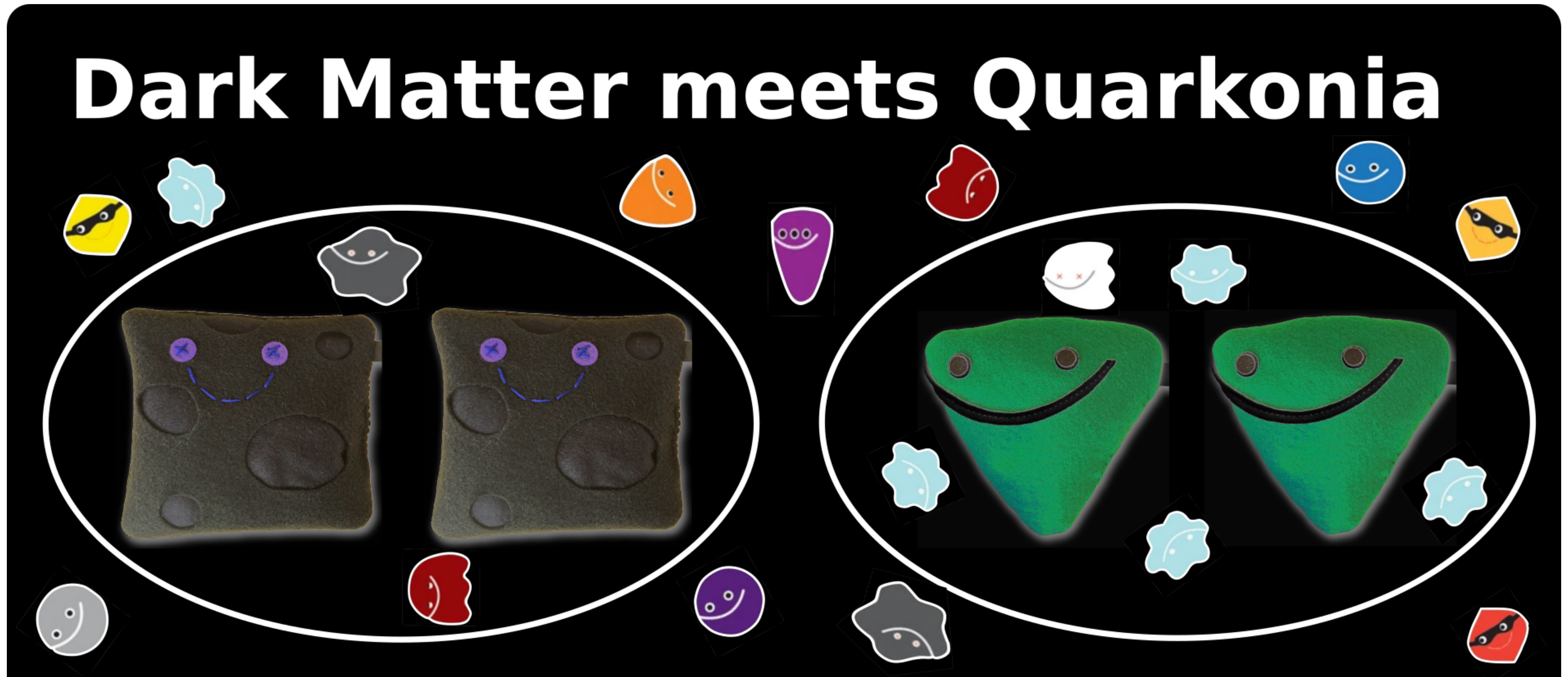
Non-Abelian Electric Field Correlator for Dark Matter Relic Abundance and Quarkonium Transport

Bound state formation and dissociation in hot non-abelian plasmas: a NLO calculation

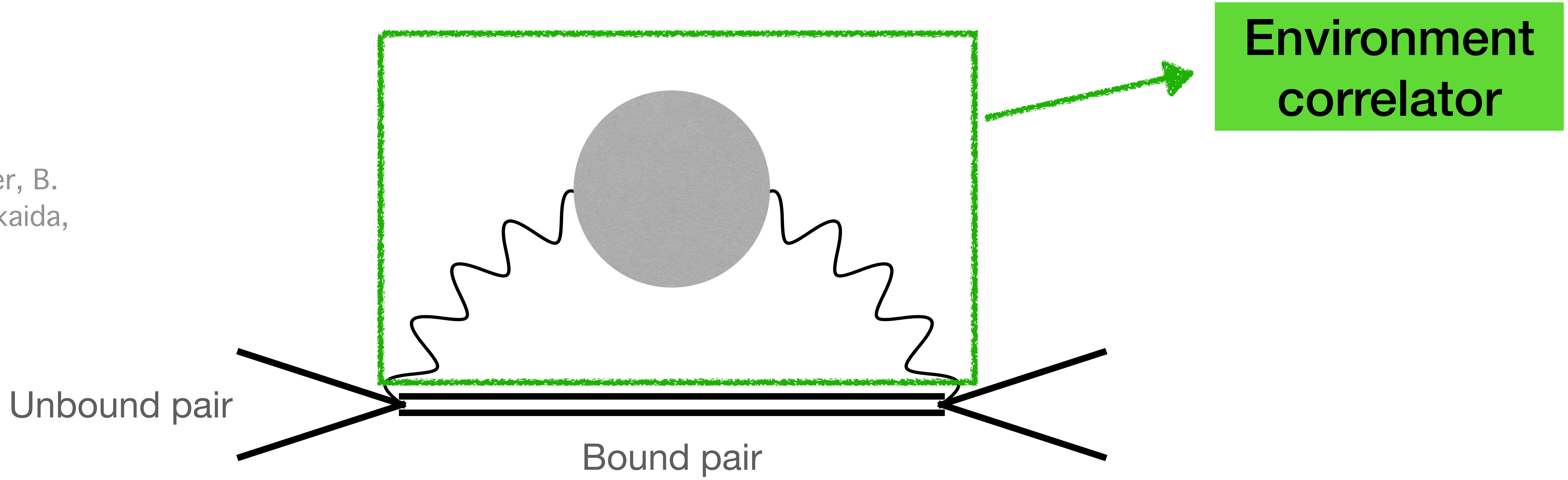
Bruno Scheihing-Hitschfeld (MIT)
in collaboration with Tobias Binder (Kavli IPMU), Kyohei Mukaida (KEK), and Xiaojun Yao (MIT)
based on 2106.XXXXX



Introduction

- Here, we focus on the effects of a $SU(N)$ non-abelian thermal environment on a pair of heavy particles, charged under $SU(N)$.

- Schematically, we can capture these by studying



Abelian case: T. Binder, B. Blobel, J. Harz, K. Mukaida, hep-ph/2002.07145

- However, this drawing is incomplete. We will see why, and why this is important.

Contents

- Open quantum systems + pNRQCD/pNREFT
- (Pre-)Lindblad equation -> Boltzmann equation
- Electric correlators of the non-abelian plasma
- The calculation at NLO:
  - Gauge invariance
  - Collinear and IR safety
  - UV divergences/Renormalization
  - Application to bound state formation

From an open quantum system to a Boltzmann transport equation

X. Yao, hep-ph/2102.01736

Unitary evolution of environment + subsystem

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Unitary evolution of environment + subsystem

⇓ Trace out the environment degrees of freedom

OQS: ρ_S has non-unitary, time-irreversible evolution

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⇓ Markovian approximation $\Longleftrightarrow$ weak coupling in H_I

OQS: Lindblad equation

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⇓ Wigner transform: $f(\mathbf{x}, \mathbf{k}, t) \equiv \int_{k'} e^{i\mathbf{k}'\cdot\mathbf{x}} \left\langle \mathbf{k} + \frac{\mathbf{k}'}{2} \middle| \rho_S(t) \middle| \mathbf{k} - \frac{\mathbf{k}'}{2} \right\rangle$

Semiclassic subsystem: Boltzmann/Fokker-Planck equation

Potential Non-Relativistic QCD (pNRQCD)
(possibly SU(N) instead of SU(3))

N. Brambilla, A. Pineda, J. Soto. A. Vairo
hep-ph/9907240, hep-ph/0410047

Consider the separation of scales $M \gg Mv \gg Mv^2, T, \Lambda_{\mathrm{QCD}}$. Then one can construct pNRQCD by systematically expanding in:

- $v \ll 1$ (nonrelativistic limit)
- $rT \sim \dfrac{T}{Mv} \ll 1$ (multipole expansion)

M: Heavy quark mass
v: Typical relative velocity of heavy quarks "inside" a quarkonium bound state
$r \sim (Mv)^{-1}$: typical size of a quarkonium state
S: field representing color singlet state
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One obtains

$$\mathcal{L}_{\text{pNRQCD}} = \mathcal{L}_{\text{light quarks}} + \mathcal{L}_{\text{gluon}} + \int d^3r \mathrm{Tr}_{\text{color}} \Big[S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O$$

$$+ V_A \left(O^\dagger \mathbf{r} \cdot g\mathbf{E} S + \mathrm{h.c.} \right) + \frac{V_B}{2} O^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, O \} + \cdots \Big]$$

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Subsystem degrees of freedom $\Longleftrightarrow H_S$

Environment degrees of freedom $\Longleftrightarrow H_E$

Interactions $\Longleftrightarrow H_I$

Summing A_0 in the octet kinetic term

- There is an issue with our power counting in the interacting theory:

$$D_0 \sim A_0 \sim \frac{p_{\text{rel}}^2}{M} \sim V_o(r),$$

so we have to sum A_0 to all orders at leading (nontrivial) power in v and r.

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- Redefine O and E_i (adjoint rep. fields):

$$O(\mathbf{R}, \mathbf{r}, t) = \mathscr{W}_{[(\mathbf{R},t),(\mathbf{R},t_0)]}\, \tilde{O}(\mathbf{R}, \mathbf{r}, t), \quad \tilde{E}_i(\mathbf{R}, t) = \mathscr{W}_{[(\mathbf{R},t_0),(\mathbf{R},t)]}\, E_i(\mathbf{R}, t)$$

where $\mathscr{W}_{[(\mathbf{R},t_f),(\mathbf{R},t_i)]} = \mathscr{P} \exp\left[ig\int_{t_i}^{t_f} ds A_0(\mathbf{R}, s)\right]$.

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X. Yao and T. Mehen, hep-ph/2009.02408

This removes the coupling to A_0 in the O kinetic term, and maintains the interaction vertex

(Pre-)Lindblad equation

One can then derive a (pre-)Lindblad equation in the quantum optical limit:

$$\rho_S(t) - \rho_S(0) = -i \sum_{a,b} \sigma_{ab}(t) \left[L_{ab}, \rho_S(0) \right] + \sum_{a,b,c,d} \gamma_{ab,cd}(t) \left(L_{ab} \rho_S(0) L_{cd}^\dagger - \frac{1}{2} \left\{ L_{cd}^\dagger L_{ab}, \rho_S(0) \right\} \right) + \mathcal{O}(H_I^3) \,.$$

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To calculate the transition rates, we need to know $\gamma_{ab,cd}$:

$$\gamma_{ab,cd}(t) \equiv \sum_{\alpha,\beta} \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \mathrm{Tr}_E \left[\langle d \,|\, O_\alpha^{(S)}(t_1) \,|\, c \rangle \langle a \,|\, O_\beta^{(S)}(t_2) \,|\, b \rangle O_\alpha^{(E)}(t_1) O_\beta^{(E)}(t_2) \rho_E \right].$$

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where $O_\alpha^{(E)} \sim \mathscr{W}_{[(\mathbf{R},t_0),(\mathbf{R},t)]} E_i(\mathbf{R}, t)$, meaning that we need to calculate (e.g. for dissociation)

$$g_{i_1 i_2}^{E++}(t_1, t_2, \mathbf{R}_1, \mathbf{R}_2) = \left\langle \mathrm{Tr}_{\mathrm{color}} \left(E_{i_1}(\mathbf{R}_1, t_1) \mathscr{W}_{[(\mathbf{R}_1,t_1),(\mathbf{R}_1,+\infty)]} \mathscr{W}_{[(\mathbf{R}_2,+\infty),(\mathbf{R}_2,t_2)]} E_{i_2}(\mathbf{R}_2, t_2) \right) \right\rangle_T$$

Boltzmann equation

Taking a Wigner transform,

$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) + \frac{\boldsymbol{k}}{2M} \cdot \nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}^{+}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}^{-}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t)$$

Recombination Dissociation

where

$$\mathscr{C}^{-}_{nl}(\mathbf{x}, \mathbf{k}, t) = \sum_{i_1, i_2} \int \frac{\mathrm{d}^3 p_{\mathrm{cm}}}{(2\pi)^3} \frac{\mathrm{d}^3 p_{\mathrm{rel}}}{(2\pi)^3} \frac{\mathrm{d}^4 q}{(2\pi)^4} (2\pi)^4 \delta^3(\mathbf{k} - \mathbf{p}_{\mathrm{cm}} + \mathbf{q}) \delta(E_{nl} - E_p + q^0)$$
$$\times d^{nl}_{i_1 i_2}(\mathbf{p}_{\mathrm{rel}}) g^{E++}_{i_1 i_2}(q^0, \mathbf{q}) f_{nl}(\mathbf{x}, \mathbf{k}, t)$$

$$\mathscr{C}^{+}_{nl}(\mathbf{x}_{\mathrm{cm}}, \mathbf{k}, t) = \sum_{i_1, i_2} \int \frac{\mathrm{d}^3 p_{\mathrm{cm}}}{(2\pi)^3} \frac{\mathrm{d}^3 p_{\mathrm{rel}}}{(2\pi)^3} \frac{\mathrm{d}^4 q}{(2\pi)^4} (2\pi)^4 \delta^3(\mathbf{k} - \mathbf{p}_{\mathrm{cm}} - \mathbf{q}) \delta(E_{nl} - E_p - q^0)$$
$$\times d^{nl}_{i_1 i_2}(\mathbf{p}_{\mathrm{rel}}) g^{E--}_{i_2 i_1}(q^0, \mathbf{q}) f^{(8)}_{Q\bar{Q}}(\mathbf{x}_{\mathrm{cm}}, \mathbf{p}_{\mathrm{cm}}, \mathbf{p}_{\mathrm{rel}}, t)$$

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Dipole transition function

Electric correlator

Heavy pairs distribution functions

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Electric correlators of non-abelian plasmas

Staple-shaped Wilson lines

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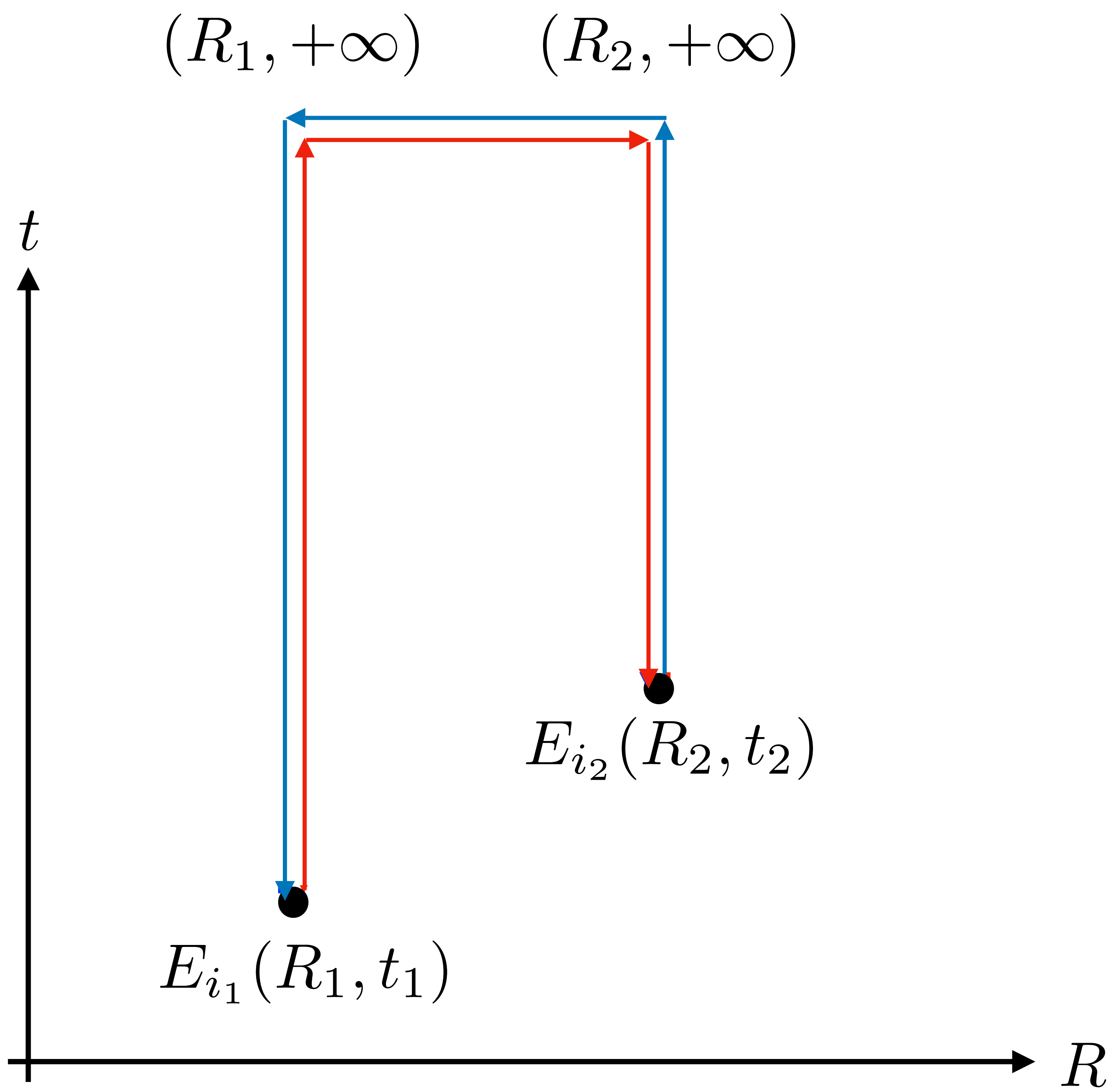
(dissociation)



$[g_E^{++}]_{i_1 i_2}(t_1, t_2, \mathbf{R}_1, \mathbf{R}_2)$

$\left\langle \mathrm{Tr}_{\mathrm{color}}\left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_1 \mathscr{W}_2 E_{i_2}(\mathbf{R}_2, t_2)\right)\right\rangle_T$

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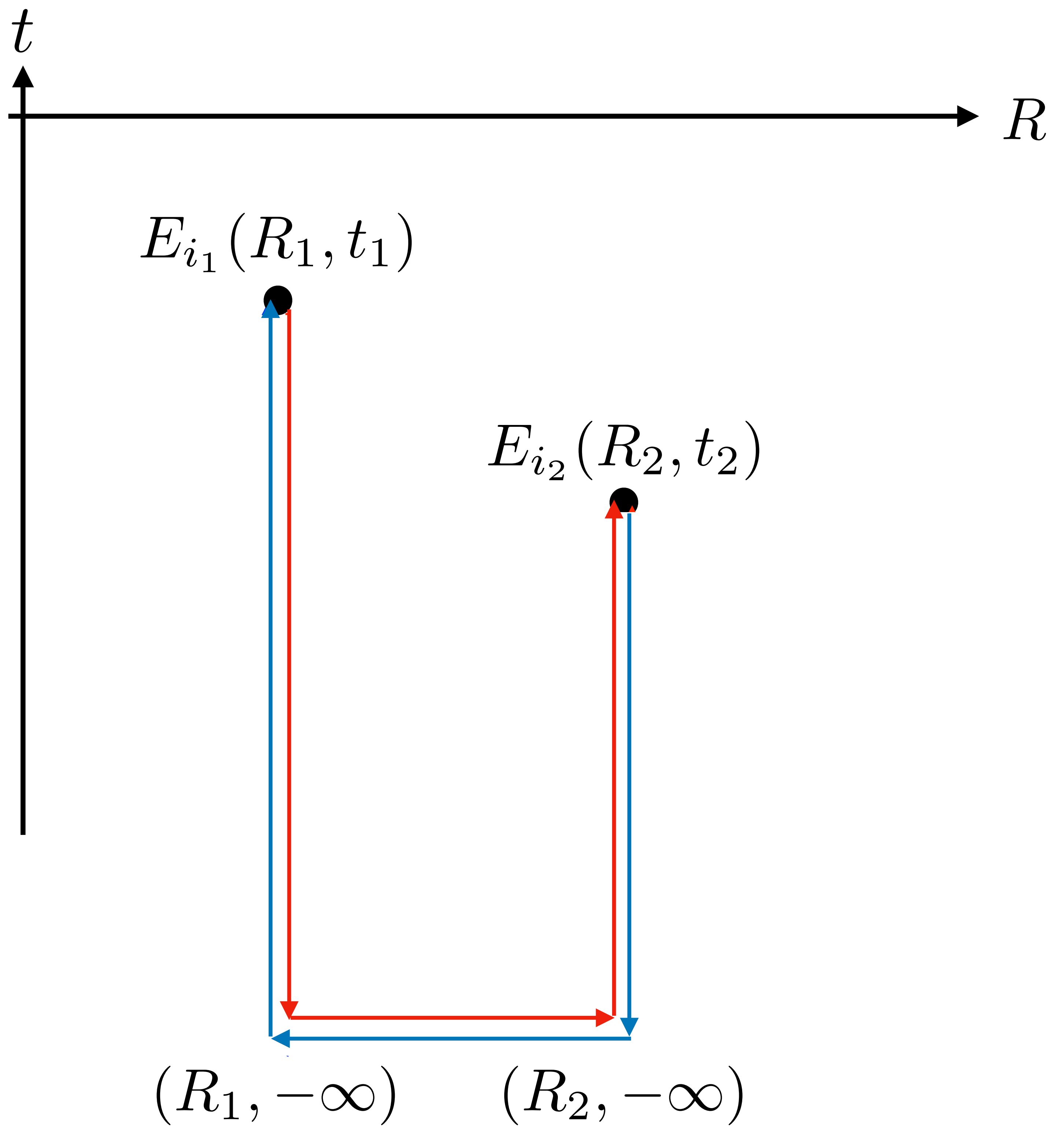


$[g_E^{--}]_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1)$

$\left\langle \mathrm{Tr}_{\mathrm{color}}\left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2) E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)\right\rangle_T$

Symmetries of the correlator

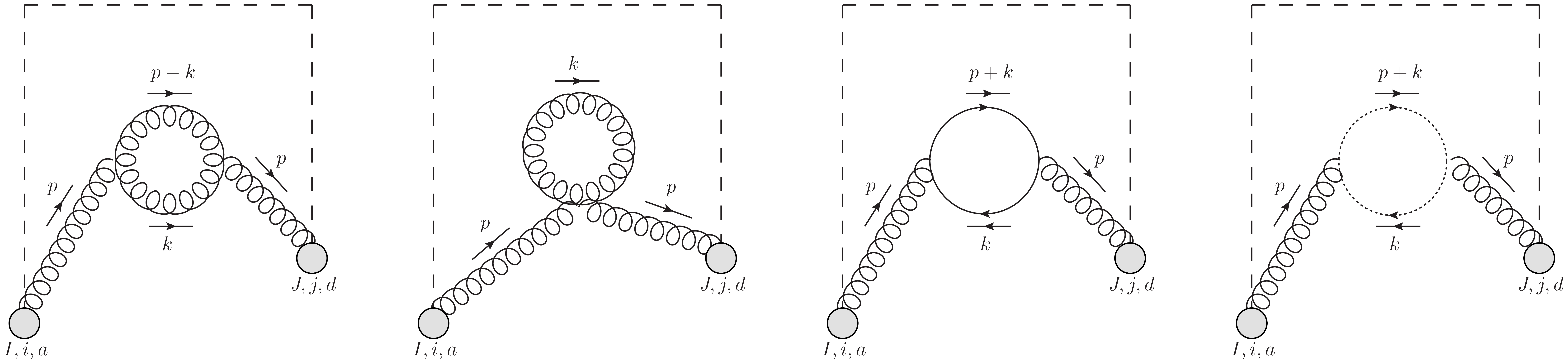
- One can show that the following KMS relations hold:
  - $[g_E^{++}]^{>}(t, \mathbf{x}) = [g_E^{++}]^{<}(t + i\beta, \mathbf{x})$
  - $[g_E^{--}]^{>}(t, \mathbf{x}) = [g_E^{--}]^{<}(t + i\beta, \mathbf{x})$
- Additionally, parity and time reversal enforce that
  - $[g_E^{++}]^{>}(t, \mathbf{x}) = [g_E^{--}]^{<}(t, \mathbf{x})$
- This means that we only need to calculate one of the electric correlators. In practice, we calculate the spectral function

$$\left[\rho_E^{++}\right](t, \mathbf{x}) \equiv \left[g_E^{++}\right]^{>}(t, \mathbf{x}) - \left[g_E^{++}\right]^{<}(t, \mathbf{x}).$$

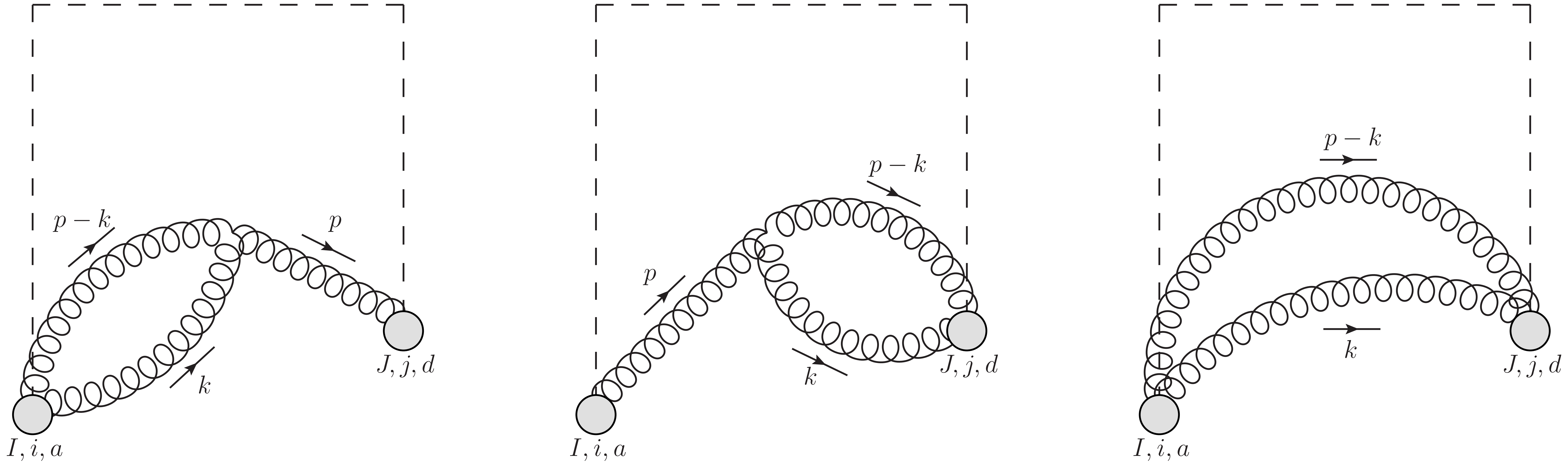
Calculating the non-Abelian electric correlator $g_E^{++}(p_0, \mathbf{p})$ at NLO

List of all contributions at NLO

We can start with the familiar diagrams

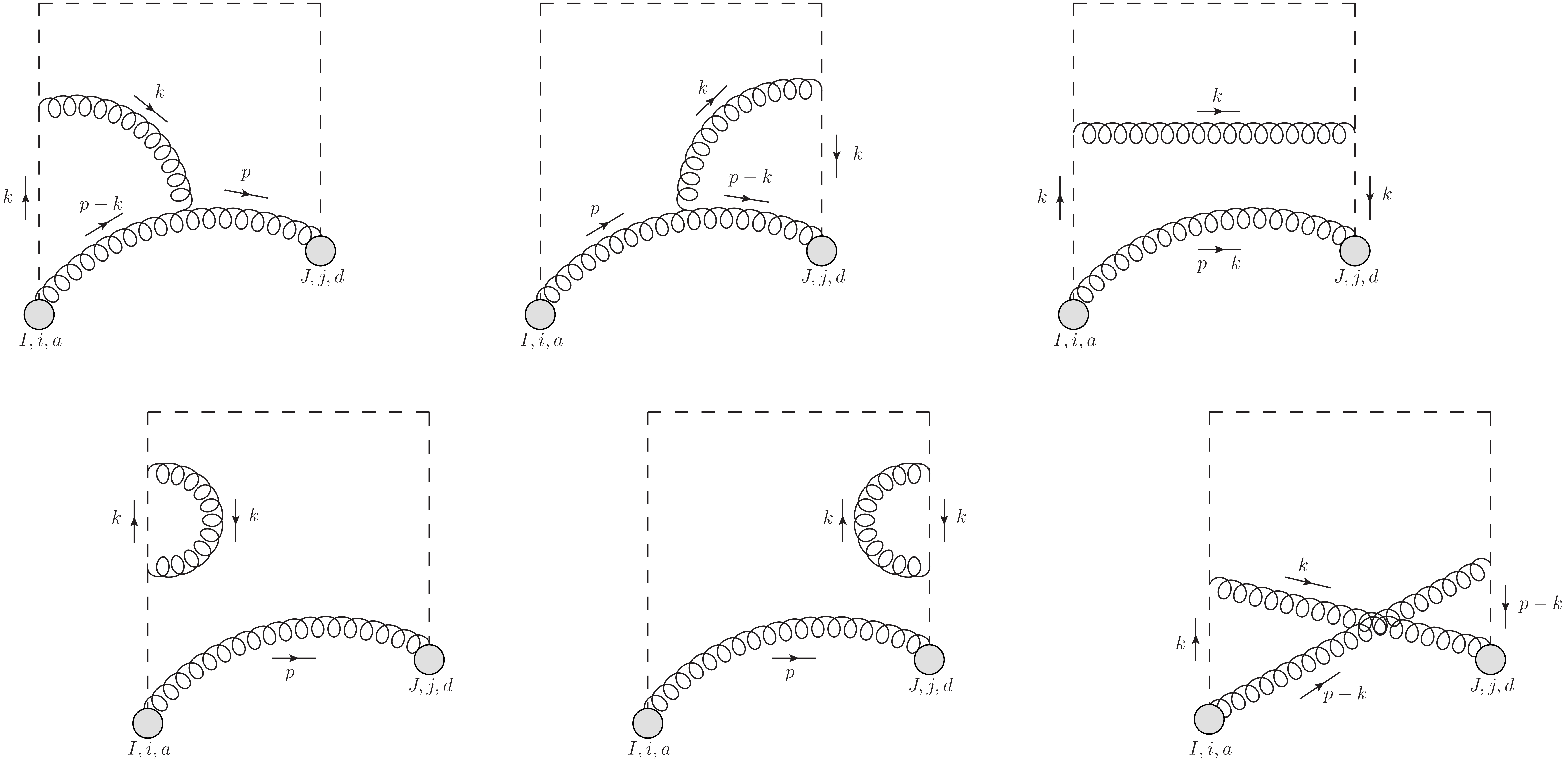


comprising the textbook Yang-Mills gauge boson self-energy.

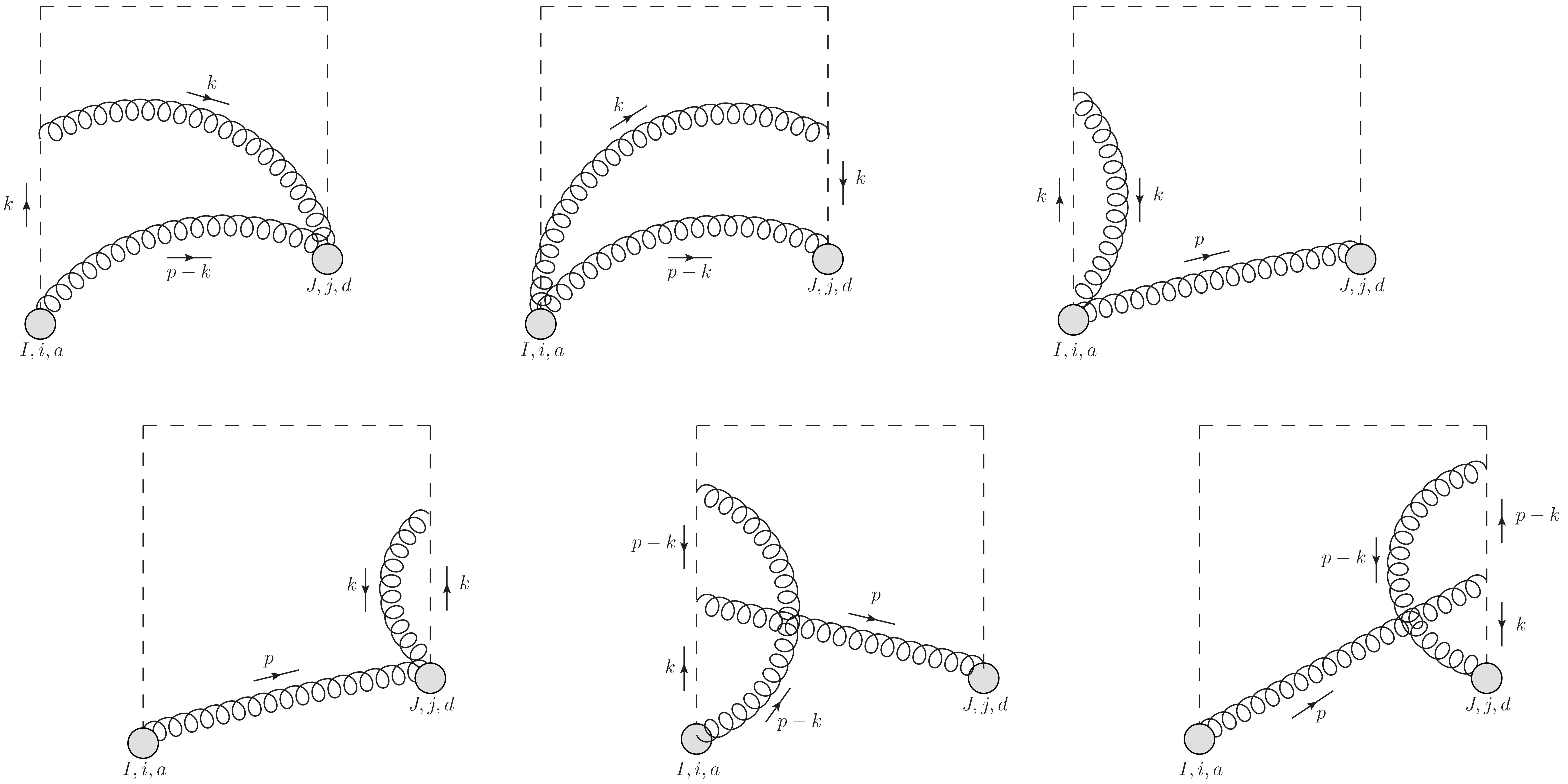


$E_i^a = \partial_0 A_i^a - \partial_i A_0^a + g f^{abc} A_0^b A_i^c \implies$ Quadratic A terms in E_i also contribute!

+ Wilson lines contributions

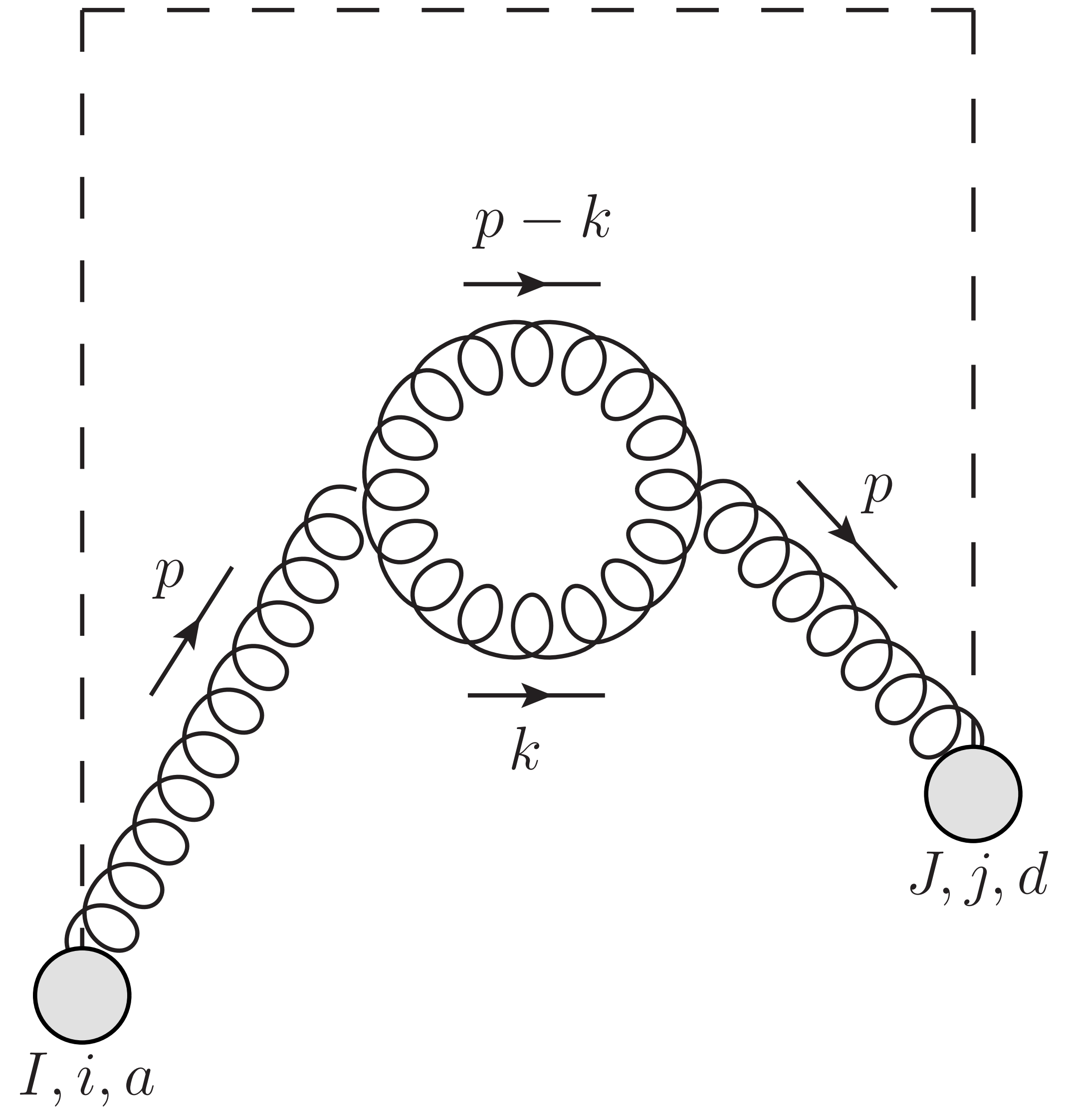


+ more Wilson lines contributions

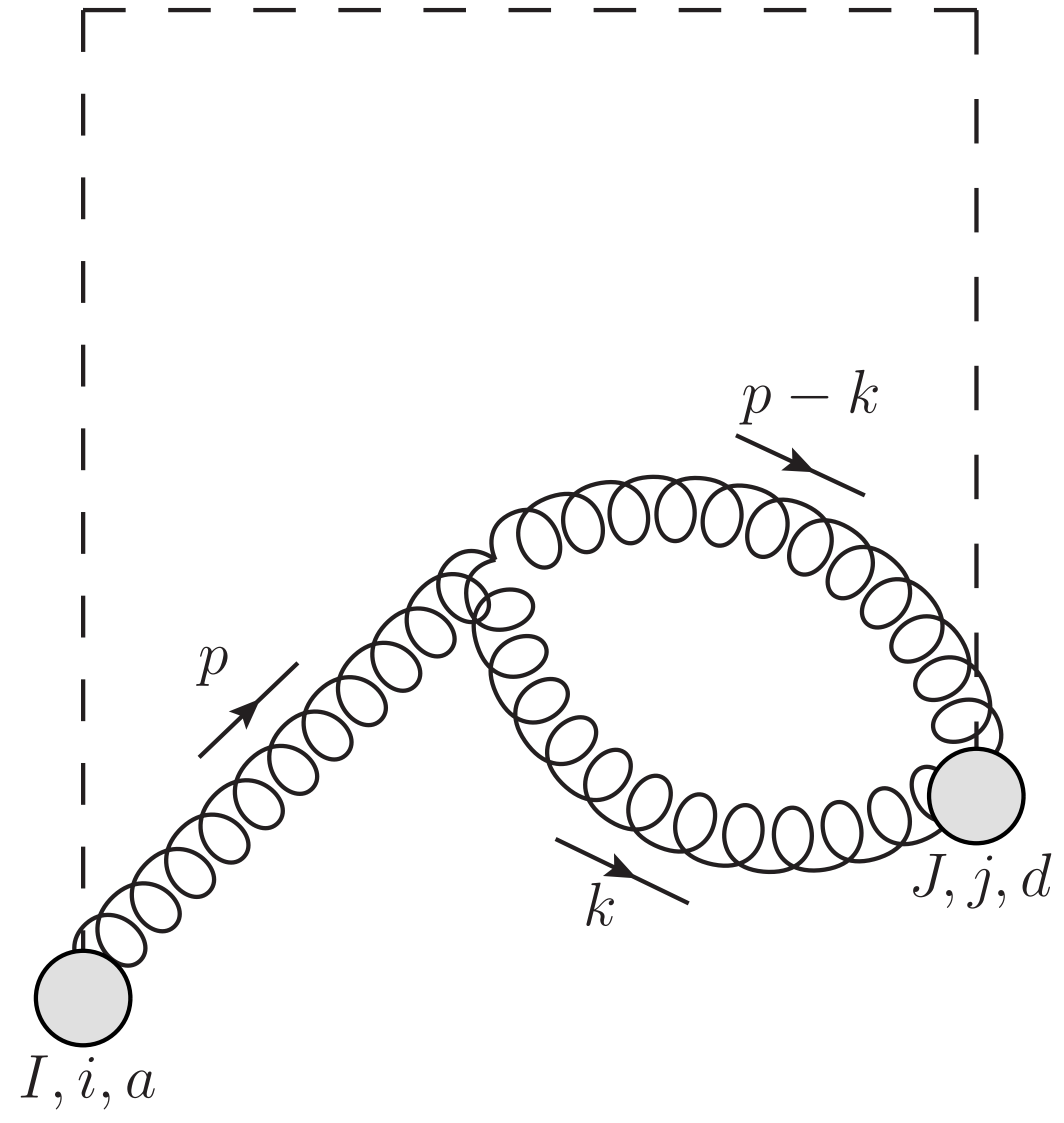


Gauge invariance (R_ξ)

- By construction, the correlators must be gauge-invariant.
- How cancellations appear:



+



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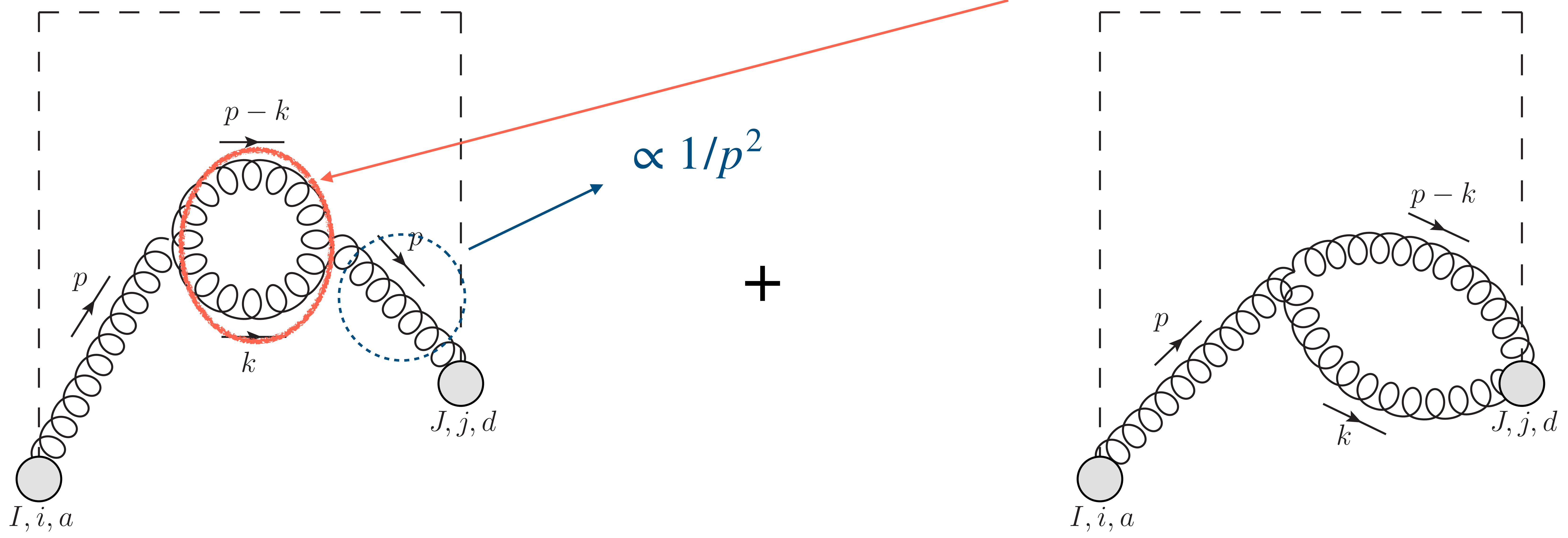
These diagrams can be added if the numerator has a factor of p^2



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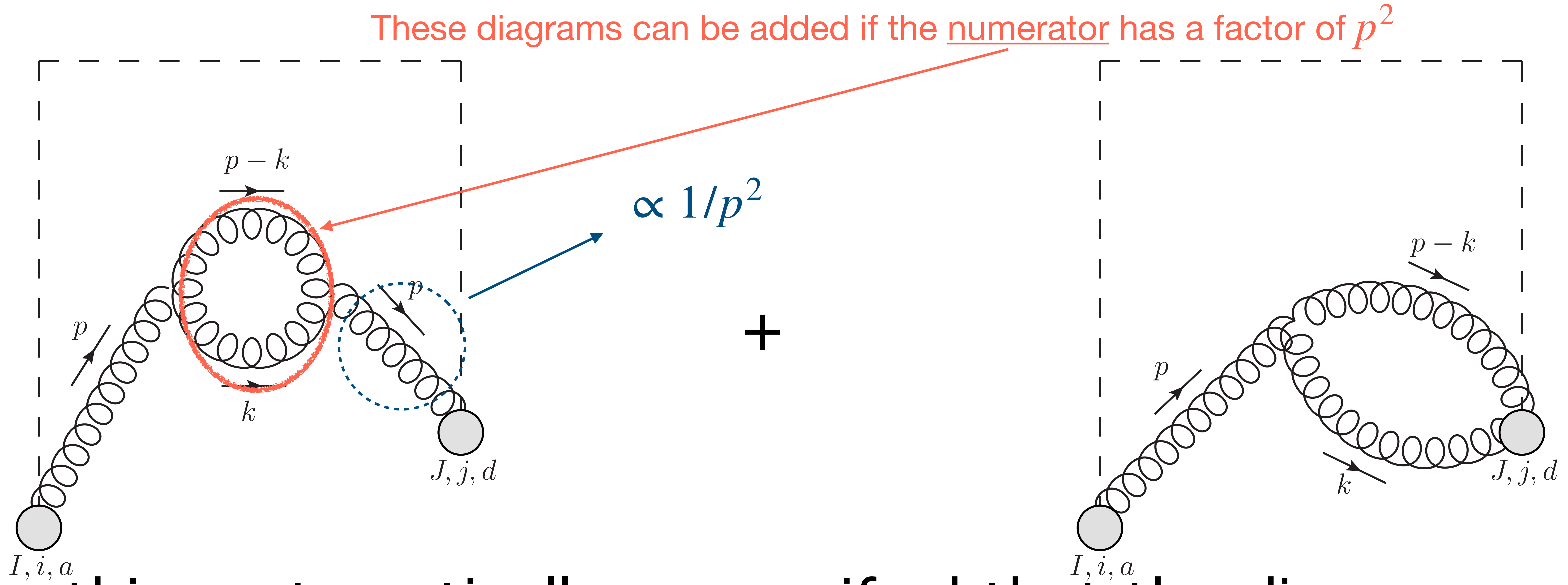
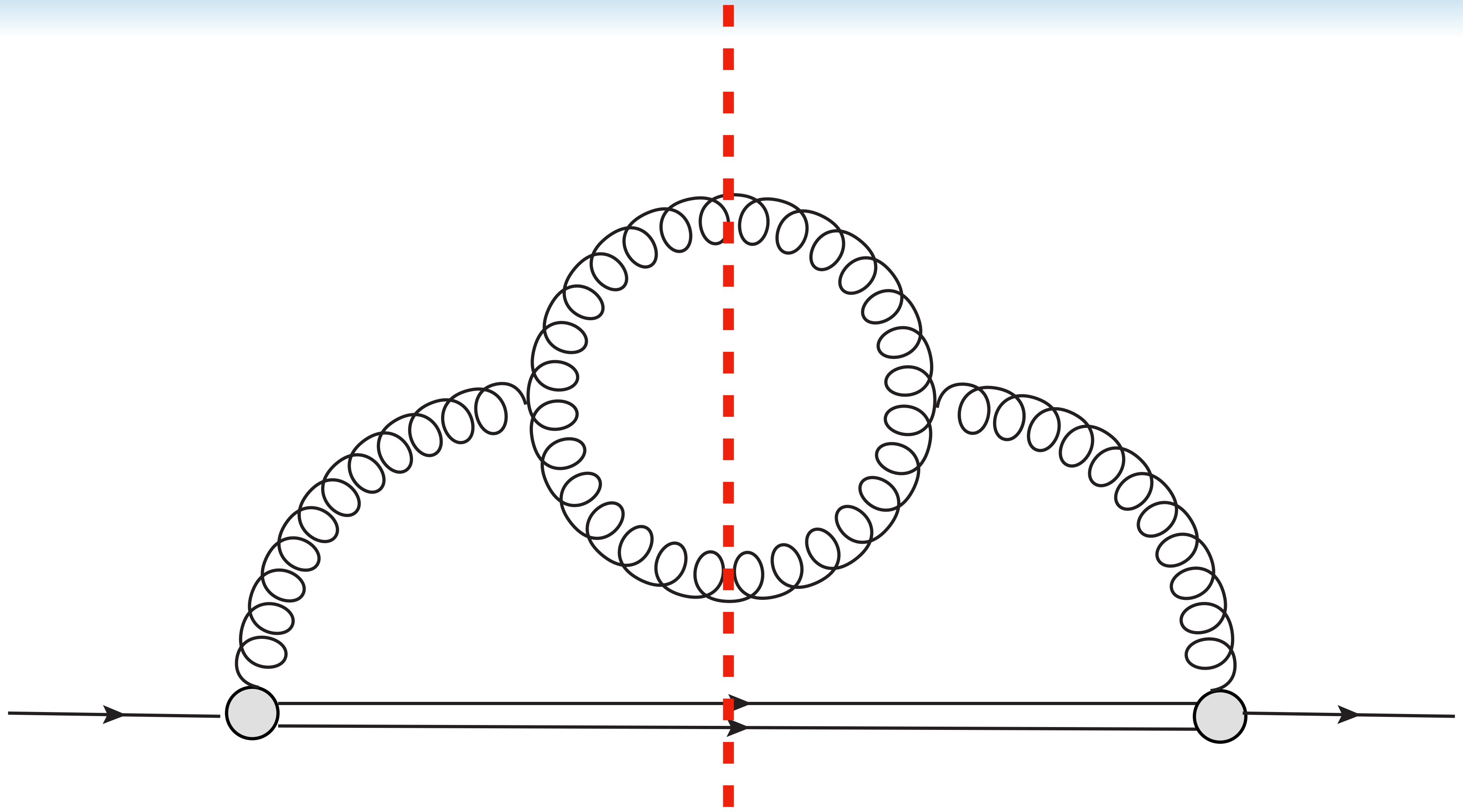
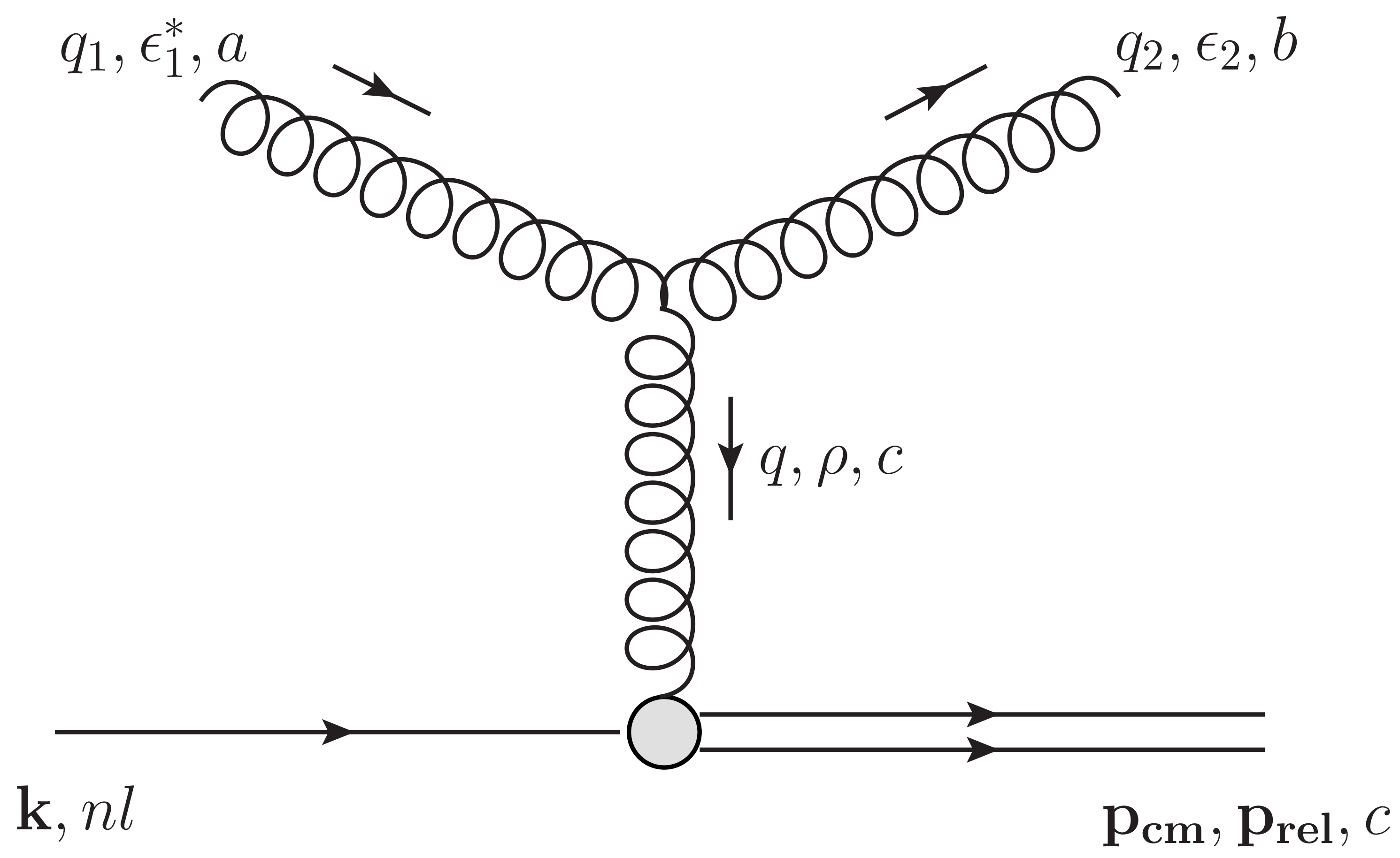


After doing this systematically, we verified that the diagrammatic calculation is explicitly gauge invariant. (the cancellation of ξ-dependent terms only happens after adding all diagrams)

Collinear safety

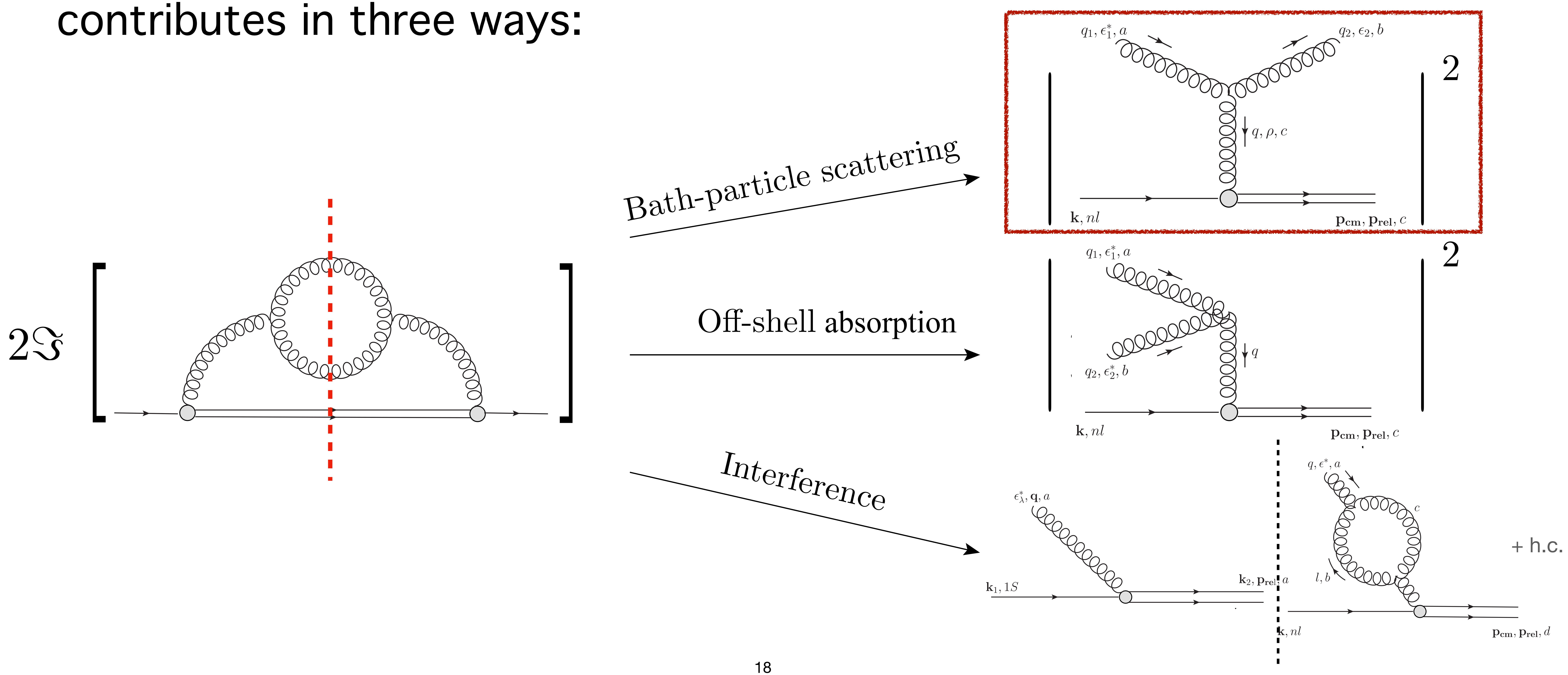
Energy flows both ways in thermal field theory cut amplitudes





- One of the reasons why we use a thermal field theory formalism is to have control over the divergences that appear in the forward scattering limit of the above diagrams

- The result is free of collinear divergences after including all terms from the electric field correlator. For example, the gauge boson loop contributes in three ways:



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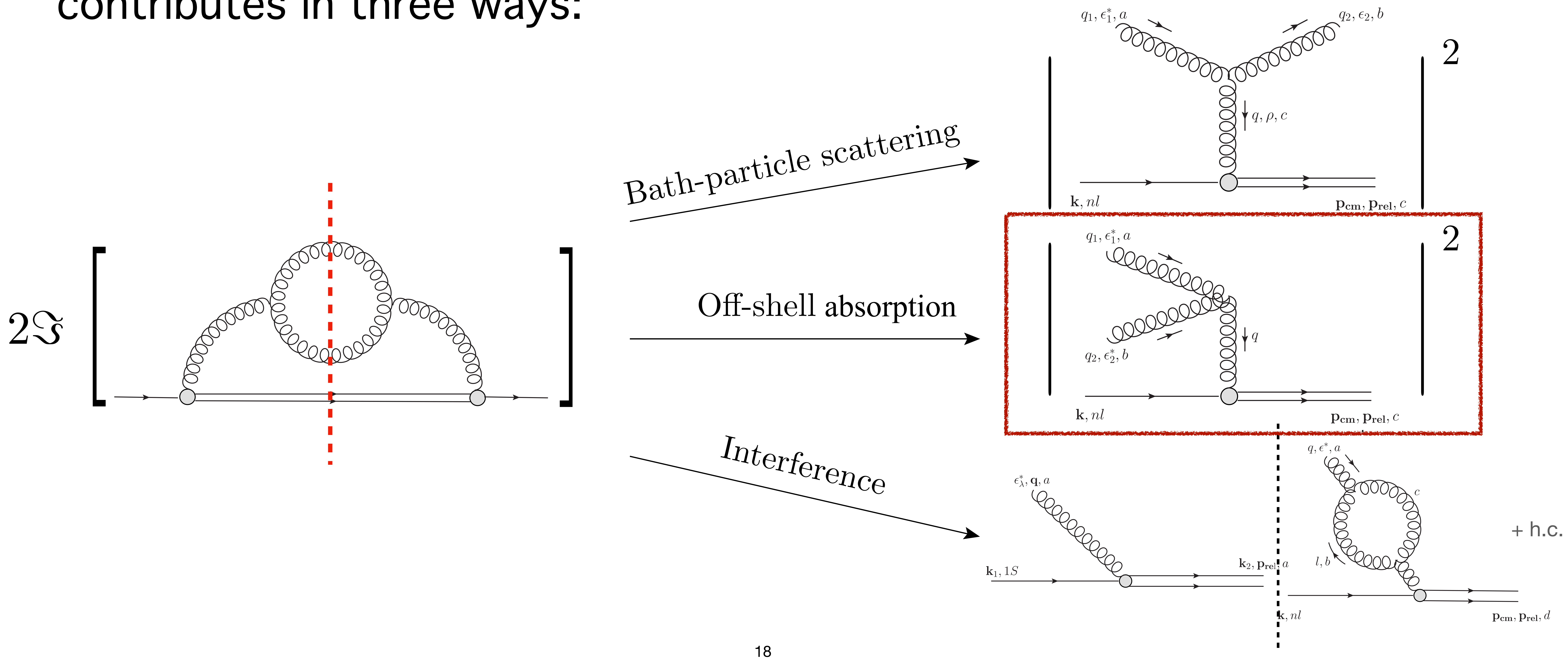

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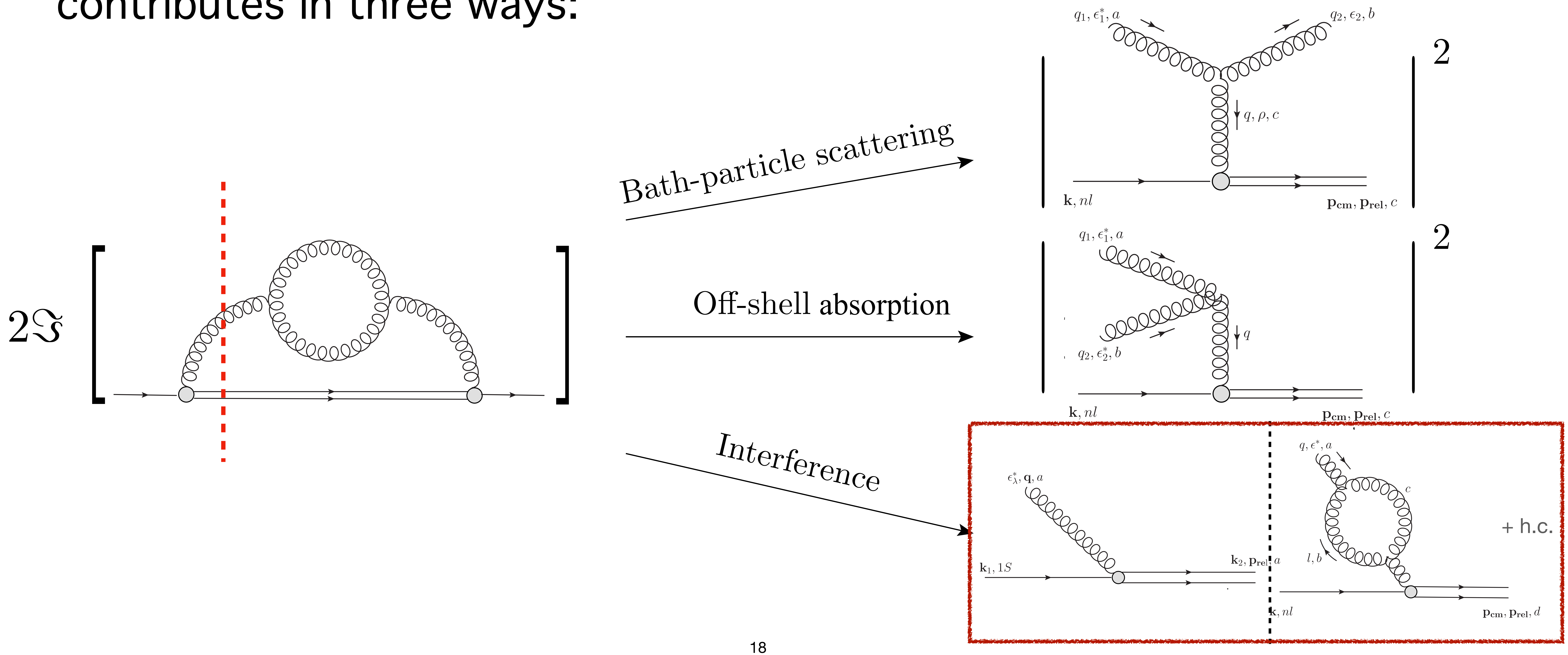
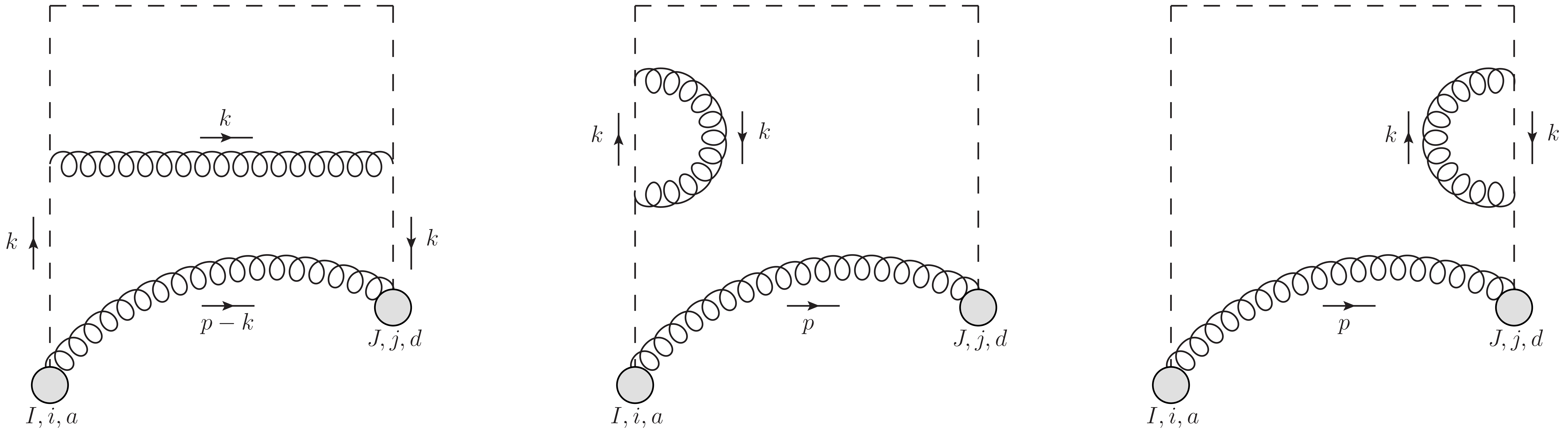


IR safety

- As the loop momentum k goes to zero, the following diagrams are individually IR divergent:



However, the sum of their IR contributions to the spectral function is finite when one integrates over the external momentum p.

UV divergences/renormalization

We can verify if our correlator needs renormalization beyond the coupling constant by calculating $\left(g^2\left[\rho_E^{++}\right](t,\mathbf{x})\right)_{\text{bare}}$ and then replacing $g^2 \to Z_{g^2} g^2$, with

$$Z_{g^2} = 1 - \frac{g^2}{8\pi^2\epsilon}\left(\frac{11N_c}{3} - \frac{4}{3}n_F T_F\right) + \left(\text{choice of scheme}\right).$$

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UV divergent contributions to the spectral function:

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UV divergent contributions to the spectral function:

Gauge boson self-energy

Contribution from Wilson lines
Crucial for gauge invariance!

UV divergences/renormalization

We can verify if our correlator needs renormalization beyond the coupling constant by calculating $\left(g^2\left[\rho_E^{++}\right](t,\mathbf{x})\right)_{\mathrm{bare}}$ and then replacing $g^2 \to Z_{g^2} g^2$, with

$$Z_{g^2} = 1 - \frac{g^2}{8\pi^2\epsilon}\left(\frac{11N_c}{3} - \frac{4}{3}n_F T_F\right) + \left(\text{choice of scheme}\right).$$

Keeping the divergent pieces only, we find (in dimensional regularization)

$$\frac{1}{2p_0^3}\int d^{d-1}\mathbf{p}\left(g^2\left[\rho_E^{++}\right](p_0,\mathbf{p})\right)_{\mathrm{bare}} = 4\pi^2 g^2 Z_g(\mu/p_0)^\epsilon + g^4\left(\left[\frac{1}{2\epsilon} + \frac{1}{2}\ln\left(\frac{\mu^2}{p_0^2}\right)\right]\left(\frac{11N_c}{3} - \frac{4}{3}n_F T_F\right) + \text{finite}\right)$$

$$= 4\pi^2 g^2 + \frac{g^4}{4}\ln\left(\frac{\mu^2}{p_0^2}\right)\left(\frac{11N_c}{3} - \frac{4}{3}n_F T_F\right) + g^4\left(\text{finite}\right)$$

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Consistent with M. Laine, hep-ph/2103.14270

Differentiating this w.r.t. $\ln\mu$ gives the non-Abelian gauge theory beta function.

Application: NLO corrections to bound state formation

- One can organize the LO + NLO terms as follows:

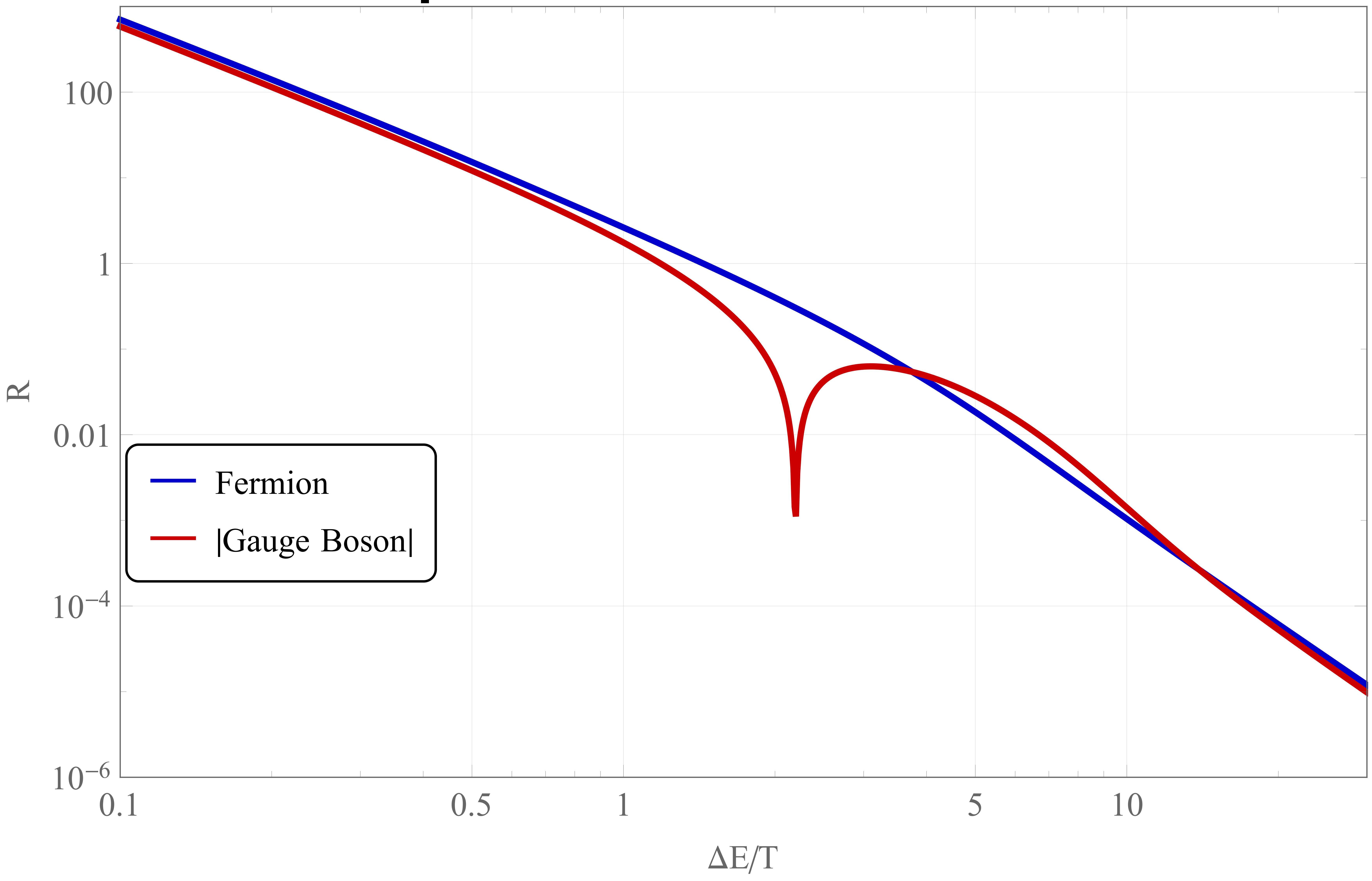
$$(\sigma v_{\rm rel})^{\rm LO+NLO}_{\mathscr{B}} = (\sigma v_{\rm rel})^{\rm LO}_{\mathscr{B}} \times \Big[1 + \alpha N_c R_g^{T=0}(\mu/\Delta E) + \alpha N_c R_g^{T\neq 0}(\Delta E/T) + \alpha N_f R_f^{T=0}(\mu/\Delta E) + \alpha N_f R_f^{T\neq 0}(\Delta E/T)\Big]$$

We find, in $\overline{\rm MS}$ (finite T plots in the next slide)

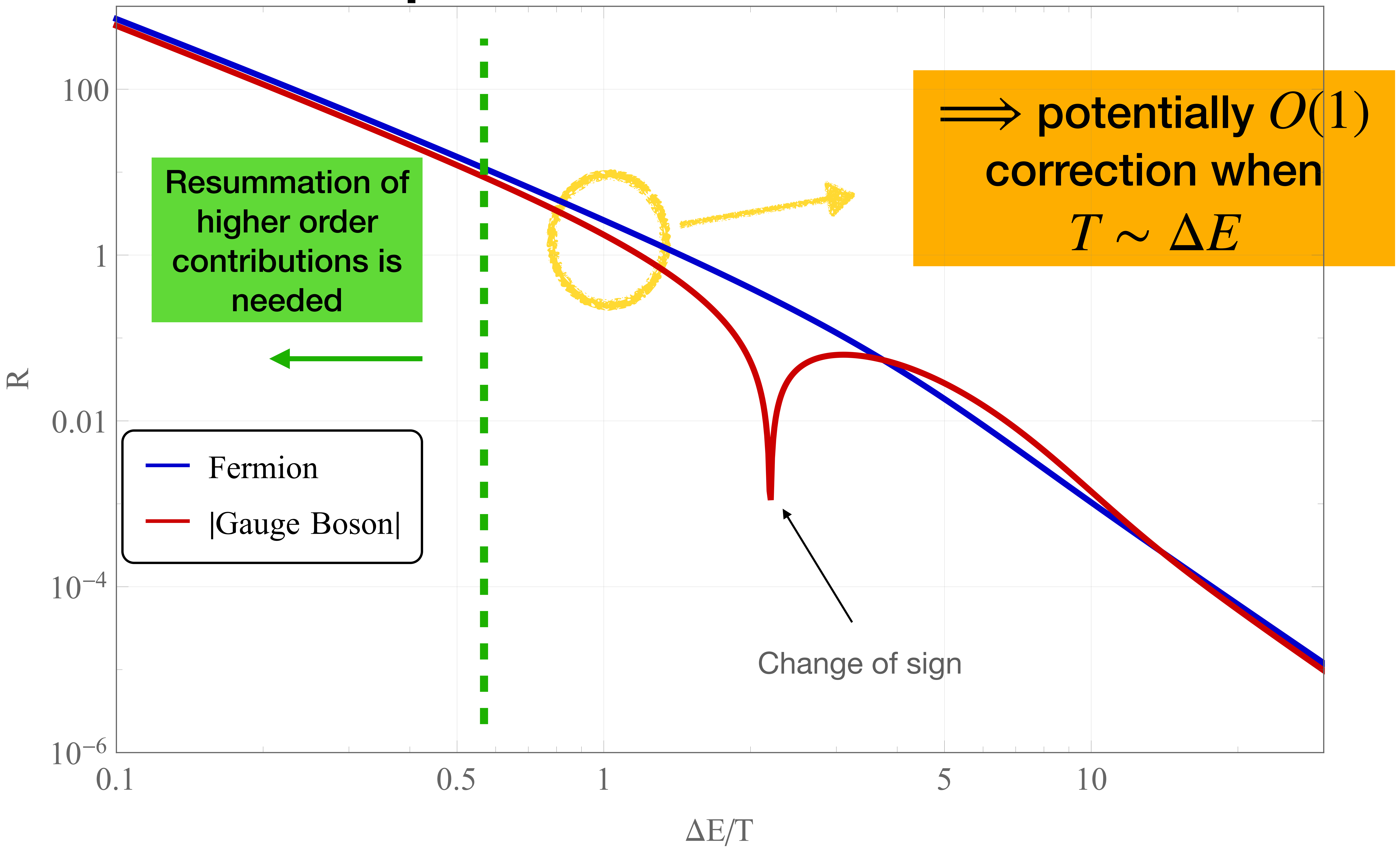
$$R_f^{T=0}(\mu/\Delta E) = -\frac{1}{\pi}\left[\frac{1}{3}\ln\left(\frac{\mu^2}{\Delta E^2}\right) + a_f\right] \qquad R_g^{T=0}(\mu/\Delta E) = \frac{1}{\pi}\left[\frac{11}{12}\ln\left(\frac{\mu^2}{\Delta E^2}\right) + a_g\right]$$

$$a_f \approx 0.64901 \qquad a_g \approx 1.22318$$

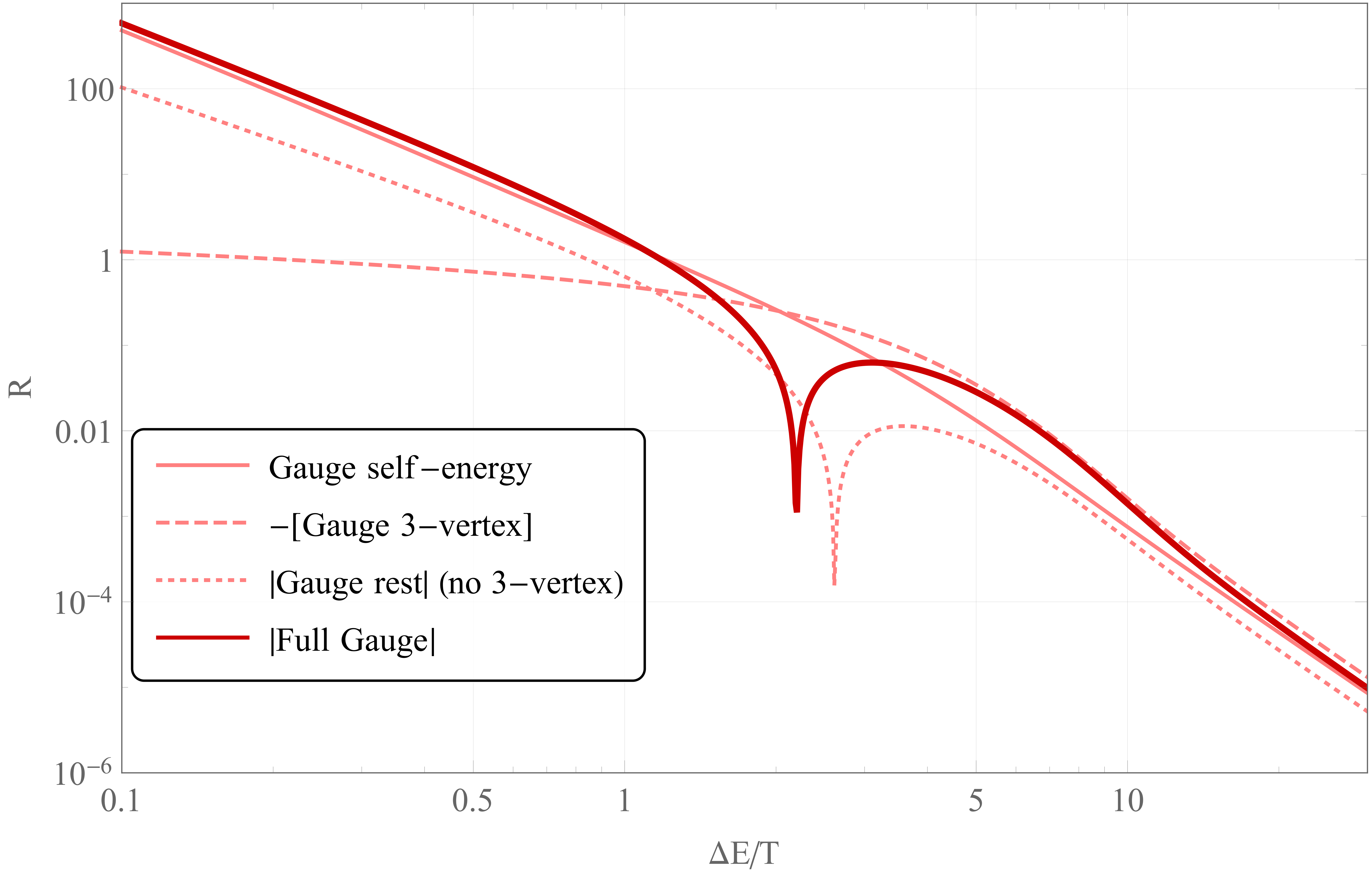
NLO finite temperature corrections



NLO finite temperature corrections



NLO finite temperature: gauge sector



Summary and outlook

- We have calculated the complete gauge-invariant electric correlator at NLO that determines the formation/dissociation rates of bound states.
- It is IR safe, collinear safe, and no renormalization beyond the gauge theory coupling constant appears.
- These results give a firm quantitative grasp on the transition rates in the temperature regime $T \approx \Delta E$ and $T \lesssim \Delta E$.
- Future work:
  - Applications to quarkonia and DM in numerical studies
  - Explore the strongly coupled regime with AdS/CFT

Thanks!