Bound-state effects in EW and colored co-annihilation and unitarity bound

Juri Smirnov

Alexander von Humboldt Fellow

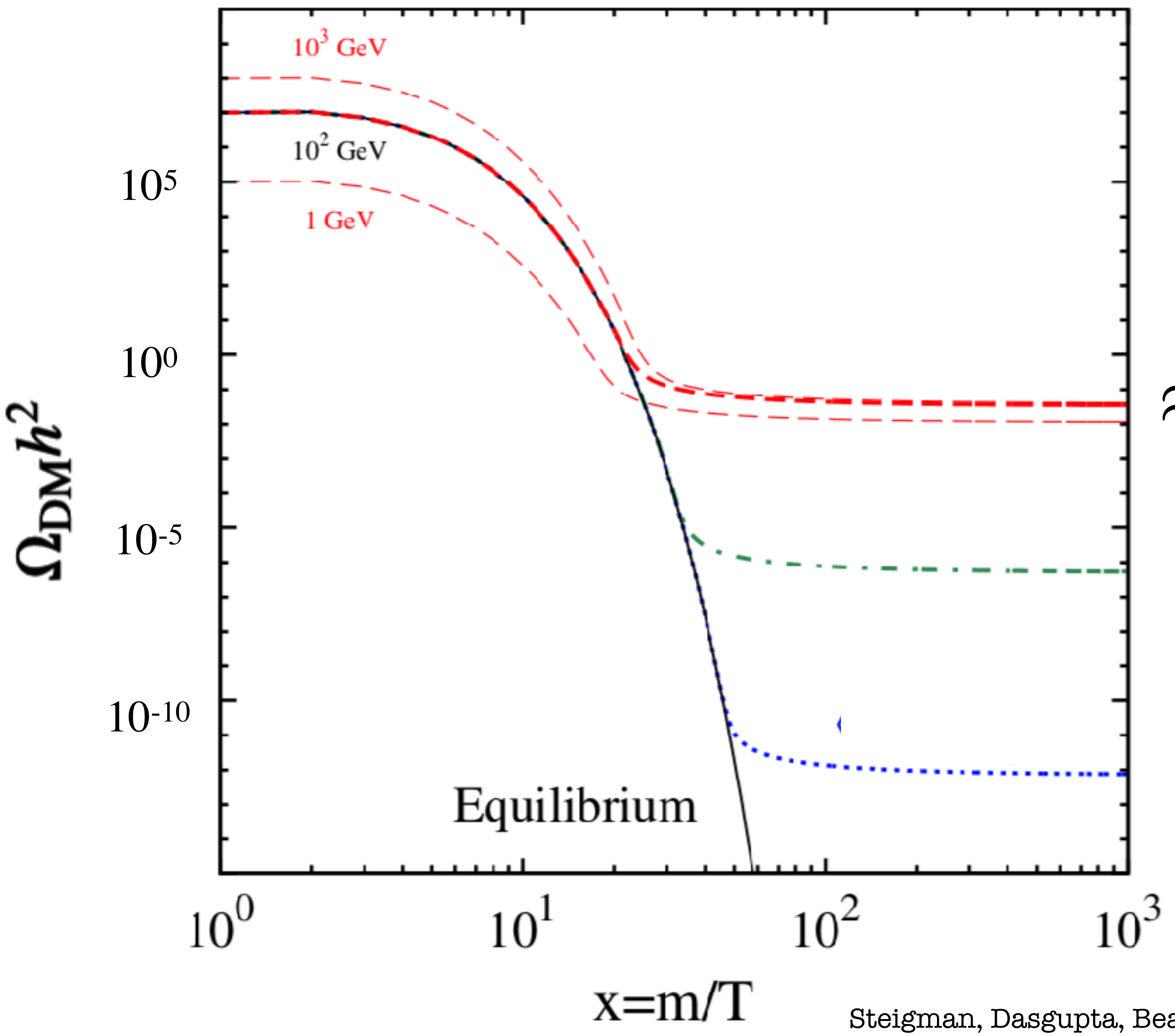
CCAPP

THE OHIO STATE UNIVERSITY

IPMU: 17/06/21

Unstable DM Bound States

WIMP Freezeout

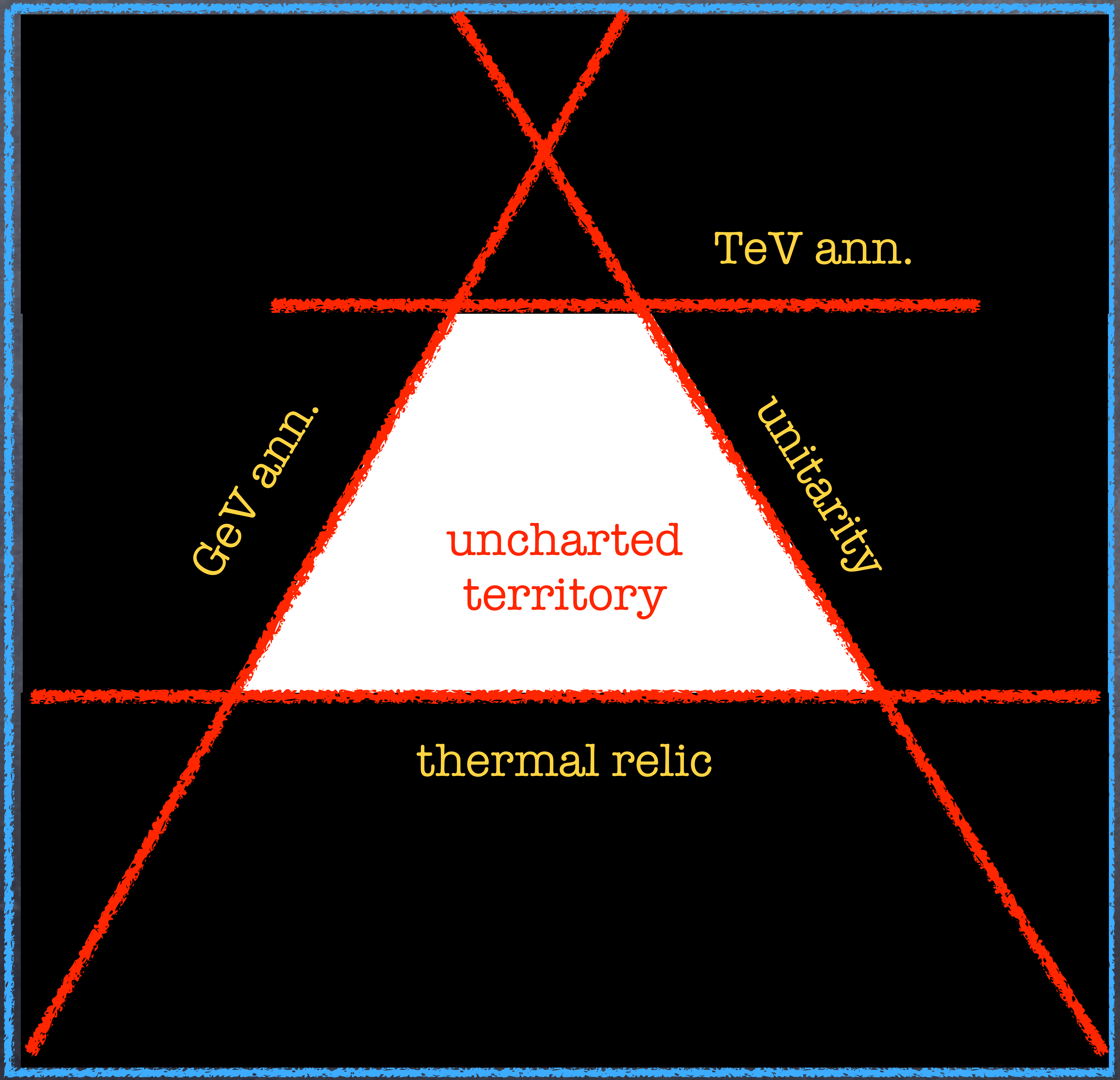


≈ 0.12

$$\Omega_{\rm DM}\, h^2 \approx \frac{0.12}{\langle \sigma v_{\rm rel.}\rangle\, [25\mathrm{TeV}]^2}$$

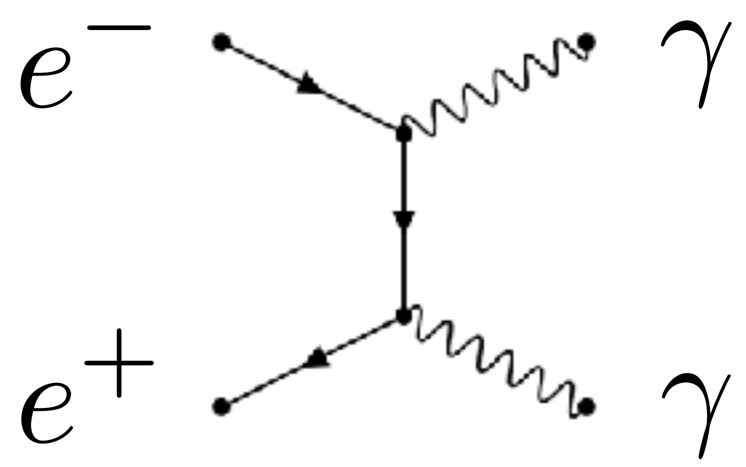
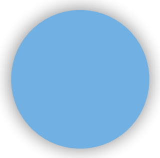
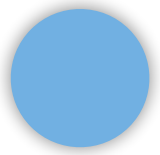
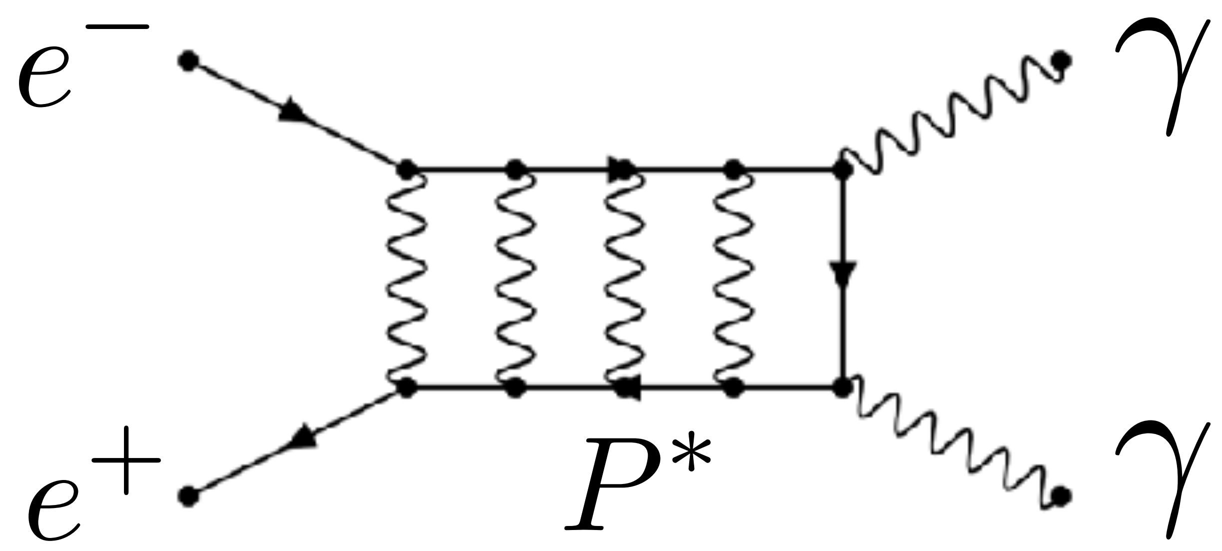
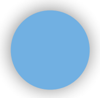
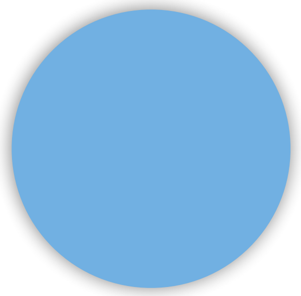
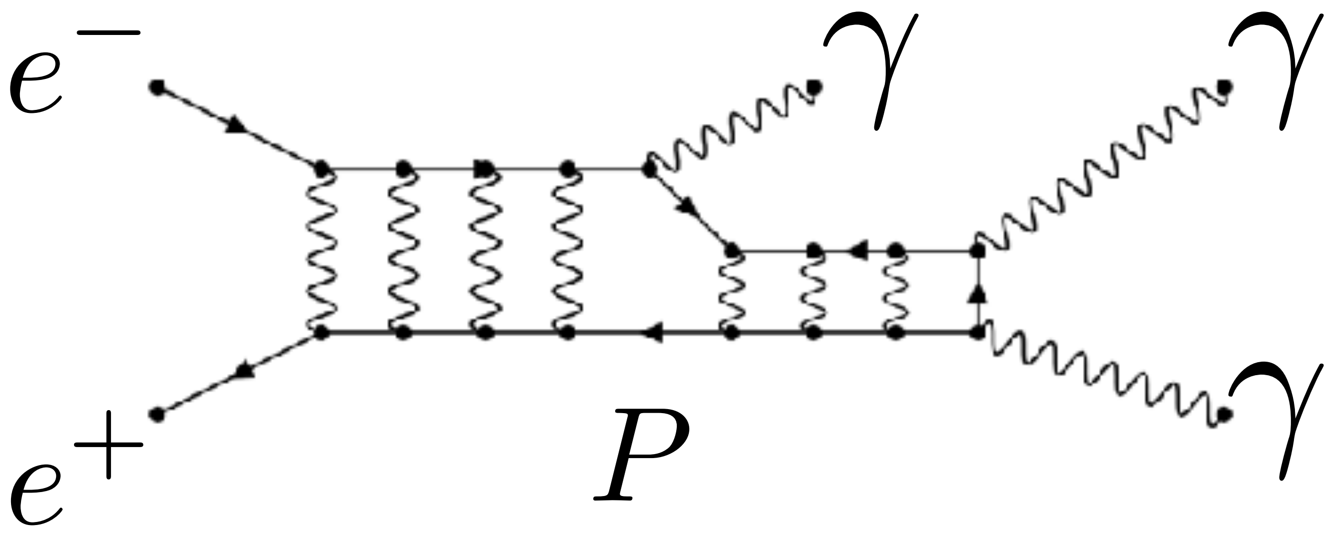
Steigman, Dasgupta, Beacom
arXiv: 1204.3622

Cartoon Overview of Approach



Dark Matter Annihilation $\langle \sigma v_{\rm rel.} \rangle$

Dark Matter Mass

Process	Diagram	Cross-Section area	
		large velocity	small velocity
$e^+ e^- \to \gamma\gamma$			
$e^+ e^- \to P^* \to \gamma\gamma$ J. Hisano			
$e^+ e^- \to P^* \to P\,\gamma$ $P \to \gamma\gamma$ K. Petraki	 Assuming Parapositronium (J=0)		

Application I: EW Annihilation

Boltzmann Equations with Bound States

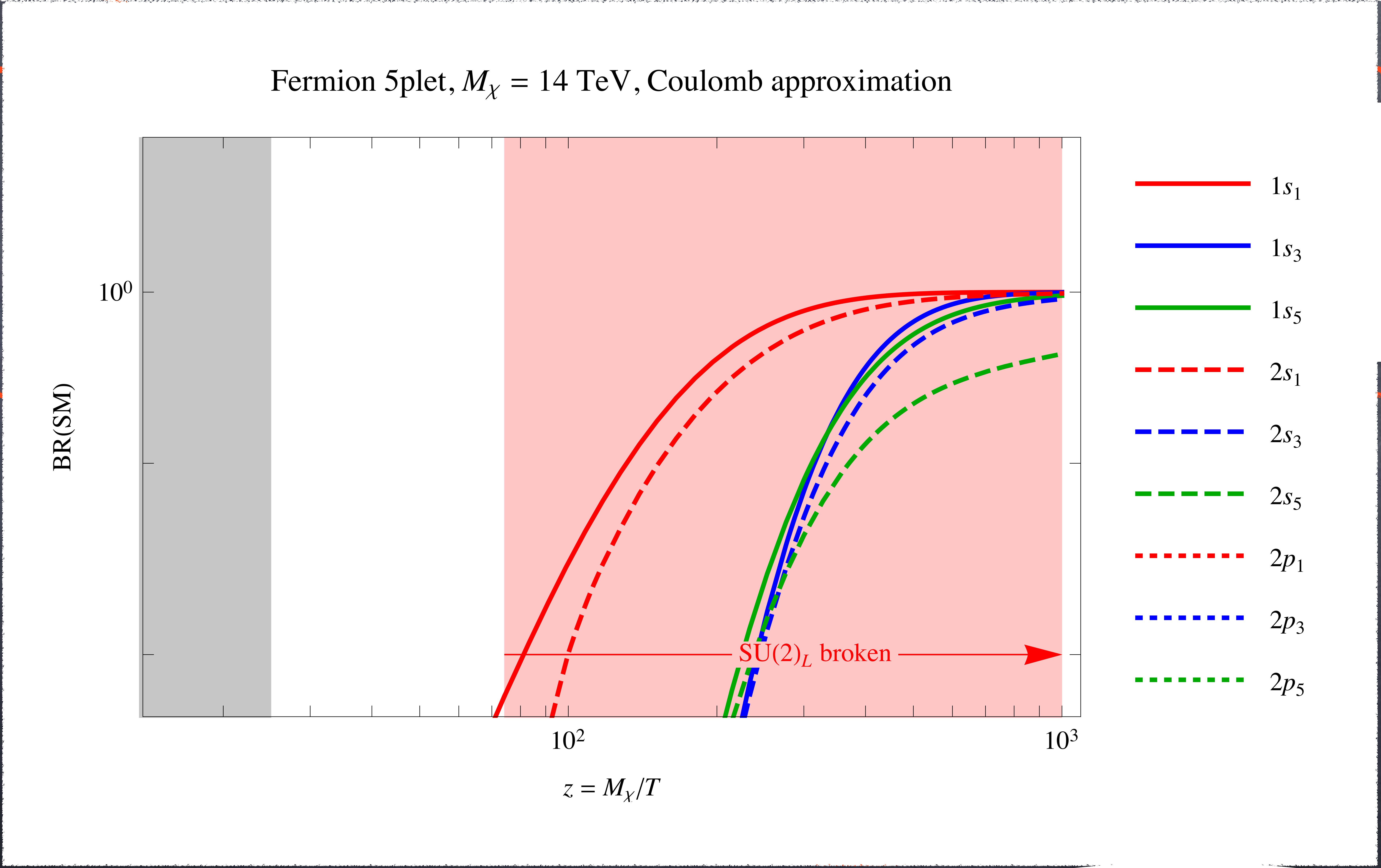
$$\frac{1}{\lambda}\frac{dY_{\rm DM}}{dz} = -\frac{\langle\sigma v_{\rm rel}\rangle_{\rm ann}}{z^2}\left(Y_{\rm DM}^2 - Y_{\rm eq.}^2\right) - \sum_I \frac{\langle\sigma v_{\rm rel}\rangle_{\rm BSF}}{z^2}\left(Y_{\rm DM}^2 - Y_B^I \frac{Y_{\rm eq.}^2}{Y_B^{\rm eq.}}\right)$$

$$\frac{1}{\lambda}\frac{dY_{\rm B}^I}{dz} = \frac{\langle\sigma v_{\rm rel}\rangle_{\rm BSF}}{z^2}\left(Y_{\rm DM}^2 - Y_B^I \frac{Y_{\rm eq.}^2}{Y_B^{\rm eq.}}\right) - \Gamma_B^I\left(Y_B - Y_B^{\rm eq.}\right) + \text{ decay of excited states.}$$

$$\frac{dY}{dz} = -\frac{\langle\sigma_{\rm eff} v_{\rm rel}\rangle s}{Hz}\left(Y_{\rm DM}^2 - Y_{\rm DM}^{\rm eq2}\right) = -\frac{\lambda\, S(z)}{z^2}\left(Y_{\rm DM}^2 - Y_{\rm DM}^{\rm eq2}\right),$$

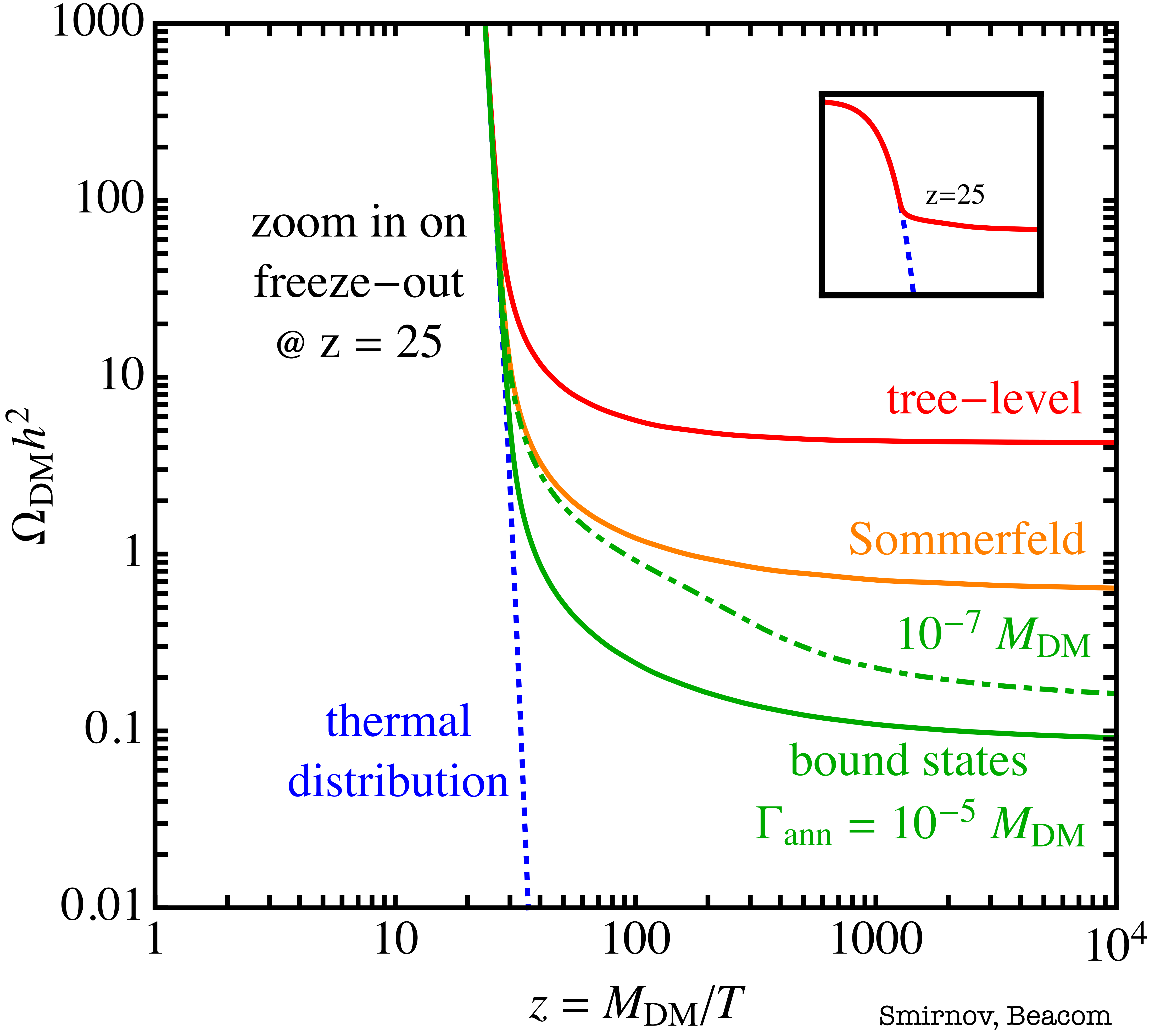
$$S(z) = S_{\rm ann}(z) + \left[\frac{\sigma_0}{\langle\sigma_I v_{\rm rel}\rangle} + \frac{g_\chi^2 \sigma_0\, M_\chi^3}{2 g_I\, \Gamma_{\rm ann}}\left(\frac{z}{4\pi}\right)^{3/2} e^{-z\, E_{B_I}/M_\chi}\right]^{-1}$$

Boltzmann Equations with Bound States



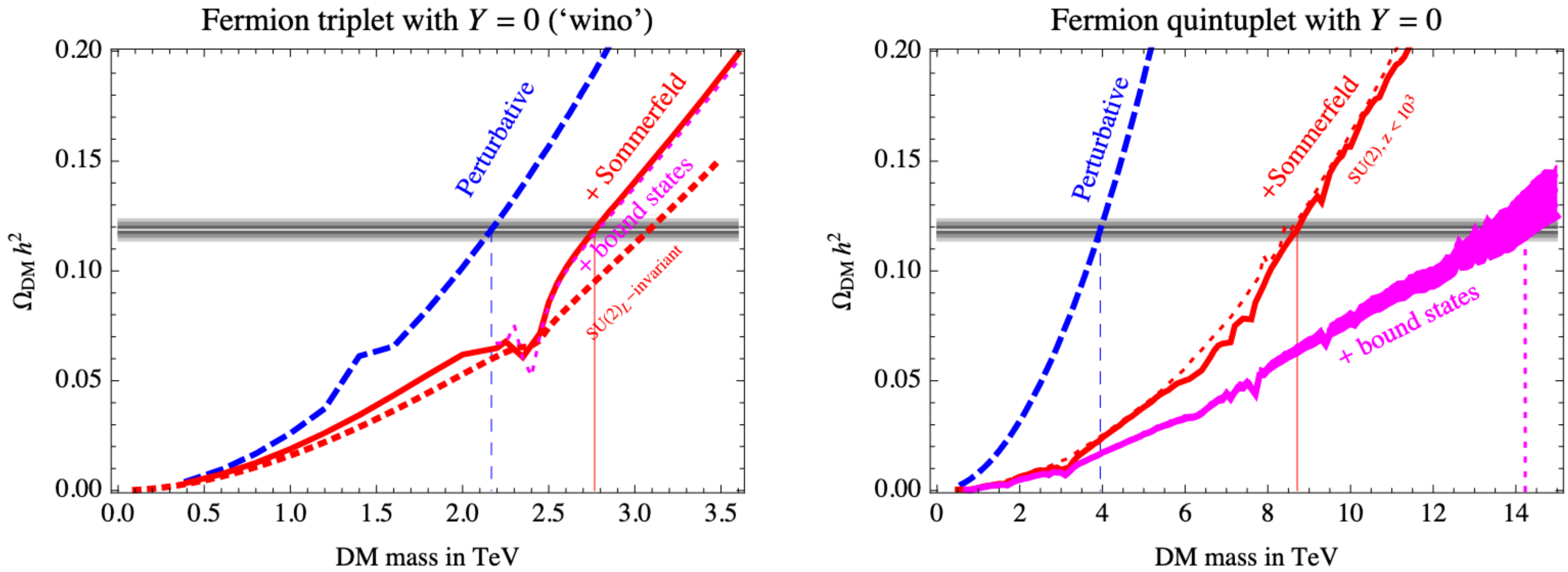
$$\langle \sigma v \rangle_{bsf} \, \mathrm{BR(B \to SM)}$$

Effect on the Freezeout



Smirnov, Beacom
arXiv: 1904.11503

Electroweak Dark Matter

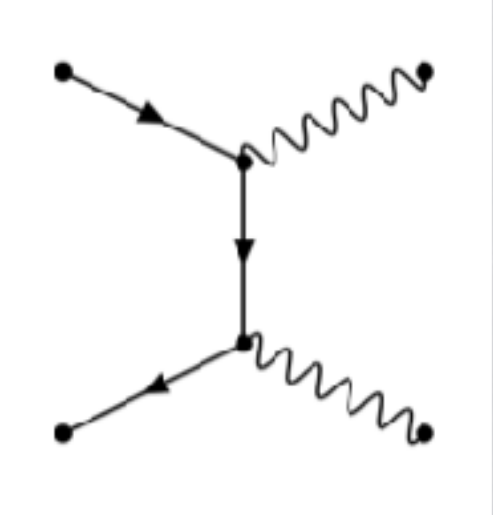


A. Mitridate, M. Redi, A. Strumia, and J. Smirnov: arXiv: 1702.01141

Conservation of Probability

$$\sigma^J_{\max} = \frac{4\pi\,(2J+1)}{p^2_{\mathrm{DM}}}$$

$$(\sigma v_{\mathrm{rel}})^J_{\mathrm{total}} < (\sigma v)^J_{\max} = \frac{4\pi(2\,J+1)}{M^2_{\mathrm{DM}}v_{\ \mathrm{rel}}}$$



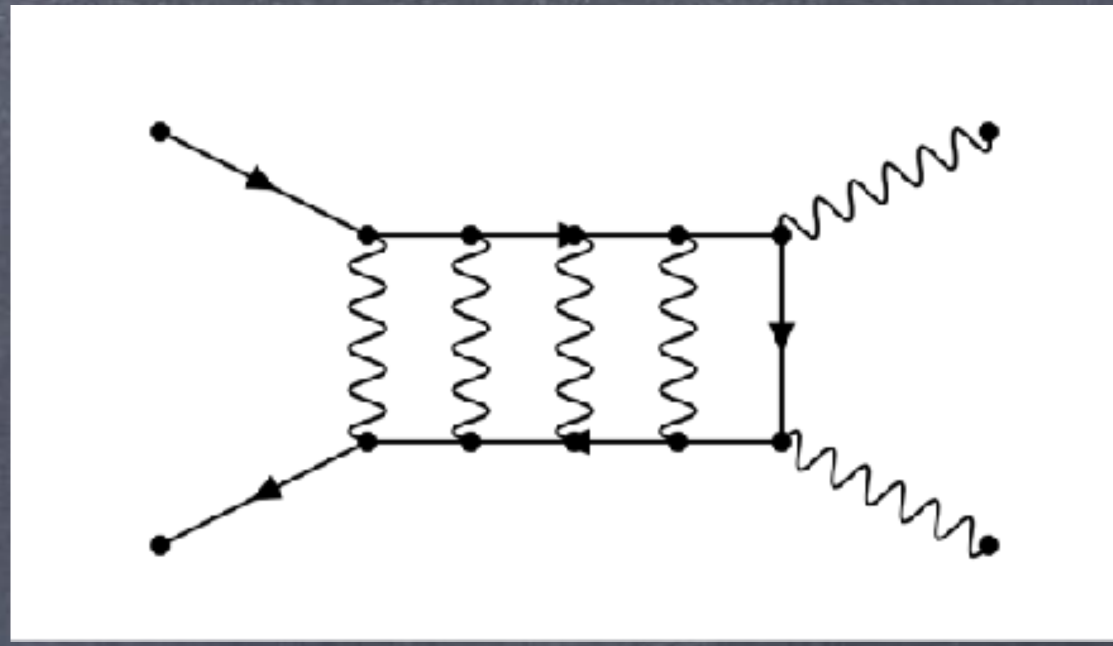
$$\Omega_\chi h^2 \geq 1.7\times 10^{-6}\sqrt{x_f}[m_\chi/(1\ \mathrm{TeV})]^2 \qquad (12)$$

M_X < 340 TeV

K. Griest and M. Kamionkowski
Phys.Rev.Lett. 64 (1990) 615

Velocity Dependence and Higher Orders

$$(\sigma v_{\rm rel})^J_{\rm total} < (\sigma v)^J_{\rm max} = \frac{4\pi(2J+1)}{M^2_{\rm DM} v_{\rm rel}}$$



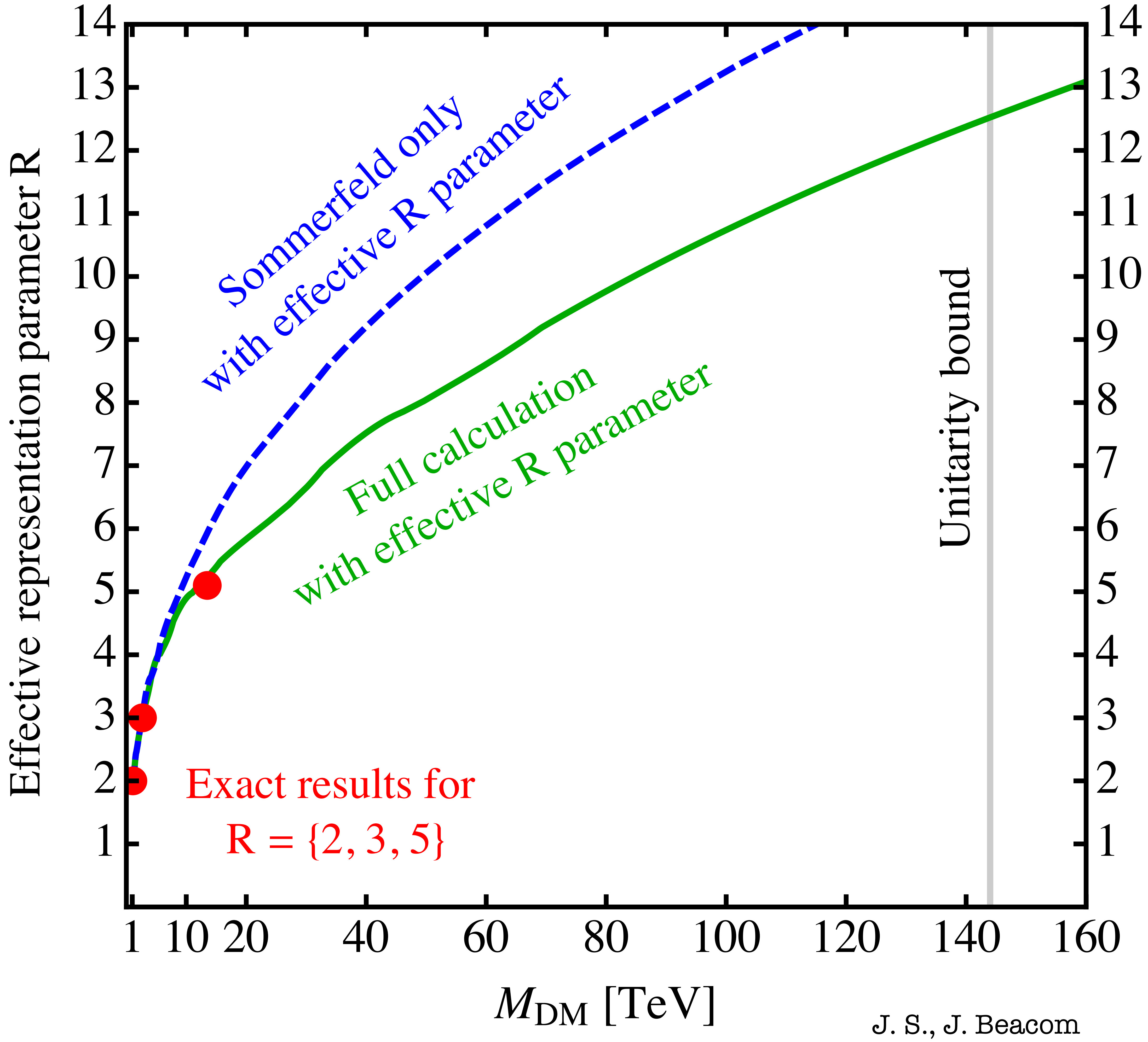
$$(\sigma_{\rm ann} v_{\rm rel}) = \frac{\pi\alpha^2}{M^2_{\rm DM}} \times \frac{2\pi\alpha/v_{\rm rel}}{1-\exp(-2\pi\alpha/v_{\rm rel})}$$

$$\approx \frac{2\pi^2\alpha^3}{M^2_{\rm DM}\, v_{\rm rel}} \qquad \alpha \gg v_{\rm rel}$$

$$(\sigma v_{\rm rel})_{\rm eff} = (\sigma v_{\rm rel})_{\rm ann} + \sum_I (\sigma_I v_{\rm rel})_{\rm BSF}\, {\rm BR}({\rm B}_I \to {\rm SM}) \leq (\sigma v_{\rm rel})_{\rm total}$$

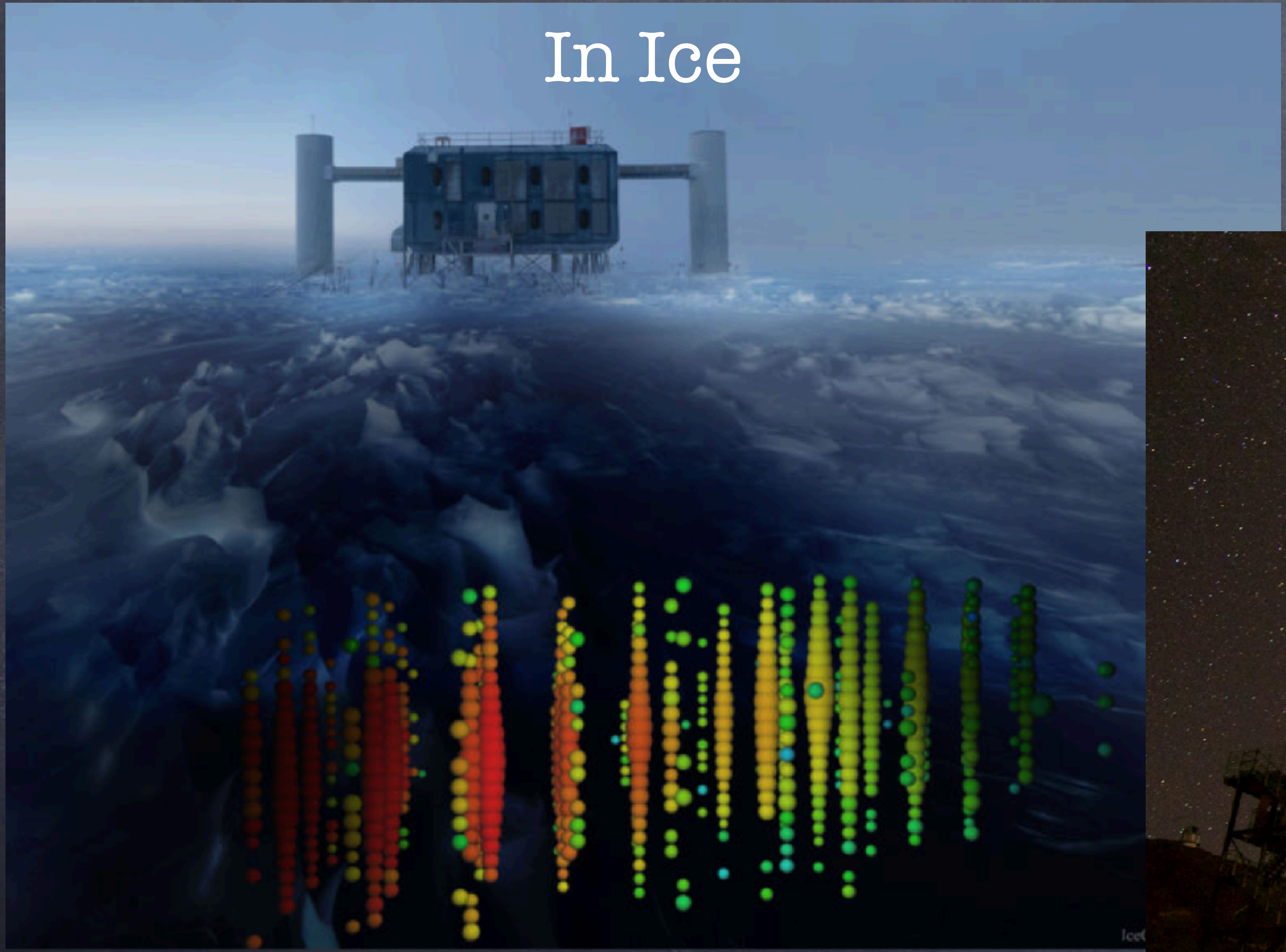
Warning: At large coupling values, higher orders need to be considered for partial wave selection

Effective Representation



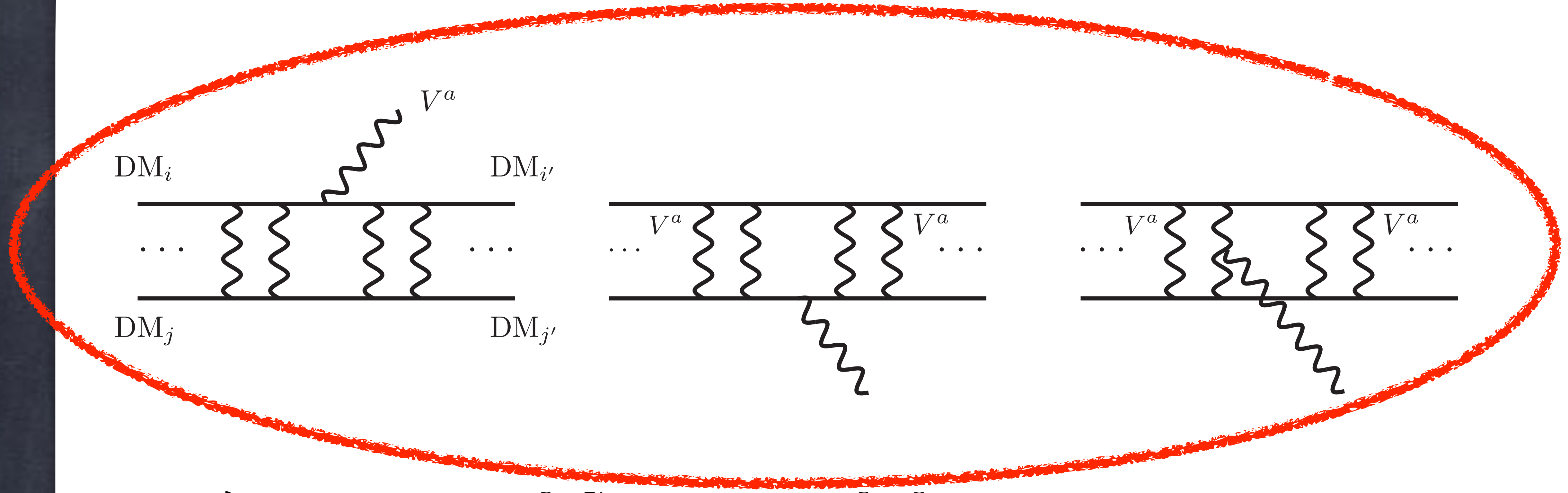
J. S., J. Beacom
arXiv: 1904.11503

New Cherenkov Detectors

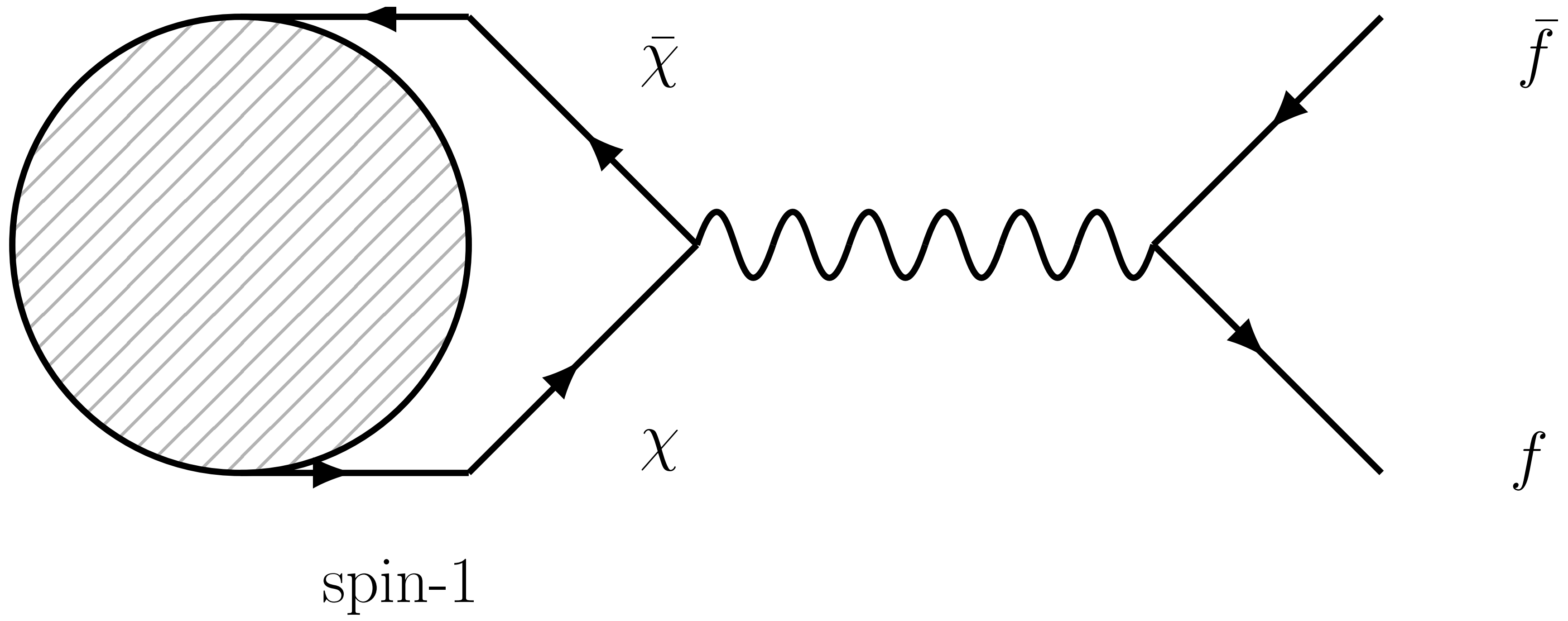


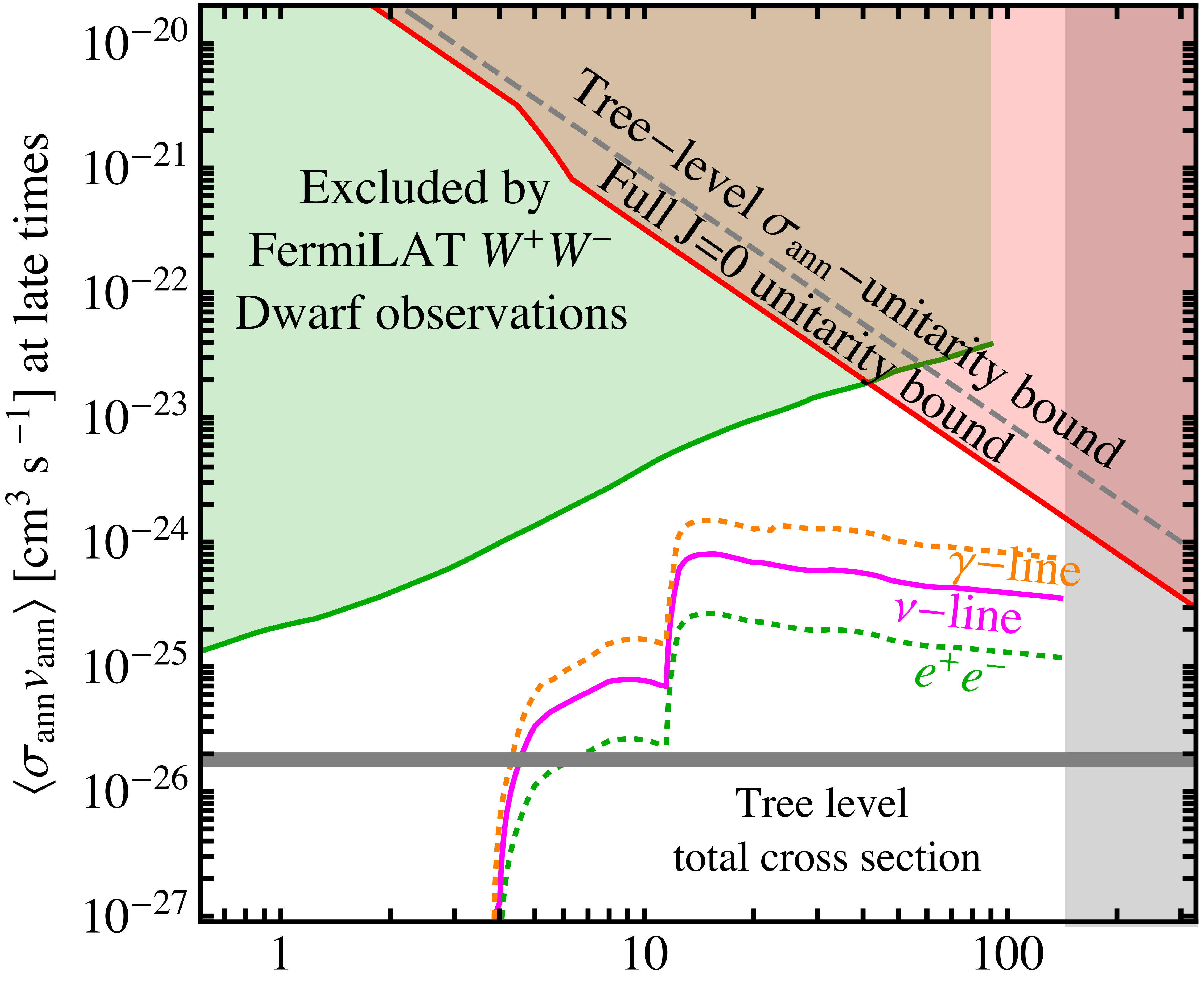
New DM Signals

A) DM Bound State forms:



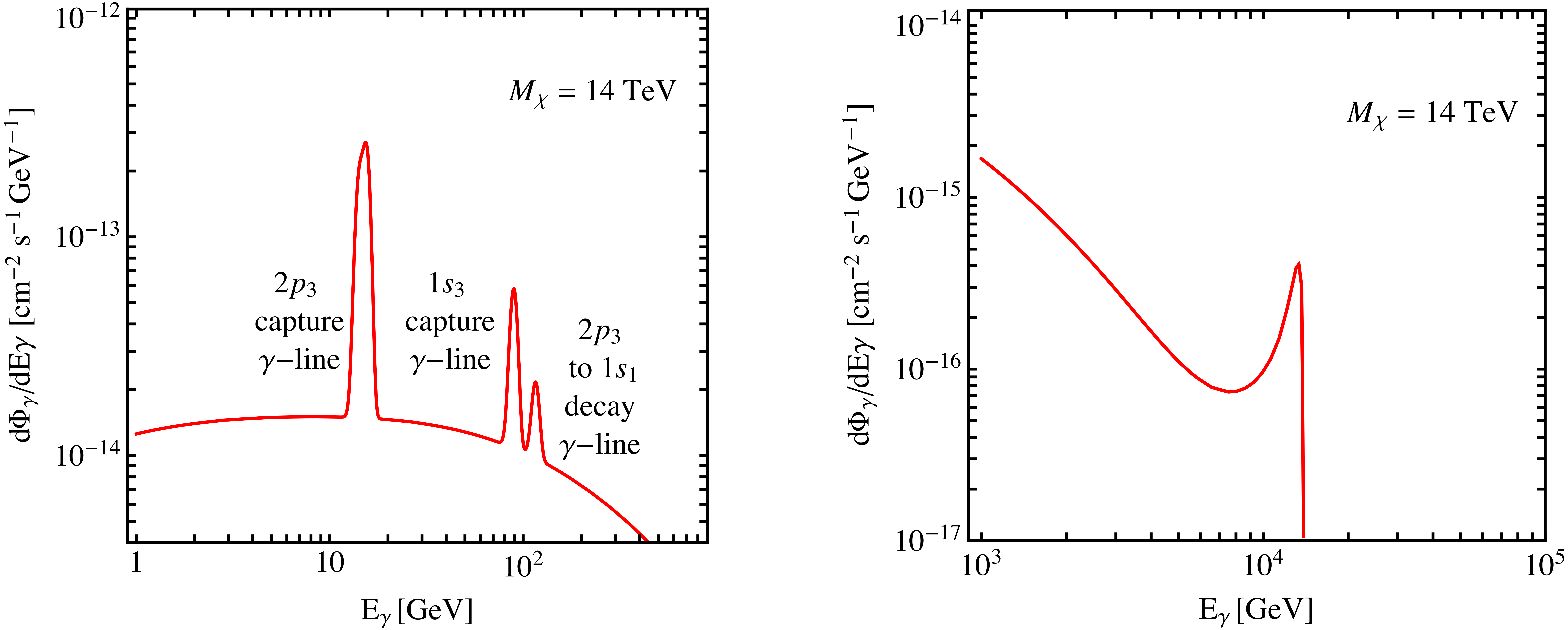
B) DM Bound State annihilates:



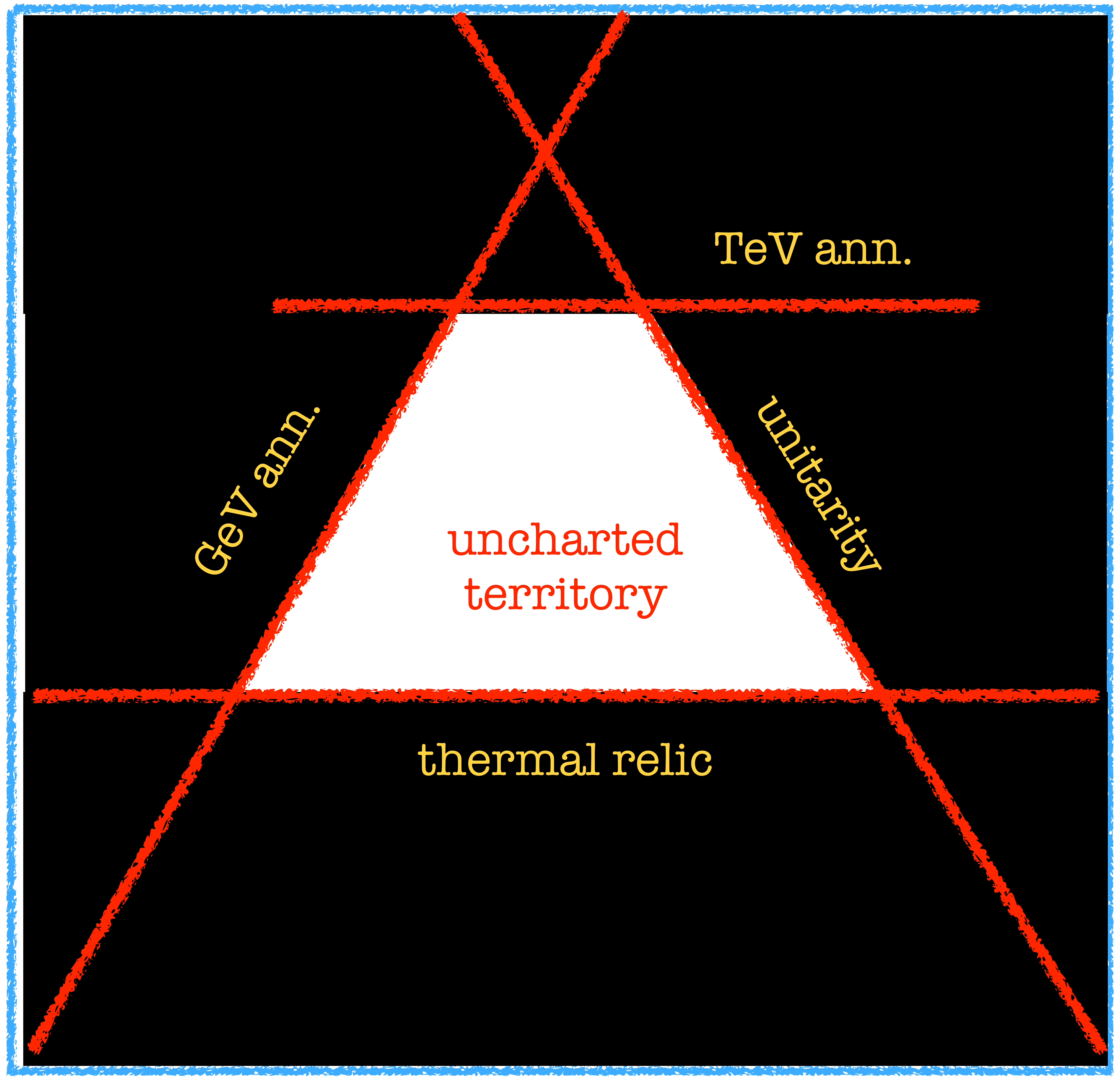


Smirnov, Beacom
arXiv: 1904.11503

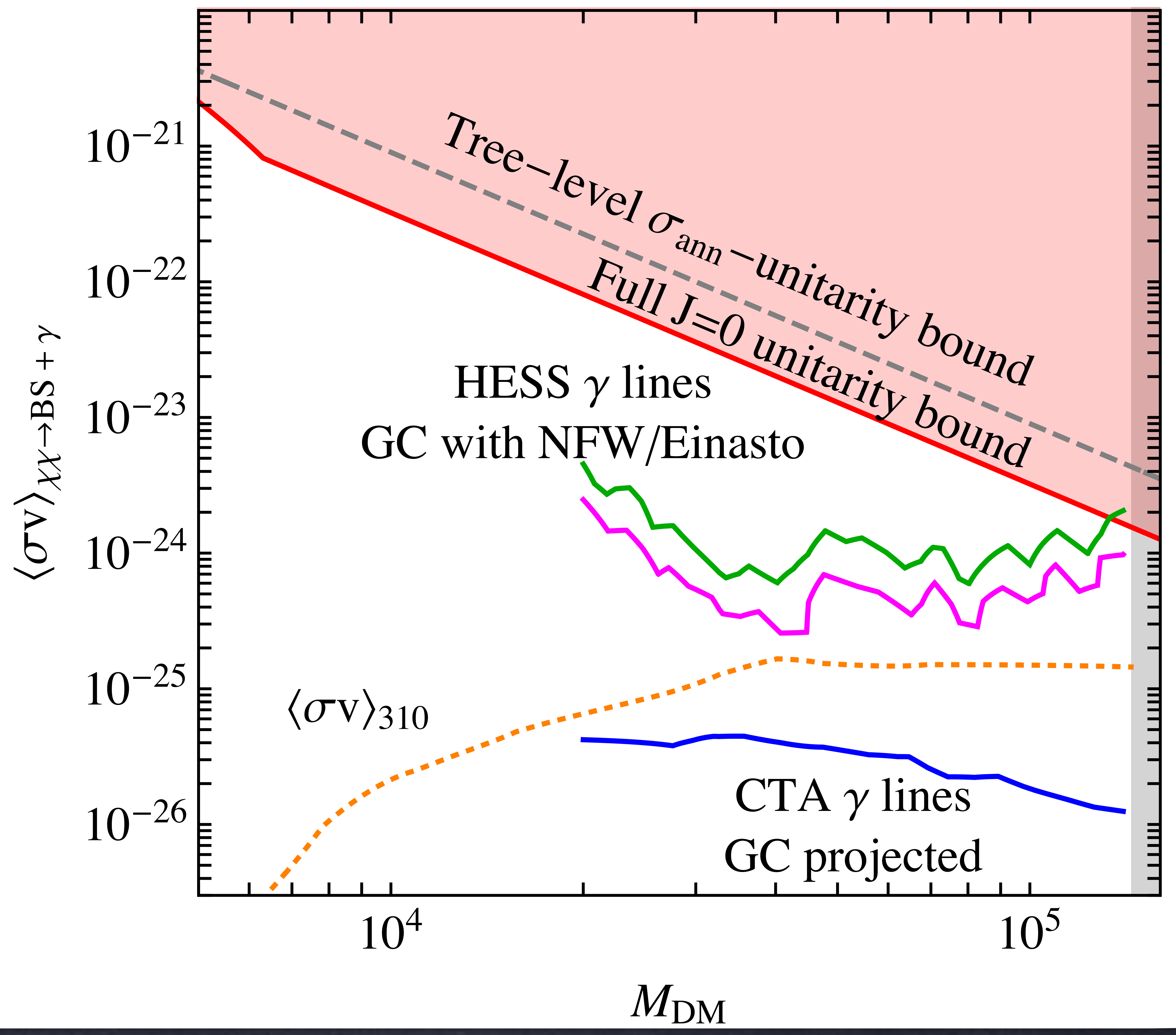
Example: DM Spectroscopy



Example: Sensitivity to Heavy Dark Matter

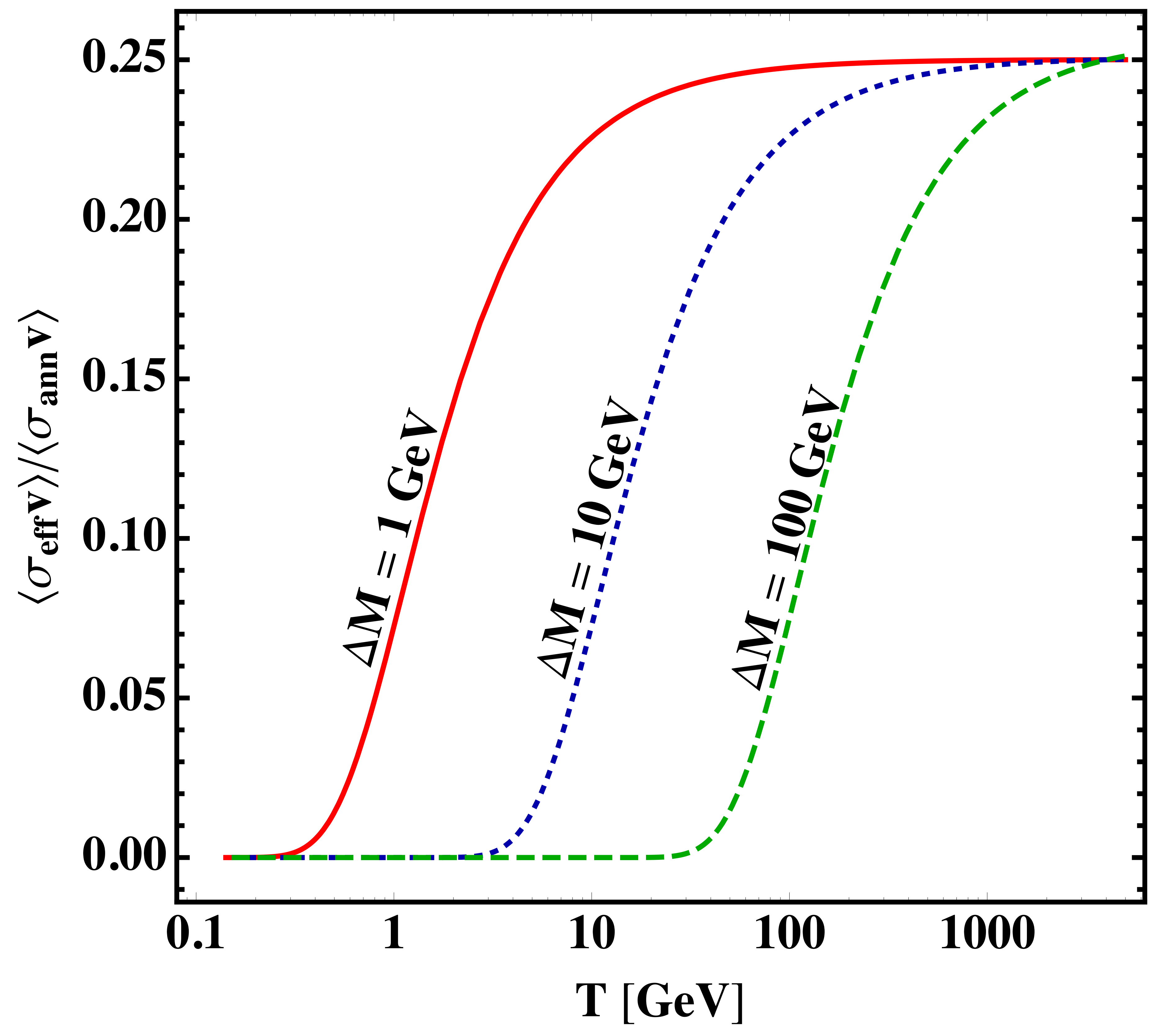


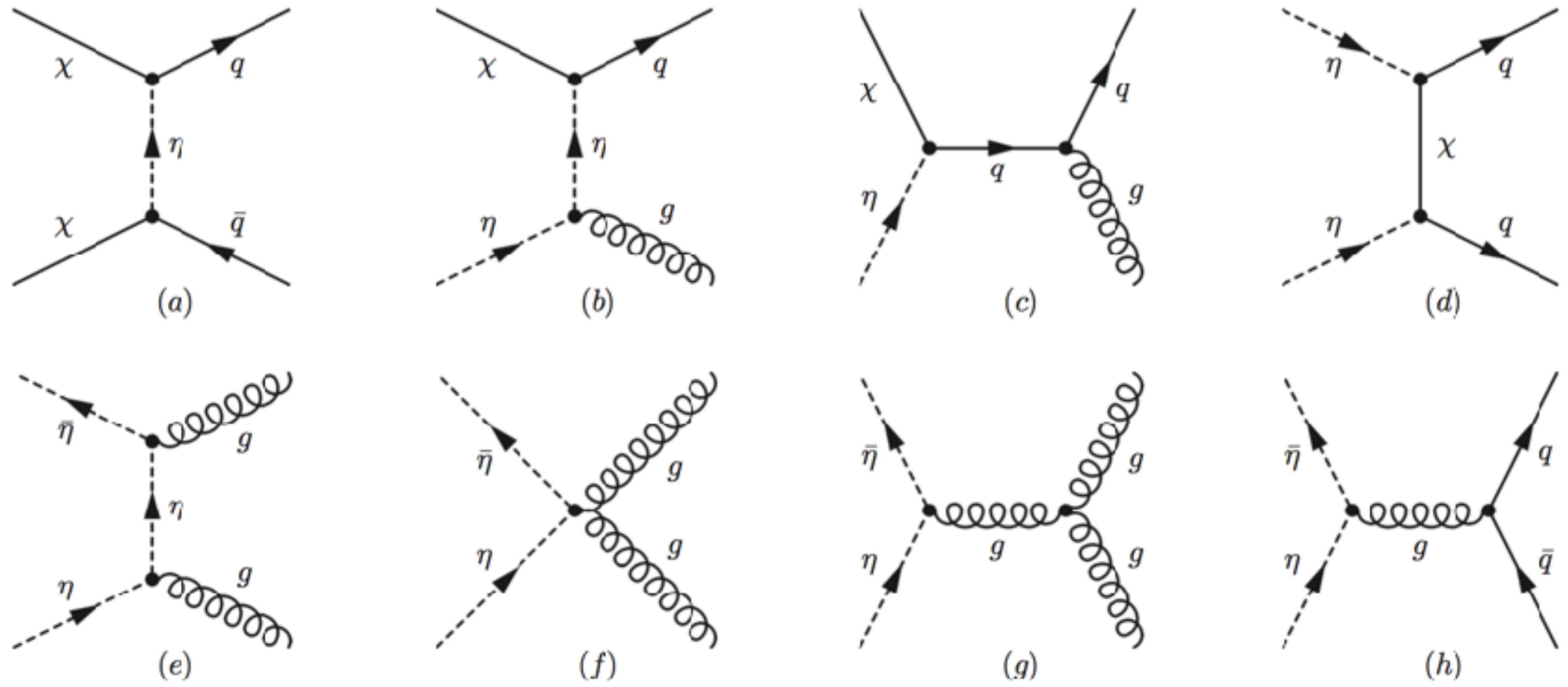
Preliminary



Application II: Annihilation with Color-charged Partners

Co-Annihilation

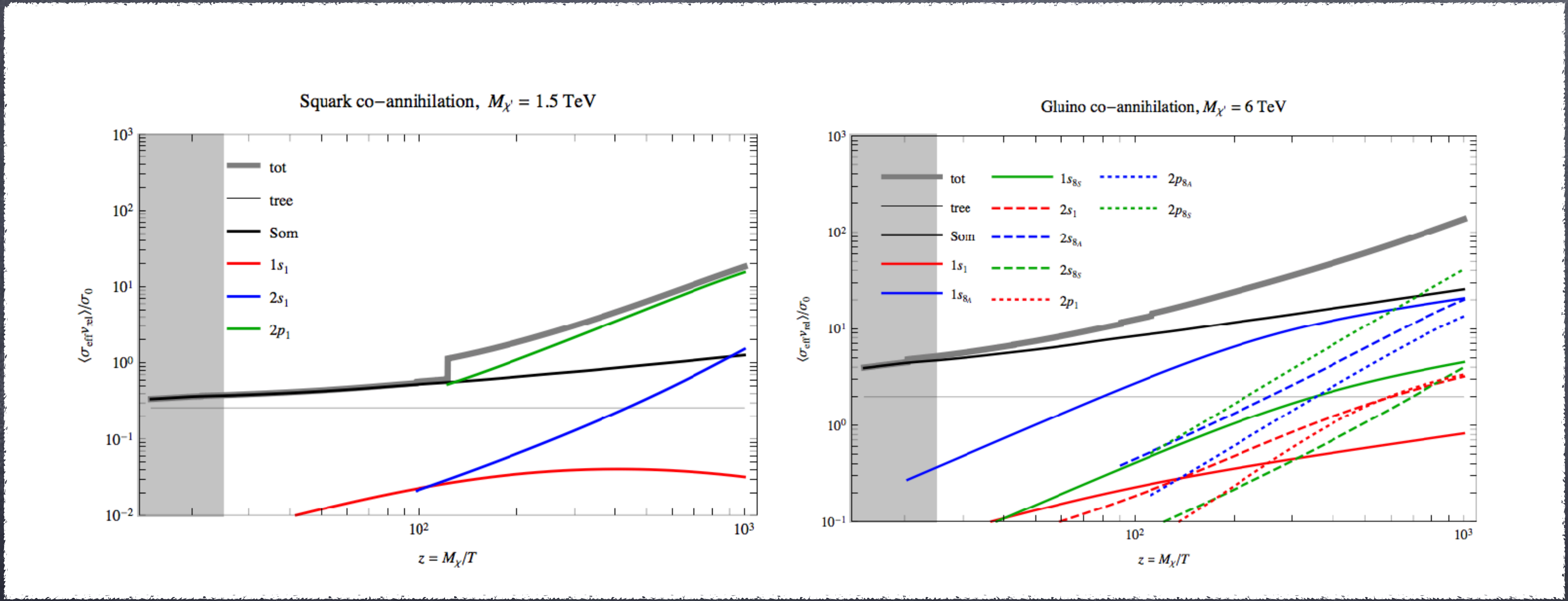




Example: Co-annihilation with a squark

$$\sigma v_{\rm rel} = \sigma(\chi'\chi' \to \text{SM particles}) v_{\rm rel} \times \left[1 + \frac{g_\chi}{g_{\chi'}} \frac{\exp(\Delta M/T)}{(1+\Delta M/M_\chi)^{3/2}}\right]^{-2} .$$

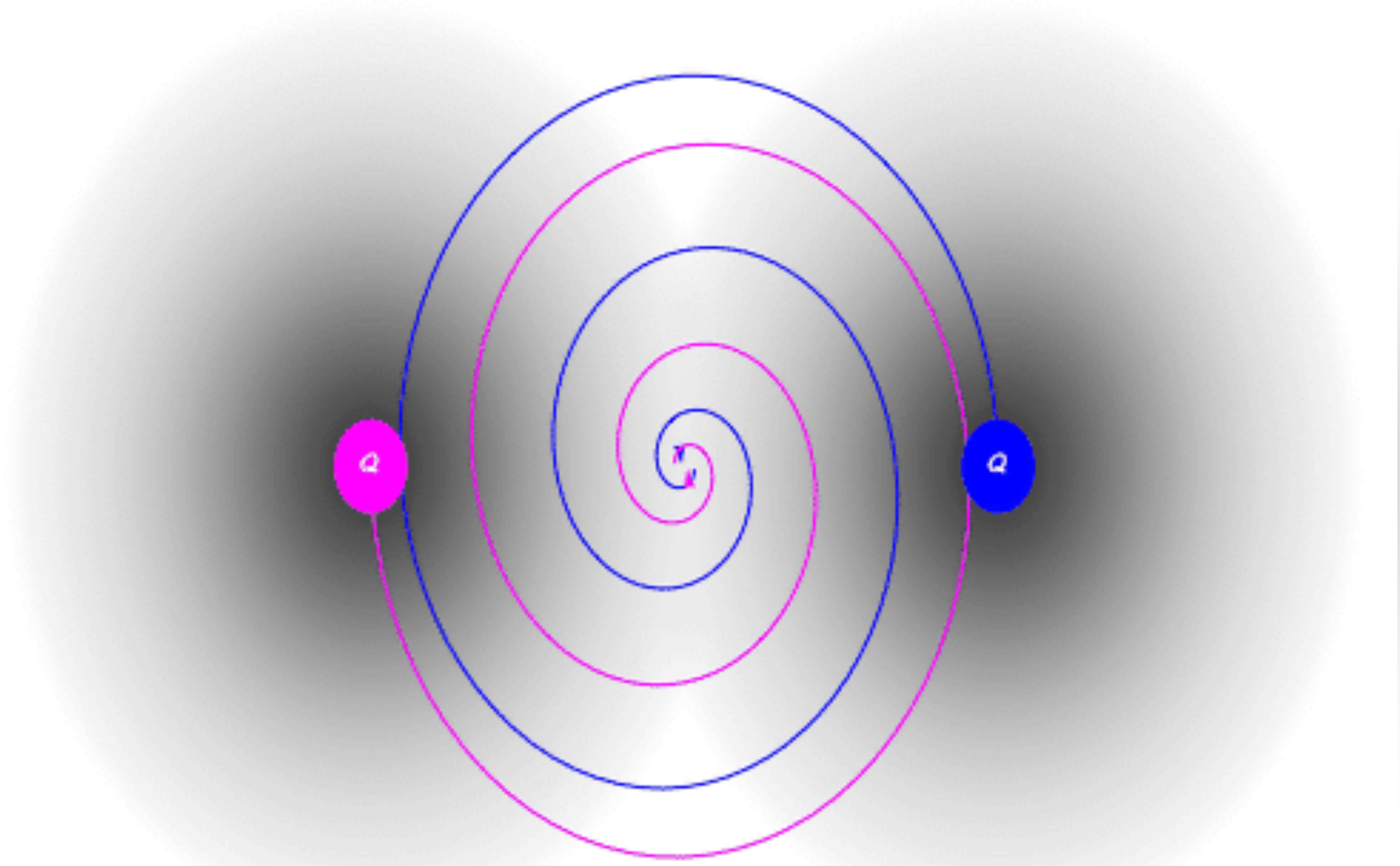
Bound-state Effects for Color-charged Relics



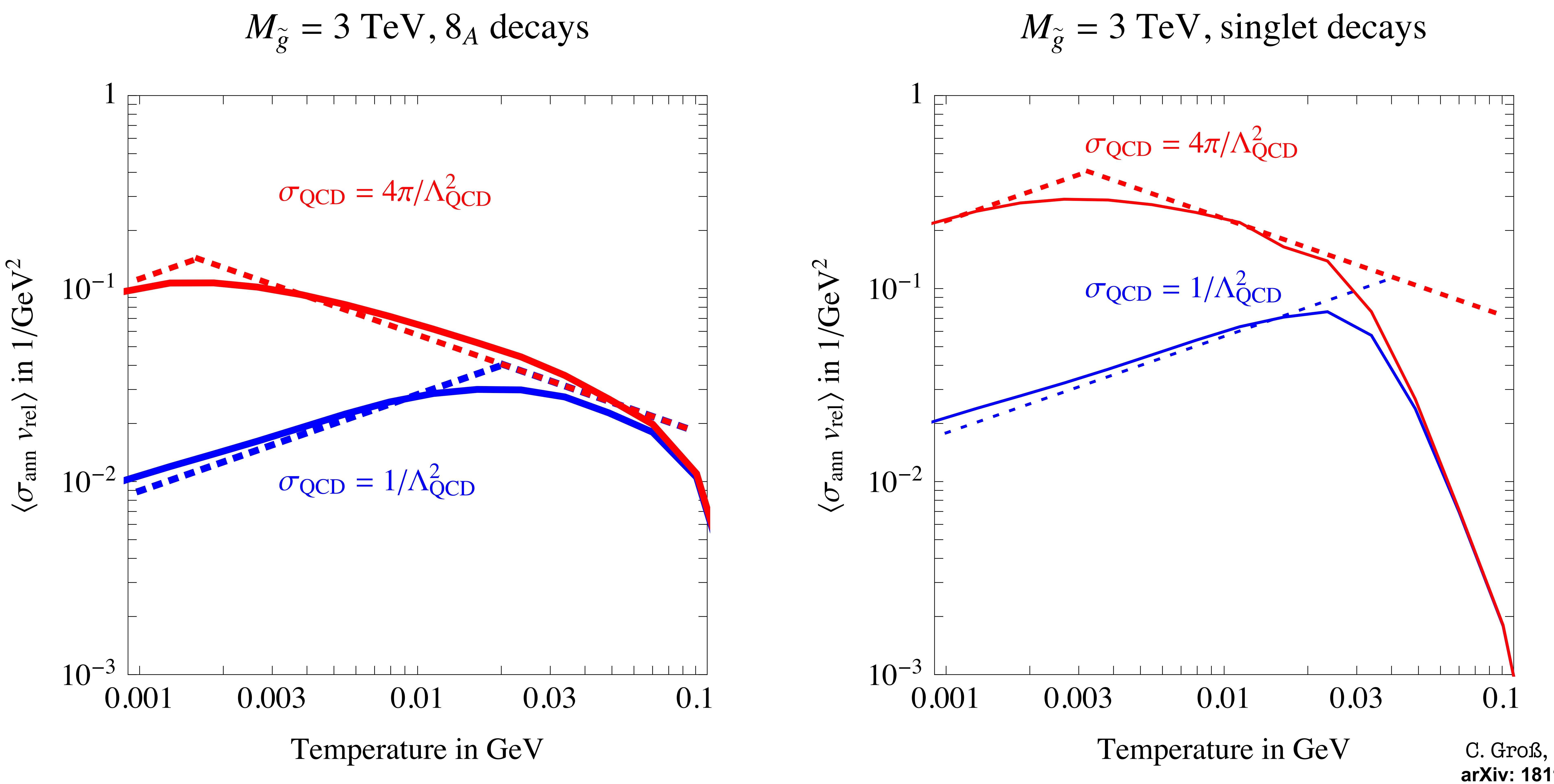
Singlet with SU(3) Triplet scalar
(Bino/Squark)

Singlet with SU(3) Qctet fermion
(Bino/Gluino)

QCD Effects on Annihilation

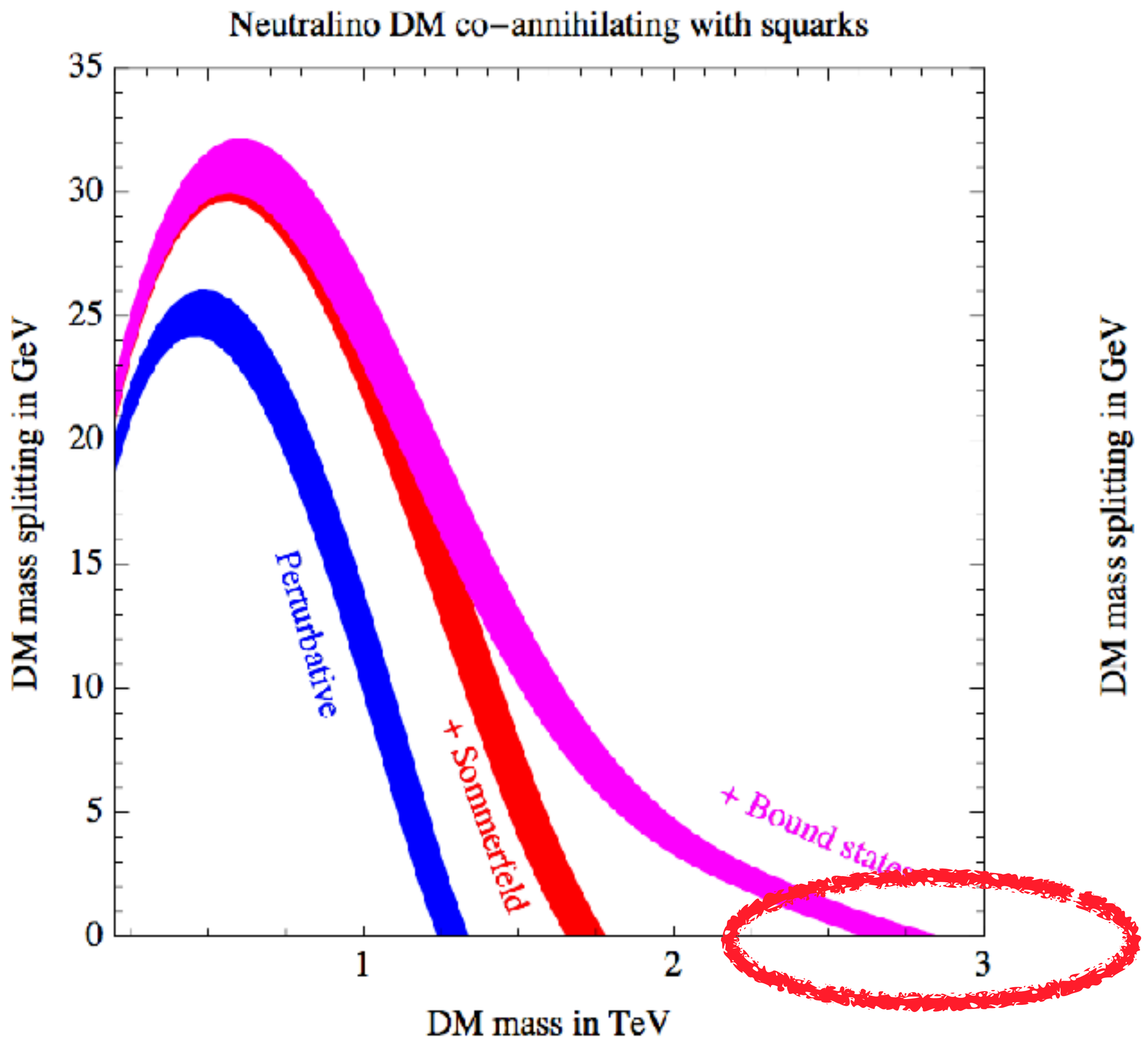


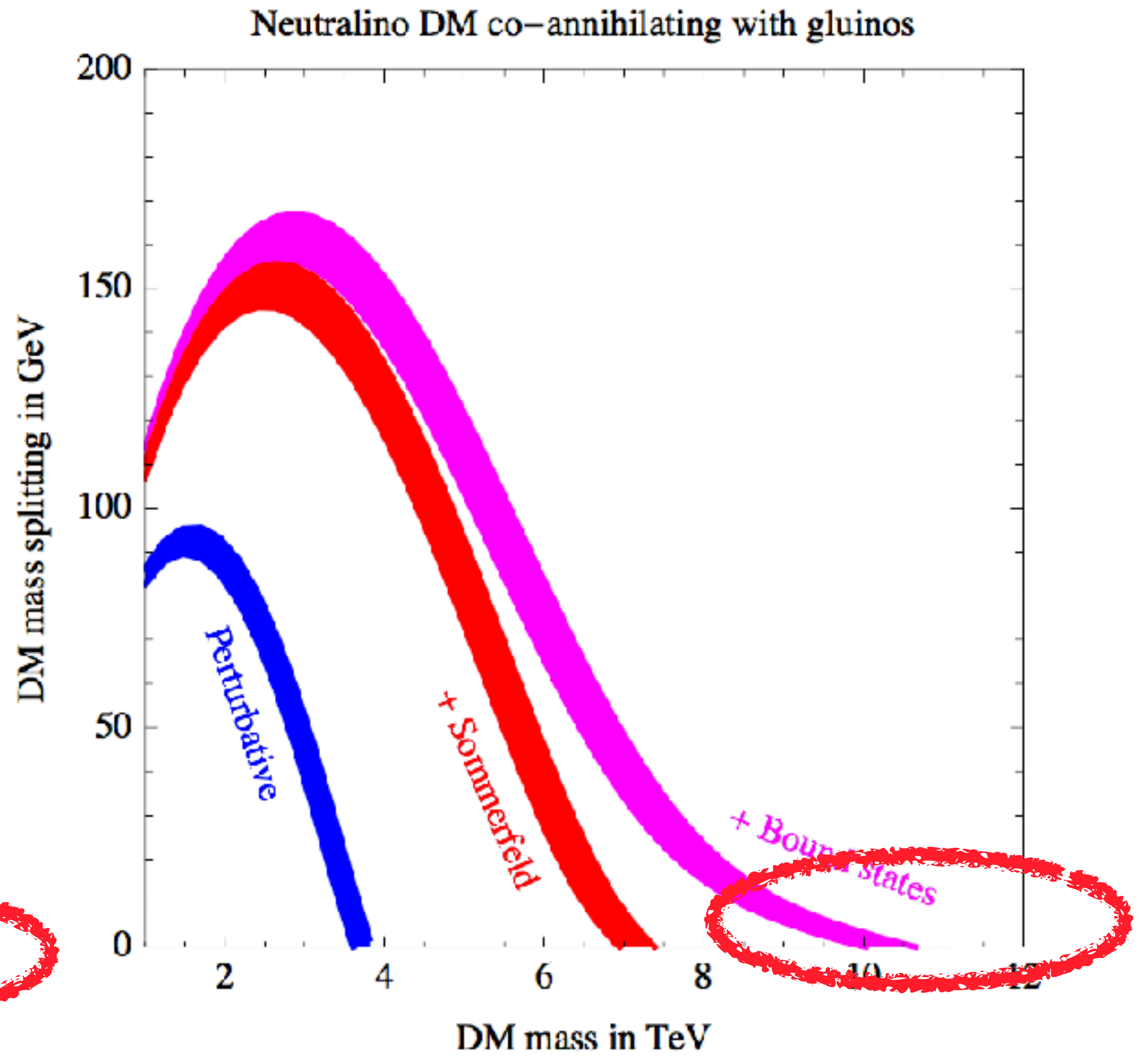
QCD Effects for Color-charged Relics



C. Groß, ..., **J.S**
arXiv: 1811.08418

Co-Annihilation: Results

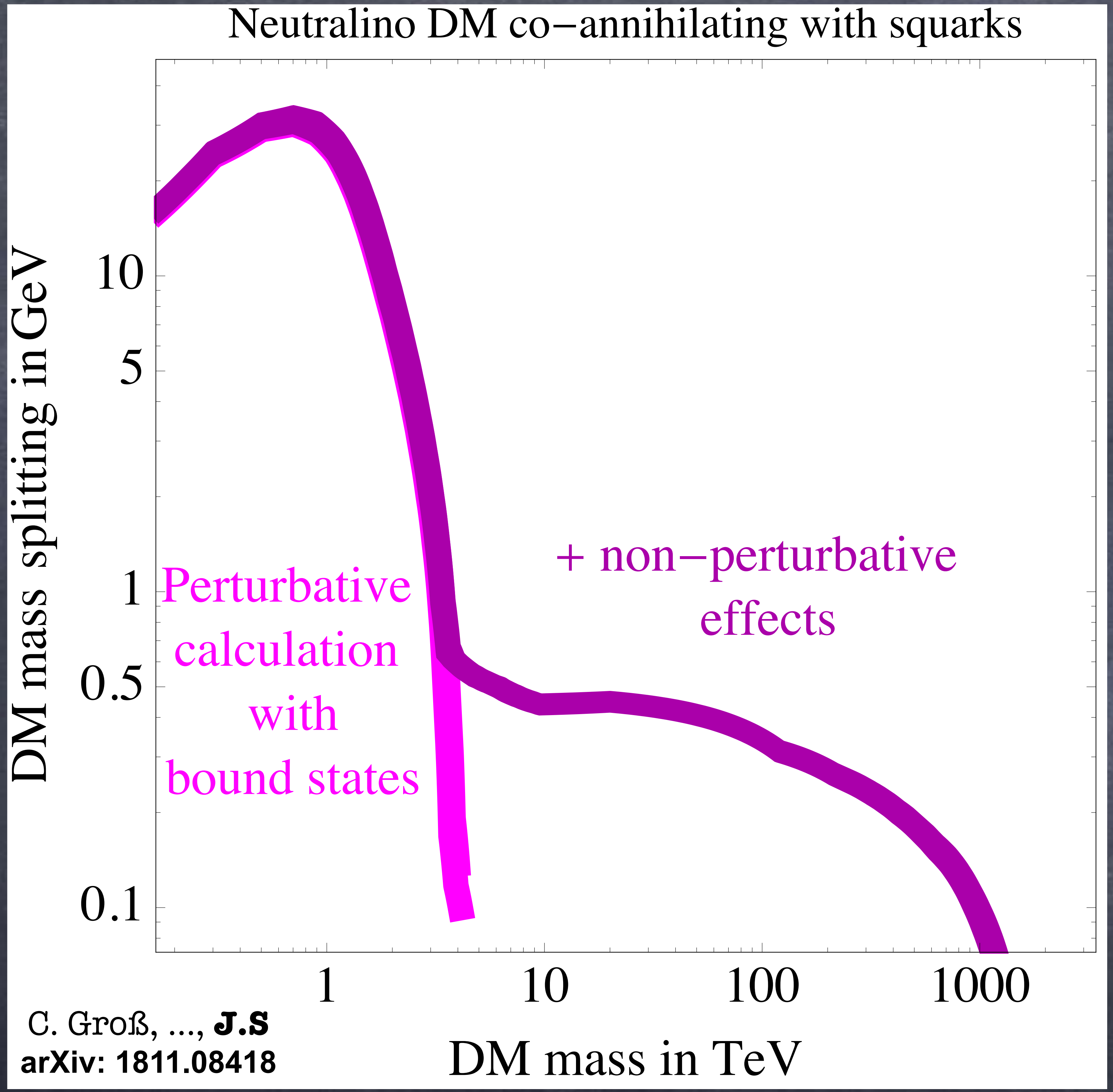




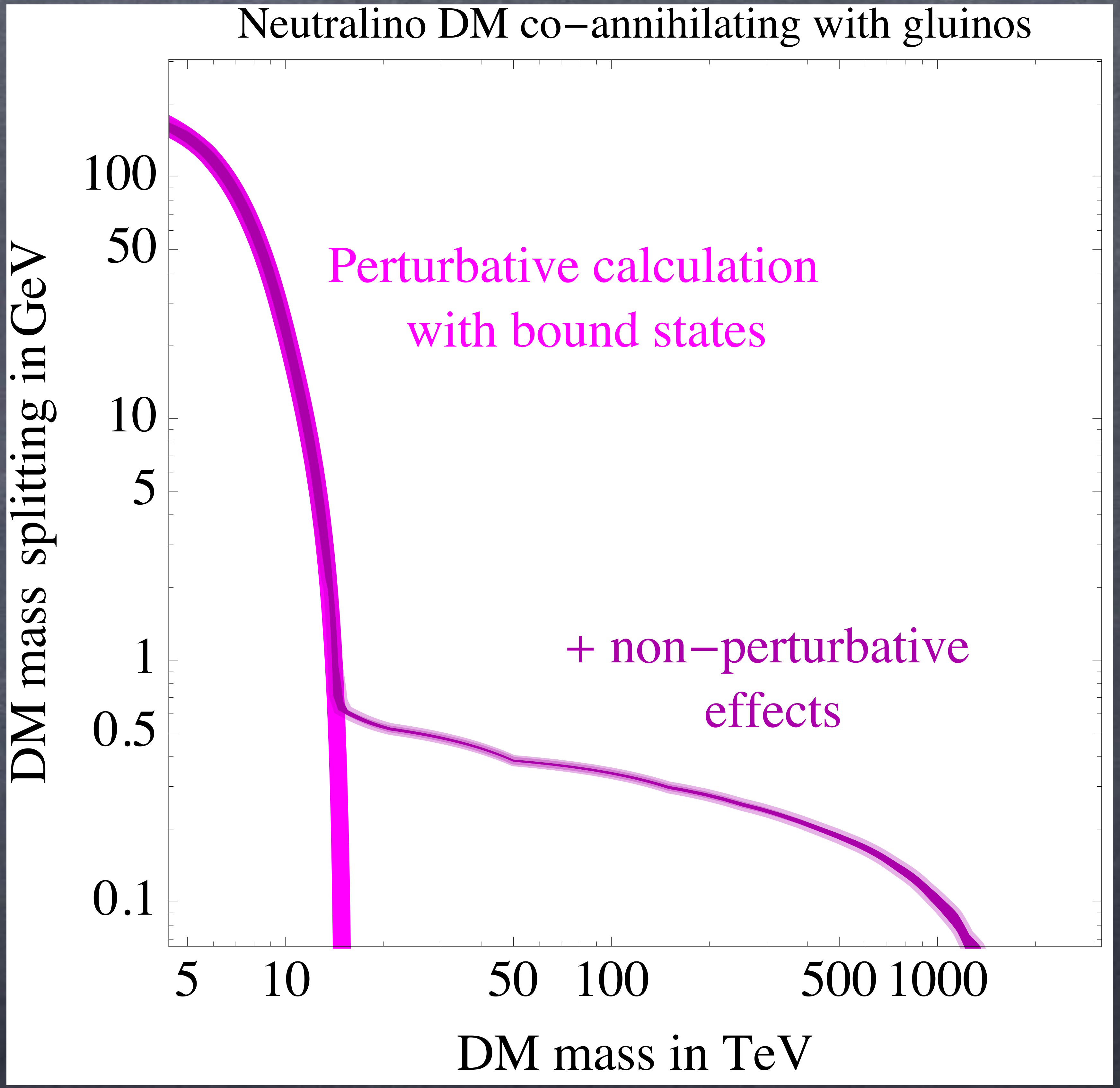
Singlet with SU(3) Triplet scalar
(Bino/Squark)

Singlet with SU(3) Qctet fermion
(Bino/Gluino)

Co-Annihilation: Results



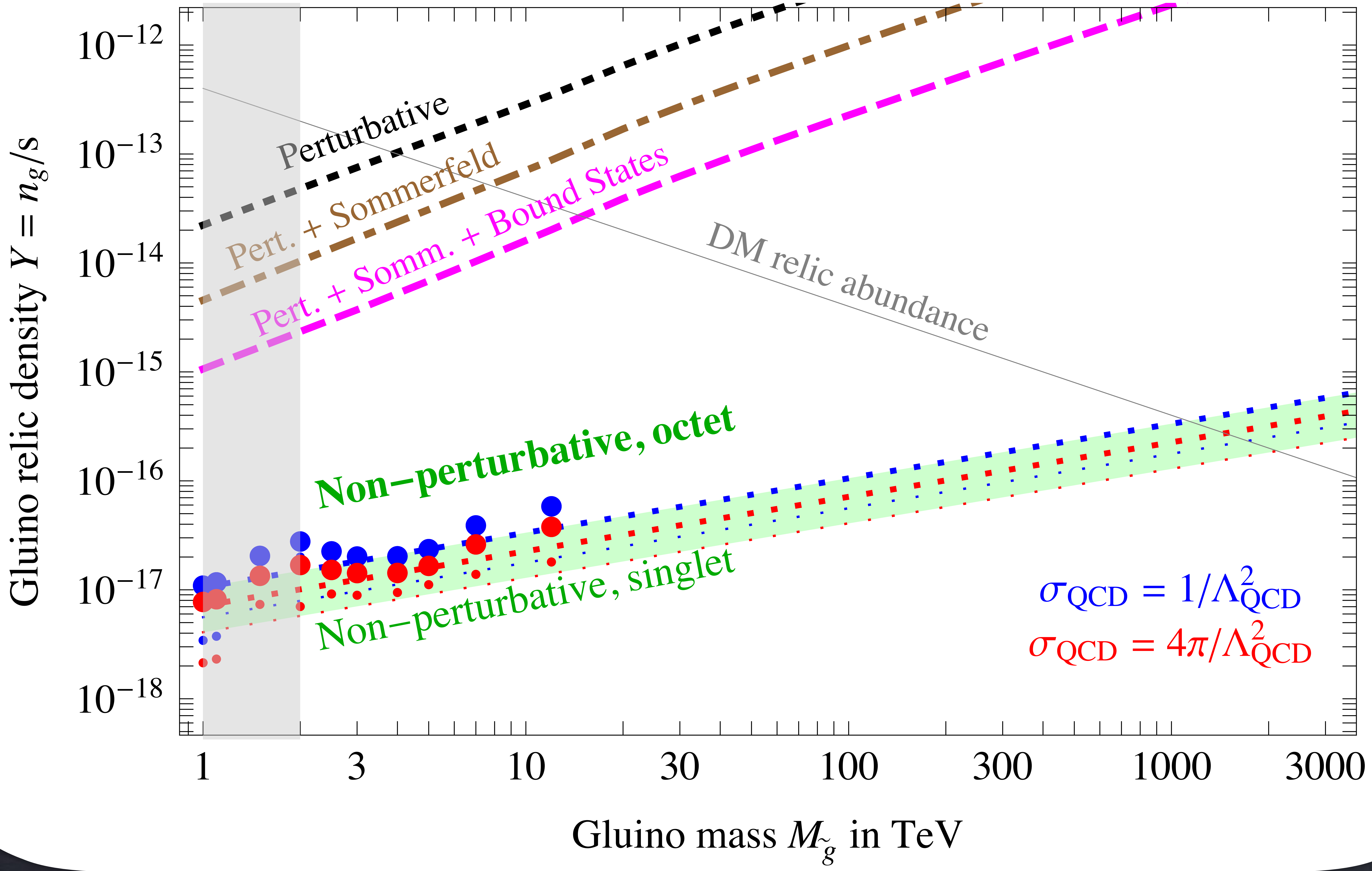
Singlet with SU(3) Triplet scalar
(Bino/Squark)



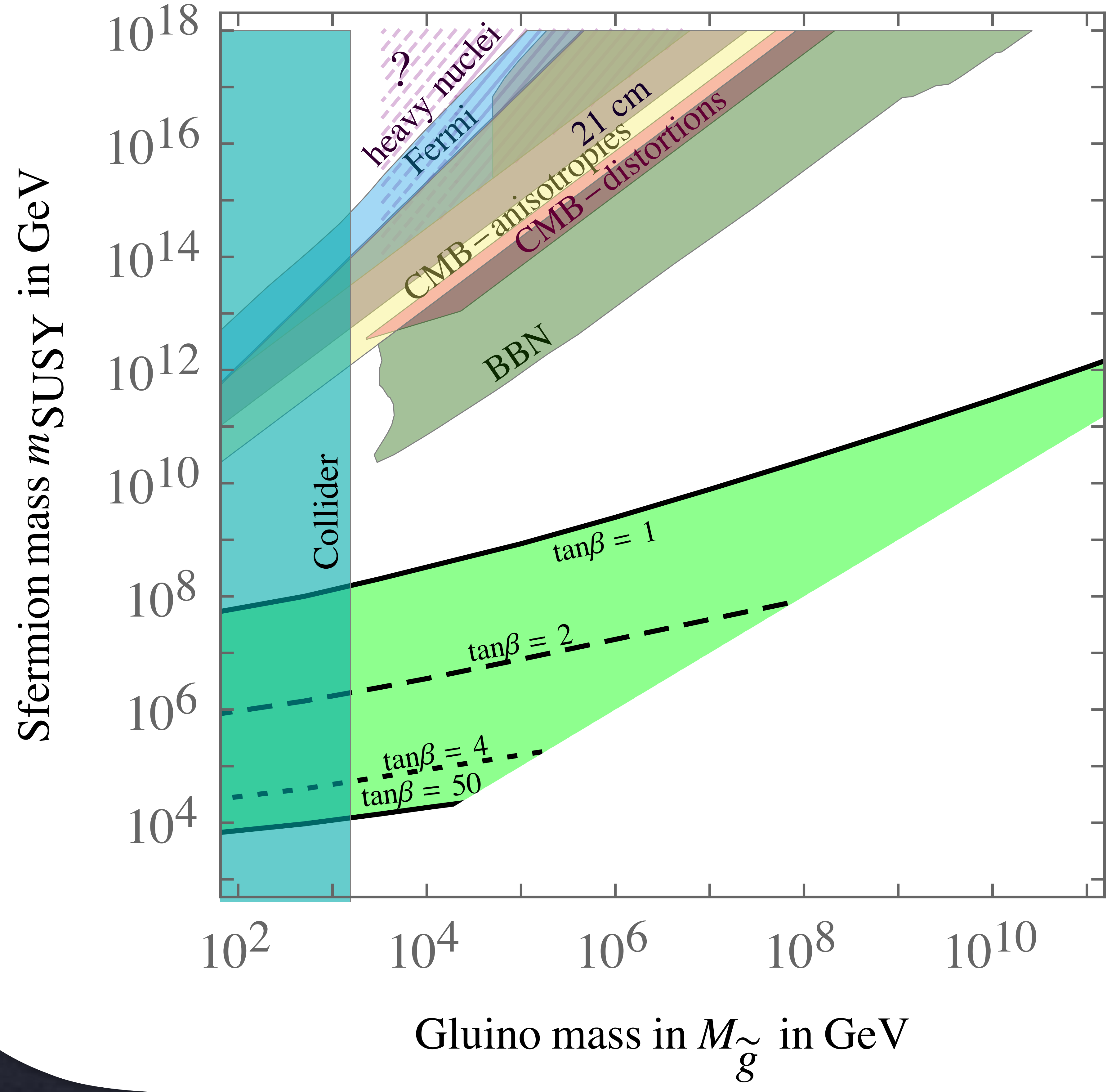
Singlet with SU(3) Qctet fermion
(Bino/Gluino)

Application III: Long Lived Color Charged Relics

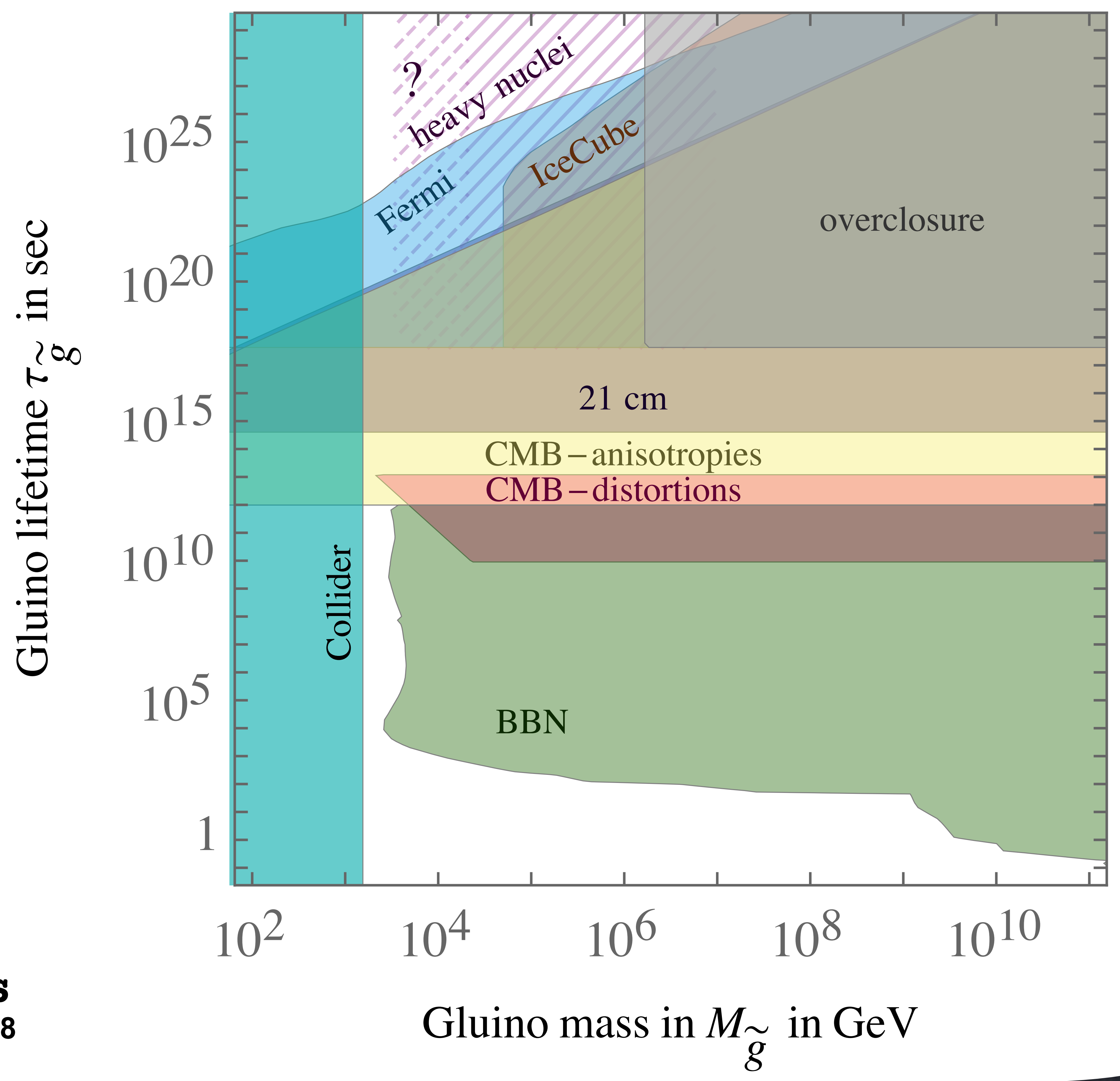
The Gluino freezeout



Bounds on split SUSY

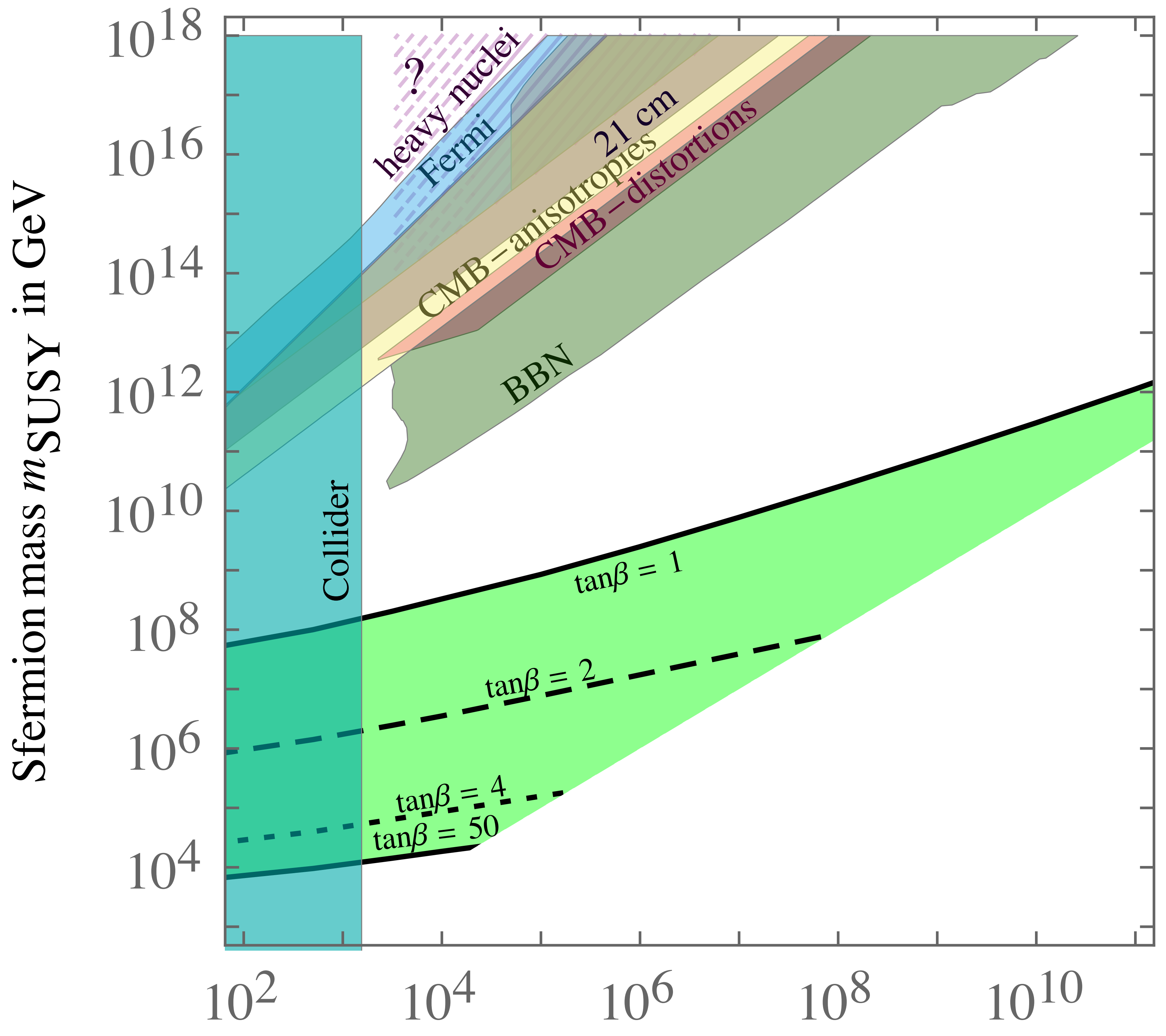


C. Groß, ..., **J.S**
arXiv: 1811.08418



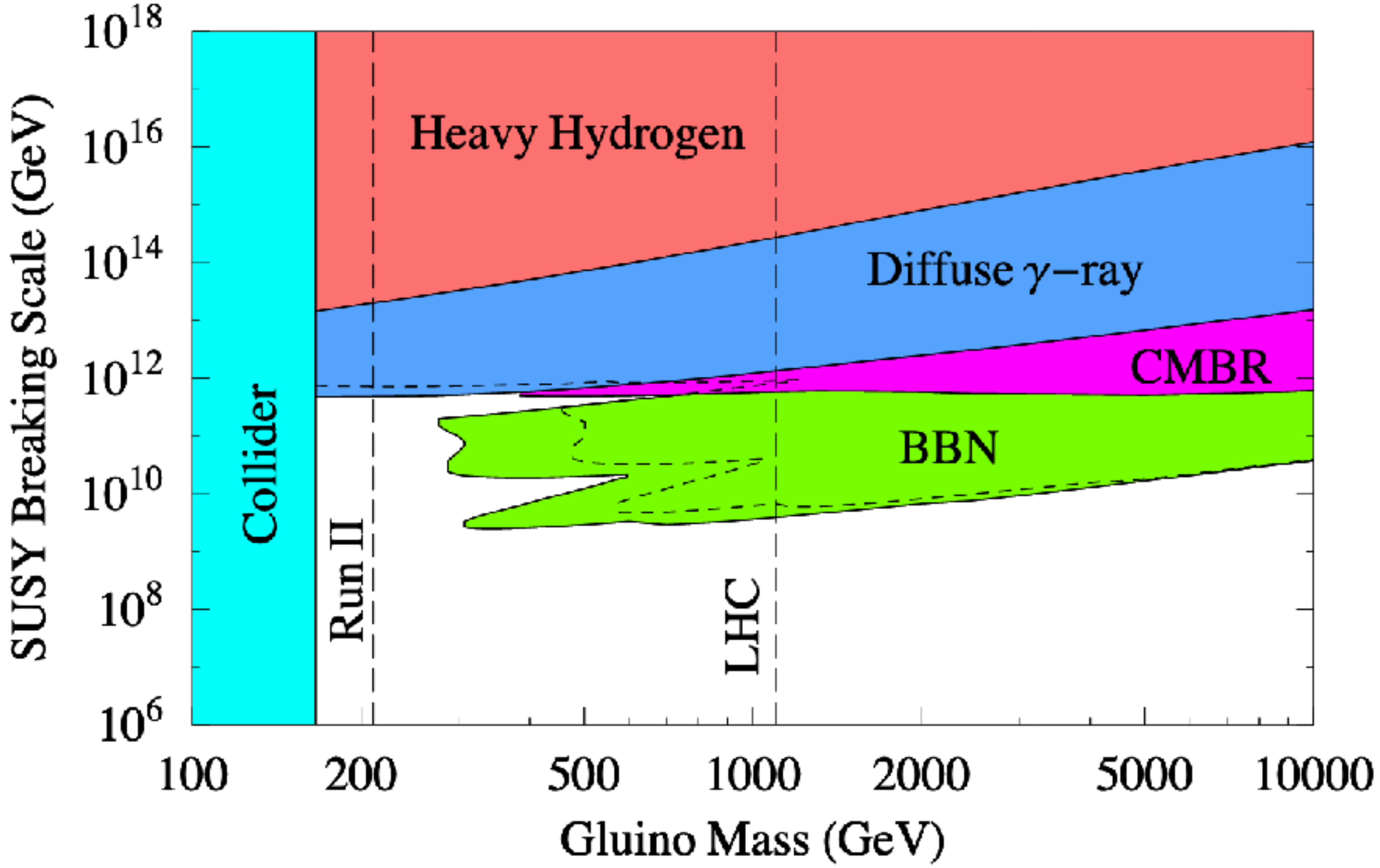
Asimina Arvanitaki et al.
hep-ph/0504210

Bounds on split SUSY



Gluino mass in $M_{\tilde{g}}$ in GeV

C. Groß, ..., **J.S**
arXiv: 1811.08418



Gluino mass in $M_{\tilde{g}}$ in GeV

Asimina Arvanitaki et al.
hep-ph/0504210

Summary

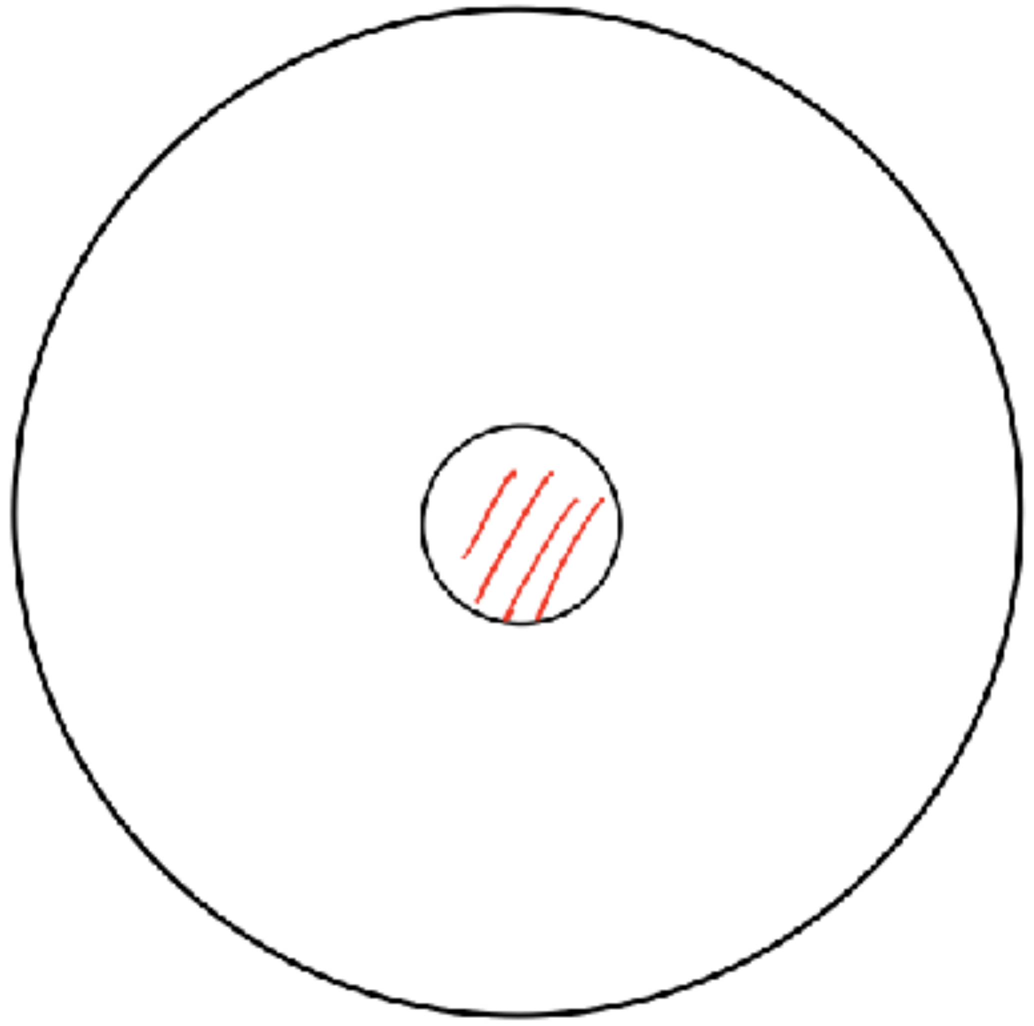
- Bound-state formation has an increasing effect for larger EW representations
- It needs to be taken into account for the estimate of the unitarity bound
- Co-annihilation with color-charged partners is strongly affected by bound-state formation, in particular at small mass splitting
- The andante of long lived color charged relic is severely affected by bound-state formation and non-perturbative QCD contributions

Thanks!

Backup

Application IV: Unitarity Bound for Extended Objects

Why are Atoms big?



$$R_A \sim \frac{1}{\alpha_{EM} m_e} \; ; \; m_e \ll M_N$$

$$M_A \sim M_N$$

$$\lambda_A \sim 1/M_A \ll R_A$$

Lower Bound from Unitarity

VOLUME 64, NUMBER 6 PHYSICAL REVIEW LETTERS 5 FEBRUARY 1990

Unitarity Limits on the Mass and Radius of Dark-Matter Particles

Kim Griest
Center for Particle Astrophysics, University of California, Berkeley, California 94720

Marc Kamionkowski
Physics Department, Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637-1433 and NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500

(Received 5 October 1989)

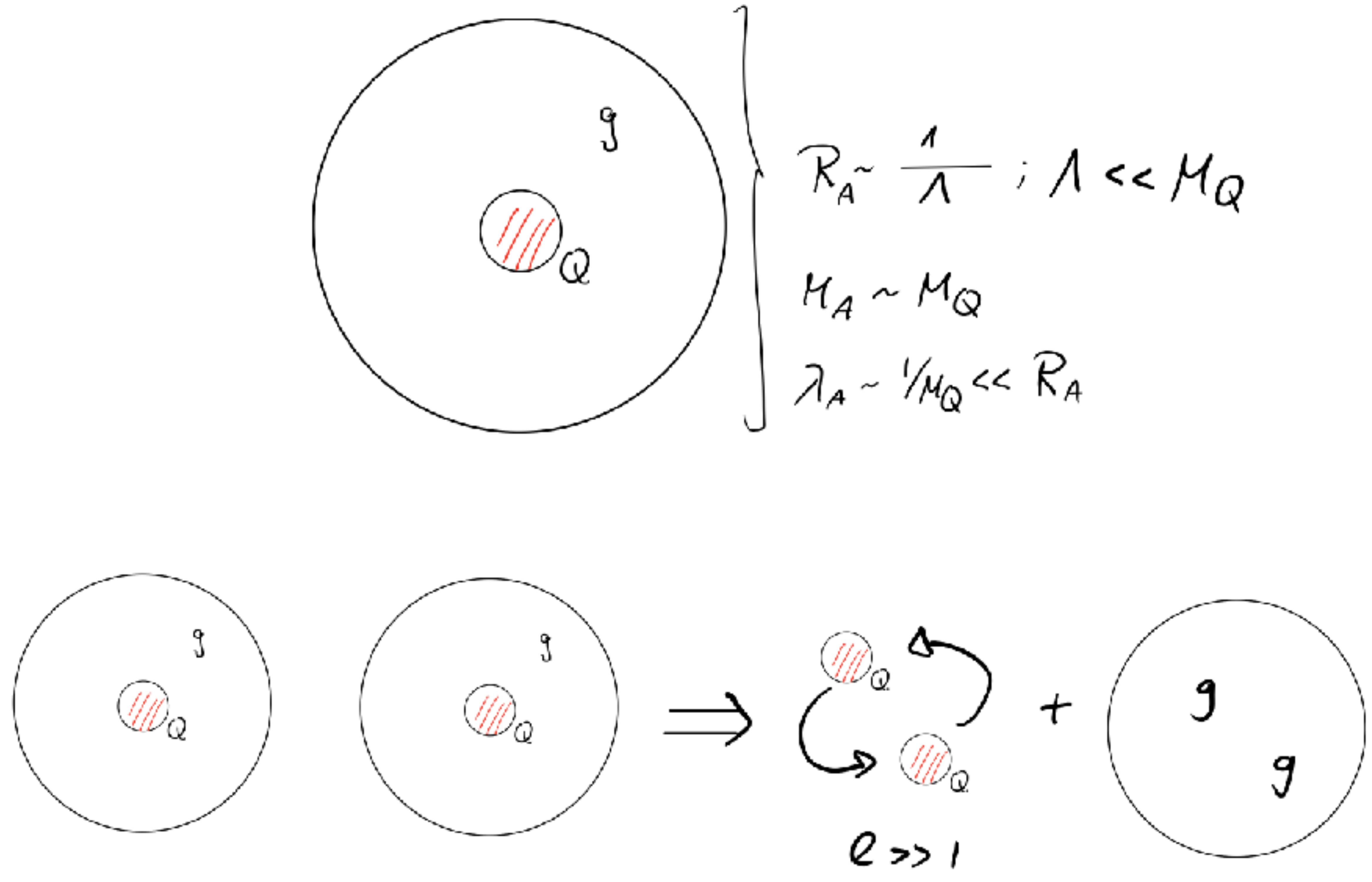
$$(\sigma)_{\text{total}} < \sum_{\ell=0}^{\ell_{\text{class}}} \frac{4\pi(2\,\ell+1)}{p_{\text{DM}}^2} \approx 4\pi R_{\text{DM}}^2$$

$R_{DM} > 7.5\ 10^{-7}$ fm = 1.7 PeV^{-1}

Limit is mass independent

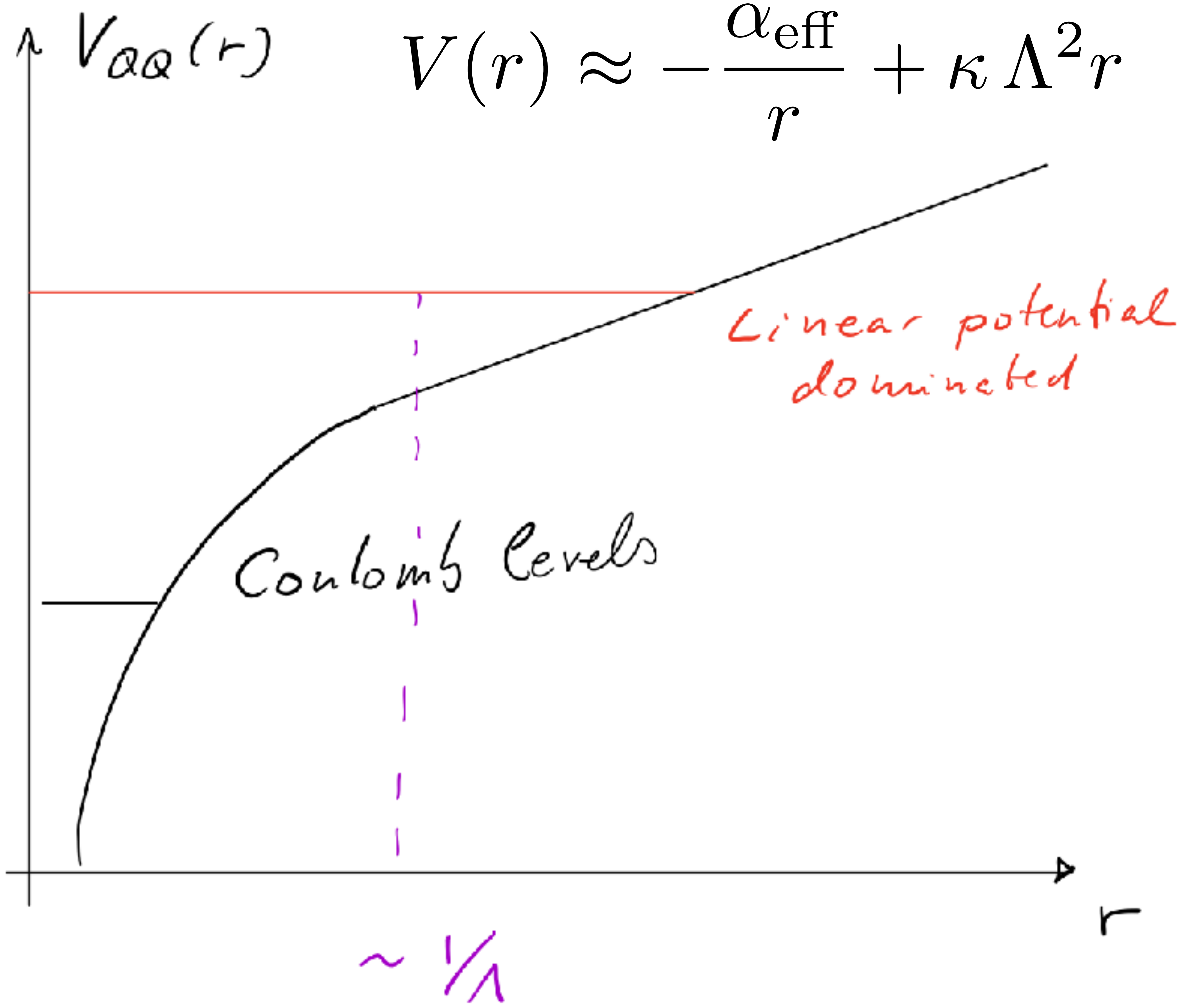
The Annihilation Process

$SU(N)_D$



Dark Quarkonium Potential

$$V(r) \approx -\frac{\alpha_{\text{eff}}}{r} + \kappa\,\Lambda^2 r$$

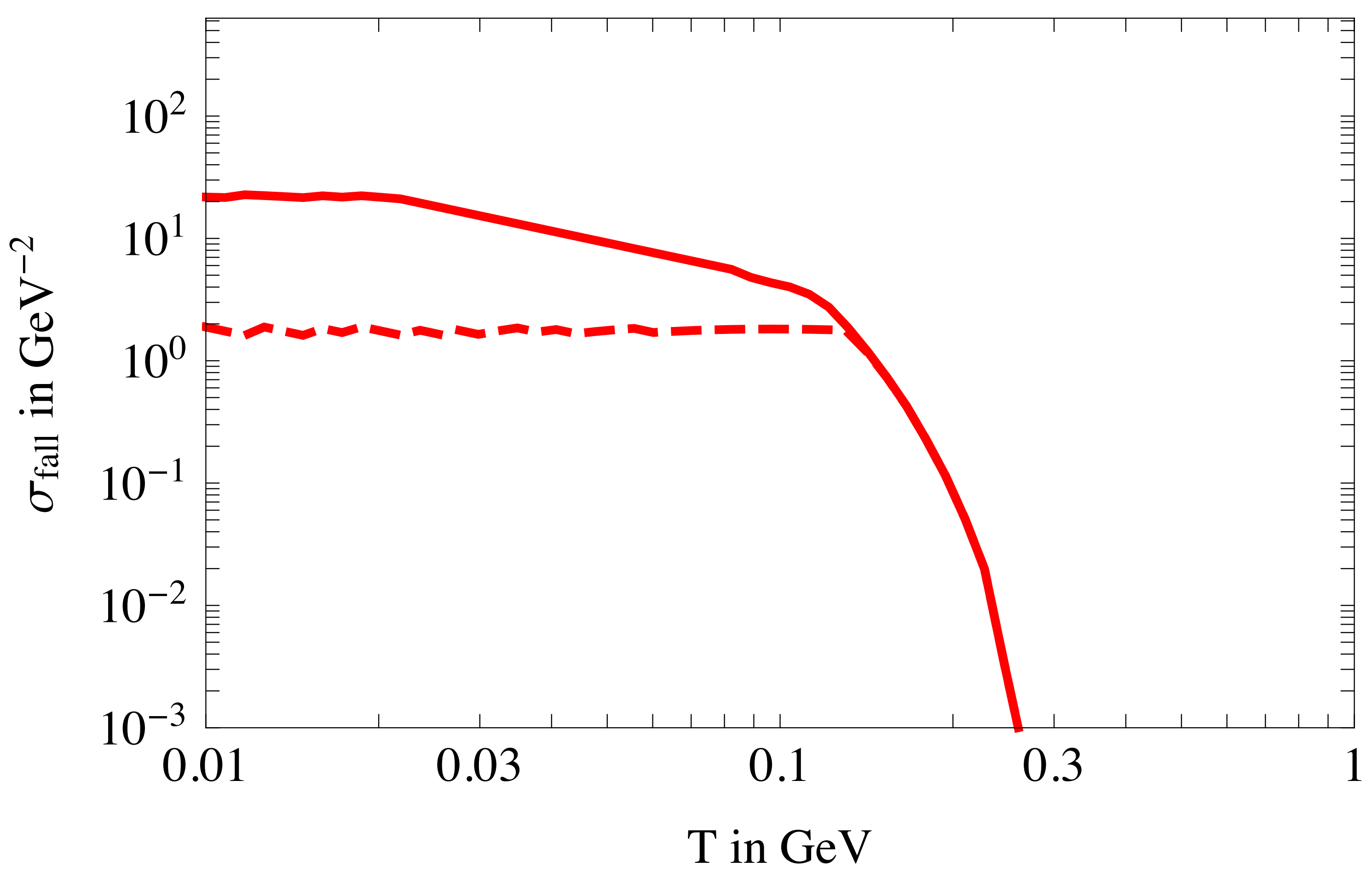


The Effective Cross Section

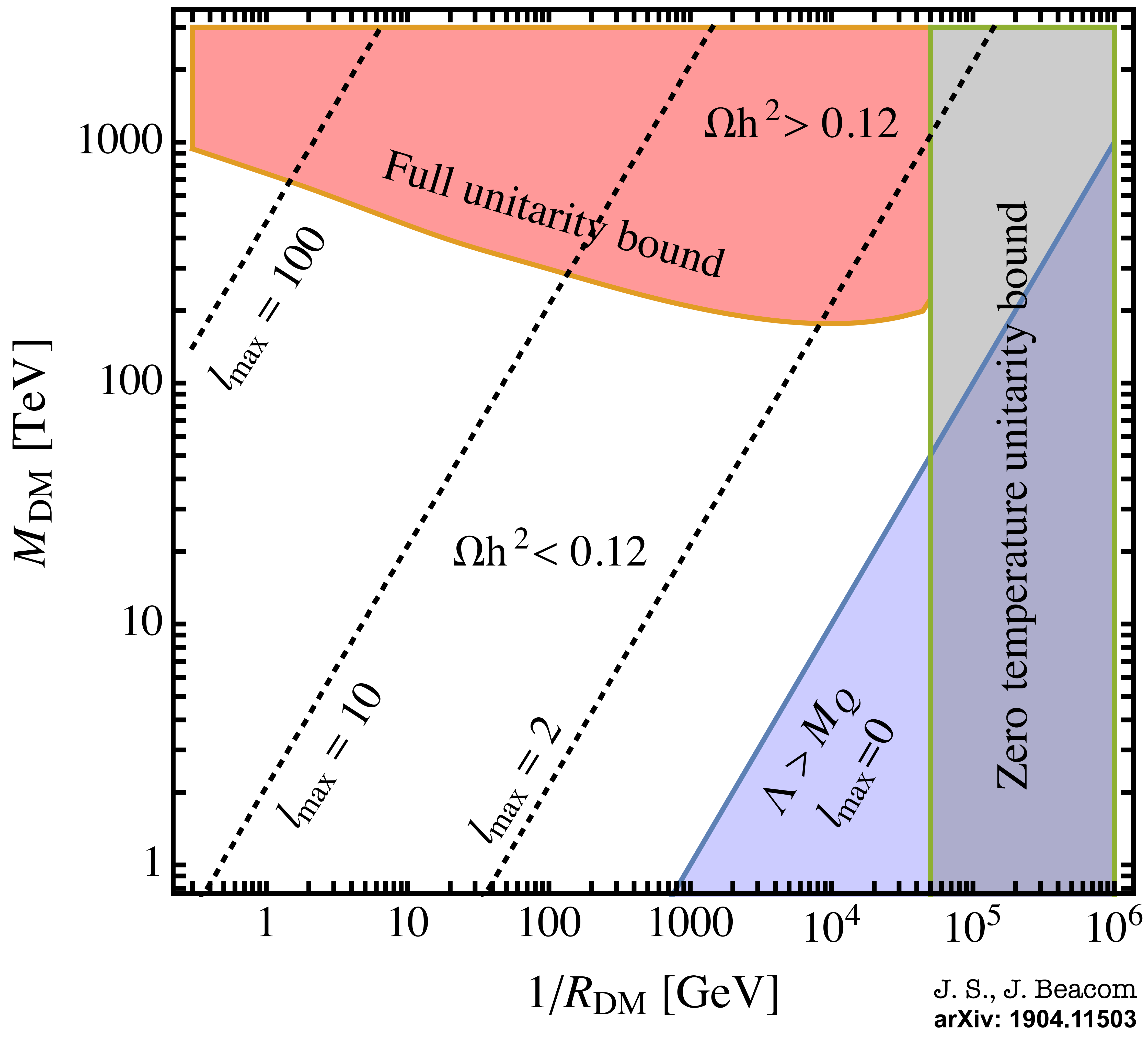
$$(\sigma^{\rm ann}) \leq \frac{4\pi\,\ell_{\rm max}(T)^2}{M_{\mathcal{Q}}^2 v_{\rm rel}^2} = \sigma_{\rm geom.} \begin{cases} 4T_c/M_{\mathcal{Q}} v_{\rm rel}^2 & ;T > T_c, \\ 1 & ;T < T_c\,, \end{cases}$$

$$T_c = \Lambda\,\lambda_R\,\alpha^{3/2}/8\sqrt{3} \qquad \sigma_{\rm geom.} = 4\pi/\Lambda^2$$

M_Q = 3 TeV



Composite DM Mass-Size Bounds



Details Bound State Formation

Non perturbative Effects

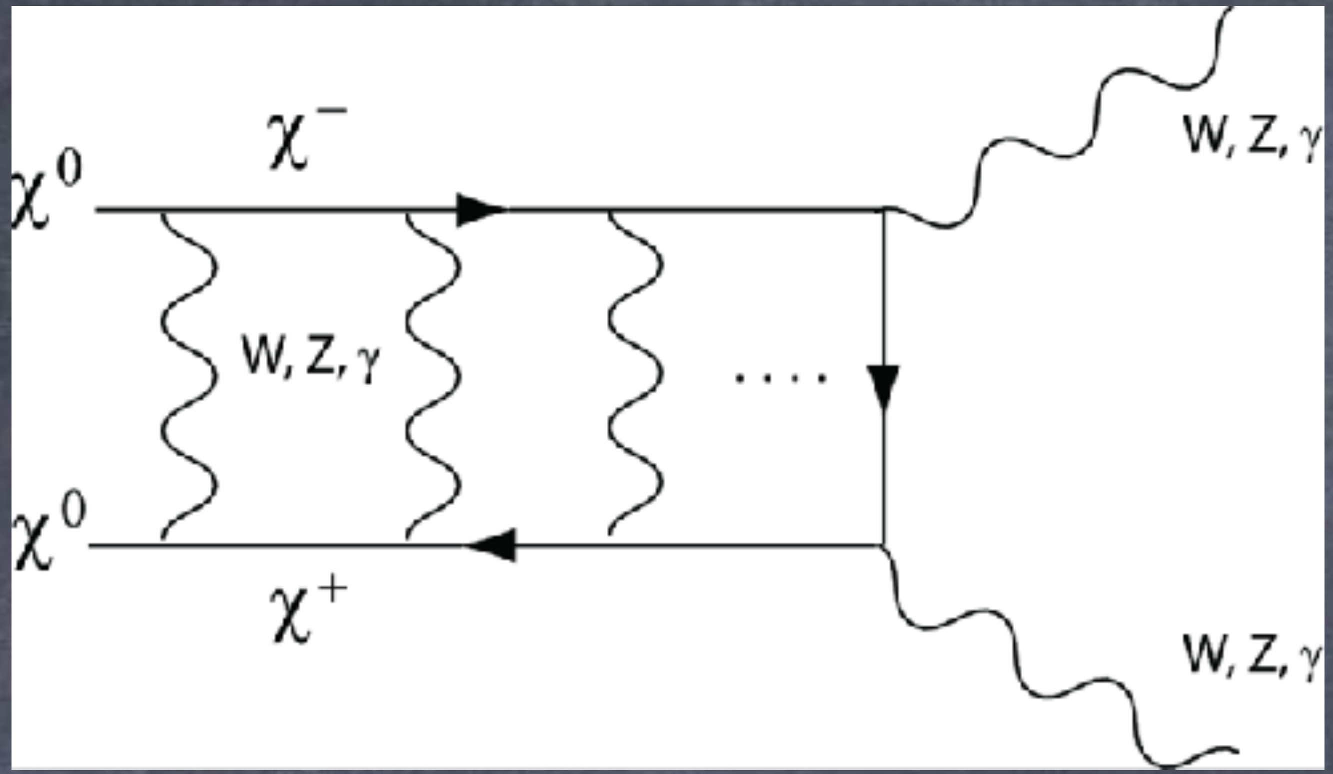
$$M_V < \alpha_{\text{eff}}\, M_\chi$$

$$R_{\text{Bohr}} < R_{\text{Yukawa}}$$

$$V(r) = -\alpha_{\text{eff}} \frac{e^{-M_V r}}{r} \approx -\alpha_{\text{eff}} \left(\frac{1}{r} - M_V \right)$$

$$E_{n\ell} \simeq \frac{\alpha_{\text{eff}}^{\cancel{2}} M_\chi}{4n^2} - \cancel{\alpha_{\text{eff}}} M_V + \mathcal{O}(M_V^2).$$

Sommerfeld Effect



$$R \otimes R' = \sum_J J$$

$$V = \alpha \frac{e^{-M_V r}}{2r}\left[\sum_J C_J \mathbf{I}_J - C_R \mathbf{I}_R - C_R \mathbf{I}_{R'}\right].$$

$$V_J = -\alpha_{\text{eff}} \frac{e^{-M_V r}}{r}, \qquad \alpha_{\text{eff}} = \lambda_J \alpha, \qquad \lambda_J = \frac{C_R + C_{R'} - C_J}{2}$$

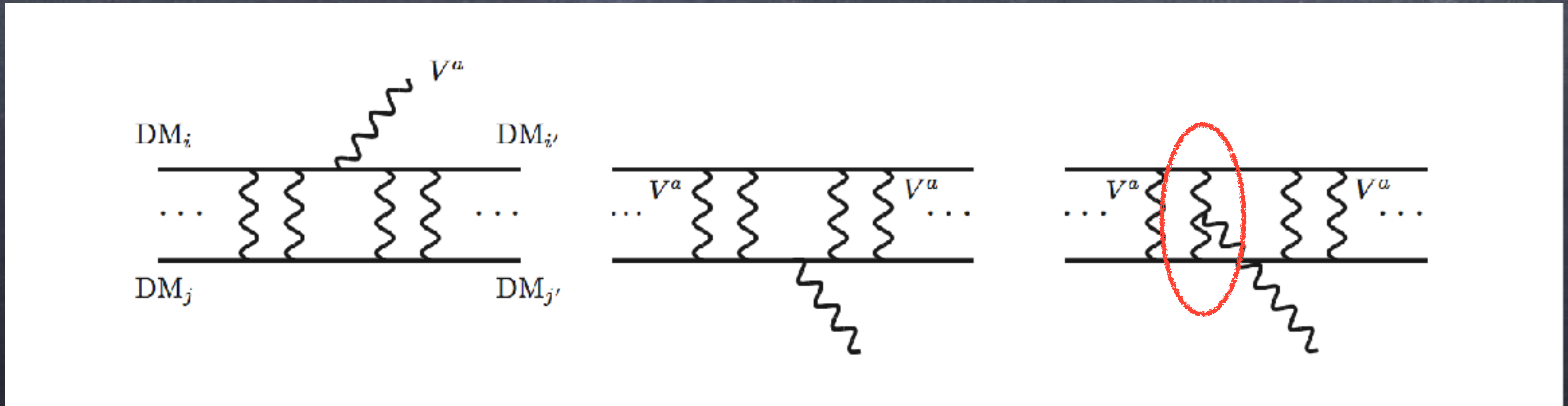
Example SU(2): $3 \otimes 3 = 1_S \oplus 3_A \oplus 5_S\,,$

$$S = \frac{2\pi\alpha_{\text{eff}}/v_{\text{rel}}}{1 - e^{-2\pi\alpha_{\text{eff}}/v_{\text{rel}}}} \qquad \text{for } M_V = 0.$$

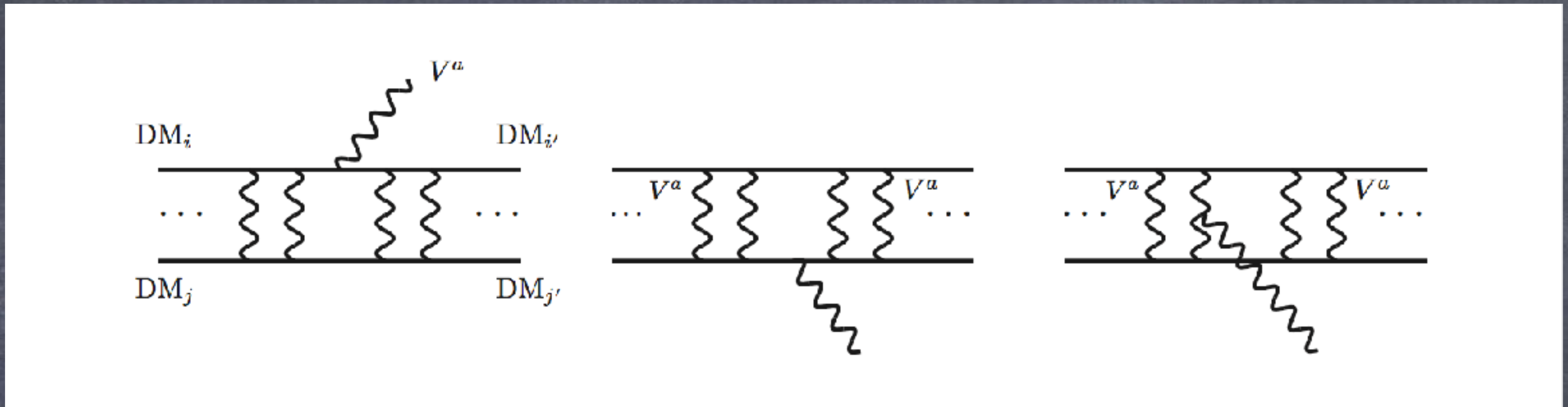
I	V i.e.	λ
1	$-2\alpha_2/r$	$+2$
3	$-\alpha_2/r$	$+1$
5	$+\alpha_2/r$	-1

Bound State Selection Rules

- The Group theory structure
- The wave function symmetry
- Angular momentum conservation $\Delta L = 1$
- Energy conservation



Bound State Selection Rules



Example SU(2):

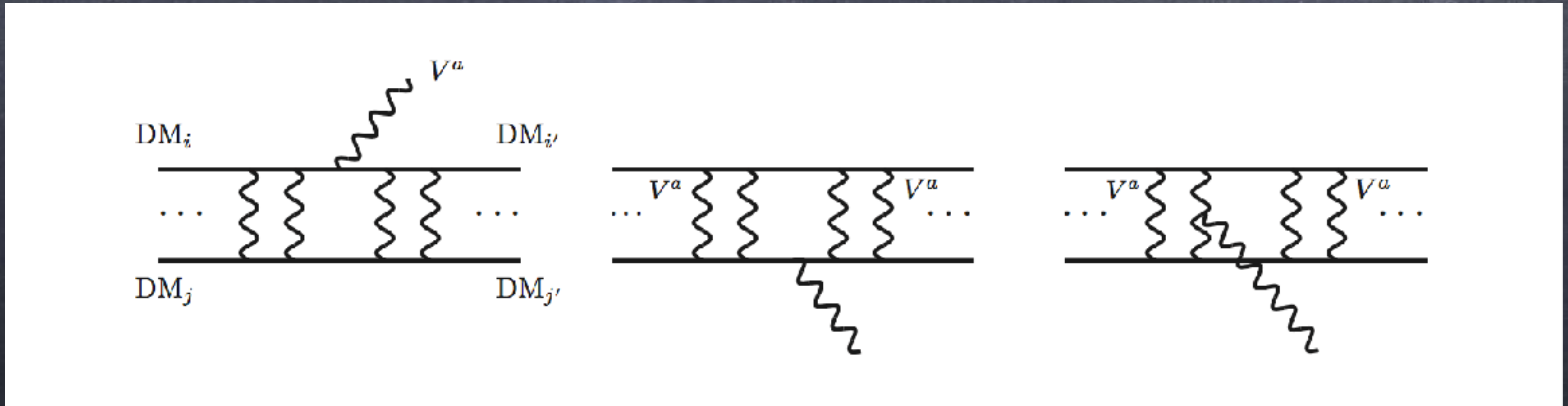
$$3 \otimes 3 = 1_S \oplus 3_A \oplus 5_S \,,$$

$I_J \rightleftarrows I_{J'}$
$1 \rightleftarrows 3$
$3 \rightleftarrows 5$

Note that effective potential changes!

Bound State Selection Rules

- The Group theory structure ✓
- The wave function symmetry
- Angular momentum conservation $\Delta L = 1$
- Energy conservation



Bound State Selection Rules wave function: antisymmetric

- Spin parity: $(-1)^S$
- Space parity: $(-1)^\ell$
- Isospin parity: $(-1)^{\tilde{I}}$ where $I = 2\tilde{I} + 1$

$$(-1)^{\ell+S+\tilde{I}} = 1$$

Example SU(2):

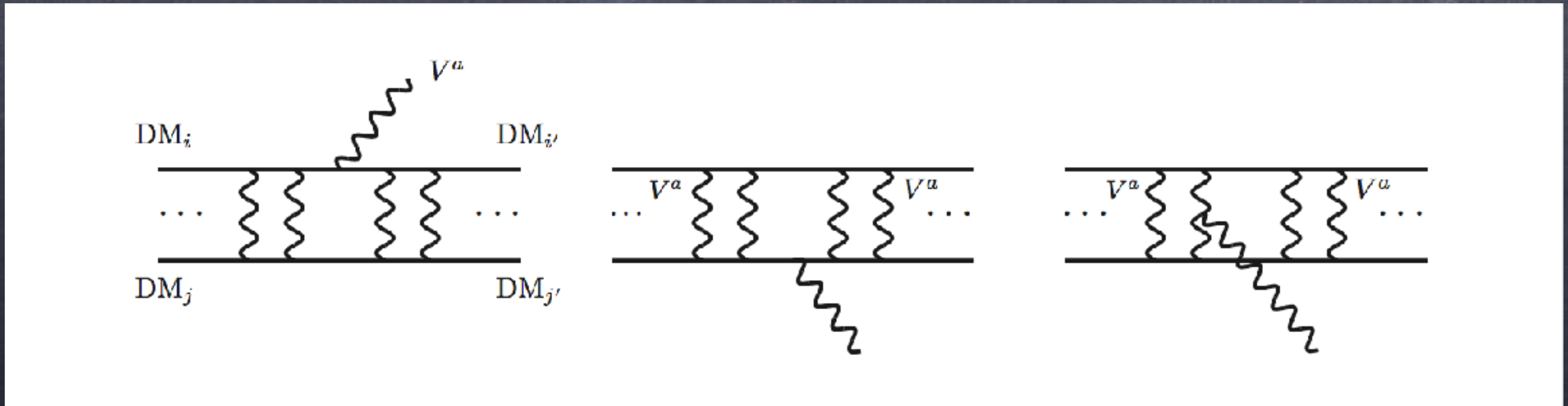
$$3 \otimes 3 = 1_S \oplus 3_A \oplus 5_S \,,$$

I	V i.e.	λ	allowed ℓ
1	$-2\alpha_2/r$	$+2$	even if $S=0$, odd if $S=1$
3	$-\alpha_2/r$	$+1$	even if $S=1$, odd if $S=0$
5	$+\alpha_2/r$	-1	even if $S=0$, odd if $S=1$

Bound State Selection Rules

- The Group theory structure ✓
- The wave function symmetry ✓
- Angular momentum conservation $\Delta L = 1$ ✓
- Energy conservation

p → s
s → p ; d → p



Binding Energy

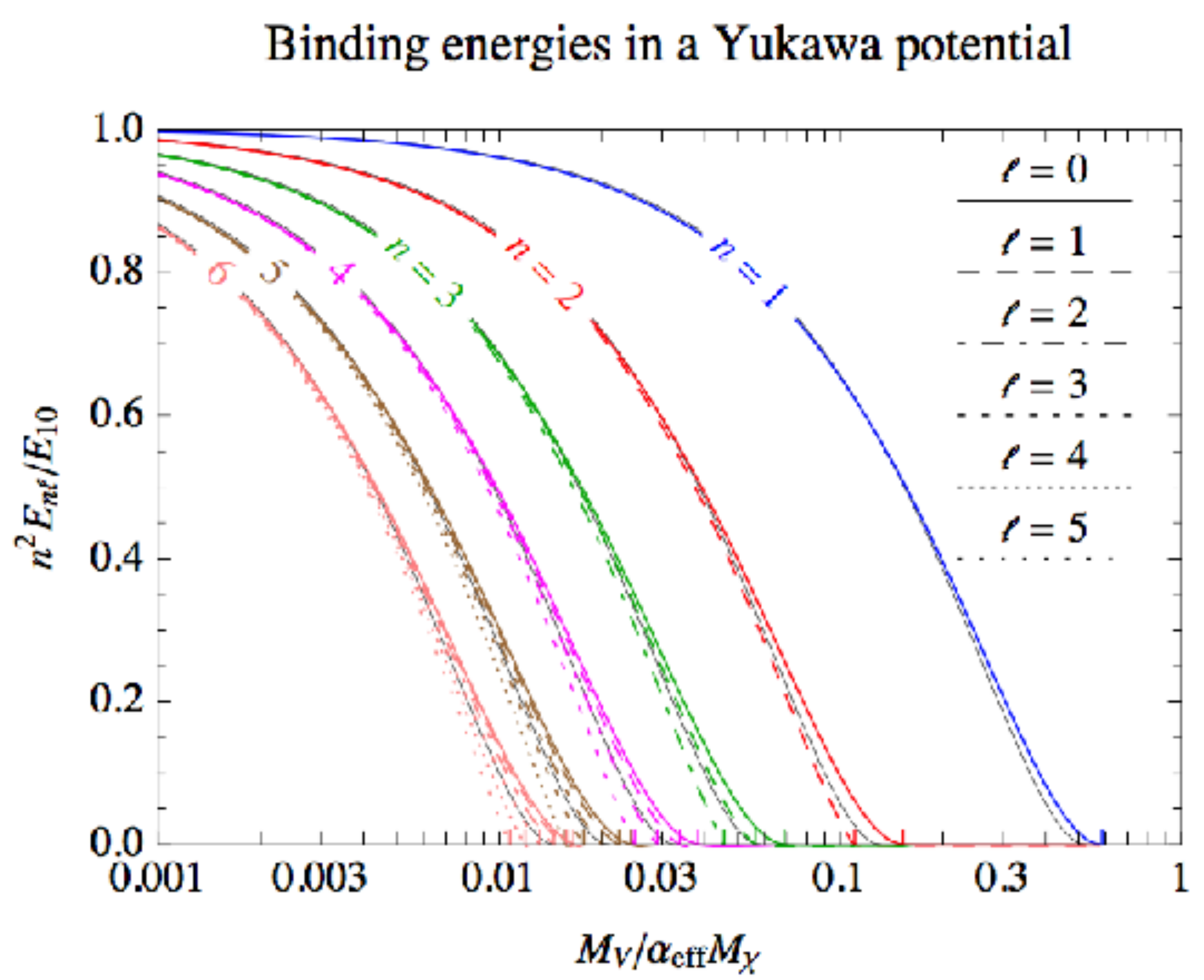


Figure 1: *Energies of bound states in a Yukawa potential (colored curves) compared to the Hulthen approximation with* $\kappa = 1.9$ *(black continuous curves).*

$$E_{n\ell} \approx \frac{\alpha_{\rm eff}^2 M_\chi}{4n^2}\left[1 - n^2 y - 0.53 n^2 y^2 \ell(\ell+1)\right]^2, \qquad y \equiv \frac{\kappa M_V}{\alpha_{\rm eff} M_\chi}$$

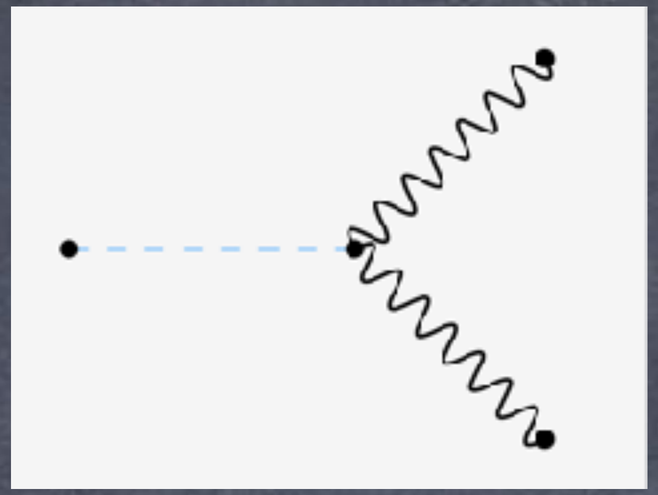
Bound State Selection Rules

- The Group theory structure ✓
- The wave function symmetry ✓
- Angular momentum conservation $\Delta L = 1$ ✓
- Energy conservation ✓

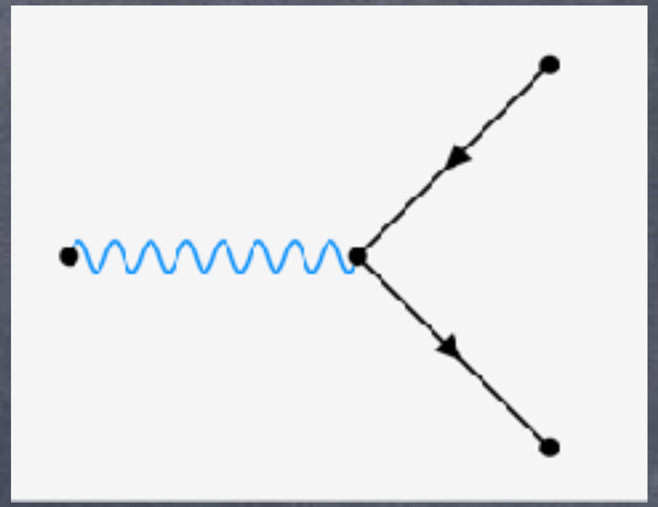
What happens to the Wimponium?

The fate of the Bound State

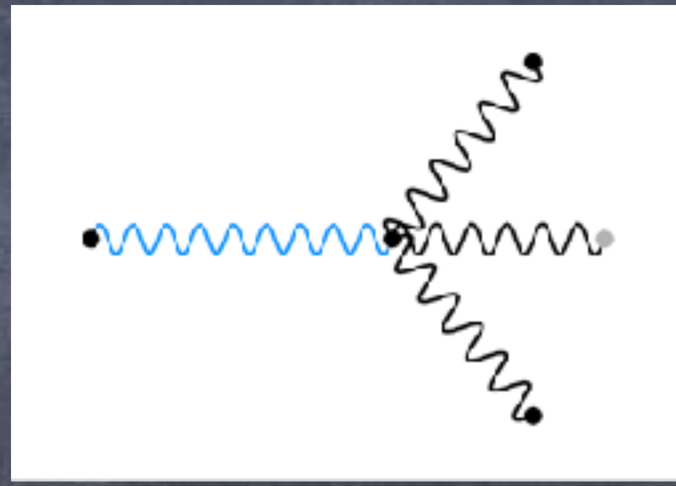
Bound states with: $\ell = 0$



spin =0



spin = 1



$$\Gamma_{\text{ann}} \propto \alpha_{\text{eff}}^5 M_\chi$$

Bound states with: $\ell > 0$
dominantly decay

$$\Gamma(2p \to 1s + \gamma) = \alpha_{\text{em}} \alpha_2^4 M_\chi \left(\lambda_f^2 - \frac{\lambda_i^2}{4} \right) \frac{512 \lambda_i^5 \lambda_f^3}{3(\lambda_i + 2\lambda_f)^8} \times \frac{1}{3 d_B} \sum_{aMM'} \left| C_{\mathcal{J}}^{aMM'} + \frac{C_{\mathcal{T}}^{aMM'}}{\lambda_f} \right|^2 .$$

$$\Gamma_{\text{ann}} \propto \alpha_{\text{eff}}^7 M_\chi$$

Details Weak Dark Matter

Effective Charge

$$\frac{\sigma v_{\rm ann}}{\sigma_0} = \frac{2R^4 + 17R^2 - 19}{32R},$$

for example $\lambda_1 = (R^2 - 1)/4$

L. Lopez Honorez, M. Tytgat, P. Tziveloglou, B. Zaldivar
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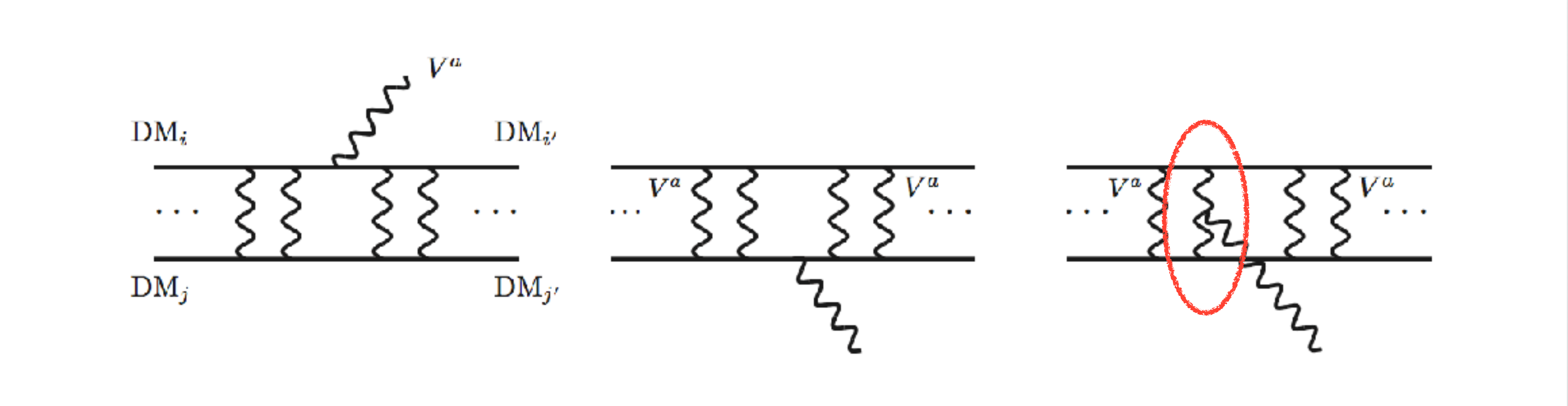
$$(\sigma v_{\rm rel})^n_\ell = (\sigma v_{\rm rel})^n_\ell(\lambda_i, \lambda_f) \times |C_J + \gamma^n_\ell C_\mathcal{T}|^2$$



A. Mitridate, M. Redi, J. Smirnov, A. Strumia
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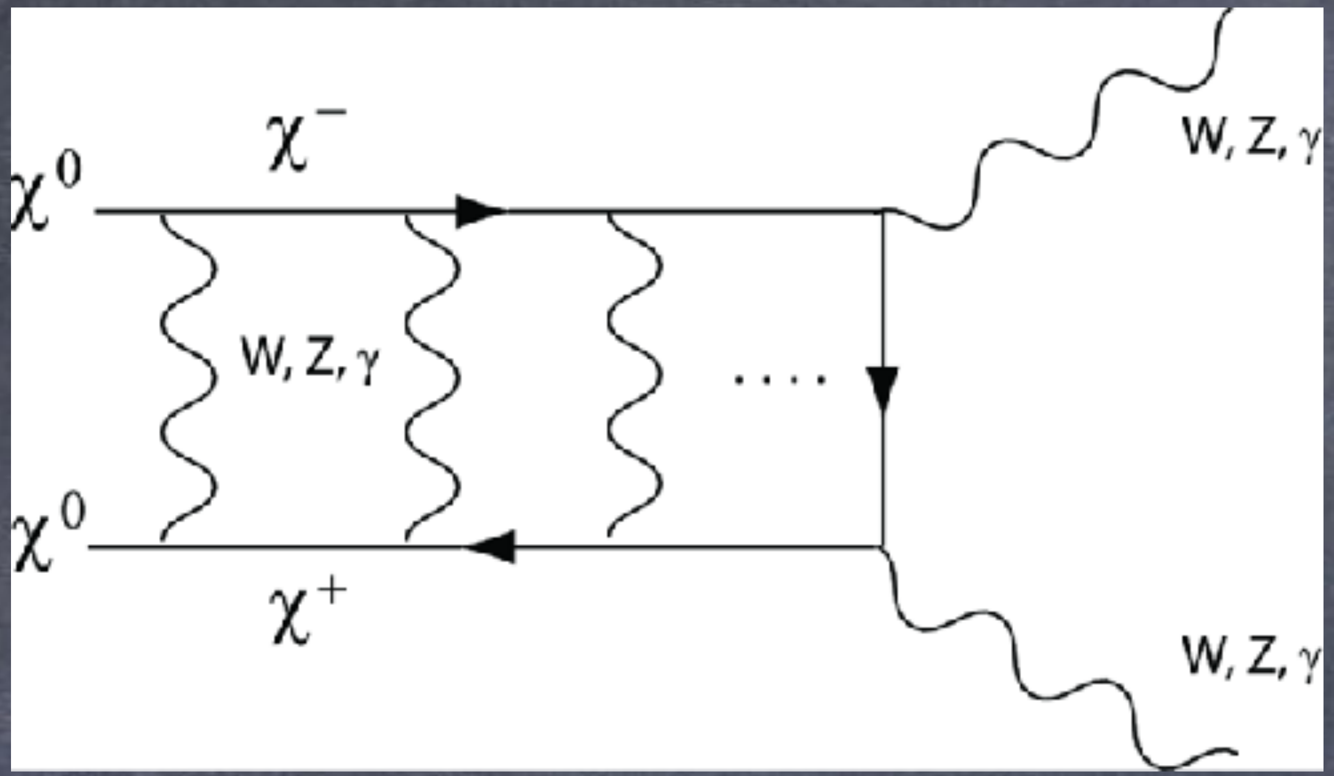
Sommerfeld



$$R \otimes R' = \sum_J J$$

$$V = \alpha \frac{e^{-M_V r}}{2r} \left[\sum_J C_J \mathbf{I}_J - C_R \mathbf{I}_R - C_R \mathbf{I}_{R'} \right] .$$

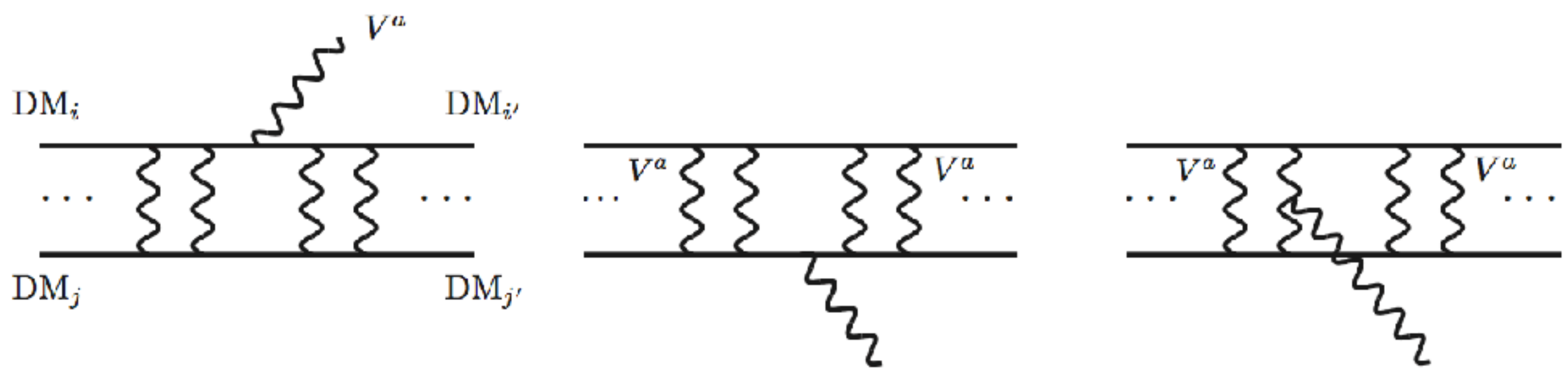
$$V_J = -\alpha_{\rm eff} \frac{e^{-M_V r}}{r}, \qquad \alpha_{\rm eff} = \lambda_J \alpha, \qquad \lambda_J = \frac{C_R + C_{R'} - C_J}{2}$$

Example SU(2): $3 \otimes 3 = 1_S \oplus 3_A \oplus 5_S$,

I	V i.e.	λ
1	$-2\alpha_2/r$	$+2$
3	$-\alpha_2/r$	$+1$
5	$+\alpha_2/r$	-1

$$S = \frac{2\pi\alpha_{\rm eff}/v_{\rm rel}}{1 - e^{-2\pi\alpha_{\rm eff}/v_{\rm rel}}} \qquad \text{for } M_V = 0.$$

Selection Rules



Example SU(2):

$$3 \otimes 3 = 1_S \oplus 3_A \oplus 5_S \,,$$

$I_J \rightleftarrows I_{J'}$
$1 \rightleftarrows 3$
$3 \rightleftarrows 5$

Note that effective potential changes!

Branching Ratio

$$\gamma_{\text{form.}} = \gamma_{\text{break}}$$

$$\langle \sigma_I v_{\text{rel}} \rangle_{\text{BSF}}\, n_{\text{DM}}^2 = \Gamma_{\text{break}}\, n_I$$

$$\text{BR}(B_I \to \text{SM}) = \frac{\Gamma_{\text{ann}}}{\Gamma_{\text{ann}} + \Gamma_{\text{break}}}$$

$$\Gamma_{\text{break}} \propto \frac{e^{-2M_{\text{DM}}/T}}{e^{-M_I/T}} \propto e^{-E_I/T}$$

$$\Gamma_{\text{ann}} \propto \frac{\alpha^2}{M_{\text{DM}}^2}\, (\alpha M_{\text{DM}})^3 \propto \alpha^5 M_{\text{DM}}$$

Binding Energy

$$\frac{E^{\gamma}_{1s_3}}{M_{\rm DM}} = \frac{\lambda_3^2\alpha_2^2}{4} = \frac{\alpha_2^2\,(R^2-5)^2}{16}\,,$$

$$\frac{E^{\gamma}_{2p_3}}{M_{\rm DM}} = \frac{\lambda_3^2\alpha_2^2}{16} = \frac{\alpha_2^2\,(R^2-5)^2}{64}\,,$$

$$\frac{E^{\gamma}_{2p_3-1s_1}}{M_{\rm DM}} = \frac{\alpha_2^2}{256}\left(3R^4+2R^2-21\right)\,.$$