Dark matter and coloured co-annihilators: from the relic density to experimental constraints

Simone Biondini

Department of Physics - University of Basel

Quarkonia meet Dark Matter - Online Workshop at Kavli IPMU

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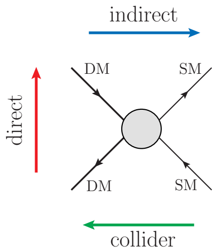
in collaboration with Mikko Laine and Stefan Vogl
1801.05821, 1811.02581, 1907.05766

Particle interpretation of DM and freeze-out



- Evidence for DM from many compelling (gravitational) observations
  - ◇ from CMB anysotropies with ΛCDM *Planck Collab. Results 2018*

- DM as a particle: many candidates (Bertone and Hooper [1605.04909])
- Any model has to comply with

$$\Omega_{\rm DM} h^2(M_{\rm DM}, M_{\rm DM'}, \alpha_{\rm DM}, \alpha_{\rm SM}) = 0.1200 \pm 0.0012$$

Thermal freeze-out

- Boltzmann equation for DM (χ)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\rm eq}^2)$$

- relevant processes $\chi\chi \leftrightarrow \mathrm{SM\,SM}$
- $\langle \sigma v \rangle$: input from particle physics with $v \sim \sqrt{T/M} < 1$

$$\langle \sigma v \rangle \approx \langle a + bv^2 + \dots \rangle \Rightarrow \boxed{\langle \sigma v \rangle^{(0)} \approx \frac{\alpha^2}{M^2}}$$

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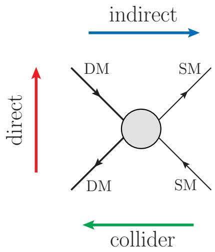

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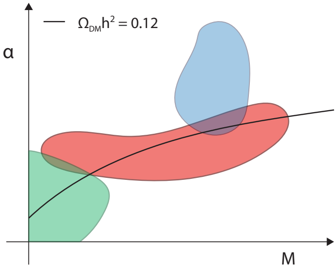
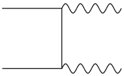
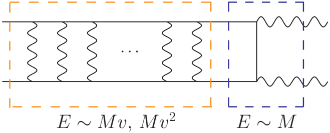


Going towards a realistic picture

- DM and/or coannihilating partners interact with gauge bosons and scalars





- repeated soft interactions: Sommerfeld enhancement and bound states Hisano, Matsumoto, Nojiri [hep-ph/0212022], [hep-ph/0307216]; B. von Harling and K. Petraki [1407.7874]; Beneke, Hellmann, Ruiz-Femenia [1411.6924]

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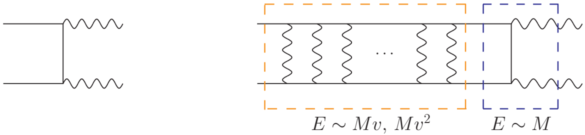

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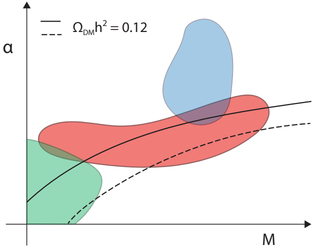
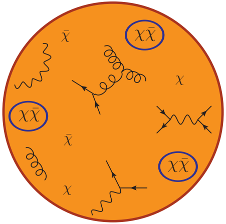
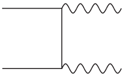
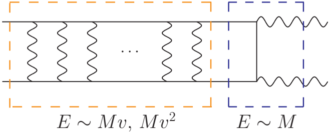




Dark matter bound states

- DM and/or coannihilating partners interact with gauge bosons and scalars





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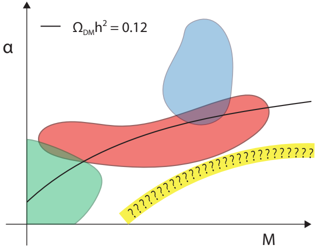
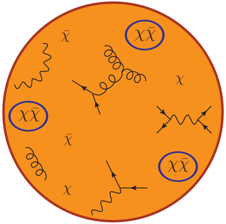
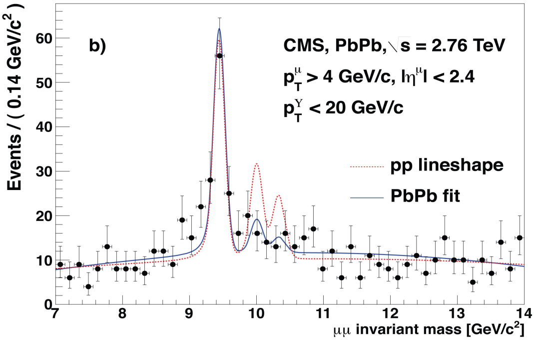


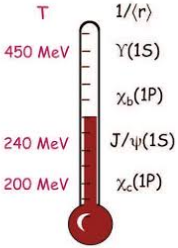


Lessons from heavy quarkonium...

- many bound states may appear in the spectrum
- their existence depends on the temperature
  $\rightarrow$**dissociation** and **recombination** processes
- bound-states calculations can be performed in NREFT/pNREFTs

(see talks by A. Vairo, M. Laine, T. Mehen and M. Escobedo and references therein!)





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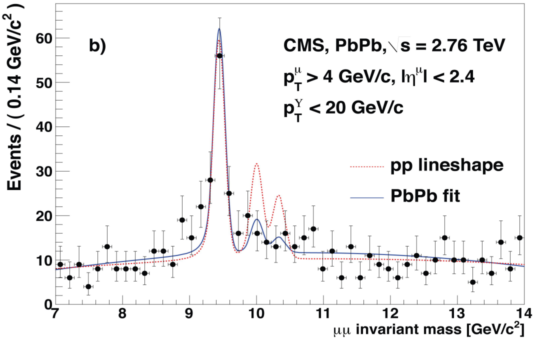
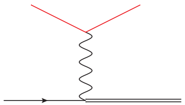
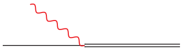


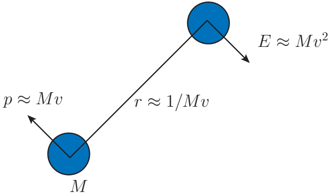




NREFTs and pNREFTs for dark matter

Apply an EFT approach

- Non-relativistic scales: $M \gg Mv \gg Mv^2$ (for bound state with Coulomb potential $v \sim \alpha$)
- Thermal scales: πT and $m_D \approx \alpha^{1/2} T$, if weakly-coupled plasma $\pi T \gg m_D$



$$\mathcal{L}_{\rm RT} = \frac{1}{2}\bar{\chi}(i\not{D} - M)\chi$$

$$\mathcal{L}_{\rm NREFT} = \psi^\dagger \left(iD^0 - \frac{\boldsymbol{D}^2}{2M}\right)\psi + \frac{c}{M^2}\psi^\dagger\chi\chi^\dagger\psi + \dots$$

$$\mathcal{L}_{\rm pNREFT} = \int d^3r \, {\rm Tr}\left\{\phi^\dagger\left[i\partial_0 - V_\phi - \delta M_\phi\right]\phi\right\} + \cdots$$

for more on pNREFTs, pNRQCD see talks by Vairo and Escobedo

- work with the natural d.o.f. at a given scale

  $E \sim Mv$ non-relativistic heavy particles

  $E \sim Mv^2$ non-relativistic pairs $\psi\psi \to \phi_s + \phi_b$ (bound states and scattering states)

$$V_\phi = V(r, T, m_D) + i\Gamma(r, T, m_D)$$

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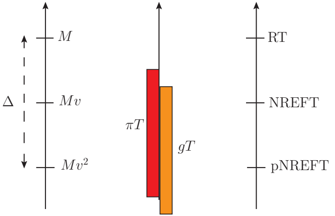

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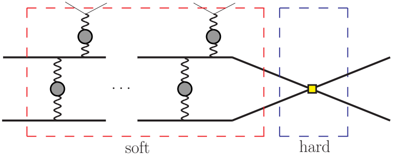
NREFTs and annihilations

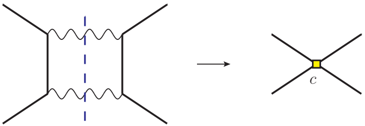


- $M \gg T, m_D, Mv, Mv^2$

- Annihilation of a heavy pair: DM-DM, with energies $\sim 2M$

$$\mathcal{O} = i\frac{c}{M^2}\psi^\dagger\chi\,\chi^\dagger\psi\,,\; c \approx \alpha^2 \quad \text{(inclusive s-wave annihilation)}$$

Caswell, Lepage (1985); Bodwin, Braaten, Lepage [hep-ph/9407339]



- $M \gg T \Rightarrow \Delta x \sim \frac{1}{k} \sim \frac{1}{M} \ll \frac{1}{T}$ local and insensitive to thermal scales

Beyond the free case: the spectral function

$$(\partial_t + 3H)n = -\Gamma_{\rm chem}(n - n_{\rm eq})\,, \quad \Gamma_{\rm chem} \approx \frac{8c}{M^2 n_{\rm eq}}\gamma \quad \text{where } \gamma = \langle \psi^\dagger \chi \chi^\dagger \psi \rangle_T$$

$$(\partial_t + 3H)n = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2) \quad \Rightarrow \langle \sigma v \rangle = \frac{\Gamma_{\rm chem}}{2 n_{\rm eq}}$$

Bodeker and Laine [1205.4987]; Kim and Laine [1602.08105]; Kim and Laine [1609.00474]

BEYOND THE FREE CASE: THE SPECTRAL FUNCTION

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- using $\Pi_<(\omega) = 2n_B(\omega)\int_{\boldsymbol{k}} \rho(\omega, \boldsymbol{k})$, with $\omega = E' + 2M + \frac{k^2}{4M}$

$$\gamma = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \Pi_<(\omega) = \int_{2M-\Lambda}^{\infty} \frac{d\omega}{\pi} e^{-\frac{\omega}{T}} \int_{\boldsymbol{k}} \rho(\omega, \boldsymbol{k}) + \mathcal{O}(e^{-4M/T})\,, \; \alpha^2 M \ll \Lambda \lesssim M$$

- $\boldsymbol{\rho}$ from the imaginary part of "Green's function" Laine [0704.1720], Burnier, Laine and Vepsälïnen [0711.1743]

$$\left[H - i\Gamma(r,T) - E'\right] G(E'; \boldsymbol{r}, \boldsymbol{r'}) = N\delta^3(\boldsymbol{r} - \boldsymbol{r'}) \qquad \lim_{\boldsymbol{r},\boldsymbol{r'} \to 0} {\rm Im}\, G(E'; \boldsymbol{r}, \boldsymbol{r'}) = \rho(E')$$

- $H = -\frac{\nabla^2}{M} + V(r,T)$, $\Gamma(r,T)$ real scatterings with plasma particles

ρ and Schrödinger equation at $T \neq 0$

- non-relativistic dynamics:

$$\omega = E' + 2M + \frac{k^2}{4M} \text{ and } H = -\frac{\nabla^2}{M} + V(r, T)$$

- the spectral function $\rho(E')$ is obtained from

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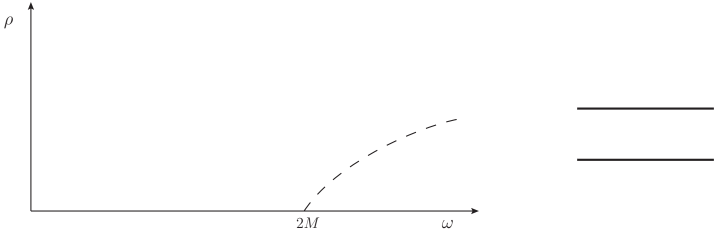
$$\gamma \approx \left(\frac{MT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{2M}{T}} \int_{-\Lambda}^{\infty} \frac{dE'}{\pi} e^{-\frac{E'}{T}} \rho_{\text{free}}(E') \Rightarrow \langle \sigma v \rangle \approx \frac{c}{M^2}$$



ρ and Schrödinger equation at $T \neq 0$

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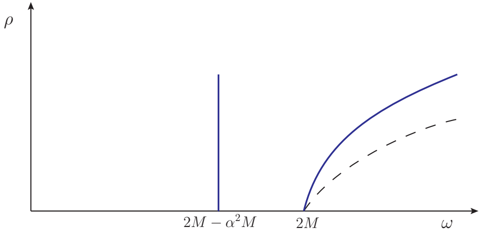

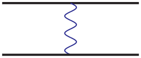

ρ AND SCHRÖDINGER EQUATION AT $T \neq 0$

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- the spectral function $\rho(E',0)$ is obtained from

$$\left[H - i\Gamma(r,T) - E'\right] G(E';\mathbf{r},\mathbf{r'}) = N\delta^3(\mathbf{r}-\mathbf{r'}) \qquad \lim_{\mathbf{r},\mathbf{r'}\to 0} \mathrm{Im}\, G(E';\mathbf{r},\mathbf{r'}) = \rho(E')$$

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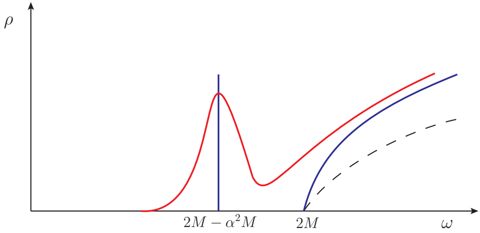

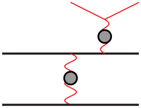

Coloured mediators: simplified models

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\chi} + \mathcal{L}_{\eta} + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{\chi} = \frac{1}{2}\bar{\chi} i\partial\!\!\!/ \chi - \frac{M}{2}\bar{\chi}\chi \,, \quad \mathcal{L}_{\eta} = (D^{\mu}\eta)^{\dagger}(D_{\mu}\eta) - M_{\eta}^2 \eta^{\dagger}\eta - \lambda_2 \left(\eta^{\dagger}\eta\right)^2$$

$$\mathcal{L}_{\text{int}} = -\mathrm{y}\,\eta^{\dagger}\bar{\chi}P_R q - \mathrm{y}^{*}\,\bar{q}P_L\chi\eta - \lambda_3\eta^{\dagger}\eta H^{\dagger}H$$

in MSSM importance of coannihilations realized ling ago J. Edsjö and P. Gondolo [hep-ph/9704361]

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{S} + \mathcal{L}_{F} + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{S} = \frac{1}{2}\partial_{\mu}S\partial^{\mu}S - \frac{M_S^2}{2}S^2 - \frac{\lambda_2}{4!}S^4 \,, \quad \mathcal{L}_{F} = \bar{F}\left(iD\!\!\!\!/ - M_F\right)F$$

$$\mathcal{L}_{\text{int}} = -y\,S\bar{F}P_R q - y^{*}S\bar{q}P_L F \; - \frac{\lambda_3}{2}S^2 H^{\dagger}H \,,$$

simplified models with co-annihilators see Garny, Ibarra, Vogl [1503.01500]; De Simone, Jacques [1603.08002]

- we look at the coannihilation scenario: $(M_{\eta} - M_{\chi})/M_{\chi}$, $(M_F - M_S)/M_S \lesssim 0.2$

Hard annihilations and NREFT

- our master equation is $\boxed{\langle \sigma v \rangle = \frac{4}{n_{\rm eq}^2} \langle {\rm Im}\, \mathcal{L}_{\rm NREFT} \rangle}$
- Non-relativistic fields $\eta = \frac{1}{\sqrt{2M}} \left(\phi e^{-iMt} + \varphi^\dagger e^{iMt} \right)$ and $\chi = (\psi e^{-iMt}, -i\sigma_2 \psi^* e^{iMt})$

$$
\begin{aligned}
\mathcal{L}_{\rm NREFT} &= i \Big\{ c_1\, \psi_p^\dagger \psi_q^\dagger \psi_q \psi_p + c_2 \left(\psi_p^\dagger \phi_\alpha^\dagger \psi_p \phi_\alpha + \psi_p^\dagger \varphi_\alpha^\dagger \psi_p \varphi_\alpha \right) \\
&+ c_3\, \phi_\alpha^\dagger \varphi_\alpha^\dagger \varphi_\beta \phi_\beta + c_4\, \phi_\alpha^\dagger \varphi_\beta^\dagger \varphi_\gamma \phi_\delta\, T^a_{\alpha\beta} T^a_{\gamma\delta} + c_5 \left(\phi_\alpha^\dagger \phi_\beta^\dagger \phi_\beta \phi_\alpha + \varphi_\alpha^\dagger \varphi_\beta^\dagger \varphi_\beta \varphi_\alpha \right) \Big\}
\end{aligned}
$$

Last term in $\mathcal{L}_{\rm NREFT}$ is relevant because for $\eta\eta \to qq$ (due to χ exchange)

$$
\begin{aligned}
c_1 &= 0\,, \quad c_2 = \frac{|y|^2 (|h|^2 + g_s^2 C_F)}{128\pi M^2}\,, \\
c_3 &= \frac{1}{32\pi M^2} \left(\lambda_3^2 + \frac{g_s^4 C_F}{N_c} \right), \quad c_4 = \frac{g_s^4 (N_c^2 - 4)}{64\pi M^2 N_c}\,, \quad c_5 = \frac{|y|^4}{128\pi M^2}
\end{aligned}
$$

S.B. and M. Laine [1801.05821]

Thermal potentials and $\bar{S}_i$

$$\langle \sigma_{\rm eff} \, v \rangle = \frac{2c_1 + 4c_2 N_c \, e^{-\Delta M_T/T} + [c_3 \bar{S}_3 + c_4 \bar{S}_4 C_F + 2c_5 \bar{S}_5 (N_c + 1)] N_c \, e^{-2\Delta M_T/T}}{\left(1 + N_c \, e^{-\Delta M_T/T}\right)^2}$$

- with $\Delta M_T \equiv \Delta M + \frac{(g_s^2 C_F + \lambda_3) T^2}{12M} - \frac{g_s^2 C_F m_D}{8\pi}$

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$$\langle \sigma_{\rm eff}\, v \rangle = \frac{2c_1 + 4c_2 N_c\, e^{-\Delta M_T/T} + [c_3 \bar{S}_3 + c_4 \bar{S}_4 C_F + 2c_5 \bar{S}_5 (N_c+1)] N_c\, e^{-2\Delta M_T/T}}{\left(1 + N_c\, e^{-\Delta M_T/T}\right)^2}$$

- with $\Delta M_T \equiv \Delta M + \frac{(g_s^2 C_F + \lambda_3) T^2}{12M} - \frac{g_s^2 C_F m_D}{8\pi}$
- the thermally modified Sommerfeld factors are defined as

$$\bar{S}_i = \frac{\int_{-\Lambda}^{\infty} \frac{dE'}{\pi} e^{-E'/T} \rho_i(E')}{\int_{-\Lambda}^{\infty} \frac{dE'}{\pi} e^{-E'/T} \rho_{\rm free,i}(E')} = \left(\frac{4\pi}{MT}\right)^{\frac{3}{2}} \int_{-\Lambda}^{\infty} \frac{\mathrm{d}E'}{\pi}\, e^{[\mathrm{Re}\mathcal{V}_i(\infty) - E']/T}\, \frac{\rho_i(E')}{N_i}$$

Thermal potentials and $\bar{S}_i$

$$\langle \sigma_{\rm eff}\, v \rangle = \frac{2c_1 + 4c_2 N_c\, e^{-\Delta M_T/T} + [c_3 \bar{S}_3 + c_4 \bar{S}_4 C_F + 2c_5 \bar{S}_5 (N_c+1)] N_c\, e^{-2\Delta M_T/T}}{\left(1 + N_c\, e^{-\Delta M_T/T}\right)^2}$$

- with $\Delta M_T \equiv \Delta M + \frac{(g_s^2 C_F + \lambda_3) T^2}{12M} - \frac{g_s^2 C_F m_D}{8\pi}$
- the thermally modified Sommerfeld factors are defined as

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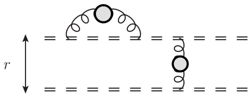
- potential from the static limit of the HTL resummed temporal gluon propagator



$$v(r) \equiv \frac{g_s^2}{2} \int_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \left[\frac{1}{\boldsymbol{k}^2 + m_D^2} - i\frac{\pi T}{k} \frac{m_D^2}{(\boldsymbol{k}^2 + m_D^2)^2} \right]$$

$$m_D = g_s \sqrt{\frac{N_c}{3} + \frac{N_f}{6}}\, T$$

pNCQD for colored scalars

Integrating out $1/r \sim m_D$ from NRQCD$_{\rm HTL}$ Brambilla, Ghiglieri, Petreczky, Vairo [0804.0993]

$$\mathcal{L}_{\rm pNRQCD_{m_D}} = \mathcal{L}_{\rm gauge} + \int d^3r {\rm Tr} \Big\{ S^\dagger \left[\partial_0 - V_s - \delta M_s\right] S + O^\dagger \left[D_0 - V_o - \delta M_o\right] O + \Sigma^\dagger \left[D_0 - V_\Sigma - \delta M_\Sigma\right] \Sigma \Big\}$$

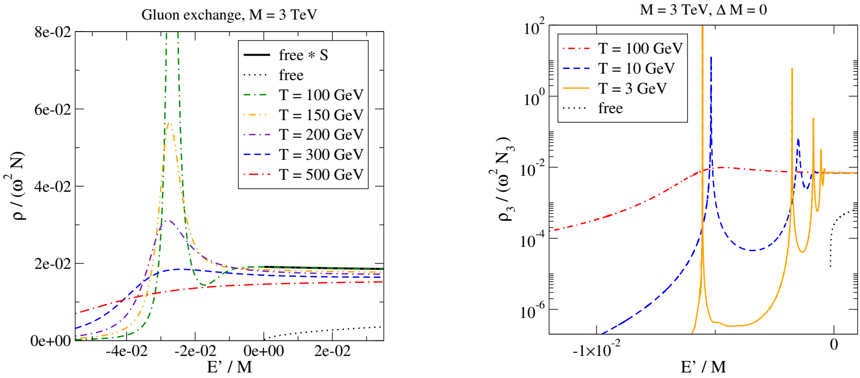
- with equal mass shifts $\delta M_s = \delta M_o = \delta M_\Sigma = -\alpha_s C_F \left(m_D + iT\right)$ and potentials

$$V_s(r) = \alpha_s C_F \left[-\frac{e^{-m_D r}}{r} + iT\,\Phi_r(m_D r) \right], \quad V_o(r) = \frac{\alpha_s}{2N_c} \left[\frac{e^{-m_D r}}{r} - iT\Phi_r(m_D r) \right]$$

$$V_\Sigma(r) = \frac{\alpha_s C_F}{N_c + 1} \left[\frac{e^{-m_D r}}{r} - iT\Phi_r(m_D r) \right]$$

- where $\Phi(m_D r) = \frac{2}{m_D r} \int_0^\infty dz \, \frac{\sin(z m_D r)}{(1+z^2)^2}$ Burnier, Laine and Vepsälïnen [0711.1743]

Spectral function, bound states and melting



- bound states start to form at $z \sim 20$, $z \equiv M_\chi / T$

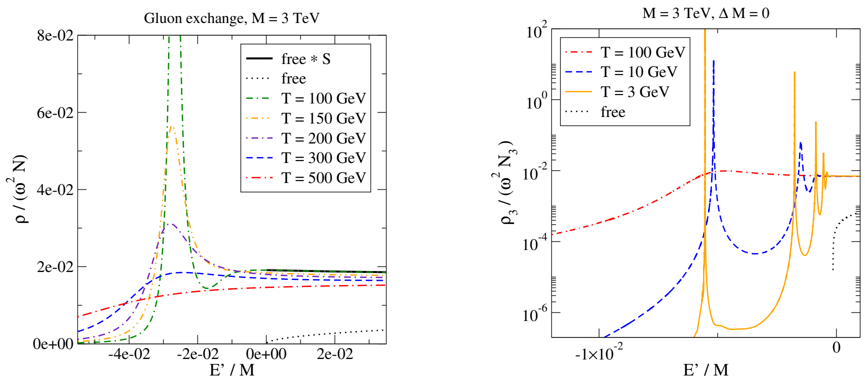
  (left plot) Laine and Kim [1609.00474]; (right plot) S.B. and M. Laine [1801.05821])

- progressive appearance of many bound states for decreasing temperature

  → similar to melting pattern for heavy quarkonia in QGP

- be aware of the stretching of the HTL potential down to small temperatures

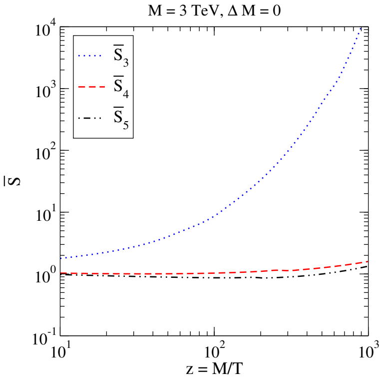
Spectral function, bound states and melting

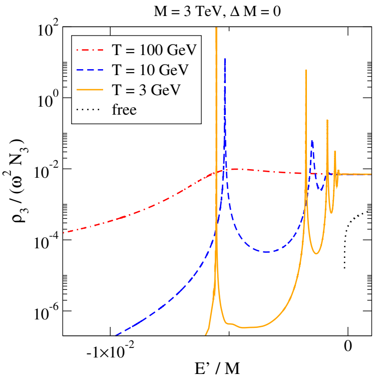


- $\rho \to \bar{S}_i \to \langle \sigma_{\mathrm{eff}} v \rangle$, relic density from Boltzmann equation ($Y = n/s$)

$$Y'(z) = -\langle \sigma_{\mathrm{eff}} v \rangle M m_{\mathrm{Pl}} \frac{c(T)}{\sqrt{24\pi\, e(T)}} \frac{Y^2(z) - Y_{\mathrm{eq}}^2(z)}{z^2} \Big|_{T=M/z}$$

Spectral function, bound states and melting





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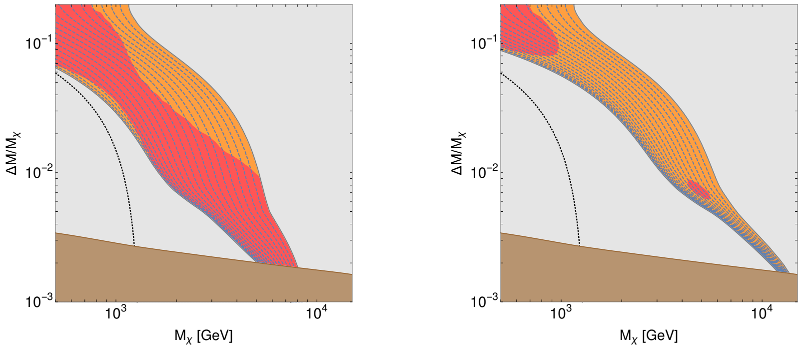
Parameter space for DM$_\chi$ + η S.B. and S. Vogl [1811.02581]

◇**valence-quark scenario**

$$\mathcal{L}_{\rm int} = -y\, \eta^\dagger \bar{\chi} P_R q - y^*\, \bar{q} P_L \chi \eta - \lambda_3 \eta^\dagger \eta H^\dagger H\,, \quad q \equiv u, d$$



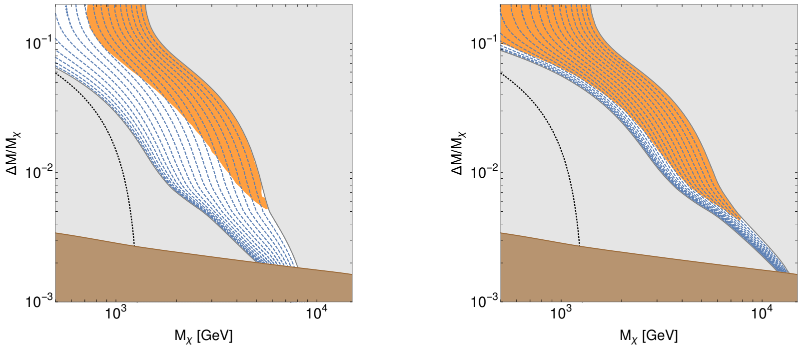
- $2\Delta M > |E_1|$ S.B. and M. Laine [1801.05821], left $\lambda_3 = 0$, right $\lambda_3 = 1.5$, $y \in [0.1, 2]$
- dotted-black $y = 0.3$ (free); λ_3 boost the annihilations because $c_3 \bar{S}_3 \approx (g_s^4 + \lambda_3^2)\bar{S}_3$
- Xenon1T sensitivity and DARWIN-like detector sensitivity

$M_\chi \simeq 1.2\,\mathrm{TeV} \to M_\chi \simeq 2.0\,(3.1)\,\mathrm{TeV}$ for $\Delta M/M_\chi = 10^{-2}$ and $\lambda_3 = 0.0\,(1.5)$

Parameter space for $\mathrm{DM}_\chi + \eta$ S.B. and S. Vogl [1811.02581]

◇ **top-quark scenario**

$$\mathcal{L}_{\rm int} = -y\,\eta^\dagger \bar{\chi} P_R q - y^*\,\bar{q} P_L \chi \eta - \lambda_3 \eta^\dagger \eta H^\dagger H\,, \quad q \equiv t$$



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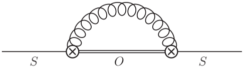
Fermion coannihilator and pNRQCD

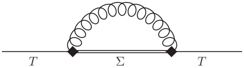
$$\mathcal{L}^{F\bar{F}}_{\rm pNRQCD} = \int d^3r \,{\rm Tr}\left\{ {\rm S}^\dagger \left[i\partial_0 - \mathcal{V}_s - \delta M_s \right] {\rm S} + {\rm O}^\dagger \left[iD_0 - \mathcal{V}_o - \delta M_o \right] {\rm O} \right\}$$
$$+ \int d^3r \left({\rm Tr}\left\{ {\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\,{\rm S} + {\rm S}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\,{\rm O} \right\} + \frac{1}{2}{\rm Tr}\left\{ {\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\,{\rm O} + {\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\,{\rm O} \right\} \right) + \cdots$$

$$\mathcal{L}^{FF}_{\rm pNRQCD} = \int d^3r \,{\rm Tr}\left\{ {\rm T}^\dagger \left[iD_0 - \mathcal{V}_{\rm T} - \delta M_T \right] {\rm T} + \Sigma^\dagger \left[iD_0 - \mathcal{V}_\Sigma - \delta M_\Sigma \right] \Sigma \right\}$$
$$+ \int d^3r \sum_{a=1}^{8}\sum_{\ell=1}^{3}\sum_{\sigma=1}^{6} \left[\left(\boldsymbol{\Sigma}^\sigma_{ij} T^a_{jk} \mathbf{T}^\ell_{ki} \right) \Sigma^{\sigma\dagger} \boldsymbol{r}\cdot g\boldsymbol{E}^a\,{\rm T}^\ell - \left(\mathbf{T}^\ell_{ij} T^a_{jk} \boldsymbol{\Sigma}^\sigma_{ki} \right) {\rm T}^{\ell\dagger} \boldsymbol{r}\cdot g\boldsymbol{E}^a\,\Sigma^\sigma \right] + \cdots$$

$\Rightarrow$ **bound states** from singlet **and** antitriplet

[for fermion-fermion pNRQCD see Brambilla, Rosch, Vairo [hep-ph/0506065]]





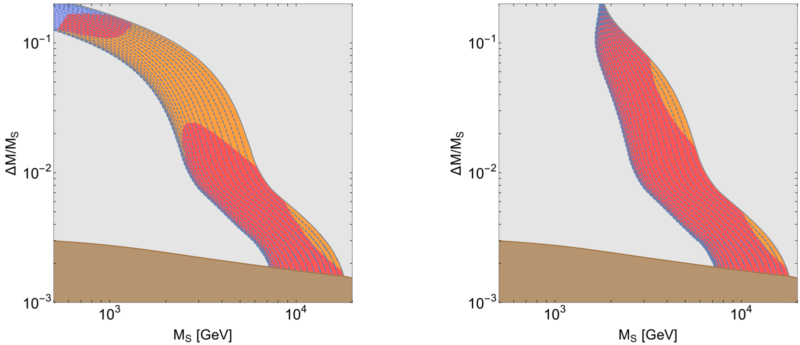
- HTL 2 $\to$ 2 complemented with the case $M_F v \sim T \gg m_D \gg E$

  imaginary part from Brambilla, Escobedo, Ghiglieri, Vairo [1303.6097], real part from S.B. and S. Vogl [1907.05766]

  for the abelian case see Escobedo and Soto [0804.0691]

Parameter space for $\mathrm{DM}_S + F$ S.B. and S. Vogl [1907.05766]

◇ **valence-quark scenario** $\mathcal{L}_{\rm int} = -y\, S\bar{F}P_R q - y^* S\bar{q}P_L F - \frac{\lambda_3}{2} S^2 H^\dagger H$

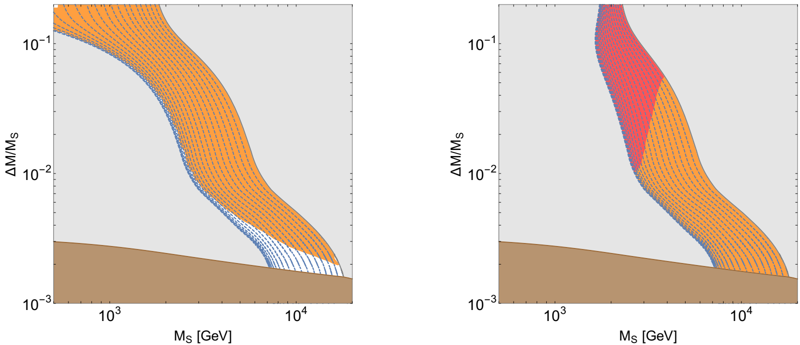


- small region sensitive to LHC searches for valence quarks
- throat-like behaviour recover the importance of $SS \to H^\dagger H$ at large ΔM;
- **bound states** from antitriplet (FF): $3 \otimes 3 = \bar{3} \oplus 6$; running for $\lambda_3(2M_S) \to \lambda_3(\mu_p)$

PARAMETER SPACE FOR DM$_S$ + F S.B. AND S. VOGL [1907.05766]

◇ **top-quark scenario**

$$\mathcal{L}_{\rm int} = -y\, S\bar{F}P_R q - y^* S\bar{q}P_L F - \frac{\lambda_3}{2} S^2 H^\dagger H$$



- no LHC sensitivity searches for top quarks
- throat-like behaviour recover the importance of $SS \to H^\dagger H$ at large ΔM; for $\lambda_3 = 1.5$, $(m_t/M_S)^2$ suppression is mild
- **bound states** from antitriplet (FF): $3 \otimes 3 = \bar{3} \oplus 6$; running for $\lambda_3(2M_S) \to \lambda_3(\mu_p)$

Conclusions and...

- the freeze-out calculation is factorized into $\langle \sigma v \rangle \approx c_i \langle \mathcal{O}_i \rangle_T$
- c_i: matching coefficients at $T = 0$ of NREFTs
- $\langle \mathcal{O}_i \rangle_T$ from heavy pair spectral function $\rho(E')$, potentials from pNRQCD

$\rightarrow$ formation/melting of (many) bound states Sommerfeld effect for above-threshold scattering states

- inclusion of Landau damping and gluo-dissociation for some hierarchy of scales

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  $\Rightarrow$ parameter space compatible with relic density **change substantially**: up to 18-20 TeV

  $\Rightarrow$ experimental prospects have to be adjusted accordingly:

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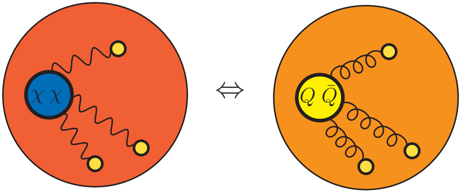
- **Limitation of the approach** (for $\Delta M \rightarrow 0$): exponential growth of the $\bar{S}$ for $T \lesssim E$
- the same issue persists in the ameliorated rate equation presented by Binder, Covi and Mukaida [1808.06472], see discussion in S.B. and M. Laine [1908.07541]

...Outlook

- NREFTs and electroweak SUSY DM, talk by M. Beneke;
- Higgs-induced bound states, talk by J. Harz; strong interacting DM, talk by T. Slatyer
- EW and coloured co-annihilations and unitarity, see talk by J. Smirnov

- improve the finite-temperature formulation of dark matter bound states by following the latest developments in heavy quarkonium [see all QCD talks!]



- Work in progress with Brambilla, Querimi, Vairo: dark matter pairs in a thermal environment with **pNREFTs** and **open quantum systems**
- pNREFT for scalar mediators S.B. and Shtabovenko 2106.06472

Back-up slides

Self-interacting dark matter

It can relax inconsistencies about

- predictions of collisionless cold DM and the observed large-scale structures
- numbers of the galactic satellite haloes
- DM density profiles in the galaxies

- figure of merit for DM self-interaction is $\sigma_{\chi\chi}/M_\chi$
- in order to relax the tensions

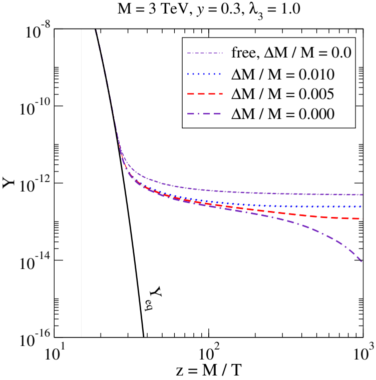
$$\frac{\sigma_{\chi\chi}}{M_\chi} \approx 1\frac{\mathrm{cm}}{\mathrm{g}} \approx 2\times 10^{-24}\frac{\mathrm{cm}^2}{\mathrm{GeV}}$$

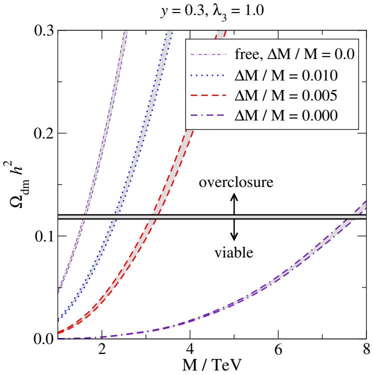
- this cross section is way larger than the one expected from electroweak physics

$$\frac{\sigma_{\chi\chi}}{M_\chi} \approx \times 10^{-38}\frac{\mathrm{cm}^2}{\mathrm{GeV}}\,, \mathrm{for}\, M_\chi \sim 100\mathrm{GeV}$$

- $\Rightarrow$ motivation for lighter scalar/gauge boson

Issue with Boltzmann and spectral function

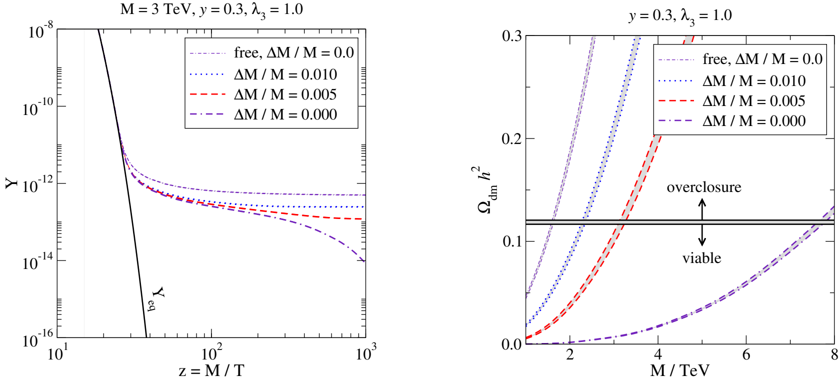




- a blind $\Delta M = 0$ brings to very large masses M, **however the splitting cannot be arbitrary small**!
- if $2\Delta M - |E_1| < 0$ the lightest two-particle states are $(\eta^\dagger \eta)$

  $\Rightarrow$ $(\chi\chi)$ rapidly convert into $(\eta^\dagger \eta)$ that are short lived and promptly decay

Issue with Boltzmann and spectral function



- What if we deal with a simpler DM model without coannihilator?
- Then exponential growth of $\bar{S}_s$ drives the DM to very small values

A different rate equation?

- Recently an alternative form of the BE has been suggested T. Binder, L. Covi and K. Mukaida (2018)

$$\dot{n} + 3Hn = -\langle \sigma v \rangle (e^{2\beta\mu(n)} - 1) n_{\rm eq}^2$$

- μ couples to the total number of dark sector particles

- number density operator, total number of particles and eq. number density

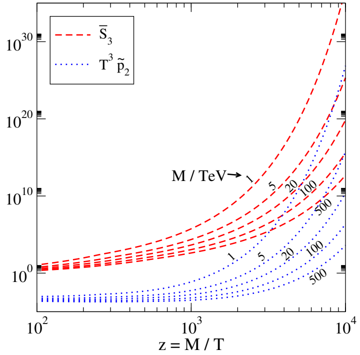
$$\hat{N} = \int_{\boldsymbol{x}} \hat{n}(\boldsymbol{x}) \,, \quad n_{\rm eq} = \lim_{\mu \to 0} \langle \hat{n} \rangle$$

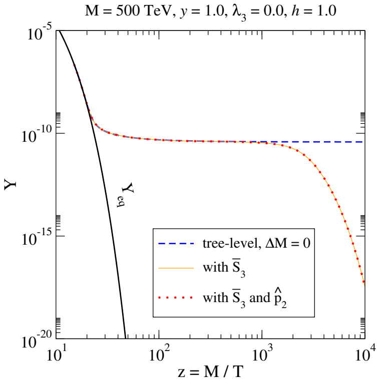
- the density matrix has the form

$$\hat{\rho} = \frac{\exp[-\beta(\hat{H} - \mu \hat{N})}{Z} \,, \quad Z = e^{p\beta V} \,, \quad n(\mu) = \frac{\partial p}{\partial \mu}$$

- expand pressure in the fugacity expansion $p = p_0 + p_1 e^{\beta\mu} + p_2 e^{2\beta\mu} + \cdots$, and obtain n

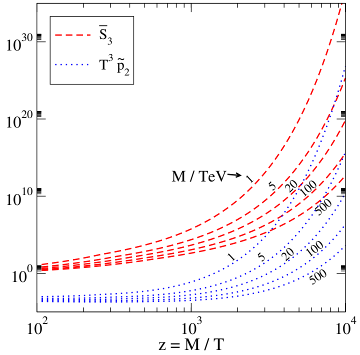
$$nT = p_1 e^{\beta\mu} + 2p_2 e^{2\beta\mu} + \cdots \,, \quad e^{\beta\mu} n_{\rm eq} \approx \frac{2n}{\sqrt{1 + 8\hat{p}_2 n}}$$

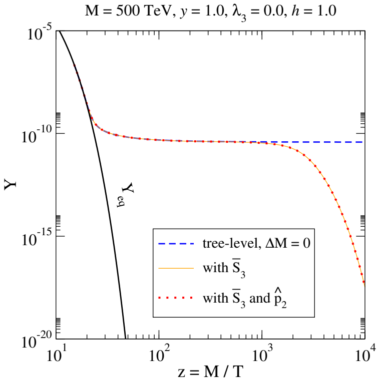




$$\hat{p}_2 \simeq \frac{N_c^2}{(N_c + e^{\beta \Delta M_T})^2}\tilde{p}_2\,, \quad T^3\tilde{p}_2 \equiv \frac{2}{N_c^2}\left(\frac{\pi T}{M}\right)^{3/2}(e^{\Delta E\beta}-1)\,, \quad \bar{S}_3 \approx \left(\frac{4\pi}{MT}\right)^{3/2}\frac{e^{\Delta E\beta}}{\pi a^3}$$

Again the same problem





$$e^{\beta\mu} n_{\rm eq} \approx \frac{2n}{\sqrt{1+8\hat{p}_2 n}}\,, \quad 8\hat{p}_2 n = 8T^3\hat{p}_2\frac{s}{T^3}Y\,, \quad \Omega_{\rm dm}h^2 = \frac{Y(z_f)M}{[3.645\times 10^{-12}\,{\rm TeV}]} \approx 0.12$$

From ρ to a Schrödinger equation

- non-relativistic dynamics:

$$E_m \equiv \omega = E' + 2M + \frac{k^2}{4M} \text{ and } H = -\frac{\nabla^2}{M} + V(r,T)$$

- the spectral function $\rho(E')$ is obtained from

$$\left[H - i\Gamma(r,T) - E'\right] G(E';\mathbf{r},\mathbf{r'}) = N\delta^3(\mathbf{r}-\mathbf{r'}) \qquad \lim_{\mathbf{r},\mathbf{r'}\to 0} \mathrm{Im}\, G(E';\mathbf{r},\mathbf{r'}) = \rho(E')$$

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- form inhomogeneous to homogeneous equation M.J. Strassler and M.E. Peskin (1991)

$$x \equiv \alpha M r \quad V \equiv \alpha^2 M \tilde{V}\,, \quad \Gamma \equiv \alpha^2 M \tilde{\Gamma}\,, \quad E' \equiv \alpha^2 M \tilde{E}'$$

- solve for the solution which is regular at the origin, $u_\ell(x) \sim x^{\ell+1}$ for $x \ll 1$

$$\left[-\frac{d^2}{dx^2} + \frac{\ell(\ell+1)}{x^2} + \tilde{V} - i\tilde{\Gamma} - \tilde{E}'\right] u_\ell(x) = 0$$

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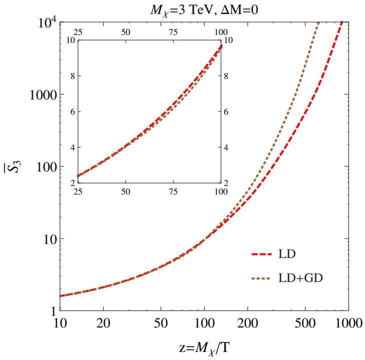
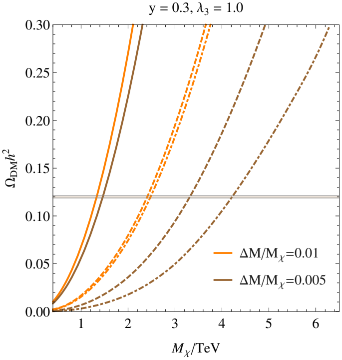
- the s-wave ($\ell = 0$) spectral function is obtained from

$$\rho(E') = \frac{\alpha\, N M^2}{4\pi} \int_0^\infty dx \mathrm{Im}\left[\frac{1}{u_0^2(x)}\right]$$

Gluo-dissociation

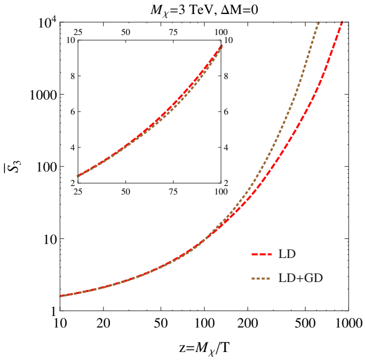




- borrow the result from heavy quarkonium $M \gg Mv \gg T \sim \Delta V$

$$\delta V_{\rm GD} = \frac{4}{3} C_F \frac{\alpha_s}{\pi} r^2 \, T^2 \Delta V f(\Delta V/T) \, , \quad \Gamma_{\rm GD} = \frac{2}{3} C_F \alpha_s r^2 (\Delta V)^3 n_B(\Delta V),$$

N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky 0804.0993

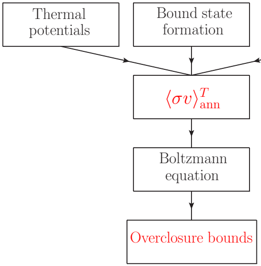


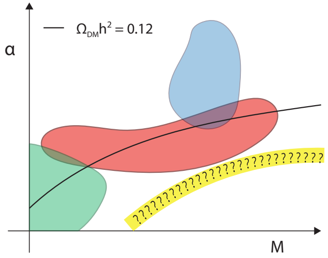
- this is not a rigorous way of implementing it: other terms are missing beyond the static limit that are of the same order in the thermal width N. Brambilla, M. A. Escobedo, J. Ghiglieri, J. Soto and A. Vairo (2010)

Summary of the theoretical framework

Relic density can be factorized in some steps
M. Laine and S. Kim 1609.00474

- Calculate the matching coefficients from the hard annihilation process, $E \sim 2M$
- Compute the static potentials and thermal widths
- Extract the spectral function $\Rightarrow$ annihilation rate
- Solve the Boltzmann equation with the **thermal** cross section





$M_F v \sim T \gg M_D \gg \Delta V$

$$\delta V^q(r,T) = -\frac{C_F}{4}\,\alpha_s\, r\, m_{D,q}^2 - C_F\,\frac{3}{2\pi}\,\alpha_s\, r^2\, T\, m_{D,q}^2\,\zeta(3) + C_F\,\frac{\alpha_s\, m_{D,q}^2}{4\,\pi^2 r T^2}\int_0^\infty \frac{dx\, F^q(xrT)}{x\,(e^{x/2}+1)}\,,$$

$$F^q(u) = \left[-4 - 3u^2 + (u^2+4)\cos(u) + u\sin(u) + (6u+u^3)\,\mathrm{Si}(u)\right],$$

$$\delta V^g(r,T) = -\frac{C_F}{4}\alpha_s\, r\, m_{D,g}^2 - C_F\,\frac{\alpha_s\, r^2\, T\, m_{D,g}^2}{\pi}\,\zeta(3) + C_F\,\frac{\alpha_s\, m_{D,g}^2}{8\,\pi^2 r T^2}\int_0^\infty \frac{dx\, F^g(x\,rT)}{x\,(e^{x/2}-1)}\,,$$

$$F^g(u) = \left[-22 - 3u^2 + (u^2+10)\cos(u) + \left(u+\frac{12}{u}\right)\sin u + (u^3+12u)\mathrm{Si}(u)\right].$$

- $m_{D,q}^2 = (g_s^2 T^2 N_f T_F)/3$ and $m_{D,g}^2 = (g_s^2 T^2 N_c)/3$
- the result agrees with known limits
  $r\,T \ll 1$ and $r\,T \gg 1$ Brambilla, Ghiglieri, Petreczky, Vairo [0804.0993]

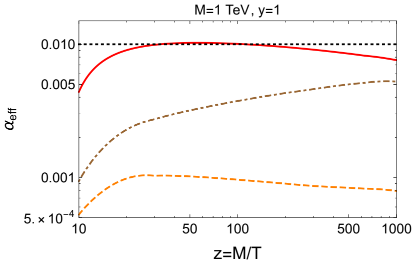
Higgs contribution

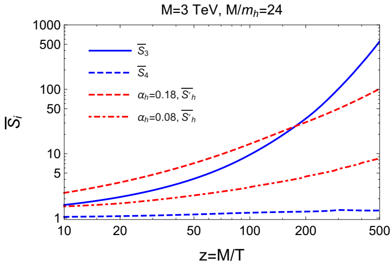
Simplified model case: $\mathcal{L}_{\rm INT} = -\lambda_3 \eta^\dagger \eta H^\dagger H$

$$\mathcal{L}^{\rm NR}_{\rm int} = -\frac{\lambda_3 v_T}{2M}(\varphi^\dagger \varphi + \phi^\dagger \phi) h \,, \quad \alpha_{\rm eff} \equiv \frac{1}{4\pi}\left(\frac{\lambda_3 v_T}{2M}\right)^2$$

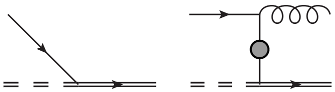
$$v_T^2 = \frac{1}{\lambda}\left[\frac{m_h^2}{2} - \frac{(g_1^2 + 3g_2^2 + 8\lambda + 4h_t^2)T^2}{16}\right]$$

- different situation if one takes $\mathcal{L}^{(2)}_{\rm int} = -g_h M_\eta \eta^\dagger \eta h + \dots$





Conversion rates: DM$_S$+F model

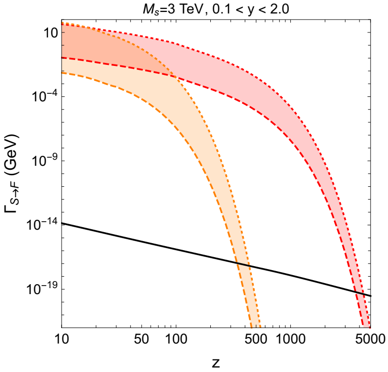


- $S + q \to F$ and $S + g + F$
- orange band: $\Delta M/M_S = 0.1$; red band: $\Delta M/M_S = 0.01$

$$\Gamma_{2\to1} = \frac{|y|^2 N_c M_S}{4\pi}\left(\frac{\Delta M}{M_S}\right)^2 n_F(\Delta M)$$

$$\Gamma_{2\to2} = \frac{|y|^2 N_c}{8 M_S}\int_{\boldsymbol{p}} \frac{\pi m_q^2 n_F\left(\Delta M + \frac{p^2}{2M_S}\right)}{p(p^2+m_q^2)}$$

$$m_q = 2 g_s^2 C_F \int_{\boldsymbol{q}} \frac{n_B(q)+n_F(q)}{q} = \frac{g_s^2 T^2 C_F}{4}$$

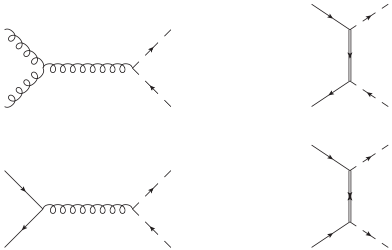


Experimental searches and relic density

- collider and direct detection experiments are sensitive to colored mediators
- relic density is almost flavour blind, whereas the quark flavour matters in the experimental searches

Collider searches at LHC: light quarks

- production channels: light quarks have significant parton luminosity



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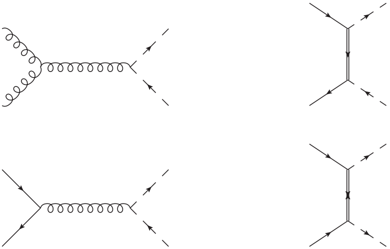
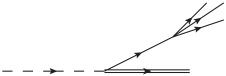


- Decay channel
  soft jets (ΔM small) and missing transverse energy



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Collider searches at LHC: top quarks

- production channels: top quarks have negligible parton luminosity

Experimental searches at colliders

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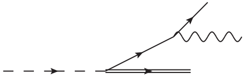
Collider searches at LHC: top quarks

- production channels: top quarks have negligible parton luminosity

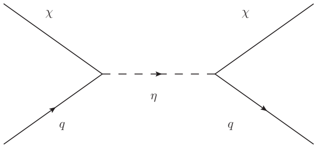
- Decay channels ($M_\eta < M_\chi + m_t$) b quarks and W (W*), then 3 or 4-body decays

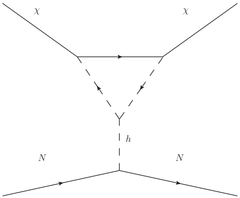


Direct detection: valence vs top quark

Valence quark

- dominant contribution to the effective DM-nucleon coupling arises from the **tree-level** exchange of η
- loop-induced coupling between the DM and the Higgs can become relevant for large λ_3, up to $\mathcal{O}(10\%)$





$$\frac{f_N}{m_N}\Big|_{\text{valence}} = -\sum_{q=u,d,s} f_{Tq}^N \left(\frac{M_\chi g_q}{2} - \frac{g_{h\chi\chi}}{2v_h m_h^2}\right) + f_{TG}^N \frac{2}{9}\frac{g_{h\chi\chi}}{2v_h\, m_h^2} - \sum_{q=u,d,s}(3q(2)+3\bar{q}(2))\frac{M_\chi g_q}{2}$$

Inert doublet model

- Supplement SM with χ SU(2) doublet, no coupling with fermions, unbroken vacuum
- We focus on the high-mass regime of the model: $M \gtrsim 530$ GeV

T. Hambye, F.-S. Ling, L. Lopez Honorez and J. Rocher, 0903.4010

$$\mathcal{L}_\chi = (D^\mu \chi)^\dagger (D_\mu \chi) - M^2 \chi^\dagger \chi - \left\{ \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 \phi^\dagger \phi \, \chi^\dagger \chi + \lambda_4 \phi^\dagger \chi \, \chi^\dagger \phi + \left[\frac{\lambda_5}{2} (\phi^\dagger \chi)^2 + h.c. \right] \right\}$$

Electroweak thermal potentials

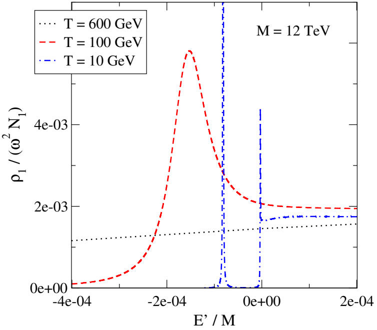
$$\mathcal{V}_{\mathrm{W}}(r) \equiv \frac{g_2^2}{4} \int_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \, i \langle W_0^+ W_0^- \rangle_{\mathrm{T}}(0, \boldsymbol{k}) \quad \text{similar for } B^\mu \text{ and } W^3$$

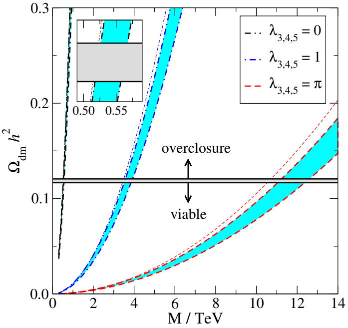
$$i \langle W_0^+ W_0^- \rangle_T = \frac{1}{\boldsymbol{k}^2 + m_{\widetilde{W}}^2} - \frac{i \pi T}{k} \frac{m_{\mathrm{E2}}^2}{(\boldsymbol{k}^2 + m_{\widetilde{W}})^2}$$

Bound states with electoweak gauge bosons

- the thermally modified Sommerfeld factors are defined as

$$\bar{S}_i = \frac{\int_{-\Lambda}^{\infty} \frac{dE'}{\pi} \rho(E') e^{-E'/T}}{\int_{-\Lambda}^{\infty} \frac{dE'}{\pi} \rho_{\text{free}}(E') e^{-E'/T}} = \left(\frac{4\pi}{MT}\right)^{\frac{3}{2}} \int_{-\Lambda}^{\infty} \frac{\mathrm{d}E'}{\pi} \, e^{[\mathrm{Re}\mathcal{V}_i(\infty) - E']/T} \, \frac{\rho_i(E')}{N_i}$$





- at small enough T bound states start to form and contribute to the annihilation cross section, up to 20% effect for large λ's S.B. and M. Laine (2017)