form a new quarkonium state.

Considering the three mechanisms

- We need a formalism in which we can include the three mechanisms consistently. This can be done within the open quantum system framework.

Considering the three mechanisms

- We need a formalism in which we can include the three mechanisms consistently. This can be done within the open quantum system framework.
- When studying screening, we need to know if for a given potential a bound state solution exists. We need quantum mechanics to describe this.

Considering the three mechanisms

- We need a formalism in which we can include the three mechanisms consistently. This can be done within the open quantum system framework.
- When studying screening, we need to know if for a given potential a bound state solution exists. We need quantum mechanics to describe this.
- In some cases, decays and recombination can be described with rate or Boltzmann equation in the semi-classical approximation. However, this is not always the case.

Considering the three mechanisms

- We need a formalism in which we can include the three mechanisms consistently. This can be done within the open quantum system framework.
- When studying screening, we need to know if for a given potential a bound state solution exists. We need quantum mechanics to describe this.
- In some cases, decays and recombination can be described with rate or Boltzmann equation in the semi-classical approximation. However, this is not always the case.
- When thermal effects are important, we need to describe all three effects taking into account quantum effects.

Quarkonium as an Open quantum system

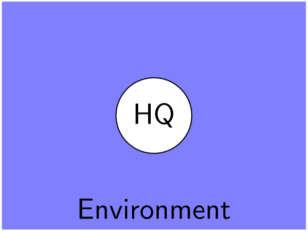
- We consider a *universe* consisting in heavy quarks (system) in a medium of light quarks and gluons (environment). The density matrix $\rho(S, E)$ describes the state of the universe. Its evolution is unitary.

Quarkonium as an Open quantum system

- We consider a *universe* consisting in heavy quarks (system) in a medium of light quarks and gluons (environment). The density matrix $\rho(S, E)$ describes the state of the universe. Its evolution is unitary.
- We define a reduced density matrix integrating out the environment degrees of freedom $\rho_S(S) = Tr_E(\rho(S, E))$. We study the evolution of ρ_S that, in general, is not unitary.

Quarkonium as an Open quantum system

- We consider a *universe* consisting in heavy quarks (system) in a medium of light quarks and gluons (environment). The density matrix $\rho(S, E)$ describes the state of the universe. Its evolution is unitary.
- We define a reduced density matrix integrating out the environment degrees of freedom $\rho_S(S) = Tr_E(\rho(S, E))$. We study the evolution of ρ_S that, in general, is not unitary.



The master equation

We call master equation the equation that describes the evolution of the reduced density matrix.

$$\frac{d\rho}{dt} = -i[H, \rho(t)] + \mathcal{F}(t, \rho(t))$$

The master equation

We call master equation the equation that describes the evolution of the reduced density matrix.

$$\frac{d\rho}{dt} = -i[H, \rho(t)] + \mathcal{F}(t, \rho(t))$$

- We can recover the Schrödinger equation and the Boltzmann equation as limits of the master equation in specific regimes.

The master equation

We call master equation the equation that describes the evolution of the reduced density matrix.

$$\frac{d\rho}{dt} = -i[H, \rho(t)] + \mathcal{F}(t, \rho(t))$$

- We can recover the Schrödinger equation and the Boltzmann equation as limits of the master equation in specific regimes.
- We need to derive the master equation from QCD. This has been done in:
  - Perturbation theory. Akamatsu (2015,2020), Blaizot and Escobedo (2017,2018).
  - Potential non-relativistic QCD (pNRQCD) in the $\frac{1}{r} \gg T$ regime. Brambilla et al. (2016,2017).

The Lindblad equation

Any master equation that is:

- Markovian
- Preserves the properties that a density matrix must fulfil (Hermitian, positive semi-definite, trace is conserve).

Can be written as a Lindblad (or GKSL) equation (Lindblad (1976), Gorini, Kossakowski and Sudarshan (1976)).

The Lindblad equation

Any master equation that is:

- Markovian
- Preserves the properties that a density matrix must fulfil (Hermitian, positive semi-definite, trace is conserve).

Can be written as a Lindblad (or GKSL) equation (Lindblad (1976), Gorini, Kossakowski and Sudarshan (1976)).

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_n \left(C_n \rho C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho\} \right)$$

The Lindblad equation

Any master equation that is:

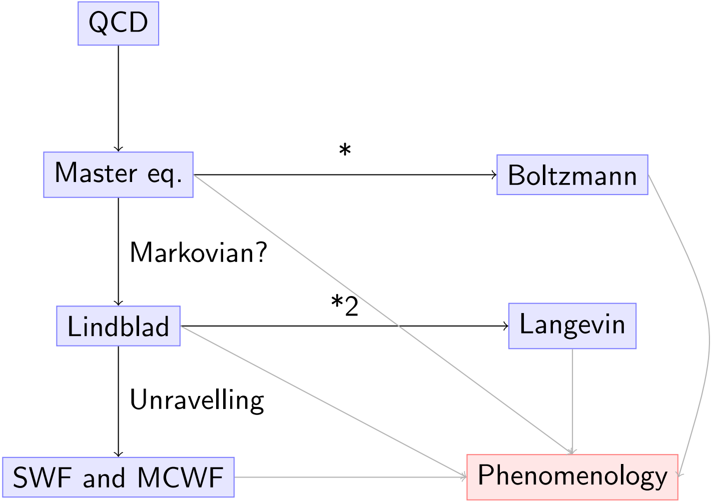
- Markovian
- Preserves the properties that a density matrix must fulfil (Hermitian, positive semi-definite, trace is conserve).

Can be written as a Lindblad (or GKSL) equation (Lindblad (1976), Gorini, Kossakowski and Sudarshan (1976)).

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_n \left(C_n \rho C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho\} \right)$$

In the case of quarkonium, the Markovian limit corresponds to the case in which the energy of the particles in the environment is larger than the binding energy.

Roadmap for OQS approach to quarkonium suppression



* Thermal effects are slow compared to the inverse of the binding energy.

*2 Heavy quarks have a well-defined (classical) position.

Integrating out the heavy quark mass

- Integrating out the scale m can be useful both to study heavy quark diffusion and quarkonium suppression.
- This step can always be done perturbatively and is not affected by the presence of the medium. $m \gg \Lambda_{QCD}, T$.

Integrating out the heavy quark mass

- Integrating out the scale m can be useful both to study heavy quark diffusion and quarkonium suppression.
- This step can always be done perturbatively and is not affected by the presence of the medium. $m \gg \Lambda_{QCD}, T$.

Classification of gluons

- Hard gluons, with energy and momentum of order m.
- Soft gluons, with energy and momentum of order mv.
- Potential gluons, with energy of order mv^2 and momentum of order mv.
- Ultrasoft gluons, with energy and momentum of order mv^2.

NRQCD

Caswell and Lepage (1986), Bodwin, Braaten and Lepage (1994)

$$\mathcal{L}_{NRQCD} = \mathcal{L}_g + \mathcal{L}_q + \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\psi\chi}$$

$$\mathcal{L}_g = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \frac{d_2}{m_Q^2} F^a_{\mu\nu} D^2 F^{\mu\nu a} + d_g^3 \frac{1}{m_Q^2} g f_{abc} F^a_{\mu\nu} F^{\mu b}_\alpha F^{\nu\alpha c}$$

$$\mathcal{L}_\psi = \psi^\dagger \left(iD_0 + c_2 \frac{\mathbf{D}^2}{2m_Q} + c_4 \frac{\mathbf{D}^4}{8m_Q^3} + c_F g \frac{\sigma \mathbf{B}}{2m_Q} + c_D g \frac{\mathbf{DE}-\mathbf{ED}}{8m_Q^2} \right.$$
$$\left. + i c_S g \frac{\sigma(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})}{8m_Q^2} \right) \psi$$

$$\mathcal{L}_\chi = c.c \; of \; \mathcal{L}_\psi$$

$$\mathcal{L}_{\psi\chi} = \frac{f_1(^1S_0)}{m_Q^2} \psi^\dagger \chi \chi^\dagger \psi + \frac{f_1(^3S_1)}{m_Q^2} \psi^\dagger \sigma \chi \chi^\dagger \sigma \psi + \frac{f_8(^1S_0)}{m_Q^2} \psi^\dagger T^a \chi \chi^\dagger T^a \psi$$
$$+ \frac{f_8(^3S_1)}{m_Q^2} \psi^\dagger T^a \sigma \chi \chi^\dagger T^a \sigma \psi$$

potential NRQCD Lagrangian at T=0

Brambilla, Pineda, Soto and Vairo, NPB566 (2000) 275

Starting from NRQCD and integrating out the scale $\frac{1}{r}$.

$$\mathcal{L}_{pNRQCD} = \int d^3\mathrm{r}\, Tr\left[S^\dagger\left(i\partial_0 - h_s\right)S \right.$$
$$\left. + O^\dagger\left(iD_0 - h_o\right)O\right] + V_A(r)Tr(O^\dagger \mathrm{r} g \mathrm{E} S + S^\dagger \mathrm{r} g \mathrm{E} O)$$
$$+\frac{V_B(r)}{2}Tr(O^\dagger \mathrm{r} g \mathrm{E} O + O^\dagger O \mathrm{r} g \mathrm{E}) + \mathcal{L}_g + \mathcal{L}_q$$

- Degrees of freedom are singlet and octets.
- Allows to obtain manifestly gauge-invariant results. Simplifies the connection with Lattice QCD.
- If $1/r \gg T$ we can use this Lagrangian as starting point. In other cases the matching between NRQCD and pNRQCD will be modified.

Relation with darm matter physics

- Similar EFTs exist for the case of heavy Majorana particles.

Relation with darm matter physics

- Similar EFTs exist for the case of heavy Majorana particles.
- The OQS framework is useful when you need to consider at the same time genuinely quantum effects and dissipation.
  - Resonant leptogenesis??
  - Bound state dark matter??

Plan

How to compute what we measure?

Experimentally, the most common way to detect quarkonium is thought its decay into leptons. What is the pNRQCD operator related with this observable?

$$Tr(J_{el}^{\mu}(t,0)J_{el,\mu}(t,0)\rho) \propto Tr(S^{\dagger}(t,0)S(t,0)\rho)$$

How to compute what we measure?

Experimentally, the most common way to detect quarkonium is thought its decay into leptons. What is the pNRQCD operator related with this observable?

$$Tr(J^{\mu}_{el}(t,0)J_{el,\mu}(t,0)\rho) \propto Tr(S^{\dagger}(t,0)S(t,0)\rho)$$

Conclusion

We need to compute the time evolution of $Tr(S^{\dagger}(t,\mathrm{x})S(t,\mathrm{x}')\rho)$ given an initial condition at $t = t_0$.

How to compute what we measure?

Experimentally, the most common way to detect quarkonium is thought its decay into leptons. What is the pNRQCD operator related with this observable?

$$Tr(J_{el}^{\mu}(t,0)J_{el,\mu}(t,0)\rho) \propto Tr(S^{\dagger}(t,0)S(t,0)\rho)$$

Conclusion

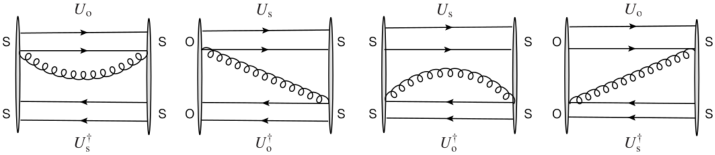
We need to compute the time evolution of $Tr(S^{\dagger}(t,\mathrm{x})S(t,\mathrm{x}')\rho)$ given an initial condition at $t = t_0$.

Reinterpretation

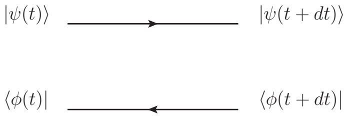
We can understand $Tr(S^{\dagger}(t,\mathrm{x})S(t,\mathrm{x}')\rho)$ as the projection of the density matrix to the subspace in which we have a singlet. Quarkonium is an open quantum system interacting with a bath.

The evolution of the density matrix

4 diagrams that connect any state at time t with a singlet at time $t + dt$.

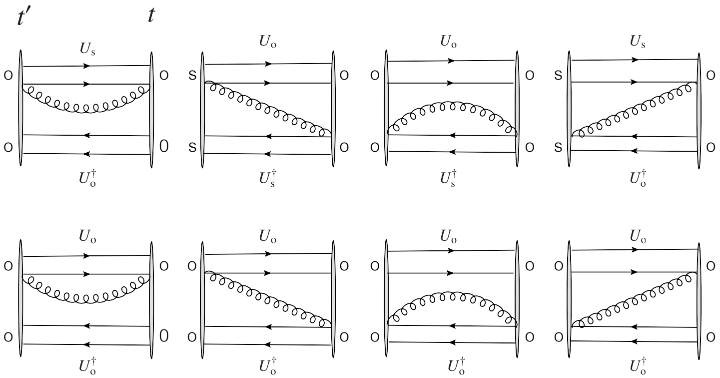


These diagrams represent the evolution of the density matrix



The evolution of the density matrix

8 diagrams that connect whatever state at time t with an octet at time $t + dt$.



The $\frac{1}{r} \gg T, m_D \gg E$ regime

Brambilla, M.A.E., Soto and Vairo (2017-2018)

Because all the thermal scales are smaller than $\frac{1}{r}$ but bigger than E the evolution equation is of the Lindblad form. [1]

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k(\kappa)\rho C_k^\dagger(\kappa) - \frac{1}{2}\{C_k^\dagger(\kappa) C_k(\kappa), \rho\})$$

$$\kappa = \frac{g^2}{6\,N_c} \mathrm{Re} \int_{-\infty}^{+\infty} ds \, \langle \, \mathrm{T} \, E^{a,i}(s,0) E^{a,i}(0,0) \rangle$$

$$\gamma = \frac{g^2}{6\,N_c} \mathrm{Im} \int_{-\infty}^{+\infty} ds \, \langle \, \mathrm{T} \, E^{a,i}(s,0) E^{a,i}(0,0) \rangle$$

[1] We use a redefined field $E(t, \mathrm{x})$ such that we do not need to write explicitly the Wilson lines.

The $\frac{1}{r} \gg T, m_D \gg E$ regime

Brambilla, M.A.E., Soto and Vairo (2017-2018)

Because all the thermal scales are smaller than $\frac{1}{r}$ but bigger than E the evolution equation is of the Lindblad form. [1]

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k(\kappa)\rho C_k^\dagger(\kappa) - \frac{1}{2}\{C_k^\dagger(\kappa) C_k(\kappa), \rho\})$$

$$\kappa = \frac{g^2}{6\,N_c} \mathrm{Re} \int_{-\infty}^{+\infty} ds \,\langle \mathrm{T}\, E^{a,i}(s,0) E^{a,i}(0,0)\rangle$$

$$\gamma = \frac{g^2}{6\,N_c} \mathrm{Im} \int_{-\infty}^{+\infty} ds \,\langle \mathrm{T}\, E^{a,i}(s,0) E^{a,i}(0,0)\rangle$$

κ coincides with the heavy quark diffusion coefficient.

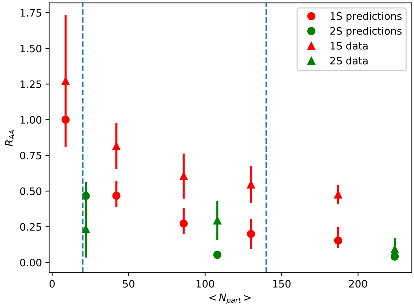
[1] We use a redefined field $E(t, \mathrm{x})$ such that we do not need to write explicitly the Wilson lines.

From κ and γ to phenomenological predictions

We can take values of κ (in this case we use lattice QCD results of Francis, Kaczmarek, Laine, Neuhaus and Ohno (2015)) and γ (we use $\gamma = 0$).

From κ and γ to phenomenological predictions

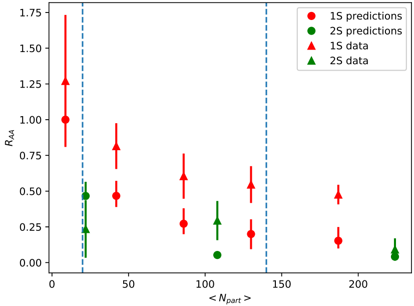
We can take values of κ (in this case we use lattice QCD results of Francis, Kaczmarek, Laine, Neuhaus and Ohno (2015)) and γ (we use $\gamma = 0$).



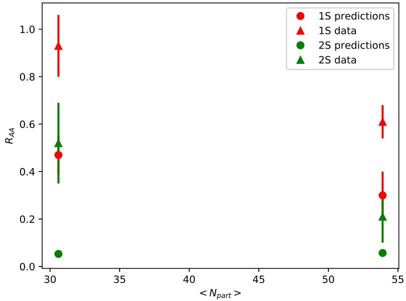
Comparison to CMS data at $\sqrt{s} = 2.76\ TeV$ (Phys.Lett. B770 (2017) 357-379), computation done in Brambilla, M.A.E., Soto and Vairo (2017-2018).

From κ and γ to phenomenological predictions

We can take values of κ (in this case we use lattice QCD results of Francis, Kaczmarek, Laine, Neuhaus and Ohno (2015)) and γ (we use $\gamma = 0$).



Comparison to CMS data at $\sqrt{s} = 2.76\ TeV$ (Phys.Lett. B770 (2017) 357-379), computation done in Brambilla, M.A.E., Soto and Vairo (2017-2018).



Comparison to CMS data at $\sqrt{s} = 5.02\ TeV$ (Phys. Lett. B 790, 270-293 (2019)), computation shown in Hard Probes 2018.

Plan

Summary

- Recently, a more precise determinations of κ became available. We have more precise information on the dependence of κ with the temperature.

Summary

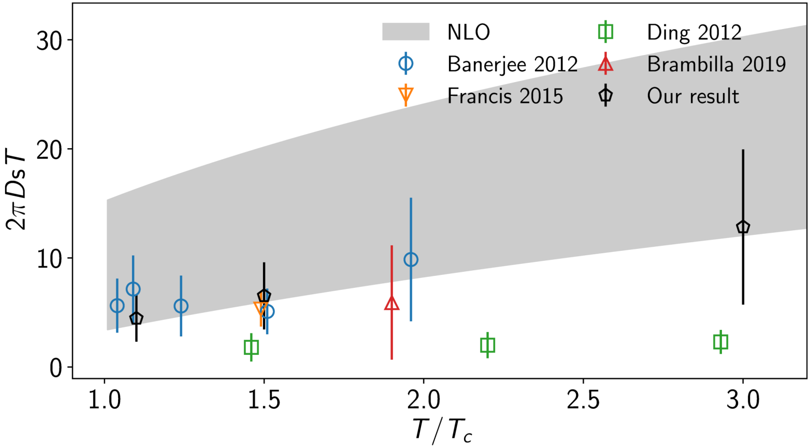
- Recently, a more precise determinations of κ became available. We have more precise information on the dependence of κ with the temperature.
- γ can be obtained from Lattice QCD information on the thermal mass shift.

Summary

- Recently, a more precise determinations of κ became available. We have more precise information on the dependence of κ with the temperature.
- γ can be obtained from Lattice QCD information on the thermal mass shift.
- To solve the Lindblad equation numerically is hard. In the past, we approximated medium evolution with a Bjorken expansion to reduce computational cost. I will discuss how we recently managed to reduce this cost.

New determination of κ. ($\kappa = 2\pi D_s T$)

Picture taken from N. Brambilla, V. Leino, P. Petreczky and A. Vairo, Phys. Rev. D **102** (2020) no.7, 074503 [arXiv:2007.10078 [hep-lat]].



Determining κ and γ from quarkonium properties

- We can reverse the logic and determine κ and γ respectively from the decay width and the thermal mass shift.

Determining κ and γ from quarkonium properties

- We can reverse the logic and determine κ and γ respectively from the decay width and the thermal mass shift.
- There is unquenched lattice QCD data on both quantities.

Determining κ and γ from quarkonium properties

- We can reverse the logic and determine κ and γ respectively from the decay width and the thermal mass shift.
- There is unquenched lattice QCD data on both quantities.
- In the case of κ, we have a determination which is based on an independent set of assumptions which can be compared with what is found studying heavy quark diffusion.

Determining κ and γ from quarkonium properties

- We can reverse the logic and determine κ and γ respectively from the decay width and the thermal mass shift.
- There is unquenched lattice QCD data on both quantities.
- In the case of κ, we have a determination which is based on an independent set of assumptions which can be compared with what is found studying heavy quark diffusion.
- In the case of γ, it is the first non-perturbative determination.

Determining κ and γ from quarkonium properties

- We can reverse the logic and determine κ and γ respectively from the decay width and the thermal mass shift.
- There is unquenched lattice QCD data on both quantities.
- In the case of κ, we have a determination which is based on an independent set of assumptions which can be compared with what is found studying heavy quark diffusion.
- In the case of γ, it is the first non-perturbative determination.
- The main assumption is that we are in the regime $\frac{1}{r} \gg T, m_D \gg E$ and that the bound states are Coulombic.

Determining κ and γ from quarkonium properties

Equations for κ and γ

$$\Gamma = \kappa \langle r^2 \rangle \qquad \delta M = \frac{1}{2}\gamma \langle r^2 \rangle$$

$\langle r^2 \rangle$ is computed assuming that the wave function is well described with a Coulombic potential.

Determining κ and γ from quarkonium properties

Equations for κ and γ

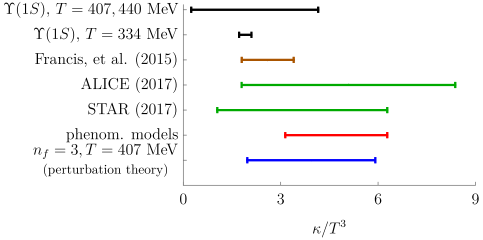
$$\Gamma = \kappa \langle r^2 \rangle \qquad \delta M = \frac{1}{2}\gamma \langle r^2 \rangle$$

$\langle r^2 \rangle$ is computed assuming that the wave function is well described with a Coulombic potential.

Lattice QCD data

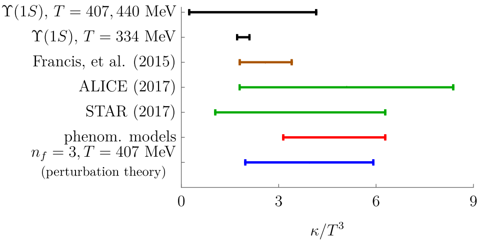
- We use the results of Kim, Petreczky and Rothkopf (2018) for the thermal mass shift and as a lower bound for the decay width.
- We use the results of Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair and Skullerud (2011) as upper bound for the decay width.
- Data at $T = 334\,MeV$, not used originally in our paper, is taken from Larsen, Meinel, Mukherjee and Petreczky (2019).

Determination of κ



Picture taken from Brambilla, M.A.E, Vairo and Vander Griend (2019)

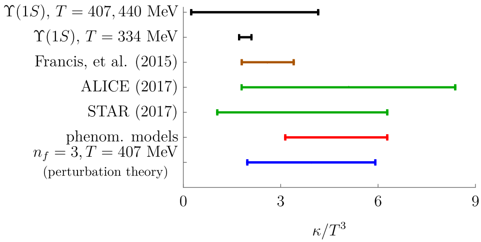
Determination of κ



Picture taken from Brambilla, M.A.E, Vairo and Vander Griend (2019)

- We took the value of Kim, Petreczky and Rothkopf (2018) at $T = 407\ MeV$ as a lower bound and the value of Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair and Skullerud (2011) at the highest temperature available as an upper bound.

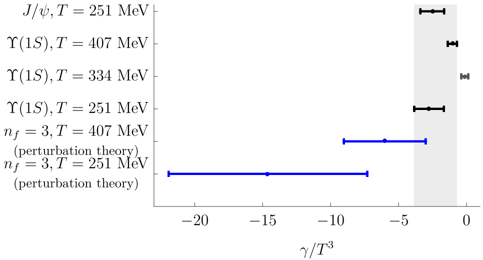
Determination of κ



Picture taken from Brambilla, M.A.E, Vairo and Vander Griend (2019)

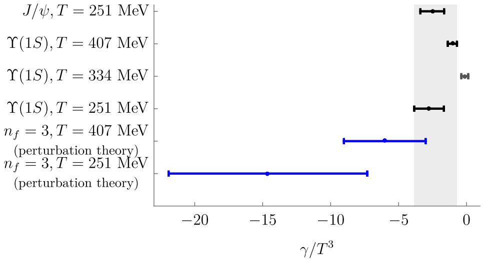
- We took the value of Kim, Petreczky and Rothkopf (2018) at $T = 407\, MeV$ as a lower bound and the value of Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair and Skullerud (2011) at the highest temperature available as an upper bound.
- Our result compares reasonably well to other determinations.

Determination of γ



Picture taken from Brambilla, M.A.E, Vairo and Vander Griend (2019)

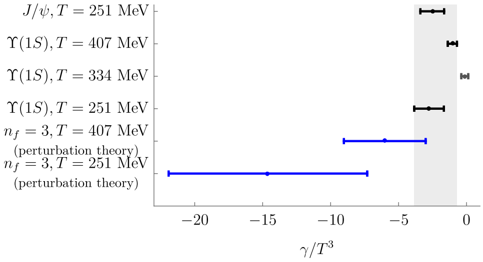
Determination of γ



Picture taken from Brambilla, M.A.E, Vairo and Vander Griend (2019)

- Results from different quarkonium state at the same temperature ($T = 251\,MeV$) are compatible with each other.

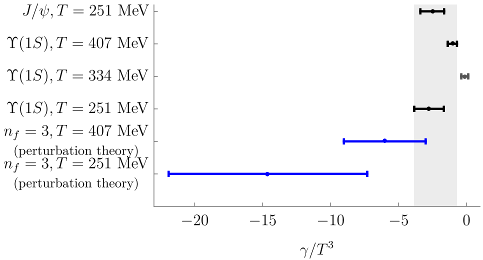
Determination of γ



Picture taken from Brambilla, M.A.E, Vairo and Vander Griend (2019)

- Results from different quarkonium state at the same temperature ($T = 251\,MeV$) are compatible with each other.
- Hint that $\frac{\gamma}{T^3}$ is not a constant.

Determination of γ



Picture taken from Brambilla, M.A.E, Vairo and Vander Griend (2019)

- Results from different quarkonium state at the same temperature ($T = 251\, MeV$) are compatible with each other.
- Hint that $\frac{\gamma}{T^3}$ is not a constant.
- Lattice extracted results are much smaller than perturbative calculations.

The Monte-Carlo Wave Function method

- Method developed by Dalibard, Castin and Molmer to efficiently solve the Lindblad equation.

The Monte-Carlo Wave Function method

- Method developed by Dalibard, Castin and Molmer to efficiently solve the Lindblad equation.
- It dramatically reduces computational cost in our case. In part because the Hamiltonian does not mix states with different color and angular momentum.

The Monte-Carlo Wave Function method

- Method developed by Dalibard, Castin and Molmer to efficiently solve the Lindblad equation.
- It dramatically reduces computational cost in our case. In part because the Hamiltonian does not mix states with different color and angular momentum.
- In our previous papers, we used a N_r size lattice to discritize the radial component and we expand in angular momentum, with l_{max} the higher l taken into account. We had to compute the evolution of a $(2N_r \cdot l_{max}) \times (2N_r \cdot l_{max})$ matrix. Doubling the lattice size multiplies the computational cost by four and l_{max} can not be infinite.

The Monte-Carlo Wave Function method

- Method developed by Dalibard, Castin and Molmer to efficiently solve the Lindblad equation.
- It dramatically reduces computational cost in our case. In part because the Hamiltonian does not mix states with different color and angular momentum.
- In our previous papers, we used a N_r size lattice to discritize the radial component and we expand in angular momentum, with l_{max} the higher l taken into account. We had to compute the evolution of a $(2N_r \cdot l_{max}) \times (2N_r \cdot l_{max})$ matrix. Doubling the lattice size multiplies the computational cost by four and l_{max} can not be infinite.
- Using MWFM, we need to simulate many times a stochastic evolution. However, the state of the system is represented by a vector of size N_r, a bit to store the color state and an integer to store the l quantum number. Doubling the lattice size only doubles the cost and l_{max} can be ∞.

The Monte-Carlo Wave Function method

Take the Lindblad equation

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k \left(C_k(\kappa) \rho C_k^\dagger(\kappa) - \frac{1}{2}\{C_k^\dagger(\kappa) C_k(\kappa), \rho\} \right)$$

Let us define

$$\Gamma_n = C_n^\dagger C_n \qquad \Gamma = \sum_n \Gamma_n$$

and

$$H_{eff} = H - i\frac{\Gamma}{2}$$

$\rho(t) = \sum_n p_n |\Psi_n(t)\rangle\langle\Psi_n(t)|$. If we know how to evolve the case $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$, it is straightforward to generalize.

The Monte-Carlo Wave Function method

The algorithm to evolve from t to $t + dt$

- With probability $1 - \langle\Psi(t)|\Gamma|\Psi(t)\rangle dt$.
  - Evolve the wave-function with $(1 - iH_{eff}\,dt)|\Psi(t)\rangle$. In our case, this implies solving a 1D Schrödinger equation because H_{eff} does not mix states with different color or angular momentum.
- With probability $\langle\Psi(t)|\Gamma_n|\Psi(t)\rangle dt$.
  - Take a quantum jump, $|\Psi(t)\rangle \to C_n|\Psi(t)\rangle$.
  - Only here transitions between different color and angular momentum are allowed.
- Normalize the resulting wave-function.

The average of this stochastic evolution of the wave-function is equivalent to the Lindblad equation for the density matrix.

Results

- Thanks to the reduction of the numerical cost, we can substitute the Bjorken by state-of-the-art hydrodynamical evolution (aHydroQP. Alqahtani, Nopoush and Strickland (2015)).

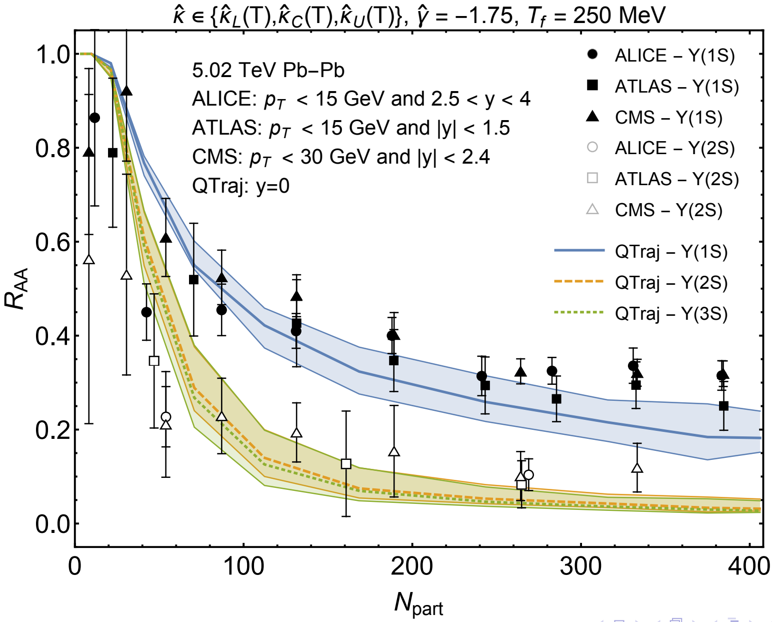
Results

- Thanks to the reduction of the numerical cost, we can substitute the Bjorken by state-of-the-art hydrodynamical evolution (aHydroQP. Alqahtani, Nopoush and Strickland (2015)).
- We know more about γ and the T dependence of κ.

Results

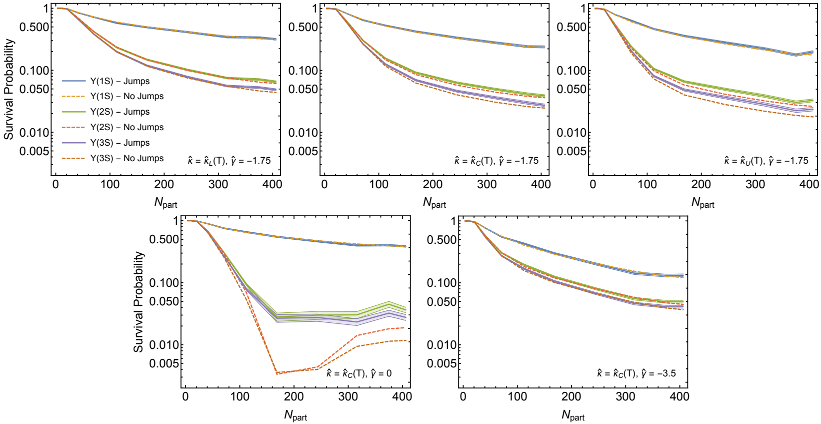
- Thanks to the reduction of the numerical cost, we can substitute the Bjorken by state-of-the-art hydrodynamical evolution (aHydroQP. Alqahtani, Nopoush and Strickland (2015)).
- We know more about γ and the T dependence of κ.
- We observe that the effect of the quantum jumps is very small. Indeed, evolving with H_{eff} is a very good approximation.

Results



$\hat{\kappa} \in \{\hat{\kappa}_L(T), \hat{\kappa}_C(T), \hat{\kappa}_U(T)\}$, $\hat{\gamma} = -1.75$, $T_f = 250$ MeV
5.02 TeV Pb–Pb
ALICE: $p_T < 15$ GeV and $2.5 < y < 4$
ATLAS: $p_T < 15$ GeV and $|y| < 1.5$
CMS: $p_T < 30$ GeV and $|y| < 2.4$
QTraj: y=0
ALICE – Υ(1S)
ATLAS – Υ(1S)
CMS – Υ(1S)
ALICE – Υ(2S)
ATLAS – Υ(2S)
CMS – Υ(2S)
QTraj – Υ(1S)
QTraj – Υ(2S)
QTraj – Υ(3S)
R_{AA}
N_{part}
1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400

H_{eff} against full evolution



Why is the evolution without jumps such a good approximation?

- Because of the large N_c limit. (See M.A.E 2020).

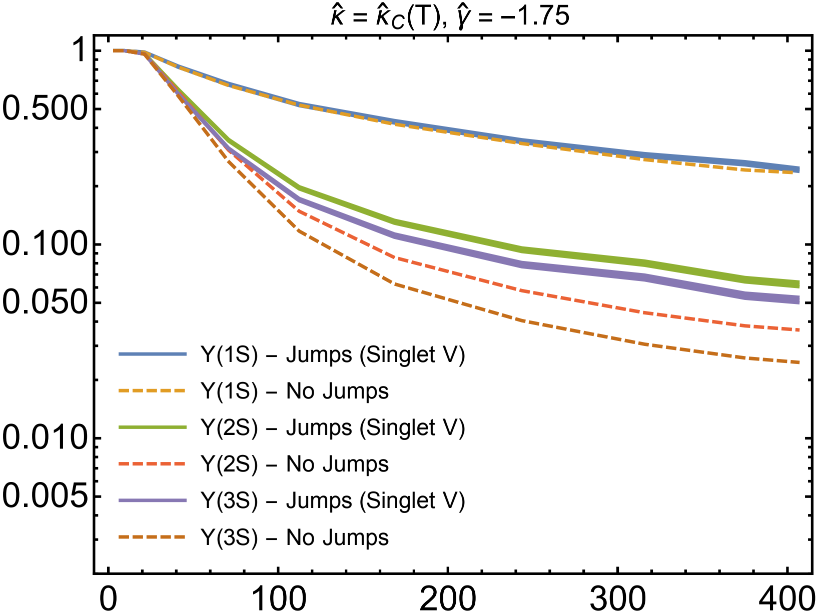
Why is the evolution without jumps such a good approximation?

- Because of the large N_c limit. (See M.A.E 2020).
- Each jump changes the l quantum number by one unit. A s-wave state will always decay to p-wave. However, a p-wave only has $1/3$ probability to decay to s-wave.

Why is the evolution without jumps such a good approximation?

- Because of the large N_c limit. (See M.A.E 2020).
- Each jump changes the l quantum number by one unit. A s-wave state will always decay to p-wave. However, a p-wave only has $1/3$ probability to decay to s-wave.
- The octet has a repulsive potential. The quark and the antiquark separate and, if they do jump back to a singlet state, they are less likely to bound.

If the octet had an attractive potential...



Plan

Conclusions

- We have computed R_{AA} exploiting the non-relativistic nature of quarkonium.

Conclusions

- We have computed R_{AA} exploiting the non-relativistic nature of quarkonium.
- In the regime $\frac{1}{r} \gg T, m_D \gg E$ the evolution is of the Lindblad form and all the information about the medium is encoded in two transport parameters, κ and γ.

Conclusions

- We have computed R_{AA} exploiting the non-relativistic nature of quarkonium.
- In the regime $\frac{1}{r} \gg T, m_D \gg E$ the evolution is of the Lindblad form and all the information about the medium is encoded in two transport parameters, κ and γ.
- κ is the heavy quark diffusion coefficient. The use of more precise Lattice data leads to a better agreement with experimental data.

Conclusions

- We have computed R_{AA} exploiting the non-relativistic nature of quarkonium.
- In the regime $\frac{1}{r} \gg T, m_D \gg E$ the evolution is of the Lindblad form and all the information about the medium is encoded in two transport parameters, κ and γ.
- κ is the heavy quark diffusion coefficient. The use of more precise Lattice data leads to a better agreement with experimental data.
- The use of the MCWF method reduces the computational cost. Thanks to this we can use a more realistic hydrodynamical description and obtain more realistic predictions.

Outlook

- The MCWF method might be useful at other temperature regimes (maybe $T \sim \frac{1}{r}$). We plan to publish our code soon.

Outlook

- The MCWF method might be useful at other temperature regimes (maybe $T \sim \frac{1}{r}$). We plan to publish our code soon.
- We are looking at other observables, like v_2 of bottomonium and R_{AA} vs p_T.

Outlook

- The MCWF method might be useful at other temperature regimes (maybe $T \sim \frac{1}{r}$). We plan to publish our code soon.
- We are looking at other observables, like v_2 of bottomonium and R_{AA} vs p_T.
- There are things that we can improve:
  - Extend the formalism to a wider range of temperature regimes. Master equations that are not of the Lindblad form.
  - Improve the initial conditions.

Thanks!